

Winner's Curse Corrections Magnify Adverse Selection*

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Abstract

The adverse-selection literature has only considered the case in which competing sellers' costs of supply are independent and privately known by the individual sellers. In contrast, the auction literature has ignored adverse selection by implicitly assuming that a bid-taker is indifferent between suppliers at a given price. We show that competition in auctions with common-value elements serves to magnify the impact of adverse selection, as a bidder supplying a higher-cost product rationally makes a heightened winner's curse correction in a procurement auction. Hence lower-cost suppliers are disproportionately likely to win the auction, potentially creating a more serious quality problem for the procurer than mainstream adverse-selection models suggest.

1 Introduction

We illustrate the interaction between two phenomena that have largely been studied disjointly, but often arise together. The first is adverse selection, for example, situations in which competing potential suppliers with similar observable characteristics nonetheless will differ substantially in the quality of the commodity or service supplied.¹ In these situations, an offer which is acceptable to a high-quality supplier (i.e., a supplier who knows privately that he will provide high quality) would be even more attractive to a low-quality type. The problem is kept interesting by assuming that quality cannot be verified by a neutral third party and hence cannot be contracted. The procurer may do best offering

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¹Such approaches as prequalifying suppliers or providing more detailed specifications presumably may ease, but do not solve, the problem under study.

different prices for different quality specifications with the specifications for the high price intolerably onerous to a low-quality supplier. These models have assumed, often unrealistically, that neither type of supplier is uncertain about his own costs of supply (Laffont & Tirole,1987; McAfee & McMillan,1986,1987b; Riordan & Sappington,1987; Vickrey,1961).

The second phenomenon is the winner's curse. Auction models have an altogether distinct interpretation of types, in which different types are competing suppliers with different estimates of the costs of fulfilling a contractual commitment. In many important markets, bidders' uncertain costs and their estimates of these costs are statistically related. Hence, bidders must take into account the winner's curse—the impact on appropriate cost estimation of the likelihood that losing bidders had information suggesting higher costs than the winning bidder's estimate. However, auction models have assumed, often unrealistically, that all bidders were competing to supply the same quality of product or service, in that the buyer has been presumed indifferent between any two suppliers who bid the same price (Rothkopf, 1969; Wilson, 1977; Milgrom & Weber, 1982; McAfee & McMillan, 1987a; Milgrom, 1989; Riley, 1989; Wilson, 1992.).

Thus, the adverse-selection literature generally ignores supplier uncertainty about their costs but recognizes possible differences in quality provided by different suppliers. The winner's curse literature recognizes suppliers' cost uncertainties but ignores cost-related quality differences across suppliers. The informational asymmetries considered in isolation in each of these literatures must be combined to model situations like the following: Consider a buyer seeking to obtain some service or product² from one of a group of competing bidders. Quality differences across bidders give the buyer non-price reasons to have preferences among them, but the traditional assumption will be maintained here: the suppliers providing preferred quality levels cannot be identified by the buyer unless predictable differences in behavior across supplier types can be inferred. Indeed, a supplier's quality level will be his own private information, also unknown to rival bidders. In practice, a variety of sources of information for inferences about quality levels would be available to the buyer, from prior experience, the grapevine, etc. Consistent with our focus (and with the tradition in the adverse-selection literature), sources of information other than the bids submitted are ignored here.

Similarly, for focus, suppliers are not allowed to choose the quality level that will be supplied, thereby setting aside moral hazard issues. Each supplier simply has his own technology of contract fulfillment; a preferred-quality supplier naturally incurs a higher cost. This assumption fits most neatly situations where quality was set by capital investment decisions that were determined in the past without reference to the preferences of this particular procurer.

Suppliers' quality levels are modeled as independently drawn, from a distribution that is assumed commonly known. However, suppliers face uncertain costs of contract fulfillment. In the tradition of common-value models, suppliers'

²We avoid distinguishing between goods and services by referring to a 'contract' with a supplier.

cost estimates are not independent; such sources of uncertainty as weather, raw materials price shifts, future labor market tightness, and subcontract disruption probabilities are common to all competing suppliers.³

The procurement problem that results in this model could be treated as a problem in multi-dimensional mechanism design. To do so, we believe, would be to overindulge in the assumption that the buyer knows the distributions of bidders' private information, and to obscure the interaction between adverse selection and the winner's curse. We consider the sort of allocation mechanism common in markets, where bidders compete in a single dimension, price. We find that the presence of adverse selection magnifies winner's curse corrections.

