İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

FUZZY PROCESS CONTROL AND DEVELOPMENT OF SOME MODELS FOR FUZZY CONTROL CHARTS

Ph.D. Thesis by

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Programme: **Industrial Engineering**

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

BULANIK PROSES KONTROLÜ VE BULANIK KONTROL DİYAGRAMI MODELLERİNİN GELİŞTİRİLMESİ

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PREFACE

In today's information-driven economy, companies may benefit a lot from suitable process control activities. One of the most powerful process control tools is the control charts. Even though the first control chart was proposed during the 1920's by W.A. Shewhart, today they are still subject to new application areas that deserve further attention. Classical process control charts are suitable when the data is exactly known and precise; but in some cases, it is nearly impossible to have such strict data if human subjectivity plays an important role. Fuzzy sets are inevitable in representing uncertainty, vagueness and human subjectivity.

In this thesis, fuzzy control charts are developed and some models are proposed. In Section 1 an introduction is given. Section 2 is about statistical process control. Basics of the statistical process control charts are presented in Section 3. Unnatural pattern analyses for the classical process control charts are explained in Section 4. Section 5 includes fundamental knowledge of the fuzzy set theory required to construct fuzzy control charts explained in Section 6. In Section 7, unnatural pattern analyses are developed for the fuzzy control charts. In Section 8, numerical examples using the data of a real case are given.

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DECEMBER 2006 Murat GÜLBAY

CONTENTS

LIST OF ABBREVIATIONS

AOQL : Average Outgoing Quality Limit,
ASA : American Standard Association,
ASF : Army Services Forces,
ASS : Average Sample Size,
ATI : Average Total Inspection,
AWS : American War Standards,
CAD : Computer Aided Design,
CAM : Computer Aided Manufacturing,
CAQ : Computer Aided Quality,
CL : Center Line,
CSP : Continuous Sampling Plan,
CuSum : Cumulative Sum,
DoD : Department of Defense (US),
EWMA : Exponentially Weighted Moving Average,
FQFD : Fuzzy Quality Function Deployment,
LCL : Lower Control Limit,
LTPD : Lot Tolerance Percent Defective,
MLP : Multi-level Inspection Plan,
NASA : The National Aeronautics and Space Agency,
SkSP : Skip-Lot Sampling Plan,
SPC : Statistical Process Control,
SPCC : Statistical Process Control Charts,
TraFN : Trapezoidal Fuzzy Number,
TriFN : Triangular Fuzzy Number,
UCL : Upper Control Limit,
VSS : Variable Sample Size.

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LIST OF SYMBOLS

$\widetilde{}$	: General sign to represent a fuzzy set/event/number,
μ	: Expected value,
σ	: Standard deviation,
$\Pr$	: Probability,
$\boldsymbol{k}, \lambda$	: Multiples of the standard deviation,
$\mu_x(A)$	: Membership degree of the fuzzy event A,
$\bar{\tilde{A}}$	: Complement of the fuzzy event $\tilde{A}$,
$S(\tilde{A})$	: Support of the fuzzy set $\tilde{A}$,
$(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d})$	: General representation of a trapezoidal fuzzy number,
μ_A^L	: Left support of the membership function,
μ_A^R	: Right support of the membership function,
$\widetilde{CL}$	: Fuzzy center line,
$\widetilde{LCL}$	: Fuzzy lower control limit,
$\widetilde{UCL}$	: Fuzzy upper control limit,
$\tilde{\leq}$	: Fuzzy less than or equal to
α	: Level of an α-cut,
$f_{\mathrm{mod}e}$	: Fuzzy mode,
f_{mr}^{α}	: α-Level fuzzy midrange,
f_{med}	: Fuzzy median,
f_{avg}	: Fuzzy average,
ξ	: Necessity index,
p	: Percent defective,
c	: Number of nonconformities.

FUZZY PROCESS CONTROL AND DEVELOPMENT OF SOME MODELS FOR FUZZY CONTROL CHARTS

SUMMARY

Even though the first classical control chart was proposed during the 1920's by W.A. Shewhart, today they are still subject to new application areas that deserve further attention. Classical process control charts are suitable when the data are exactly known and precise; but in some cases, it is nearly impossible to have such strict data if human subjectivity plays an important role. It is not surprising that uncertainty exists in the human world. To survive in our world, we are engaged in making decisions, managing and analyzing information, as well as predicting future events. All of these activities utilize information that is available and help us try to cope with information that is not. A rational approach toward decision-making should take human subjectivity into account, rather than employing only objective probability measures. A research work incorporating uncertainty into decision analysis is basically done through the probability theory and/or the fuzzy set theory. The former represents the stochastic nature of decision analysis while the latter captures the subjectivity of human behavior. The fuzzy set theory is a perfect means for modeling uncertainty (or imprecision) arising from mental phenomena which is neither random nor stochastic. Fuzzy sets are inevitable in representing uncertainty, vagueness and human subjectivity.

In this study, process control charts under linguistic, vague, imprecise, and uncertain data are developed in the light of the Fuzzy Set Theory. Linguistic or uncertain data are represented by the use of fuzzy numbers. Fuzzy control charts for the linguistic data are proposed and integrated with the α-cut approach of fuzzy sets in order to set the degree of tightness of the inspection.

