İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

MODELLING OF PERIODIC PERFECTLY CONDUCTING ROUGH SURFACES IN TERMS OF HIGHER ORDER IMPEDANCES

M.Sc. Thesis by

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PERİODİK MÜKEMMEL İLETKEN ENGEBELİ YÜZEYLERİN YÜKSEK MERTEBEDEN EMPEDANSLAR İLE MODELLENMESİ

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ABBREVATIONS

MoM	:	Method of Moment
FEM	:	Finite Element Method
FDTD	:	Finite Difference Time-Domain
IBC	:	Impedance Boundary Condition
SIBC	:	Standard Impedance Boundary Condition
GIBC	:	Generalized Impedance Boundary Condition
HIBC	:	Higher Order Impedance Boundary Condition
HPIBC	:	Higher Order Periodic Impedance Boundary Condition

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LIST OF SYMBOLES

$\vec{E}$	:	Total Electric field vector
$\vec{E}^i$	:	Incident field vector
$\vec{E}^s$	:	Scattered field vector
μ_0	:	Permeability of the free space
ε	:	Dielectric permittivity
σ	:	Conductivity
k	:	Wavenumber
ω	:	Angular frequency
λ	:	Wavelength
ϕ_0	:	Incidence angle
α	:	Maximum value of the amplitude of the rough surface
L	:	a Period of the periodic rough surface
β	:	Propagation Constant
Z	:	Surface Impedances
Ω	:	Unit Cell (a single period of the periodic geometry)
$\hat{\Omega}$	:	Unit Cell of the Equivalent Structure
Γ_0	:	Perfectly conducting periodic surface

MODELLING OF PERIODIC PERFECTLY CONDUCTING ROUGH SURFACES IN TERMS OF HIGHER ORDER IMPEDANCES

SUMMARY

In this study, a method is presented for the equivalent representation of a perfectly conducting (PEC) periodic rough surface in terms of planar boundary with higher order periodic impedance boundary condition (HPIBC). In the proposed approach, the periodic rough surface is replaced by a flat one having higher order periodic impedance boundary condition (HPIBC). For the sake of simplicity the analysis is carried out for one-dimensional (1-D) surfaces. The explicit relations between the inhomogeneous surface impedances appearing in HPIBC and the surface variation of the periodic PEC surface are derived through the Taylor expansion of the total field. This direct relation between the surface impedances and the variation of the periodic surface also causes the impedances to be periodic. The surface impedances are independent of the incidence angle, and yield to an universal equivalent boundary condition for perfectly conducting periodic rough surfaces. Therefore, by representing the perfectly conducting(PEC) rough surface in terms of a plane one characterized by the above mentioned inhomogeneous impedance boundary condition, one can achieve a simpler formulation of the corresponding scattering problem. On the other hand, it has to be remarked that the method allows to computing the field scattered by the original surface only in the region exterior to the fictitious plane.

The resulting scattering problem related to a planar surface with HPIBC is solved by using the Floquet mode expansion, which reduces the problem to the solution of a linear system of equations. The results of the proposed method are compared with those of Finite Difference Time Domain (FDTD) Method for a number of different surfaces. Numerical simulations show that the method yields accurate results and it is computationally effective. From the numerical implementations it is observed that the MSE error between two methods is always less than 1%.

PERİYODİK MÜKEMMEL İLETKEN ENGEBELİ YÜZEYLERİN YÜKSEK MERTEBEDEN EMPEDANSLAR İLE MODELLENMESİ

ÖZET

Bu çalışmada, mükemmel iletken periyodik engebeli yüzeylerin, yüksek mertebeden periyodik empedans sınır koşuluna sahip düzlemsel bir yüzey ile eşdeğer olarak modellenmesini sağlayan bir metod sunulmaktadır. Önerilen metod, periyodik engebeli yüzeyin yüksek mertebeden periyodik empedans koşuluna sahip düzlemsel bir yüzey ile gösterilimine dayanmaktadır. Sadelik amacıyla, analiz tek boyutlu yüzeyler için gerçekleştirilmektedir. Yüksek mertebeden periyodik empedans sınır koşulunda bulunan homojen olmayan yüzey empedansları ile mükemmel iletken periyodik yüzeyin değişimi arasındaki açık ilişki, toplam elektrik alanın Taylor serisi açılımı ile elde edilirler. Yüzey empedansları ile periyodik yüzeyin değişimi arasındaki direkt ilişki, empedansların da periyodik olmasını sağlamaktadır. Yüzey empedansları aydınlatma açısından bağımsızdırlar ve mükemmel iletken periyodik engebeli yüzeyler için evrensel bir eşdeğer sınır koşulu oluşturmaktadırlar. Dolayısıyla, mükemmel iletken engebeli yüzeyleri yukarıda anlatıldığı gibi homojen olmayan empedans sınır koşuluna sahip bir düzlemsel yüzey ile modelleyerek, söz konusu saçılma problemi için daha basit bir formülasyon elde etmek mümkündür. Öte yandan, göz önünde bulundurulmalıdır ki; burada önerilen metod ile, orjinal yüzeyden saçılan alanı sadece yüzeyin maksimum noktasına teğet olan farazi bir düzlemin üzerinde kalan bölgede hesaplamak mümkündür.

Elde edilen, yüksek mertebeden sınır koşuluna sahip düzlemsel yüzeyle ilişkili eşdeğer saçılma problemi; problemi, lineer bir denklem sisteminin çözümüne indirgeyen Floquet teoremi ile çözülmüştür. Önerilen metodun sayısal sonuçları, Sonlu Farklar Zaman Domeni(FDTD) Metodu ile elde edilen referans sonuçlar ile karşılaştırılmaktadırlar. Sayısal simülasyonlar göstermektedir ki, önerilen metod ile tatmin edici sonuçlar alınmaktadır ve metod hesaplama süresi açısından etkindir. Uygulamalardan görüldüğü üzere, referans yöntem ve önerilen yöntemin sonuçları arasındaki ortalama kare hata (MSE), %1 'in altında kalmaktadır.

1. INTRODUCTION

1.1 Scattering from Periodic Rough Surfaces

Rough surface models are divided into two classes: surfaces with random irregularities and surfaces with given profiles. Random surfaces met in nature are treated with statistics because of the absence of details. Surfaces with given profiles are known exactly and do not involve any statistics. However, for another class of rough surfaces,rough surfaces with periodic irregularities, a non-statistical approach is possible and desirable. Although periodic surfaces are less often met in practice than random surfaces, there is at least two reasons why they merit close study. Firstly, it is possible to give a general indication of the general behavior of rough surfaces.Secondly, if a surface is to be manufactured for a specific purpose, it is easier to make the roughness periodic than make it of a random nature with a recommended probability distribution [1].