A straightforward intuition underlies the magnification effect. Even if a bidder's cost estimate were unbiased *ex ante*, he needs to make a winner's curse correction: his cost estimate will no longer be unbiased if he wins, as winning will imply a lower bound on the cost estimates of losing bidders. In equilibrium, all bidders make such winner's curse corrections. This implies that a bidder with a higher-cost technology knows that he will only win if all rivals with lower-cost technologies are sufficiently more pessimistic about contract fulfillment costs than he is. Consequently, if he wins the bid, he may have seriously underestimated his costs. Understanding the differential magnitude of his potential winner's curse effect, he will rationally add a corrective premium to his bid which is larger than the premium added by lower-cost bidders. As a result, the differences between the bids of higher-cost and lower-cost suppliers is greater than the cost differences between their technologies. This magnification effect implies that lower-cost-technology bidders are disproportionately likely to submit the lowest bid.

Elucidation of the magnification effect is most transparent in a second-price auction, analyzed here. Extension to an English auction is straightforward, discussed at the end.

2 The Model

Competition to fulfill an indivisible contract involves n bidders. Bidder i 's "quality" type t_i is his own private information. If a quality type t bidder wins the auction and the second-lowest bid is p , his payoff is $p - C - \delta(t)$; here C is the unknown common element to contract fulfillment costs and $\delta(t)$ is the difference in costs resulting from the type-specific technologies that introduce quality differences.⁴ The function is assumed increasing and differentiable, with $\delta(0)$ normalized to 0. Each bidder's quality type is assumed to be an independent draw; without further loss of generality, this draw can be assumed to be uniform on $[0, 1]$. Losing bidders have a 0 payoff regardless of type.

³The frequency with which such common cost uncertainties arise limits applicability of both auction models and adverse-selection models that assume independently distributed private information.

⁴Assuming these two cost terms to be additive is simply a convenient normalization, at no loss of generality.

Each bidder i privately observes a cost estimate $X_i \in \mathsf{X} \subset [x_L, x_H] \subset \mathfrak{R}_+$, with $\{x_L, x_H\} \subset \mathsf{X}$; X may be an interval or a finite set. It is convenient to view X_i as an estimate of C , the cost he would have if he were quality type 0. Conditional on C , the cost estimates X_1, \dots, X_n are independent and identically distributed. Unconditionally, X_1, \dots, X_n are affiliated. Roughly, this means that a higher value of any X_i makes higher values of C and of other X_j 's more likely; see Milgrom & Weber (1982) for a detailed discussion.⁵ A prior distribution on C is assumed to be nonatomic and commonly known. Let the support of this distribution be denoted $[c_L, c_H] \subset \mathfrak{R}_+$.

In the usual game-theoretic sense, bidder i 's type is two-dimensional: $\tau_i = (x_i, t_i)$, his cost estimate and his quality type. The type space for a bidder is then $T = \mathsf{X} \times [0, 1]$, with $\mathsf{T} = T^n$. Let $\tau = (\tau_1, \dots, \tau_n)$ and let Ψ be the measure on T that generates a marginal ψ on T .

Consider a second-price auction. Strategies of bidder i are functions $s_i : T \rightarrow B$, specifying that $s_i(x, t)$ is the bid i submits when i observes estimate $X_i = x$ and is of quality type t ; here $B = [c_L, p_H] \subset \mathfrak{R}_+$ is the set of permissible bids, with $p_H = c_H + 2\delta(1)$ for convenience.