In the literature, there exist few papers on fuzzy control charts, which use defuzziffication methods in the early steps of their algorithms. The use of defuzziffication methods in the early steps of the algorithm makes it too similar to the classical analysis. Linguistic data in those works are transformed into numeric values before control limits are calculated. Thus both control limits as well as sample values become numeric. This transformation may cause biased results due to the loss of information included by the samples. A new approach called *direct fuzzy approach* to fuzzy control charts is modeled in order to prevent the loss of information of the fuzzy data during the construction of control charts. It directly compares the linguistic data in fuzzy space without making any transformation. Finally, fuzzy unnatural pattern analyses are developed to monitor the abnormal patterns of the fuzzy data on the control charts. Numerical examples using the data of a real case are also given to highlight the practical usage of the proposed approaches.

BULANIK PROSES KONTROLÜ VE BULANIK KONTROL DİYAGRAMI MODELLERİNİN GELİŞTİRİLMESİ

ÖZET

Klasik kontrol diyagramları, W.A. Shewhart tarafından 1920'lerde geliştirilmiş olmasına rağmen yeni uygulama alanları ile günümüzde hala gelişimini sürdürmektedir. Verilerin tam ve kesin olduğu durumlarda klasik kontrol diyagramlarının kullanılması uygundur; ancak subjektifliğin önemli bir rol oynadığı durumlarda bu kadar kesin verilere sahip olmak neredeyse imkansızdır. İnsan yaşamında belirsizliklerin olması sürpriz bir durum değildir. Hayatın devamı için, gelecekteki olayları tahmin etmenin yanı sıra, kararlar vermek, bilgiyi analiz etmek ve yönetmek zorundayız. Bütün bu aktivitelerde, eldeki bilgiler kullanılabilir biçimde derlenerek bunlardan sonuçlar elde edilmeye çalışılır. Karar vermede gerçekçi yaklaşımlar sadece nesnel olasılık ölçüleri ile değil insan subjektifliğini de dikkate almalıdır. Belirsizlik altındaki durumlarda karar analizleri genellikle olasılık teorisi ve/veya bulanık kümeler teorisi kullanılarak yapılmaktadır. Bunlardan birincisi karar vermenin stokastik yapısını diğeri ise insanın düşüncesinin subjektifliğini temsil eder. Bulanık kümeler teorisi, ne rassal ne de stokastik olan insanın zihinsel yapısından kaynaklanan belirsizliğin modellenmesinde mükemmeldir. Belirsiz, kesin olmayan ve dilsel anlatımlar içeren durumlarda bulanık kümeler teorisinin kullanılması kaçınılmazdır.

Bu çalışmada, bulanık kümeler teorisi kullanılarak belirsizlik içeren dilsel verilerle kontrol diyagramlarına yeni yaklaşımlar geliştirilmiştir. Belirsizlik içeren dilsel veriler, bulanık sayılarla ifade edilmiştir. Dilsel veriler için bulanık kontrol diyagramları α-kesim yaklaşımı kullanılarak geliştirilmiş ve bu suretle muayene sıklığı tanımlanmıştır.

Literatürde, ilk adımlarında durulaştırmanın temel alındığı bazı bulanık kontrol diyagramları modelleri mevcuttur. Durulaştırma metotlarının en başta kullanılması, klasik kontrol diyagramlarına aşırı derecede benzer modeller geliştirilmesine neden olmuştur. Bu çalışmalardaki dilsel veriler, kontrol limitlerinin hesaplanmasından hemen önce nümerik değerlere dönüştürülmüştür. Bu dönüştürme ile veriler karakteristik özelliklerini kaybettiğinden kontrol diyagramlarında yanıltıcı durumlarla karşılaşılmasına neden olmaktadır. Bulanık kontrol diyagramlarının oluşturulmasında, bulanık verilerin taşıdığı bilgilerin kaybolmasını önlemek amacıyla "Direkt Bulanık Yaklaşım" geliştirilmiştir. Belirsizlik içeren dilsel ifadeler durulaştırma kullanılmadan bulanık ortamda değerlendirilmiştir. Aynı zamanda, bulanık verilerin kontrol diyagramındaki normal olmayan davranış testleri için bulanık bir yaklaşım geliştirilmiştir. Önerilen yaklaşımların pratik kullanımlarının yansıtılması açısından gerçek verilere dayalı nümerik örnekler sunulmuştur.

1 INTRODUCTION

1.1 History and Evolution of Quality Control

Every act by an individual, a group of individuals or an organization to ensure that a product or service meets a desired or specified standard can justifiably be seen as a quality control activity. Viewed in this way, quality control is almost, if not exactly, as old as the human race. It is quite logical to reason that, in the earliest times, quality control acts were not conscious, but rather were performed subconsciously as part of everyday activities, in isolation, and were restricted to the single individual. The history and evolution of quality control are therefore linked with the technological advances of the human race.

We should start by defining some terms. The Glossary and Tables for Statistical Quality Control defines the following terms [1]:

Nonconformity: A departure of a quality characteristic from its intended level or state that occurs with a severity sufficient to cause an associated product or service not to meet a specifications requirement.