Periodic structures often appear in the applications such as antenna design, microwave systems, metamaterials, radomes, EMC applications etc and the analysis of electromagnetic wave propagation in such structures has an important place in the electromagnetic theory. Correspondingly, random rough surfaces are often treated as periodic structures constructed by repeating a suitable unit cell [2–6]. Scattering of electromagnetic waves from perfectly conducting periodic surfaces are of importance and one can find several research activities in this direction. The development of broadband absorbers, study of sea surface scattering, design of uniform antenna arrays, microwave lenses, and artificial dielectric media are a few examples. The most common methodology for solving scattering problems related to periodic surfaces is based on the Rayleigh Hypothesis which is only valid for surfaces having a sinusoidal variation and small slopes compared to wavelength. In such a case the scattered field is assumed to be represented in terms of discrete spectrum of outgoing plane waves. The method was first investigated by Rayleigh in the case of normal incidence onto a sinusoidal surface in 1895. Rayleigh obtained an approximation of the first few scattered modes [7–13]. His method is easily generalized for oblique incidence by LaCasce and Tamarkin in 1956 and the general Rayleigh approach was developed by Rice in 1951 for slightly rough and random surfaces [14,15]. Deryugin extended the method to the solutions for periodic rectangular corrugations in 1960 [16]. Schouten and De Hoop developed the extension of the Rayleigh method for the solutions of any analytically given rough surface in 1957 [17]. The boundary element method and the perturbation methods [18, 19] are the other frequently used approaches in the solution of scattering problems related to periodic surfaces. Besides, the Method of Moments (MoM) [20] was used to analyze perfectly conducting surfaces with a profile of sinusoidal shape, where a unit cell of the periodic surface is treated by means of a Green's function for periodic arrays [21, 22]. The finite element method (FEM) [23] is an useful alternative because of its capabilities to model complex structures and inhomogeneous materials. Another popular method is finite difference time-domain (FDTD) method. However, especially for oblique incidence of a plane wave on a periodic surface, a time-domain formulation becomes challenging with respect to the analysis of the periodic structures; FDTD solution is a very satisfying method [24]. It is also used as a reference solution of the original problem presented in this work.

1.2 Approximate Boundary Conditions

Approximate boundary conditions can be very helpful in simplifying the solutions of wave problems involving complex structures such as electromagnetics, hydrodynamics, and acoustics etc. In electromagnetics, approximate boundary conditions have been widely used in scattering, propagation, and waveguide analysis to simulate the material and geometric properties of the surfaces involved. Since the boundary conditions themselves are approximate, the difficulties are eliminated at the expense of obtaining only an approximate solution of the scattering problem. The approximate boundary conditions is based on the relationship between the tangential electric field and tangential magnetic field at any point on the boundary between the exterior of the scatterer and the free space. In many cases acceptable accuracy may be attained, and the approximate boundary conditions may be used to obtain solutions to scattering problems that cannot be solved using other methods. Accordingly, a considerable amount of computational cost is reduced when the approximate boundary conditions are employed. Assuming the approximate boundary condition accurately models the electromagnetic behavior of the surface, the loss in accuracy relative to the exact solution can be minimal. The accuracy of the approximate boundary condition depends on the complexity of the given geometry [25–28]. Among these methods, Impedance Boundary Conditions (IBC) are widely used to simplify the mathematical and numerical complexities in the solution of scattering problems in electromagnetic theory. Along this line, effective impedance boundary conditions have been developed for such objects as the earth surface, thin layers of dielectrics and multilayered dielectric structures.The simplest form of Impedance Boundary Conditions is the first-order (or standard) impedance boundary condition (SIBC) which was first described by Leontovich in 1948. In this case, tangential components of the electric field are related to those of the magnetic fields by a simple multiplication factor. In particular, when the reflection characteristics of the boundary are essentially the same for all angles of the incidence the SIBC is an excellent approximate boundary condition. The applications of this method was recognized after 1950s with the works of J.R. Wait. Wait used the boundary condition to simulate the land in studies of ground wave propagation over the earth [29–33]. As a result of their simplicity and ease of use, more general versions of these conditions (improved or higher order versions) are now considered for electromagnetic applications. These improved impedance conditions are derived as a result of a restriction, namely the behavior of the coating or the surface treatment must be independent of the angle of incidence. These higher order impedance boundary conditions(HOIBC), often referred to as generalized impedance boundary conditions(GIBC), permit the simulation of more complicated material and composite surfaces with greater accuracy. By using HOIBC, the virtue of the derivatives and the additional degrees of freedom helps to simulate the material properties better. The restriction that the properties of the coating or surface treatment not be a function of the angle of incidence is removed by incorporating derivatives of the field components in the impedance boundary condition [34–38]. Agreeably, the coefficients appearing in the differential equations of the HOIBC depend on the local parameters of the scatterer and can be determined in a number of ways [39]. On the other hand, although the generalized impedance boundary conditions have been used in a number of analytical and numerical solutions of scattering problems, difficulties may arise if the surface has an edge or a discontinuity. Accordingly, different boundary conditions can be derived for specific geometries, such as rough surfaces or buried objects, in order to give a simple solutions to the scattering problems..

The determination of the IBC for a given scatterer constitutes an important class of problems in the electromagnetic theory and various approximate methods have been established in the literature for special kind of geometries and surfaces. The equivalent impedance boundary concept can also be used for the scattering of electromagnetic waves from periodic surfaces. As far as I know, not much work have been done in this direction except [40]. In this work, a standard impedance boundary condition is derived for a perfectly conducting periodic rough surface.

1.3 The Aim of the Work

The main objective of this study is to derive the higher order inhomogeneous impedance boundary condition for the perfectly conducting periodic rough surfaces and to give a simple method for the solution of the equivalent problem. In [41], a higher order inhomogeneous impedance boundary condition is given for the perfectly conducting cylindrical objects. In the present work, the idea given in [41] is extended to the periodic rough surface case. In the proposed approach, the periodic rough surface is replaced by a flat one having higher order periodic impedance boundary condition (HPIBC). For the sake of simplicity the analysis is carried out for one-dimensional (1-D) surfaces. Through the Taylor expansion of the total field it is shown that the surface impedances which appear in the impedance boundary condition are directly related to variation of the periodic surface, which also causes the impedances to be periodic. The surface impedances are independent of the incidence angle, and yield to an universal equivalent boundary condition for perfectly conducting periodic rough surfaces. Therefore, by representing the perfectly conducting(PEC) rough surface in terms of a plane one characterized by the above mentioned inhomogeneous impedance boundary condition, one can achieve a simpler formulation of the corresponding scattering problem. On the other hand, it has to be remarked that the method allows to computing the field scattered by the original surface only in the region exterior to the fictitious plane.

The resulting scattering problem related to planar surface with HPIBC then can be solved by applying the Floquet Theorem [42], which reduce the problem to the solution of a linear system of equations. The results of the proposed method are compared with those of Finite Difference Time Domain (FDTD) Method for some surfaces and quite good agreements have been observed.

The organization of the paper is as follows: in section II an equivalent higher order periodic impedance boundary condition is derived for the given perfectly conducting rough surface. A method based on the Floquet representation of the scattered field for equivalent problem is given in Section III, while the numerical results are presented in Section IV. Concluding remarks are addressed in Section V. A time factor $e^{-i\omega t}$ is assumed and omitted throughout the paper.

2. DERIVATION OF THE IMPEDANCE BOUNDARY CONDITION

Consider the two-dimensional scattering problem related to the periodic structure given in Figure 2.1. In this configuration, Γ_0 is a perfectly conducting periodic surface having a period L. In each period the surface is represented by the equation $x_2 = f(x_1 + pL)$, $x_1 \in (0,L)$, $p = 0,\pm1,\pm2,...$, where $f(x_1)$ is a single-valued periodic function and has continuous first-order derivatives for all $x_1 \in R$. The half-space above the perfectly conducting surface Γ_0 is filled with a non-magnetic simple lossless dielectric material whose dielectric permittivity is ε.