3 A Price-Dependent Expected-Cost Function

Let the bid functions of bidders $2, \dots, n$ be fixed and assumed continuous and increasing. Let bidder 1 be of type t and observe cost estimate $X_1 = x$. Given Ψ and the bid functions of rivals, the lowest rival bid will be a random variable, W . Temporarily imagine that bidder 1 is informed of the realization $W = w$ before he makes his bid. Bidder 1 could then update his cost estimate based on the knowledge that one rival (of unknown type) bid w , and all other rivals bid higher. Let his updated cost estimate be denoted:

$$c(x, w, t) = E[C + \delta(t) | X_1 = x, W = w].$$

By construction, c is continuous, and trivially increasing in t . By affiliation, c is increasing in x . We temporarily assert the natural property that c is increasing in w . This assertion will be shown below to be justified in symmetric equilibrium. It is possible to bound the responsiveness of $c(\cdot)$ to w . Fix $w_L < c_L$; then $w_L < c(x, w_L, 0) \leq c(x, w_L, t) \forall t$. Correspondingly, fix $w_H > c_H + \delta(1)$; then $w_H > c(x, w_H, 1) \geq c(x, w_H, t) \forall t$. Let $b(x, t)$ be defined by

$$b(x, t) = c[x, b(x, t), t]. \tag{1}$$

Given x, t , from the inequalities involving w_L, w_H , the intermediate value theorem implies there must exist a b satisfying (1). We claim such a b is unique. To demonstrate this claim, fix x, t, b satisfying (1), and consider a real number ε near 0. By affiliation,

$$\text{sign}[c(x + \varepsilon, b, t) - b] = \text{sign}[\varepsilon].$$

⁵Riley(1988) explains that the assumption that the X_i are strictly stochastically ordered by C is nearly equivalent. It is equally suitable for our purposes.

The intermediate value theorem implies a $\gamma(\varepsilon)$ so that $b + \gamma(\varepsilon) = c[x + \varepsilon, b + \gamma(\varepsilon), t]$. As c is increasing, $\text{sign}[\gamma(\varepsilon)] = \text{sign}[\varepsilon]$. As c is continuous, $\gamma(\varepsilon)$ can be made arbitrarily close to zero by choosing ε close enough to 0. This construction implies

$$\text{sign}[c(x, b + \gamma(\varepsilon), t) - b - \gamma(\varepsilon)] = \text{sign}[\gamma(\varepsilon)]. \quad (2)$$

Treating $c(x, b, t)$ as a function of b , (2) implies that where it crosses the 45° line, it crosses from above; hence b satisfying (1) is unique so (1) serves to specify a well-defined function $b : T \rightarrow B$.

That b is increasing follows straightforwardly from increasingness of c . Let $(\hat{x}, \hat{t}) \geq (x, t)$, distinct; then $c[\hat{x}, b(x, t), \hat{t}] > c[x, b(x, t), t] = b(x, t)$ implying $b(\hat{x}, \hat{t}) > b(x, t)$.

4 A Best-Response Function

Consider given strategies of rival bidders yielding a w for which $w > c(x, w, t)$. Then bidder 1's expected profit is zero if his bid is more than this w , and strictly positive if his bid is less: he would win, be paid the second-lowest bid w , and an estimate of his contract fulfillment cost that correctly accounts for these events would be $c(x, w, t)$. For a w for which $w < c(x, w, t)$, however, bidder 1's expected profit is strictly negative if he wins. Thus if bidder 1 could observe w before he bid, to bid $c(x, w, t)$, that is, to bid $b(x, t)$, would be a best response to the profile of rival strategies yielding w . As $b(x, t)$ is a best response among all functions of x, w and t , it is necessarily a best response among all functions of x and t , and the assumption that bidder 1 observed w before bidding is harmless in equilibrium. The preceding argument has not depended in any way on the identity of bidder 1.

5 The Magnification Effect

We analyze the model when all bidders submit the bid $b(x, t)$. This would appear to be a mutual best response, but the issue of equilibrium existence in auction models with multi-dimensional types is quite complex; to focus on the interplay between adverse-selection and the winner's curse, this issue is removed to the last section.

This equilibrium extends a familiar characterization of second-price equilibria in affiliated-values auction models: a bidder is submitting a bid equal to the price at which he is indifferent between winning and losing in the event that he ties for lowest bid.