Nonconforming unit: A unit of product or service containing at least one nonconformity,

Defect: A departure of a quality characteristic from its intended level or state that occurs with a severity sufficient to cause an associated product or service not to satisfy intended normal, or reasonably foreseeable usage requirements.

Defective (Defective Unit): A unit of product or service containing at least one defect, or having several imperfections that in combination cause the unit not to satisfy intended normal, or reasonably foreseeable usage requirements. Note: The word defective is appropriate for use when a unit of product or service is evaluated in terms of usage (as contrasted to conformance to specifications).

Ancient Developments

As human evolved, so did the nature of their activities. Eventually humans were no longer content with simply filling their stomachs for the day. Ancient history indicates that as early as several thousand years before the common era, humans had embarked on complex technical endeavors. Inevitably, the erstwhile subconscious and isolated quality control gave way to a more formal approach.

It is not known precisely when this subconscious and uncoordinated quality control came to an end. However, archaeological findings and the remains of ancient structures indicate that by the time of the construction of Egypt's pyramids, conscious efforts at quality control had emerged. The perfection of the pyramids, the flawlessness of the classical Greek master works, and the endurance of Roman structures attest to a conscious effort to control quality [2]. Ancient Egyptians were involved in the earliest known formalized efforts to control quality. Their chief contribution was in engineering [3]. The bare struggle for existence resulting from the annual inundation by the Nile River forced the Egyptians to acquire knowledge of engineering, arithmetic, geometry, surveying, and mensuration [4]. From all these endeavors, the basic decimal system was developed. The Egyptians also devised measures of length (the cubit) and area (squared cubit) [5].

The computation of the area of a circle and of the value of pi by the early Egyptians was more accurate than that of any other ancient civilization. The Egyptians produced elementary geographical maps and star maps and used a simple form of theodolite. They discovered and developed the concept of a 365 ¼ day year. By their calendar, the year was divided and thus standardized into 12 months, each consisting of 30 days [4]. The concept of the 24-hour day (12 hours of day and 12 hours of night) also came from them [5]. The bearing of all these developments and inventions on quality control does not seem direct and therefore may not be immediately clear. However, their contribution becomes clear when it is considered that these mathematical and engineering inventions found use in the construction of the pyramids. In connection with the work on the pyramids, the "royal cubit" was accepted and used as the master standard for linear dimensions [6]. The high quality of these pyramids, both in their mathematical precision and in the material used for their construction, is attested to not only by the fact that they still stand after thousand of years, but also by the fact that their magnificence is still marveled at. The calendar in use today is basically the same as the one invented by the early Egyptians. This, in itself, indicates the high quality of that invention.

Apart from their interest in the principles and theories of science, the ancient Greeks also left a legacy in quality control. Apparently motivated by trade and commerce, they produced high-quality pottery and enhanced the art of vase making, both in the development of various types of vases and in their decoration [4]. Ancient Greek contributions to precision and quality are also noticeable in their architecture. The culmination of Greek architecture in the fifth century BCE was the perfect development and highest artistic expression of column-and-lintel construction. These edifices were believed to have inspired the later architectural constructions of ancient Rome, the Renaissance, and modern times [4].

Ancient Romans also left a legacy in quality, especially in architecture and engineering. Roman architecture, which flourished between 100 BCE and the mid fourth century CE was by far the most important form in terms of its grandeur and its influence on later times.

In structural engineering, the ancient Romans developed high quality reinforced concrete, which was used in perfectly constructed hemispherical domes and in many other lasting structures [4]. Some of the splendid early Roman aqueducts and bridges can still be observed.

Further evolution and development of current quality control occurred in several basic stages. Feigenbaum (1983) identifies these stages as operator quality control, foreman quality control, inspection quality control and statistical quality control, total quality control, and organization-wide total quality management [7]. Each stage is a broad grouping of developments that occurred over a long period of time. A more detailed delineation of the evolution of quality control requires that these developments be considered in smaller time frames.

Middle Ages

In the Middle Ages and up to the 1800s, the supply of services and the production of goods were essentially limited to single individuals or, at most, to a group of several persons. The individual worker or workers controlled the quality of products. A peculiarity of this era was that the individual was both the producer and the inspector. The result was that quality standards were self-established. The decisions on conformance between the quality of the product or service and the needs of the customer were made by the individual.

This era, however, was not totally lacking in organized control of quality. It was in this period that craft guilds were most active in Europe. These guilds were medieval associations of master craftsmen organized for the protection and economic and social gain of their members. They regulated local urban economies by establishing monopolies over trade; maintaining stable prices under stable conditions; and specifying standards for the quality of goods [5]. In their efforts to manage quality, the guilds set standards, stipulated working conditions and wages, and protected their members from governmental abuse and unfair competition [4]. They also regulate every detail of manufacture, from raw material to finished product [8]. This regulation of manufacturing activities may have been one of their most direct efforts at quality control.