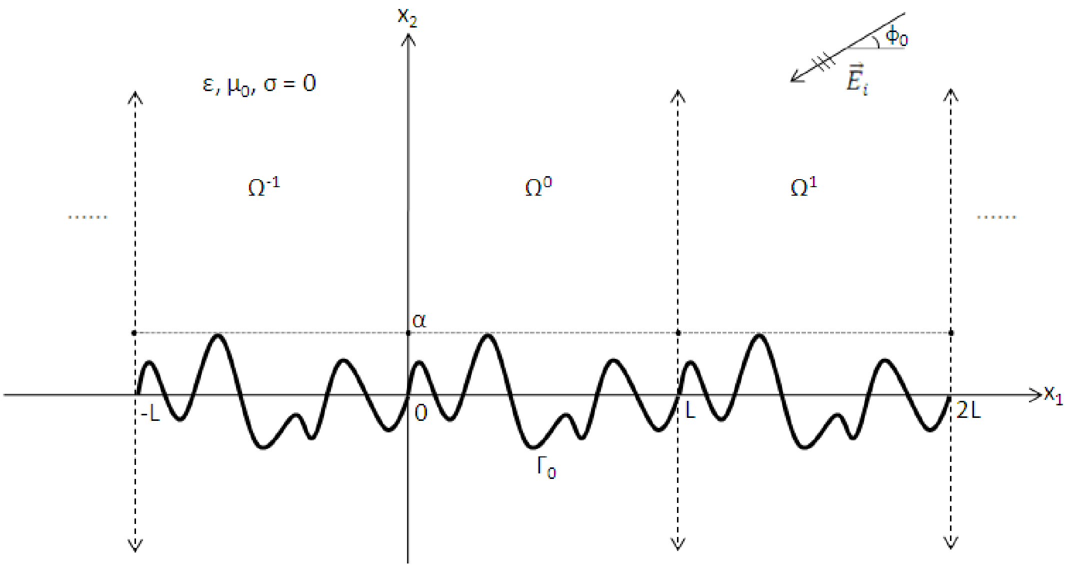

Figure 2.1: Perfectly conducting periodic surface

The scattering problem related to configuration given in Figure 2.1 is to determine the effect of Γ_0 on the propagation of electromagnetic waves in the upper half-space, more precisely, to obtain the scattered field from the periodic rough surface. In the present work the surface Γ_0 is assumed to be illuminated by a TM polarized time-harmonic quasi-periodic wave whose electric field vector is given as $\vec{E}^i = (0,0,E^i(x))$ where $x = (x_1,x_2)$ is the position vector in R^2. The quasi-periodicity [43] implies that $E^i(x_1+L,x_2) = e^{-i\beta L}E^i(x_1,x_2)$ where β is the propagation constant in the x_1-direction. Then according to the Floquet theorem the scattered field distribution of a periodic structure as in Figure 2.1 remains unchanged under a translation of the observation point in the x_1-direction through a period L while its amplitude is multiplied by a complex constant $e^{-i\beta L}$ which corresponds to the variation of the incident field with x_1. Note that due to the homogeneity in the Ox_3 direction the problem is reduced to a two-dimensional ($2D$) scalar one in terms of the total field function $E(x)$, where the total electric field vector is defined by $\vec{E} = (0,0,E(x))$.

The total electric field $E(x)$ within the region $\Omega^p := \{x = (x_1,x_2) : pL < x_1 < (p+1)L, \quad x_2 > f(x_1)\}$ satisfies the reduced wave equation

$$\Delta E + k^2 E = 0, \qquad x \in \Omega^P \tag{2.1}$$

with the boundary condition

$$E(x) = 0 \quad on \;\; \Gamma_0 \tag{2.2}$$

where $k = w\sqrt{\varepsilon\mu_0}$ is the wave-number of the upper half-space. The scattered field which is defined by $E^s = E - E^i$ satisfies the classical quasi-periodic radiation condition expressed by the Floquet series of the scattered wave [44, 45]. Invoking Floquet theorem, the problem can be readily reduced to a consideration of the fields over a single period, sometimes is called a unit cell. For that reason, in the following, the analysis will be carried out for $p = 0$.

The problem stated by (2.1) and (2.2) can be treated by using one of the common methodologies such as surface integral methods, small-perturbation approach, Kirchoff approximation etc [46–52]. In the following a new method is introduced which is based on the representation of the periodic surface Γ_0 in terms of a HPIBC on a plane above the surface. To do this, one first has to obtain required impedance boundary condition for a given periodic surface Γ_0.

In order to derive an equivalent impedance boundary condition for the problem stated above, the idea in [41] is extended to the present problem, i.e.: we first consider the plane $x_2 = \alpha$, $\alpha \geq max(f(x_1))$ and expand the total electric field E into a Taylor series with respect to x_2 around this plane, namely,

$$E(x_1,x_2) = \sum_{m=0}^{\infty} \frac{1}{m!} \frac{\partial^m E(x_1,\alpha)}{\partial x_2^m} (x_2-\alpha)^m, \qquad x \in \Omega^0. \tag{2.3}$$

Since the total field $E(x_1,x_2)$ is a regular function of x_2, the series (2.3) is convergent down to the surface Γ_0 [53].

For that reason (2.3) can be considered as an explicit expression of the total electric field in the region $f(x_1) < x_2 < \alpha$ and has to satisfy the boundary condition (2.2) on the surface $x_2 = f(x_1)$, which yields

$$\sum_{m=0}^{\infty} \frac{1}{m!} \frac{\partial^m E(x_1,\alpha)}{\partial x_2^m} (f(x_1)-\alpha)^m = 0. \tag{2.4}$$

On the other hand the general expression of the $M'th$ order impedance boundary condition given on the plane $x_2 = \alpha$ with inhomogeneous surface impedances $Z_0(x_1) = i\omega\mu_0$ and $Z_m(x_1)$, $m = 1,2,..M$ is in the form

$$\sum_{m=0}^{M} Z_m(x_1) \frac{\partial^m E(x_1,\alpha)}{\partial x_2^m} = 0, \qquad x \in \Omega^0. \tag{2.5}$$

Then, by taking M terms in the Taylor series appearing in (2.4) and comparing the resulting expression with the boundary condition (2.5), one can easily conclude that the surface impedances are related the surface function $f(x_1)$ as follows:

$$Z_m(x_1) = i\omega\mu_0 \frac{1}{m!} (f(x_1)-\alpha)^m, \qquad m = 0,1,...,M. \tag{2.6}$$

Note that since $f(x_1)$ is periodic, the resulting surface impedances $Z_m(x_1)$'s are also periodic. This property together with the quasi-periodicity of E yields the boundary condition (2.5) to be periodic which we call higher order periodic impedance boundary condition (HPIBC).

(2.5) furnishes an equivalent representation of the periodic PEC surface in terms of a flat one having inhomogeneous surface impedances. Therefore, for $x_2 > \alpha$, the scattering problem is then reduced to the solution of (2.1) under the boundary condition (2.5). In the next section we will give a solution for this equivalent problem in terms of Floquet mode expansion of the scattered field.

3. SOLUTION OF THE EQUIVALENT PROBLEM

The scattering problem stated above is now equivalently reduced to the solution of the scattering of electromagnetic waves from a plane having a HPIBC given in (2.5) (see Figure 3.1). This latter problem which is also periodic can be treated by using Floquet theorem and we will concentrate the scattered field in a single period $\hat{\Omega}^0$ where $\hat{\Omega}^0 := \{x = (x_1, x_2) : 0 < x_1 < L, \;\; x_2 > f(x_1)\}$.