In terms of the notation set up here, the magnification effect corresponds to the inequality $b_t := \partial b / \partial t > \delta'(t)$. That is, bids increase faster than the increase in costs due to an increase in types. To investigate this, define

$$\xi(x, s, t) = E[C + \delta(s) | X_1 = x, W = b(x, t)].$$

In contrast to (1), ξ isolates the equilibrium effect on expected project cost of the inferences about rivals' cost estimates drawn from tying for lowest bid, as a function of bidder 1's type. That is, with s fixed, varying t merely varies the lower bound on rivals' bids, and influences the expectation solely through its influence on C . By construction,

$$\xi(x, s, t) = \delta(s) + E[C|X_1 = x, W = b(x, t)],$$

and $b_t = \delta'(t) + \xi_t$. By affiliation, $E[C|X_1 = x, W = w]$ is increasing in w , so $\xi_t > 0$, and thus $b_t > \delta'(t)$.

Hence, in any equilibrium of the second-price auction with adverse-selection, differences in bids across types will exceed type-specific cost differences. In other words, cost differences are **magnified** in bid differences.

As a result, a relatively low-cost bidder is much more likely to win. If there were no magnification, a type t bidder 1 would only outbid a type $\hat{t} < t$ bidder 2 in the event $\theta_0 := \{X_1 + b_x(X_1 - X_2) + \delta(t) < X_2 + \delta(\hat{t})\}$, with $\Pr[\theta_0] < \Pr[X_1 < X_2]$. However, with the magnification of cost differences,⁶ to a first-order approximation, the event in which bidder 1 outbids bidder 2 is

$$\theta_1 := \{X_1 + b_x(X_1 - X_2) + \delta(t) + \xi_t(t - \hat{t}) < X_2 + \delta(\hat{t})\},$$

so that $\Pr[\theta_1] < \Pr[\theta_0]$.

6 The Effect of Increased Competition

Suppose the actual competition is only among the first $m \leq n$ bidders, with some ad hoc bar on the remaining bidders allowing us to examine varying numbers of bidders without having to adjust our underlying probability measure. An implication of affiliation is Theorem 5 in Milgrom & Weber (1982):

$h(\{a_i, b_i\}_{1,n}) := E[C|a_i \leq X_i \leq b_i, i = 1, \dots, n]$ is nondecreasing in all arguments.

If affiliation is semi-strict, cases in which $a_i < b_i$ and yet h fails to be increasing can be viewed as pathological.

Consider any realization of types t_2, \dots, t_n ; let σ_i be the inverse bid function specific to type t_i : $k \stackrel{def}{=} b[\sigma_i(k), t_i]$. Then, in the m -bidder auction, bidder 1's conditioning event $W = w$ becomes

$$\theta_m := \{\sigma_i(w) \leq X_i, i = 2, \dots, m; x_L \leq X_i, i = m + 1, \dots, n\}$$

with one of the first $m - 1$ inequalities exact. The effect of increasing the number of bidders from $\hat{m} < m$ to m is to make $E[C|X_1 = x, W = w]$ more responsive to w , by (2). Thus ξ_t is an increasing function of m . The implication is then that, for any fixed difference in costs, increasing the number of bidders exacerbates the resulting bid differences.

⁶Note that if costs were independent, as is assumed in most adverse selection models, ξ_t would be zero and there would be no magnification effect.

7 Efficiency Implications

In the absence of correlated cost uncertainties, the $\Pr[\theta_1]$ above would be driven to 0, as happens, for example, in the models of Gabor & Granger (1966), Gardner (1971), Monroe (1973), Akerlof (1979), Leland (1979), Wolinsky (1983) and Cooper & Ross (1984). The fact that $\Pr[\theta_1]$ is positive sharply illustrates the richness of this model: auction outcomes reflect bid magnifications, but incompletely. Here, a higher-cost-technology bidder has private information about common cost uncertainties, which for nonpathological distributions yields positive equilibrium expected profit.