Late 1800s to the 1920s

With the advent of industrialization in the late nineteenth and early twentieth centuries, the complexity of manufacturing increased. The growing technology resulted in a need to form group of workers that performed either similar or specific tasks. With this, the era of the supervisor began. Industrial firms were comparatively small, and the owner was physically present. Thus, the owner knew what was happening in the firm. Therefore standards were set and key decisions on quality control were made by the owner.

As the nineteenth century progressed, the complexity of production and of manufacturing enterprises and techniques grew. The number of workers reporting to each supervisor increased. Organizations soon began to realize the need for individuals who, although not directly involved in the actual manufacturing and production processes, were active in inspecting the quality of the product. This ushering in of quality control inspection lessened the burden on the supervisor. As a result, the supervisor and the worker were finally able to devote most of their time and concern to the actual manufacture and production.

Toward the end of the nineteenth century, the need for the dissemination of technical knowledge through technical publications was recognized. In this era the *Journal of the American Statistical Society*, began publication. This journal, which published many of the major technical papers on quality and reliability, represented a source of current technical knowledge and developments [9].

The routine quality checks provided by inspectors in the early 1900s were not good enough for some companies. Companies like Western Electric, under contract from the American Bell Telephone Company, sought more rigorous quality control methods that would engender confidence in their instruments and appliances. It was this need that eventually led in 1924 to the formation of the Inspection Engineering Department of Western Electric's Bell Telephone Laboratories. The early membership of these laboratories consisted of Harold F, Dodge, Donald A. Quarles, Walter A. Shewhart, George D. Edwards, R. B. Miller, and E.G.D. Peterson, Harry G. Roming, M.N. Torrey, and P.S. Olmstead later became members.

It was in connection with their development of theories and methods of quality control and assurance that the first control charts emerged. In response to "problems connected with the development of an acceptable form of inspection report which might be modified from time to time, in order to give at a glance the greatest amount of accurate information" [10]. Shewhart designed control charts in 1924 that have come to be referred to as first Shewhart control charts.

Yet more developments were forthcoming from this group of pioneer quality controllers. Prior to the 1900s, there was a dearth of terms to describe adequately various nations and concepts. Between 1925 and 1926 the Western Electric group defined various terms that are associated to this day with acceptance sampling. These include *consumer's risk*, *producer's risk*, *probability of acceptance*, *operating characteristic (OC) curves*, *lot tolerance percent defective (LTPD)*, *average total inspection (ATI)*, *double sampling*, and *type A* and *type B risks*. The basic concepts of sampling inspection by attributes were presented by Dodge in 1925. In 1927, average outgoing quality limit (AOQL) sampling tables and the concepts of multiple sampling were developed by the Western Electric group. The demerit rating system joined the list in 1928.

The 1930s

A major development in the 1930s was the increased application of acceptance sampling techniques in industry as the methods developed at Western Electric spread throughout the United States and abroad. This era saw not only industrial applications of these techniques but also the dissemination of Shewhart's ideas.

By the mid-1930s, international interest in quality control had emerged. In 1935 Pearson developed the British Standards Institution Standard Number 600, entitled "*Application of Statistical Methods to Industrial Standardization and Quality Control*." In 1939, the article "*The Control of Proportion Defective as Judged by a Single Quality Characteristic Varying on a Continuous Scale*" laid the foundation for variable sampling [11].

Meanwhile, in the United States, more developments were occurring. In 1939 H. Romig presented his work on variable sampling plans in his PhD. Dissertation "Allowable Averages in Sampling Inspection" [12].

The 1940s

The 1940s saw the birth of what is referred to as statistical quality control [7]. In 1940, the American Standards Association (ASA), acting on the request of the War Department, became involved in the application of statistical quality control to manufactured products. From this work, the American War Standards AWS Z1.1: "*Guide to Quality Control*" and AWS Z1.2 "*Control Chart Methods of Analyzing Data*" emerged [13].

Dodge and Romig presented LTPD protection sampling schemes that were based on fixed consumer risks. They also offered AOQL protection schemes consisting of rectifying inspection plans that guaranteed some stated protection after 100 percent inspection of rejected lots. These acceptance sampling plans were published in an article in 1941 [14], and in book form in 1944 [15, 16]. These tables are part of what has come to be known as the *Dodge-Romig system*.

It was no surprise that after the concept of a consumer's risk was identified and considered, the notion of a risk of an opposite kind arose. This other kind of risk related to the consumer's refusal to accept, that is, the consumer's rejection of something good. The notion of a numerical producer's risk emerged and was incorporated with that of a *consumer's risk* [17].

As part of the war effort, other groups were formed to conduct research on quality control. In 1943, while working as a member of the Statistical Research Group based at Columbia University. A. Wald put forth the theory of sequential sampling. This group also made other valuable advances in variables and attributes sampling and in sequential analysis [18]. The results of the work of this group were considered to be so important to the war effort that they were classified for the duration of the war. In 1948, the group's work on sampling inspection was published [19]. The Joint Army-Navy Standard JAN-105, developed in 1949, was based on this article [18].