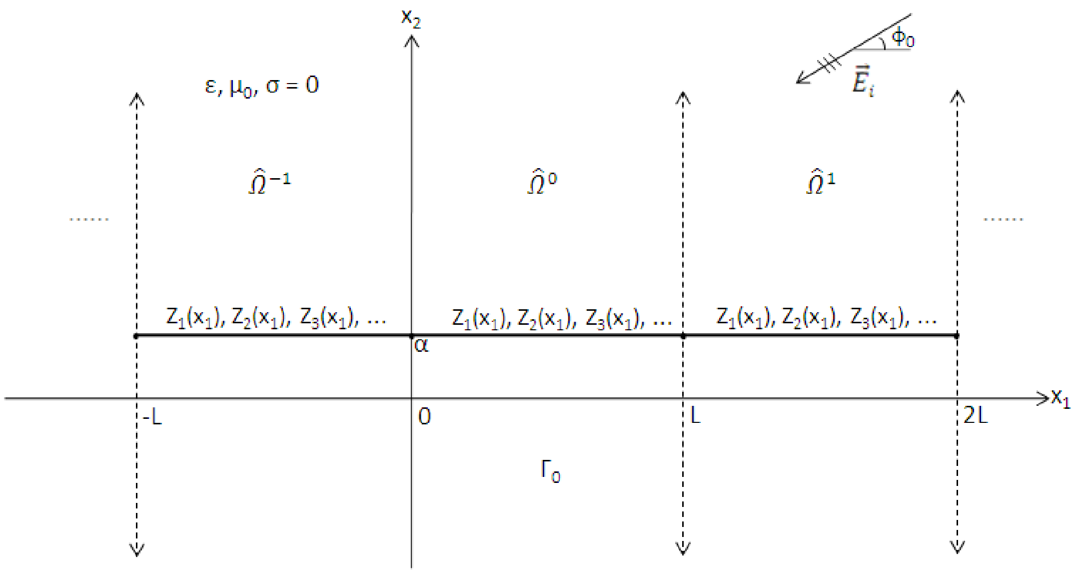

Figure 3.1: Geometry of the Reduced HPIBC Problem

Consider now the case where the incident field is a plane wave of the form

$$E^i(x_1, x_2) = e^{-i\beta x_1} e^{\gamma x_2} \tag{3.1}$$

where $\beta = k\cos\phi_0$ and $\gamma(\beta) = \sqrt{\beta^2 - k^2}$, with $\phi_0 \in (0, \pi)$ is the incidence angle. The square root function appearing in γ is defined as $\gamma(0) = -ik$.

Then the scattered field defined by $E^s = E - E^i$ satisfies the reduced waves equation

$$\Delta E^s + k^2 E^s = 0, \qquad x \in \hat{\Omega}^0 \tag{3.2}$$

and the boundary condition

$$\sum_{m=0}^{M} Z_m(x_1)\frac{\partial^m E^s(x_1,\alpha)}{\partial x_2^m} = -\sum_{m=0}^{M} Z_m(x_1)\frac{\partial^m E^i(x_1,\alpha)}{\partial x_2^m} \tag{3.3}$$

under the appropriate radiation condition. The scattering problem given by (3.2) and (3.3) can now be solved by expressing the unknown scattered field E^s in terms of Floquet modes, namely,

$$E^s(x_1,x_2) = \sum_{n=-\infty}^{n=\infty} B_n e^{-i\beta_n x_1} e^{-\gamma_n x_2}, \qquad x_2 > \alpha \tag{3.4}$$

where $\beta_n = \beta + (2n\pi/L)$ and $\gamma_n = \sqrt{\beta_n^2 - k^2}$. In (3.4), B_n's, $n = 0, \pm 1, \pm 2, \ldots$ are the coefficients to be determined. It has to be remarked that according to Rayleigh hypothesis, it is allowed that the unknown scattered field can be expressed as only outgoing waves [13].

The substitution of (3.1) and (3.4) into (3.3) results in

$$\begin{aligned}&\sum_{n=-\infty}^{\infty} i\omega\mu_0 B_n e^{-\gamma_n\alpha} e^{-i2\pi\frac{n}{L}x_1} + \sum_{n=-\infty}^{\infty}\sum_{m=1}^{M} B_n Z_m(x_1)(-\gamma_n)^m e^{-\gamma_n\alpha} e^{-i2\pi\frac{n}{L}x_1} \\ &= \{-i\omega\mu_0 e^{\gamma\alpha} - \sum_{m=1}^{M} Z_m(x_1)(\gamma^m e^{\gamma\alpha})\ \}\end{aligned} \tag{3.5}$$

which can be reduced to a simple one by using the orthogonality properties of the functions $e^{-i2\pi\frac{p}{L}x_1}$, $p = 0, \pm 1, \pm 2, \ldots.$ To this end one first multiplies both sides of (3.5) by $e^{i2\pi\frac{p}{L}x_1}$, $p = 0, \pm 1, \pm 2, \ldots$ and integrate over the interval $(0, L)$ to lead a compact form as follows:

$$[K]B \ = \ f \tag{3.6}$$

$$B \ = \ [\ldots, B_{-1}, B_0, B_1, \ldots]^T \tag{3.7}$$

$$K \ = \ [k_{pq}],\ \ p, q = 0, \pm 1, \pm 2, \ldots \tag{3.8}$$

$$k_{pq} = \begin{cases} i\omega\mu_0 e^{-\gamma_q\alpha} + \sum_{m=1}^{M} (\gamma_q)^m e^{-\gamma_q\alpha}\hat{Z}_{m,0} & ,q = p \\ \sum_{m=1}^{M} (\gamma_q)^m e^{-\gamma_q\alpha}\hat{Z}_{m,q-p} & ,q \neq p \end{cases} \tag{3.9}$$

and

$$\hat{Z}_{m,q-p} = \frac{1}{L}\int_0^L Z_m(x_1) e^{-i2\pi\frac{(q-p)}{L}x_1} dx_1 \tag{3.10}$$

$$f_p = \int_0^L \{-i\omega\mu_0 e^{\gamma_p\alpha} - \sum_{m=1}^{M} Z_m(x_1)(\gamma_p^m e^{\gamma_p\alpha})\} e^{i2\pi\frac{p}{L}x_1} dx_1 \tag{3.11}$$

(3.6) constitutes a system of linear equations for the unknowns $B_p, p = 0, \pm 1, \pm 2, \ldots$ which can be solved by truncating the infinite summation in the left hand side of (3.5) to an appropriate interval $(-N, N)$. Then the substitution of $B_{-N}, \ldots, B_{-1}, B_0, B_1, \ldots, B_N$ into (3.4) yields the required scattered field.

4. NUMERICAL IMPLEMENTATION

The validity of the surface impedance modeling given in the previous section is tested by considering some illustrative examples. In all cases the half-space above the PEC surface is assumed to be free space. The reference solution of the original scattering problem is obtained via Finite Difference Time Domain (FDTD) method [54]. In order to make a precise comparison between the results of both methods, the mean square error (MSE) defined by

$$MSE = \frac{\sum_{n=1}^{N_c} |E^{s,n} - E_r^{s,n}|^2}{N_c} \tag{4.1}$$

is considered. Here N_c is the number of observations, while $E^{s,n}$ and $E_r^{s,n}$ are the scattered fields on the corresponding observation point n, obtained via HPIBC modeling and FDTD, respectively. The operating frequency is chosen as 300 MHz.