Efficiency analyses require a specification of the costs to the buyer of obtaining a lower-quality product. Let $V + \Delta(t)$ be the value to the buyer of a contract fulfilled in accordance with technology t ; $\Delta(t)$ is the buyer's value of the quality the seller brings to contract fulfillment. There need not be any particular relationship between the orders of magnitude of $\delta(t)$ and $\Delta(t)$ or their respective responsiveness to t .⁷ Maskin (1992) and Goeree and Offerman (2003) have found that an efficient allocation is not guaranteed when bidders' private information is multi-dimensional, but have not modeled a bid-taker as facing adverse selection with quality—and thus efficiency—consequences.

Suppose that $\Pr[V + \Delta(0) > C + \delta(1)] = 1$, and that $\Delta(t) - \delta(t)$ is non-constant, so that efficiency requires the contract be fulfilled, and simply breaks down to the nontrivial issue of whether the preferred type is likely to win the auction. If $\Delta'(t) < \delta'(t)$ on $(0, 1)$, then efficiency is enhanced by the magnification effect; the buyer should be buying from the lowest-quality seller, who is disproportionately likely to win the auction.

Adverse selection is usually thought, however, to have the opposite implication: that high-quality sellers would be the efficient choice if they could be discerned, which would result from $\Delta'(t) > \delta'(t)$ on $(0, 1)$. In this case, the magnification effect is deleterious to efficiency, as the preferred higher-quality sellers are disproportionately less likely to win the auction. It is in this sense that the prior literature on adverse selection may be said to have understated the problem.

Recall that the magnification effect increases with more bidders. If, as in standard auction models, there is some number of bidders past which the expected price paid for the contract falls but slightly with further added competition, then it will pay the buyer to employ a credible precommitment to consider only the first m bids submitted. This is despite a restriction to anonymous policies in the model; in practice, when a buyer has information external to the auction about potential sellers' technologies, and can influence the identities of the buyers given serious consideration, it is common to do so.

⁷Two examples: About a decade ago, an innovation in producing printer paper (adopted by all major suppliers within about fifteen months) reduced costs by nearly 10%; users were said not to detect the quality deterioration. The cost difference between a mirror on the Hubble space telescope which had passed a skewness test and one which skipped that test was reported to be less than \$25,000; the difference in value to NASA appears to have been approximately one billion dollars.

8 Relevance of Magnification for Other Auction Forms

Consider an English auction, as that mechanism is modelled in Milgrom & Weber(1982). To maintain the viewpoint above, we continue to assume that bidders are competing to sell, so the English auction takes an unfamiliar but logically equivalent form: the price falls continuously from an initial price that is clearly too high, while bidders can irrevocably exit if the price becomes unacceptably low (i.e., when the contract price falls below their ‘exit price’) with the contract being awarded to the last remaining bidder.⁸ As Milgrom & Weber demonstrate, this auction consists of an initial phase in which the $n - 2$ bidders with the highest willingnesses-to-accept publicly reveal their exit prices, followed by a second-price auction among the remaining two bidders, equipped with this additional information. Hence, the above analysis implies that, at a minimum, cost differences between the final two bidders are magnified in differences in their planned exit prices.⁹ In fact, a corresponding argument can be applied at each state: planned exit prices of remaining bidders after k bidders have exited magnify cost differences.

First-price auctions are more complicated. Since an equilibrium bid function in a second-price auction is a price at which the bidder is indifferent between winning and losing, this price can be interpreted as an equilibrium willingness-to-accept. A bidder who bid as low as his willingness-to-accept in a first-price auction could not hope to profit. His equilibrium bid adjusts upward from his willingness-to-accept, continuing to increase his bid so long as the incremental increase in his expected profitability in the event that he wins, associated with being paid more, exceeds the impact on his expected profit of winning incrementally less often. Since differences in costs are magnified in differences in willingness-to-accept, this incremental profitability/probability tradeoff begins with the magnification effect already in place.¹⁰

⁸Herodotus reports the auctions for wives in ancient Greece sometimes took this form, when a particular woman could only be wed if her family provided a dowry, and the family wished the dowry payment to be minimized by auction competition (Cassady [1967]).