The 1950s

Although statistical quality control continued into this period, the era was marked by increased activity in the development and modification of quality control standards. In 1950, a committee formed by the military issued MIL-STD-1O5A which was a compromise military quality control standard between the Army Service Forces (ASF) tables of 1944 and JAN-105. Later modifications of MIL-STD-105A resulted in MIL-STD-105B, MIL-STD-105C, and MIL-STD-105D [18]. MIL-STD-414 came into being in 1957. This last-mentioned military standard dealt with acceptance sampling by variables.

Not surprisingly, the U.S. Department of Defense (DoD) was also active in this area. The DoD issued Handbook H107 for Single-Level Continuous Sampling Procedures and Tables for Inspection by Attributes (Inspection and Quality Control Handbook (Interim) H107, 1958). This handbook was followed by Handbook H108, which contained multilevel continuous sampling procedures and tables for inspection by attributes (Inspection and Quality Control Handbook H108, 1959). A section of Handbook H108 also has tables for life and reliability testing. These military-related standards were not concerned with suppliers' detailed quality program requirements or inspection techniques. The correction of this flaw was, however, not long in coming. Military standards to this effect, MIL-O-9858A and MIL-I-45208A were soon released. However, these two standards went beyond specifying programs for suppliers; in addition, they presented comprehensive quality-control and quality-assurance programs [13]. It seemed as though most of the government agencies had suddenly become aware of the significance of quality control and quality assurance.

The National Aeronautics and Space Agency (NASA) released the standards NHB 5300.4(1B). They were comparable in comprehensiveness to MIL-O-9858A and MIL-I-45208A [13]. The standards AWS Z1.1 and AWS Z1.2, which had been produced earlier on the request of the War Department, were revised and adopted in 1958 by the ASA as American Standard Z1.1 and American Standard Z1.2. According to the [13], these revised standards made reference to methods of collecting, arranging, and analyzing inspection and test records to detect lack of uniformity of quality and to apply the control chart technique in order to ascertain the quality of materials and manufactured products were given.

By the 1950s awareness of the importance of quality control had spread beyond the United States. The introduction of quality control courses and quality control charts had a late start in Japan. Deming was instrumental in the dissemination and popularization of quality control in Japan [20]. In 1950, he started teaching a series of courses on statistical methods in that country. Talks to influential industry leaders in Japan were subsequently added to the courses; it was only in 1950 that the renowned Japanese quality control expert K. Ishikawa began his studies of quality control concepts.

The 1950s, however, also witnessed further advances in and contributions to new statistical quality control techniques. One such contribution came from Britain when Page (1954) introduced the Cumulative Sum (Cusum) Chart. On the Cusum Chart, the individual values of the statistic of interest are not plotted; instead, the cumulation of these values is formed and charted. The Cusum technique therefore accounts for the effect of historical data on current data. A distinctive characteristic of the Cusum technique is that it gives equal weight to all the data, both past and present. The effect of this equal weighting of all data is that old data have the same significance as the most recent data [21].

Continuing his earlier work on the Continuous Sampling Plan (CSP-1, [22]) developed Skip-Lot Sampling Plans (SkSP) and Chain Sampling Plans (ChSP) [23].

A modification of CSP-I was proposed by Lieberman and Solomon (1955). The plan referred to as Multi-Level Inspection Plan (MLP) allows for multiple level of inspection instead of the single level used in the CSP-I scheme. MLP starts with 100 percent inspection [24].

Soon variants on the CSP and MLP appeared, including CSP-2, CSP-3, CSP-F, CSP-T, CSP-V, and MLP-T. The conception of the CSP-2 was motivated by experiences in the application of the CSP-1 to military items during World War II. It was thought that for sampling cases, where an appreciable number of nonconforming units are permissible. It might be logical not to revert to 100 percent inspection every time a nonconforming unit is found. Instead CSP-2 calls for a return to 100 percent inspection only when the spacing between nonconforming units is smaller than some prescribed minimum. CSP-3 was suggested by an inspection planning organization of the Western Electric Company as a refinement of the CSP-2 pan [25]. It was designed to be used for cases where single sample units are selected one at a time from a product comprising a now of individual units CSP-3 calls for the inspection of four additional sample units whenever an allowed nonconforming unit is found during sampling and for the immediate return to 100%, inspection if one of the four is found to be nonconforming. In this way, it provides extra protection against spotty quality.

CSP, ChSP, SkSP, and MLP are sampling plans based on the attributes of the items being inspected. Because of the lack, of information carried by attributes these plans tend to use large samples, making them expensive to operate. As early as 1957, alternative schemes had been developed, MIL-STD-414, issued in 1957, contained variable acceptance sampling plans. The variables of an item contain more information about the quality of the item than the attributes. Therefore, variable sampling uses comparatively smaller samples than its attributes-based counterpart.

Another important development was the application of the exponentially weighted moving average (EWMA) in quality control [26]. This concept was presented by Roberts (1959) when he compared the average run lengths of the "geometric moving average chart" to the Shewhart chart [27].

The 1960s

A new phase in quality control dawned in the 1960s. This was the beginning of an era that Feigenbaum [7] described as total quality control. Prior to the 1960s, quality control activities were essentially associated with the shop floor. The decision-making structures of businesses could not utilize effectively the results and recommendations, emanating from the statistical techniques being applied. The techniques were not applied to those serious quality control problems in which management was most interested.