The first results are related to a periodic rough surface of

$$f(x_1) = (\frac{2x_1 - L}{20L}).\cos(3\pi\frac{2x_1 - L}{2L}).(\frac{2x_1}{L}).\exp(|L/2 - x_1| + \frac{-5(x_1 - L/2)^2}{L}) \tag{4.2}$$

where the period $L = 12\lambda$ (see Figure 4.1(a)). The surface is illuminated by a plane wave of incidence direction $\phi_0 = 110^o$. Figure 4.1(b) and 4.1(c) illustrate the amplitudes and phases of the scattered fields on the line $x_2 = 0.75\lambda$ computed via FDTD and HPIBC modeling. The truncation number N is chosen as 40. The results for the impedance modeling are obtained for only one impedance term in the HPIBC (2.5), i.e.: $M = 1$, which is nothing but the standard impedance boundary condition (SIBC). For the surface given in (4.2) the SIBC yields very accurate results, this is because the surface has a smooth variation, where maximum rate of change with respect to x_1 is approximately $max(\frac{df(x_1)}{dx_1}) = 0.025$, and a single impedance is enough to represent this variation on the chosen impedance plane. The MSE is %0.68 and it does not change when higher order impedance terms are included in the solution.

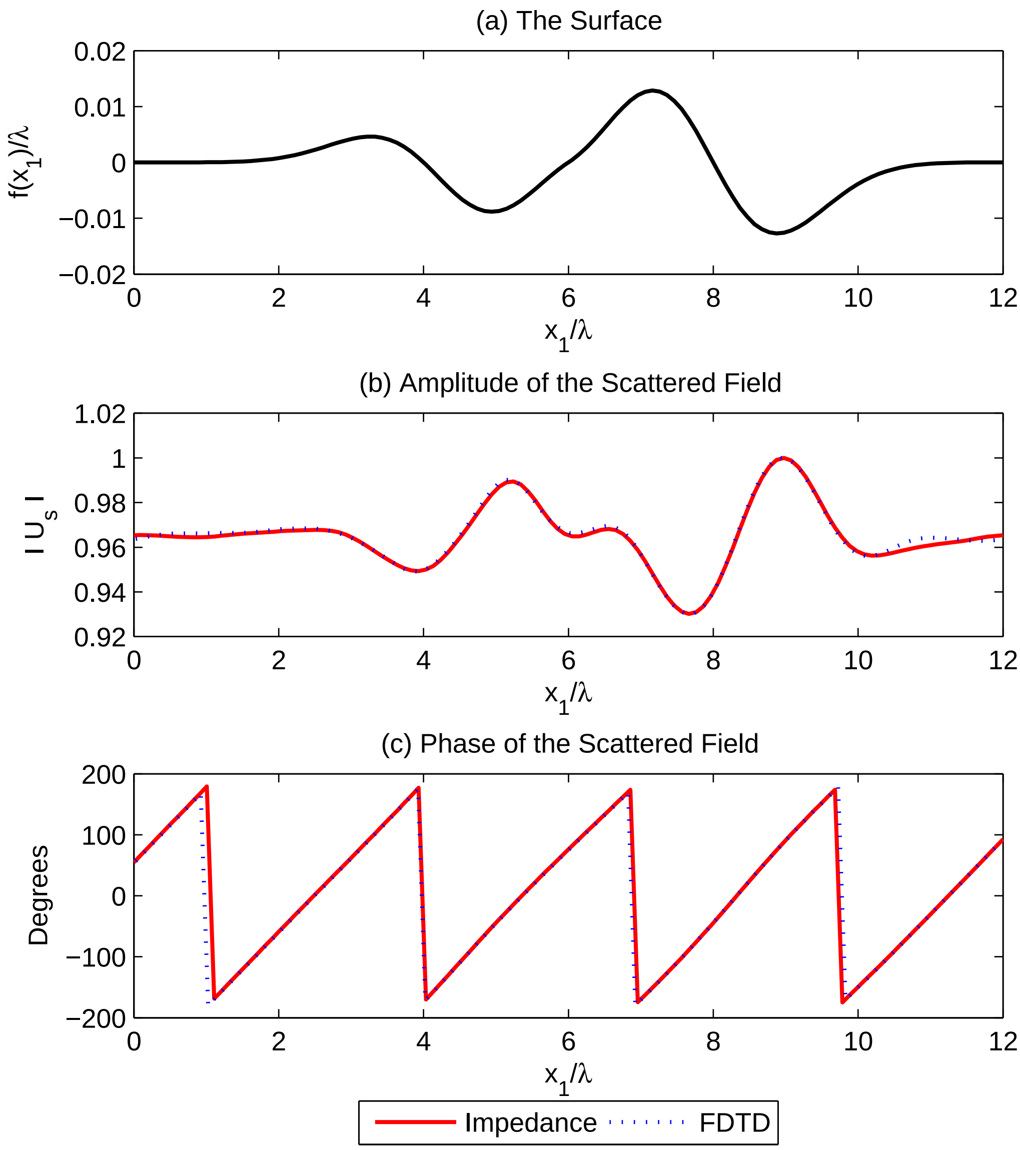

Figure 4.1: (a) The surface (b) Amplitudes (c) phases of the scattered fields obtained by HPIBC modeling and FDTD approach

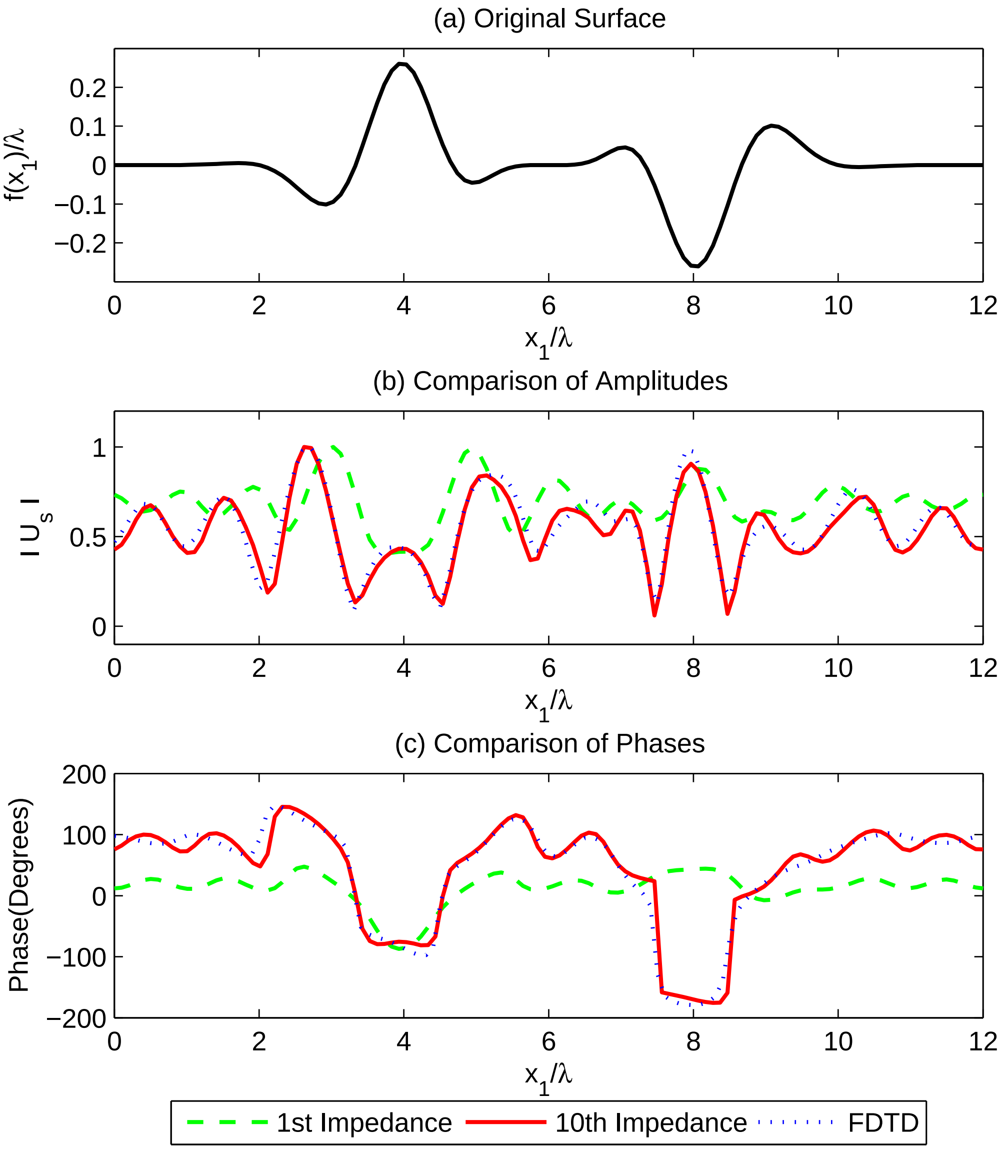

Figure 4.2: (a) The surface (b) Amplitudes (c) phases of the scattered fields obtained by HPIBC modeling and FDTD approach