⁹There is reason to suspect that Milgrom & Weber’s analysis may well carry over to the adverse selection case, implying that the buyer’s expected cost would be lower than for a second-price auction. However the public information inferred from exit prices would have much less impact. An exiting bidder would determine his exit price via a function $b(x, t, p)$, where p is the vector of earlier exit prices. While the function being used could be calculated, the exit price would only allow inference of an iso-bid contour in (x, t) space. Only x would be of interest to bidders still competing. Moreover remaining bidders could not be certain that bidders exited in decreasing order of their cost estimates. A bidder would rationally update his cost estimate by integrating across the iso-bid contour with respect to a density that reflected his current beliefs across types of the exiting bidder. However, these beliefs would differ from the prior marginal across types, as they would rationally incorporate available information about the probable rank-order of the exiting bidder’s cost estimate. A similar calculation is discussed in Harstad & Rothkopf (2000,section 7). Their analysis suggests that the expected contract fulfillment cost for a second-price auction would be a good estimate of that for an English auction.

¹⁰Let bidders $2, \dots, n$ use increasing bid functions B . Consider the situation in which two different quality types of bidder 1, $t_L < t_H$, determine a best response for different

Hence adverse selection exacerbates winner’s curse corrections across a broad range of auction types.

9 On Equilibrium

This section supplies information about the existence of equilibrium in the model introduced here, and similar models. To our knowledge, the material provided here has not been compiled in a single source.

The mathematical aspects of this model relevant to the existence of an equilibrium correspond very closely to those in Pesendorfer and Swinkels (2000). Their concern is with asymptotic efficiency and asymptotic information aggregation; their characterization—that for any $\nu > 0$, there is a large enough number of bidders such that the function they analyze is within ν of being a best response—applies to the $b(x, t)$ function above. Their signals corresponding to x are discretely distributed, those corresponding to t are continuously distributed.

Jackson (1999,2009) has provided an example auction in which multi-dimensional private information yields a nonexistence theorem. His example is very close to a variant of the model above when both types of private information are drawn from discrete distributions and $\delta(t)$ is the identity function. Jackson’s discussion indicates that his example does not readily extend to nonexistence of equilibrium when both types of private information are nonatomic, but that he is concerned that very similar issues arise.

Goeree and Offerman (2003) find an equilibrium for a quite similar model that avoids Jackson’s nonexistence problems. Their model has both signals drawn from nonatomic distributions, has every signal (common and private) drawn independently from every other random variable in the model, and achieves a common-value element by assuming that the auctioned asset’s common value is the average of the independent private estimates. They then find an equilibrium under the assumption that both signal distributions are log-concave (and that $\delta(t)$ is the identity function). Adapting our model to meet these assumptions would be sufficient for their Proposition 2 (which establishes the equilibrium).

cost estimates $x_L > x_H$ chosen so that both would have the same willingness to accept: $b_0 := b(x_L, t_L) = b(x_H, t_H)$. Now consider the impact upon expected profits of an increase in the bid from $\beta > b_0$ to $\beta + d\beta$:

$$\Pr [B(x_j, t_j) > \beta, j = 2, \dots, n | X_1 = x] - (\beta - b_0) \Pr \left[\beta = \min_{j=2, \dots, n} B(x_j, t_j) | X_1 = x \right].$$

The first term indicates the effect of being paid incrementally more when bidder 1 still wins; the second the effect of winning incrementally less often. Notice that, with the same willingness-to-accept, this expression depends on bidder 1’s quality type only through the different signals that would lead different quality types to have the same willingness-to-accept. For the expression to be larger for type t_L than for type t_H , for $b_0 < b \leq B(x_H, t_H)$, would be sufficient to imply that the magnification effect is even greater for first-price than for second-price auctions. As $x_L > x_H$, affiliation implies that the first term in the above expression is greater for t_L than for t_H . Adverse selection has made the second term much more complicated than in mainstream models. If the density of the lowest rival bid translates smoothly with increases in X_1 , then the second term above is approximately the same size for t_L and t_H , so then the well-behaved first term dominates the expression.

Tian (2009) provides a necessary and sufficient condition for existence of a pure strategy equilibrium. If the condition is not satisfied for a given game, the game has no equilibrium in pure strategies. While significant, it does not appear easy to determine whether this condition is satisfied or falsified in a game of incomplete information, as is the case with our model.