Other concepts that attempted to involve all employees of the organization, in the quality control function began to emerge. In the same year that Feigenbaum [7] put forward his concept of total quality control, the concept of zero defects (ZD) was born.

The 1960, was the beginning of the race for space. Since space exploration is risky and costly, quality control was a great concern. It was realized that a multi-million-dollar missile could be destroyed and lives could be lost by the failure or malfunction of a S2 part. The elimination of defective components in missile construction had, therefore, always been a goal. With this objective in mind, the Martin Marietta Corporation sought new ways or detecting discrepancies and defects in the parts used in missile construction. In December 1961 the company was finally able to deliver a missile with zero defects [28] and the term zero defects was coined. It was an idea that achieved its objectives through worker motivation and involvement.

The concept of quality circles, another major development in total quality control and management, had its early beginnings in Japan. At the dawn of the 1960s, Japanese industries strongly felt the need for a more through education of the supervisor, who was the liaison between management and workers.

The 1970s

In the 1970s, quality control entered another phase. Ishikawa referred to this stage as companywide quality control [29]. Feigenbaum [7] identified the same phase as total quality control organization wide. This phase was marked by emphasis on the involvement in quality control of every worker, from the company president to the machine operator. The significant point here was that the highest level of management must be actively involved in quality control. Quality thereby became the responsibility of each individual. Quality system eventually came to be used as an all-embracing term to describe the collective plans, activities, and events that are provided to ensure that a product, process, or service will satisfy given needs. Feigenbaum [7] defines quality system as the agreed on company-wide and plant-wide operating work structure, documented in effective, integrated technical and managerial procedures, for guiding the coordinated actions of the people, the machines, and the information of the company and plant in the best and most practical way to assure customers quality satisfaction and economical costs of quality.

Inseparably linked with assurance and control of quality is the concept of quality cost. The ASQC recognized the importance of quality cost in the overall quality structure. In 1971, it defined the various categories of quality cost. Wadsworth et al. (1986) classified these costs as preventive, appraisal, internal, and external [9]. Feigenbaum [7] divided them into two broader categories: preventive costs and appraisal costs as belonging to costs of control, and internal costs and external costs as belonging to costs of failure of control.

Drifts and variations in the values of manufacturing process parameters give rise to loss of quality of the manufactured product. Yet, it is more costly to control the causes of manufacturing variations than to make a process insensitive to these variations [30]. It was in regard to this aspect or quality that Taguchi [31,32] made his contributions to quality control. He promoted the use of statistical methods for product design improvement. The Taguchi methods embrace both off-line and on-line quality control functions. They include parameter design, tolerance design, the quality loss function, on-line quality control, design of experiments using orthogonal arrays, and methodology applied to evaluate measuring systems [33].

The implementation of various statistical quality control methods in industry was enhanced by the use of computers. The general use of computers in quality control is relatively recent, but by the middle lo late 1970s computers had come to be used in automated testing, in computer-aided design (CAD), in computer-aided manufacturing (CAM), in computer-aided process control, and in data acquisition, storage, and analysis. Computer-aided quality (CAQ) represents the totality of the application of computers to quality control. CAQ, according to Feigenbaum [7], integrates the engineering database that designed the part and the product and guided its manufacture with the inspection and testing of the part and product. Thus, CAQ could be operated from the same data bases as CAD and CAM.

The 1980s

If each era is markedly by a major quality control activity, then the 1980s appropriately be termed the era of quality slogans. Although these slogans themselves do not impart quality to the items, they have, if nothing else, succeeded in increasing the public's awareness of the importance of quality. .

A big push in quality control in industry during the 1980s has been toward quality management particularly its human aspect. The problem now confronting industry is how to ensure that quality control procedures are adhered to, if the shop-floor worker rails, for some reason, to record the process parameter values at the right time, then the statistical quality control techniques that require these values cannot be applied without the danger of their giving a false indication of the state of the process. Therefore, a significant portion of quality management addresses this human aspect. However, the concerns of quality management are much more extensive than this concern with the performance of the shop-floor worker. They embrace the whole organization.

As in most other fields of technology, quality control and quality assurance have experienced tremendous growth in the area of computer applications. It is not known exactly when computers were first used for these purposes. Due to the proprietary nature of technological developments, it is also difficult to identify precisely the first computer applications in quality control and quality assurance. A 1969 issue of the

Journal of Quality Technology contains a computer program [34] for data analysis in quality control.

More Recent Developments and Ongoing Events

Activities such as product design assurance, procurement quality assurance, production quality control, and product quality audit are of very recent origin, and are ongoing. Product design assurance acknowledges the important role of design in the final quality of the item. Poor design may result in erroneous specifications that ultimately leave their mark on the quality of the final product.

Procurement quality assurance deals with the quality of raw material. The rationale behind procurement quality assurance is straightforward. The manufacture of a quality product requires the use of quality raw materials.

Production quality control consists of the entire range of activities that are performed in the production process to achieve desired quality. Therefore, these activities include the use of computers in process control and manufacturing, preventive and corrective maintenance; process performance and capability tests; in-factory control of nonconformities; quality and quality control of in-process inventories; periodic survey of process control programs; and a system to establish and control applicable specifications and related instructions. A discussion of these activities can be found in Wadsworth et al. [9].