A more rapidly changing surface defined by the function

$$f(x_1) = (0.1).(\frac{2x_1 - L}{2}).\sin(9\pi\frac{2x_1 - L}{2L}).\exp(-\frac{(x-L/2)^2}{2(L/3)^3} - \frac{5(x_1 - L/2)^2}{L}) \qquad (4.3)$$

is considered as a second numerical application. For this surface $max(\frac{df(x_1)}{dx_1}) = 0.55$ and the peak-to-peak amplitude 0.53λ are quite greater than those of the surface given in previous example. Therefore, it is expected to include higher order impedance terms in HPIBC method. In Figure 4.2(b) and 4.2(c), the amplitudes and phases of scattered field obtained for $M = 1$ and $M = 10$ are compared with the FDTD solution where the incidence angle of the plane wave and observation height are chosen as $\phi_0 = 90$ and $x_2 = 0.75\lambda$, respectively. Although SIBC modeling produces incompatible results compared to the FDTD solution, the inclusion of higher order terms for that kind of rapidly changing rough surfaces improves the quality of the results and makes it possible to obtain quite satisfactory solutions as clearly seen in Figure 4.2. The effect of higher order impedance terms can also be observed via error analysis. In Figure 4.3, the MSE error as a function of the number of terms in the impedance boundary condition is plotted and it is observed that; although the MSE is %61 while $M = 1$, it decreases rapidly by increasing number of higher order impedance terms. For $M = 10$, MSE becomes %0.8 and remain still for $M > 10$.

Although the theory is developed for the smoothly varying surfaces it is worth to analyze its behavior for different kind of surfaces such as corrugated one. To this aim we consider the surface given in Figure 4.5(a). As it is seen from Figure 4.4(a),the surface is a triangular sawtooth function which has four edges within a single period and the maximum rate of change with respect to x_1 is approximately $max(\frac{df(x_1)}{dx_1}) = 0.07$. The amplitudes and phases of the scattered field calculated at $x_2 = 0.9\lambda$ are illustrated in Fig. 4.5(b) and 4.5(c) for the illumination angle of $\phi_0 = 95^o$. As it is is observed from the figures, $M = 1$, which corresponds to SIBC, is not fairly enough to model the rough surface, while $M = 7$, (i.e., HPIBC) gives more accurate results compared to the FDTD solution. The corresponding MSE's for $M = 1$, $M = 4$, $M = 7$ and $M = 12$ are %16, %0.7, %0.65 and %0.65 respectively and does not change for $M > 7$ (see Figure 4.4).

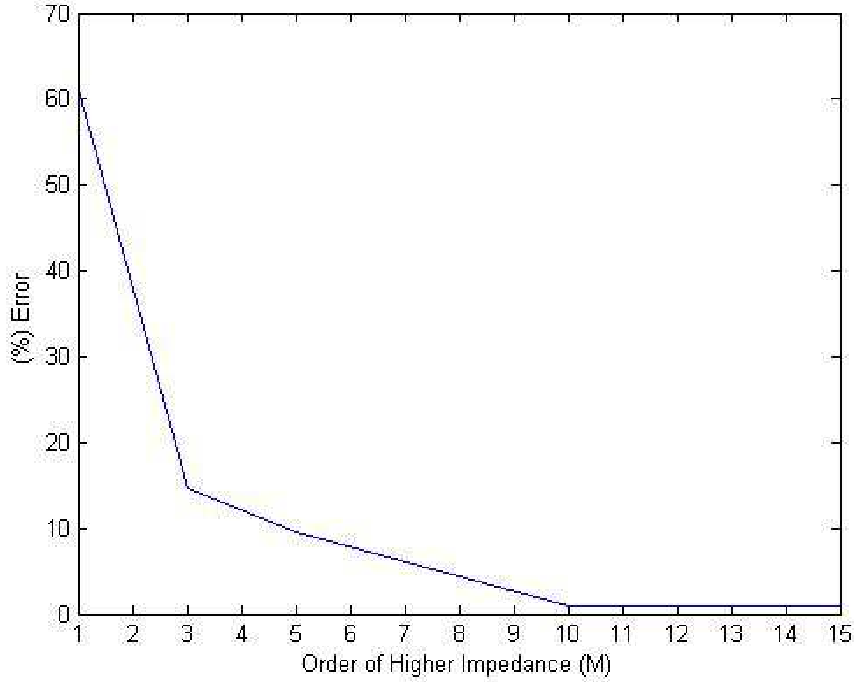

Figure 4.3: Variation of the MSE versus number of terms in HPIBC for the solution according to surface given in Figure 4.2(a)

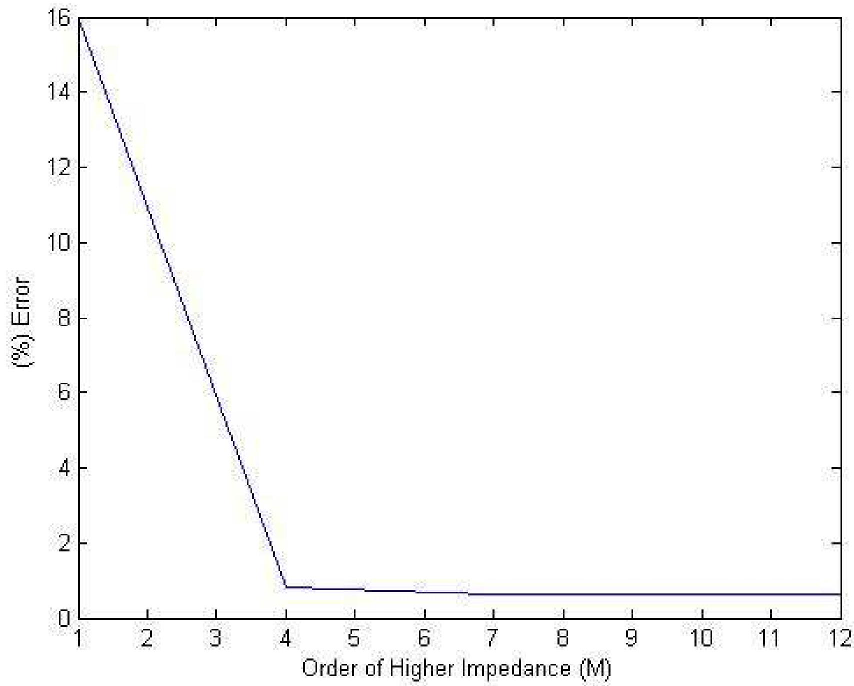

Figure 4.4: Variation of the MSE versus number of terms in HPIBC for the solution according to surface given in Figure 4.5(a)