The ability to demonstrate existence of an equilibrium in increasing pure strategies has been greatly enhanced by Reny's Theorem (2008), which differs from prior approaches by not requiring convexity of best responses. Since his approach is quite different and in ways more promising, we outline the relationship between this model and Reny's results. Readers are referred to Reny (2008), section 2 for definitions of terms used in the remainder of this section.

Let S be the set of monotone strategies; that is, S is the set of monotone functions mapping domain T into range B . For any $(s_0, s_1) \in S \times S$, define the metric

$$\mu(s_0, s_1) := \int_T |s_0(\tau) - s_1(\tau)| d\psi(\tau),$$

let $\mathbf{s} = (s_1, \dots, s_n)$, $\mathbf{S} = S^n$, and treat (\mathbf{S}, μ) as a metric space. As it is straightforward to establish that (\mathbf{S}, μ) is closed and convex, it is an absolute retract.

The other key element to define is the correspondence $G : \mathbf{S} \rightarrow \mathbf{S}$ of monotone best responses. For any strategy profile $\mathbf{s} \in \mathbf{S}$ and any $i = 1, \dots, n$, $G_i(\mathbf{s})$ is the subset of S consisting of all profiles of monotone functions that are each best responses to the profile of strategies s_{-i} of i 's rivals, $i = 1, \dots, n$. Naturally, G is nonempty-valued. Suppose any convergent sequence $\{(s^k, \hat{s}^k)_{k=1,2,\dots}\} \rightarrow (\varsigma, \hat{\varsigma})$, with $(s^k, \hat{s}^k) \in \mathbf{S} \times \mathbf{S}$ and $s^k \in G(\hat{s}^k)$. It is straightforward that $(\varsigma, \hat{\varsigma}) \in \mathbf{S} \times \mathbf{S}$. Reny's approach can be used if $\varsigma \in G(\hat{\varsigma})$; then G would be upper-hemicontinuous. There is reason to doubt that ς can be proven to lie in $G(\hat{\varsigma})$. (An alternative to needing upper-hemicontinuity of G would be to treat the set of permissible bids as a finite set. Then Reny's Theorem 4.1 could be applied directly, to show the existence of a monotone pure-strategy equilibrium.)

The key step to use Reny's approach is to show that G is contractible-valued. Fix strategies $(s_2, \dots, s_n) \in S^{n-1}$ and suppose $s_1 \in S$ and $\hat{s}_1 \in S$ are both best responses to the fixed profile s_{-1} . Then it is clear that, for almost every type τ , if both $s_1(\tau)$ and $\hat{s}_1(\tau)$ are both expected-payoff maximizing responses for player 1's type τ , so is $\max\{s_1(\tau), \hat{s}_1(\tau)\}$. Hence images of G are join-closed, and the mapping h in equation (6.1) in Reny (2008) is a contraction map for any image of G .

The setup just outlined allows demonstrating existence of a monotone pure-strategy equilibrium via Reny's Theorem 2.1, which restated in terms of the current notation is:

Theorem: *Suppose that a compact metric space (\mathbf{S}, μ) is an absolute retract and that $G : \mathbf{S} \rightarrow \mathbf{S}$ is an upper hemicontinuous, nonempty-valued, contractible-valued correspondence. Then G has a fixed point.*

10 Concluding Remarks

If any of many common procurement situations are modeled via either a mainstream common-value auction model or a standard adverse-selection model, the model contains only part of the critical strategic elements. The interplay between the winner's curse and adverse selection, illustrated here via the magnification effect, points to fundamental misconceptions and faulty prescriptions that can arise in either model if analyzed separately. Markets where sales occur through auctions or auction-like competitions can call for a similarly integrated analysis if the seller's payoff is affected by technological differences among buyers in ways not fully reflected in revenue; one example among many is a government auctioning airwaves rights without restricting the uses to which buyers may put them.

A combined model cannot sensibly be much simpler than that introduced here; nonetheless, the last section has pointed to serious complexities. Confronting such complexities (at the probable sacrifice of some mathematical elegance) is necessary if useful prescriptions are to arise (cf. Rothkopf and Harstad, 1994). This paper has but begun to point the way, partly by showing that it means settling for smaller characterizations.

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