A quality time line is given in Table 1.1 that is a reference to the point in time of the occurrence of each of the major quality control events. It shows the order of occurrences of the events in the evolution of quality control and the types of quality control activities that were predominant in each era.

Table 1.1: Quality Time Line

Era	Development
Ancient period	Early Egyptians "Royal cubit" area cubit Basic decimal system Area of a circle, value of pi Division of time Early Greeks High quality and standards of art High precision and quality of architecture High-quality literature Early Romans Architecture High quality in masonry Structural engineering
Middle Ages	Operator quality control Craft guilds in Europe Regulated economies Established trade monopolies maintained stable price Specified standard for good Set workmanship standards Stipulated working conditions Regulated detail of manufacture
1900s	*Journal of the American Statistical Society* Supervisor quality control
1920s	Inspection quality control First Shewhart control charts Consumer risk, producer's risk Probability of acceptance OC curves, LTPD ATI, double sampling Type A and type B risks LTPD sampling tables AOQL sampling tables Demerit rating system

Table 1.1: Quality Time Line (*continued*)

Era	Development
1930s	Joint Committee for the Development of Statistical Applications in Development and manufacturing Development of British Standards Institution Standard 600, “Application of Statistical Methods to Industrial Standardizations and Quality Control” Variable sampling plan Scanlon Plan U.S. Food, Drug and Cosmetic Act
1940s	Statistical quality control Dodge-Romig Sampling inspection tables (LTPD protection) Rectifying inspection (AOQL protection) Army “Standard inspection procedures” (AQL) Rectifying inspection on continuous sequence of products (AOQL) Sequential sampling Advances in variables and attributes sampling and sequential analysis Sampling inspection (AQL) American War Standards AWS Z1.1 “Guide to Quality Control” AWS Z1.2 “Control Chart Methods of analyzing data” *Industrial Quality Control* published by the Society of Quality Control Engineers and the University of Buffalo American Society for Quality Control formed
1950s	Quality control training courses in the United State Australian Laboratory Accreditation System (for testing) JAN-105 Multivariate quality control Average sample number (ASN) Grubb’s sampling table MIL-STD-105A 'Formation of Advisory Group on Reliability of Electronic Equipment (AGREE) MIL-M-26512A Cusum control charts Freund’s acceptance control charts MIL-STD-414 *Inspection and Quality Control Handbook (Interim) H107 and H108 for Single-Ievel and Multi-Level Continuous Sampling Procedures and Tables for Inspection by Attributes, respectively* MIL-O-9858A

Table 1.1: Quality Time Line (*continued*)

Era	Development
1950s (Continued)	MIL-I-45208A NHB 5300.4(IB) ASA guidelines for treating problems concerning economic control of quality of materials and manufactured products, ZI.1 and Z1.2 Exponential weighted moving averages *Applied Statistics* published Quality control charts in Japan Quality control training courses in Japan Chain sampling inspection plans Skip-Lot sampling plan . Additional continuous sampling inspection plans Sampling plans for inspection by variables Multilevel continuous sampling plans Continuous inspection schemes Poultry Products Inspection Act
1960s	Total quality control Zero defects *Quality Progress* published *Journal of Quality Technology* published Quality circles U.S. Consumer Product Safety Act U.S. Food, Drug and Cosmetic Act Amendments on manufacturing, processing, packaging and handling of human food. Radiation Control for Health and Safety Act
1970s	Categories of quality costs defined by the ASQC U.S. laboratory accreditation Quality system Cause-and-effect (Ishikawa) diagrams Taguchi methods Quality improvement through Statistically designed experiments Participative quality control Quality defined by ANSI/ASQC Standard A3 U.S. Meat Inspection Act Medical Device Amendments Organization wide quality control and total quality management

Table 1.1: Quality Time Line (*continued*)

Era	Development
1980s	Plethora of quality slogans Plethora of quality control software and computer programs
Recent Developments	Product design assurance Procurement quality assurance Production quality control Product quality audit Increasing customer requirements for quality Industry adjustment to customers' higher awareness of quality

Finally, Table 1.2 is a list of those individuals who are considered to be the pioneer, in quality control and their contributions to the field. This table can therefore be used as a quick reference to the major contributions or accomplishments of each pioneer.