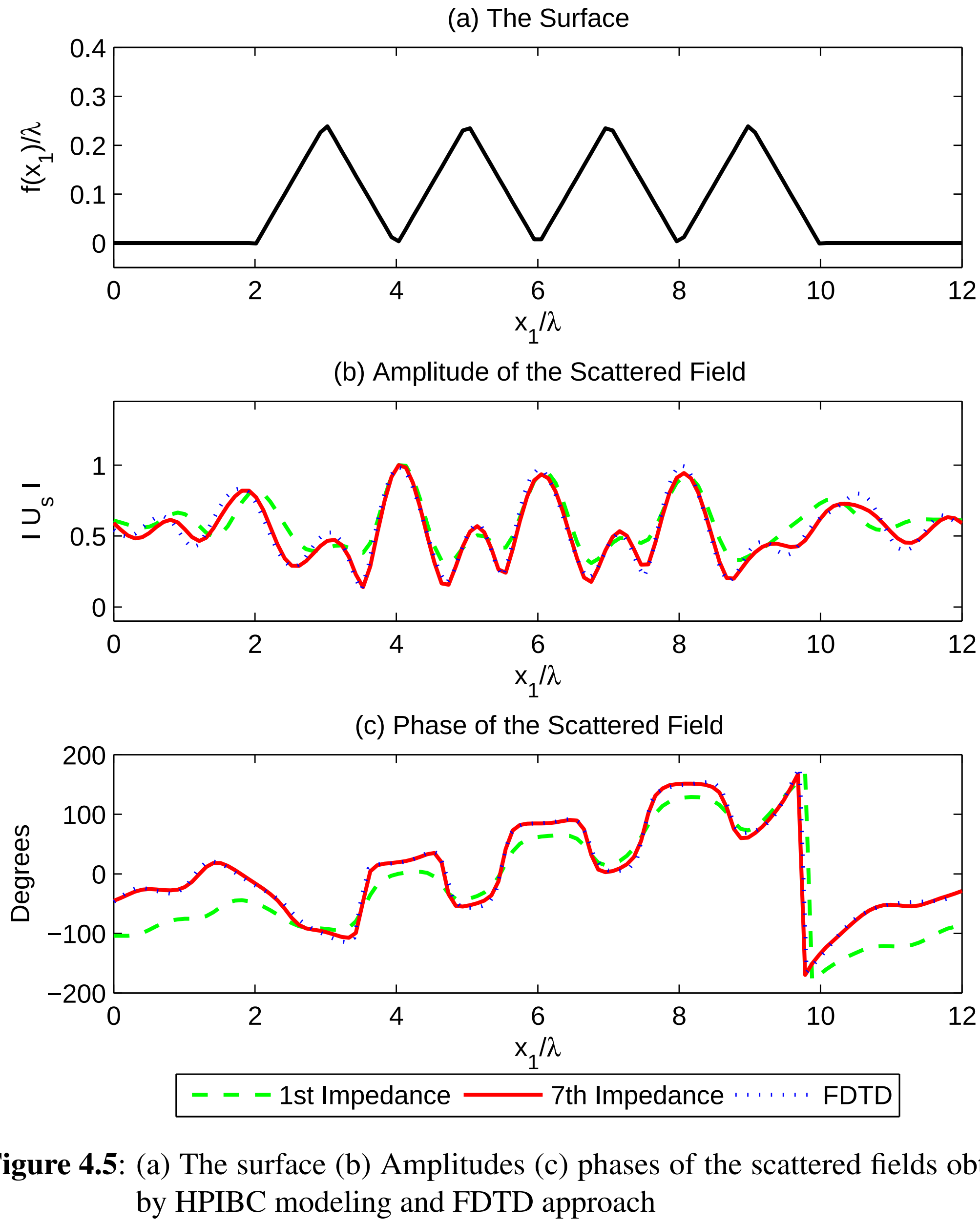

Figure 4.5: (a) The surface (b) Amplitudes (c) phases of the scattered fields obtained by HPIBC modeling and FDTD approach

As it is stated in Chapter 2, the periodic rough surface is replaced by a flat one having HPIBC on the plane $x_2 = \alpha$, $\alpha \geq max(f(x_1))$. This plane is chosen as the one tangential to the maximum point of the original surface. If a rough surface with a quite greater peak-to-peak amplitude is chosen to be examined, the flat surface having HPIBC ,which replaces the original one, should be placed relatively high with respect to the minimum value of the amplitude of the original surface. In order to investigate the effect of this, a rough surface by the function

$$f(x_1) = (0.1).(\frac{x_1}{L}).\sin(3\pi\frac{x_1}{L}).(\frac{1+x_1}{L/2}) \tag{4.4}$$

is considered (see Figure 4.6(a)). Although variation of the the rough surface is not rapidly changing, the peak-to-peak amplitude is relatively greater than the ones of the previous examples. The peak-to-peak amplitude of the original surface chosen for this example is approximately 0.7. It yields that the flat surface having HPIBC of the equivalent problem is placed relatively higher. In Figure 4.6(b) and 4.6(c), the amplitudes and phases of scattered field obtained for $M = 1$ and $M = 4$ are compared with the FDTD solution where the incidence angle of the plane wave and observation height are chosen as $\phi_0 = 100$ and $x_2 = 1\lambda$, respectively. As it is shown by this example, by taking the more derivatives in the impedance boundary condition into the account, one can achieve to model more rough surfaces with sharp peaks. The corresponding MSE's for $M = 1$ and $M = 4$ are %1.3 and %0.5 respectively and does not change for $M > 4$.

According to previous examples, the method yields accurate results and effective to model the periodic rough surfaces. The last analysis is carried out to show the effect of the operating frequency. The surface is chosen as the same one used in previous example(see Figure 4.7(a)). The amplitudes and phases of the scattered field calculated for the operating frequencies of 300MHz, 450MHz and 900MHz are illustrated in Fig. 4.7(b) and 4.7(c) and the results are obtained for $M = 4$, the incidence angle $\phi_0 = 95$ and observation height $x_2 = 1\lambda$. As the frequency is increased, dimensions of the rough surface is going to be greater according to wavelength and effects of more details emerge on the scattered field, so that the variation of the scattered field is rapidly changing as expected.

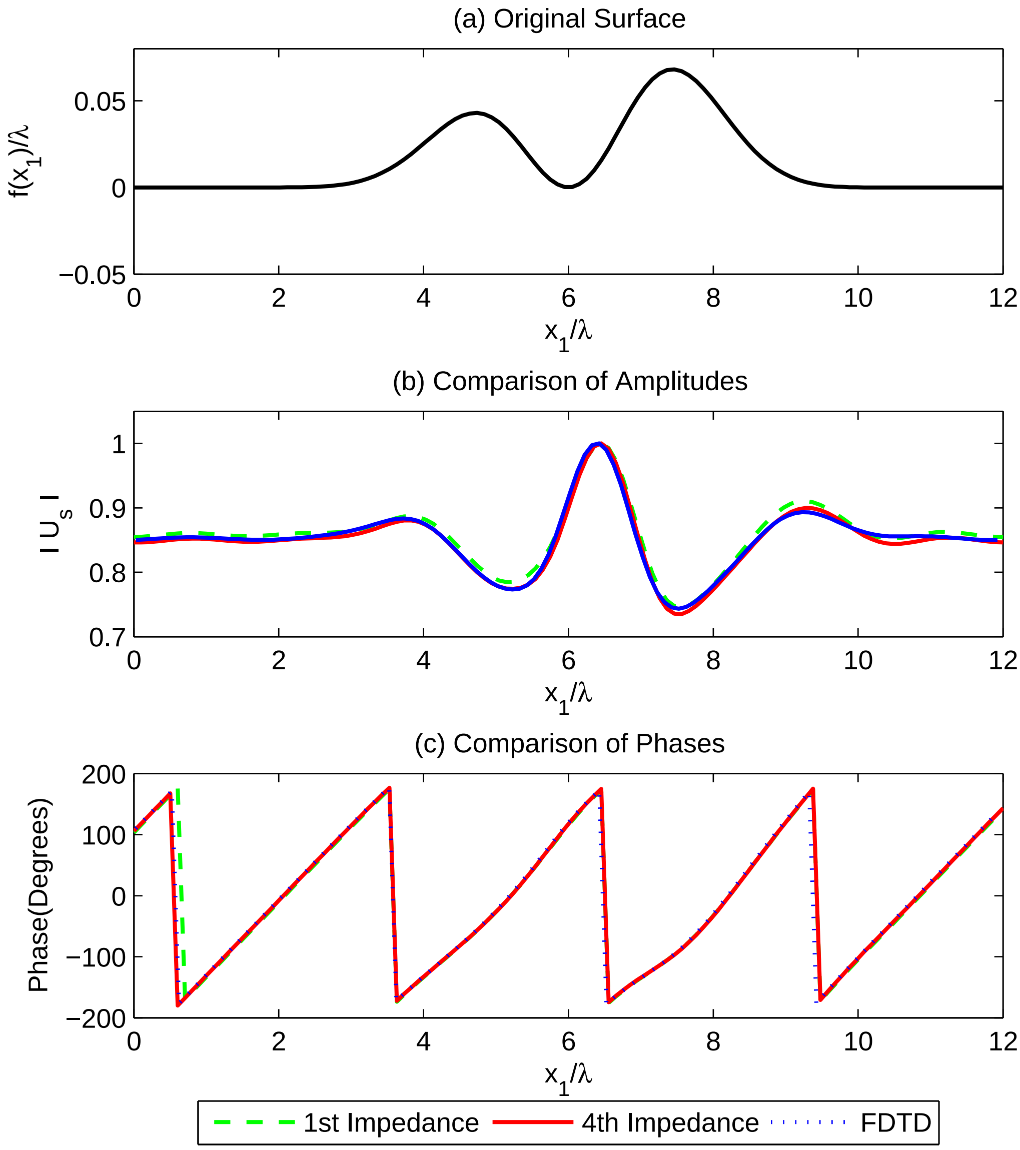

Figure 4.6: (a) The surface (b) Amplitudes (c) phases of the scattered fields obtained by HPIBC modeling and FDTD approach

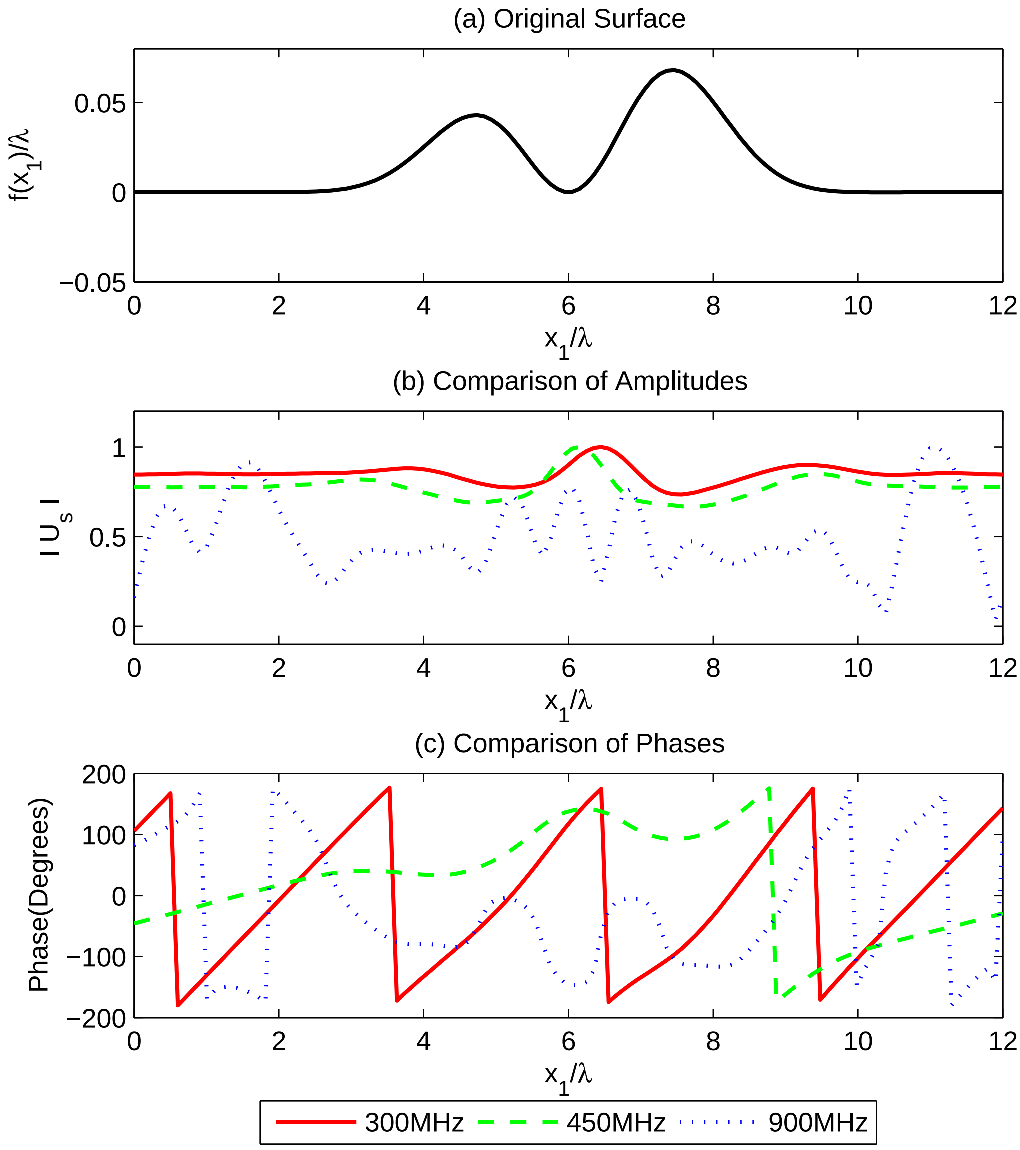

Figure 4.7: (a) The surface (b) Amplitudes (c) phases of the scattered fields obtained by HPIBC modeling for the frequencies 300MHz, 450MHz and 900MHz

5. CONCLUSION

An equivalent representation of a periodic PEC surface is derived in terms of a HPIBC on a planar one. It has been shown that there are explicit relations between the surface variation and surface impedances. The equivalent problem with HPIBC is first formulated through the Floquet mode expansion of the scattered field and then reduced to a simple system of linear equations. The validity of the presented method is tested by comparisons with FDTD simulations and quite satisfactory results are obtained. From the numerical implementations it is shown that the MSE error between two methods is always less than 1%. It has been observed that for surfaces having smooth variations and small peak to peak values, SIBC modeling is quite enough to obtain satisfactory results. On the other hand it is also shown that higher order impedances have to be included to improve the accuracy for the rapidly changing surfaces having relatively higher peak to peak values. The method is also applied to the corrugated surfaces having sharp edges and again very satisfactory results are obtained. Future studies will be devoted to the extension of the method to the two-dimensional periodic surfaces.

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