Table 1.2: Pioneers in Quality Control

Pioneer	*Accomplishment*
Crosby, P.B.	Founded Quality College, Winter Park, Florida Initiated the quality cost reduction program "Buck a Day (SAD)" Developed the "14-Step Quality Improvement Program" Originated a widely used definition of quality Wrote *Quality Is Free* and numerous other popular books on quality Developed "Zero Defects-30'" a 30-day quality program for a supervisor and 8 to 10 of the supervisor's employees
Deming, W.E.	Developed quality control training during World War II Researched the use of statistics in quality control for the War II Brought statistical methods in quality control to Japan after World War II Originated a definition of statistical quality control that emphasizes statistical aspects and economic goals of quality control Developed "14 points" (or obligations) of management's responsibility for quality and management of an enterprise Identified two separate causes ("special" and "common") for poor quality and responsibilities for their correction

Table 1.2: Pioneers in Quality Control (c*ontinued*)

Pioneer	*Accomplishment*
Dodge, H.F.	Founding member of the Western Electric inspection Department (the department developed theories and methods of quality control and quality assurance) Developed basic concepts of sampling inspection by attributes Defined consumer's risks and producer's risks Member of a group of statisticians and engineers formed by the War Department to conduct research in the use of statistics in quality control (the group developed standard inspection procedures and sampling tables) Initiated widespread applications of control chart techniques throughout Western Electric Prepared the ASTM manual on presentation of data Chairman of ASA Committee ZI Developed the Dodge-Romig Sampling inspection Tables on attribute acceptance sampling Developed first continuous sampling plans Developed skip-lot sampling plans Developed chain sampling plans
Edwards, G.D.	Founding member of Western Electric Inspection Department Taught courses on the use of statistical quality control throughout manufacturing plants in the United States during World War II
Feigenbaum, A.V.	Developed the concept of total quality control Identified five stages in the history and evolution of quality control
Freund, R.A.	Member of the ASQC committee for precision in terminology which prepared "Delineations Symbols, Formulas and Tables for Control Charts" Developed an acceptance control chart for samp1e or subgroup variability
Grubbs, F.E.	Developed tables for attributes sampling plans
Gryna, F.M.	Developed together with Juran the concept of operator self-control (by this concept, control must be delegated to the operator in the workplace)
Hotelling, H.	Member of the Statistical Research Group at Columbia University during World War II (the group developed sequential analysis and multivariate analysis in quality control) Developed the t^2 statistic

Table 1.2: Pioneers in Quality Control (*Continued*)

Pioneer	*Accomplishment*
Ishikawa, K.	Introduced control chart methods to Japan Developed cause-and-effect diagram Acclaimed as the "father of quality circles" Suggested intervals in construction of histograms used in quality control indicated the use of paired barplots in quality control
Juran, J.M.	Renowned international consultant in quality control Member of a group of engineers associated the Western Electric Inspection Department Developed many concepts in quality (his work is credited as being the basis of Japan's postwar management Developed one of the general definitions of quality Espoused the application of the Pareto principle in quality control Developed the alternative designations sporadic and chronic for the causes of poor quality Developed in conjunction with Gryna, the concept of operator self-control
Pearson, E.	Developed British standards on the application of statistical methods to industrial standardization and quality control Developed estimation curves Indicated the use of range and its properties in quality control
D. A. Quarles	Founding member of the Western Electric Inspection Department
Romig, H.G.	Developed, along with Dodge. the Dodge-Romig Sampling Inspection Tables on attributes acceptance sampling
Scanlon, J.	Developed the Scanlon Plan for employee motivation
Shewhart, W.A.	Founding member of the Western Electric Inspection Department Developed the first control charts Formed one of the groups sponsored by the War Department during World War II to conduct research on the use of statistics in quality control Developed the concept of assignable causes Developed basic concepts of type 1 and type II error
Taguchi, G.	Developed methods for quality improvement studies using experimental design procedures (explored the concept of off-line)
Torrey, M.N.	Later member of the Western Electric Inspection Department Further developed CSP. in conjunction with Dodge

Table 1.2: Pioneers in Quality Control (*Continued*)

Pioneer	*Accomplishment*
Wald, A.	Member of the Statistical Research Group at Columbia University during World War II Developed sequential-sampling plans, procedures. and tables Proposed truncation value in sequential sampling plans Developed general expression for average sample numbers (ASN) Developed parametric equations for OC curves for sequential sampling plan

1.2 Probability Theory used in Statistical Quality Control

Statistical methods can be used to summarize or describe a collection of data that is called *descriptive statistics*. In addition, patterns in the data may be modeled in a way that accounts for randomness and uncertainty in the observations, to draw inferences about the process or population being studied; this is called *inferential statistics*. Both descriptive and inferential statistics can be considered part of applied statistics.

A control chart is a run chart of a sequence of quantitative data with five horizontal lines drawn on the chart:

- A *centre line*, drawn at the process mean;
- An *upper warning limit* drawn two standard deviations above the centre line;
- An *upper control-limit* (also called an *upper natural process-limit* drawn three standard deviations above the centre line;
- A *lower warning limit* drawn two standard deviations below the centre line;
- A *lower control-limit* (also called a *lower natural process-limit* drawn three standard deviations below the centre line.

Shewhart set *3-sigma* limits on the following basis of the probability theory.

- The coarse result of Chebyshev's inequality that, for any probability distribution, the probability of an outcome greater than k standard deviations from the mean is at most $1/k^2$.

Chebyshev's inequality: Let X be a random variable with expected value μ and finite variance σ^2. Then for any real number $k > 0$,

$$\Pr\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}. \tag{1.1}$$

- The finer result of the Vysochanskii-Petunin inequality, that for any unimodal probability distribution, the probability of an outcome greater than k standard deviations from the mean is at most $4/(9k^2)$.

Vysochanskii-Petunin inequality: Let X be a random variable with unimodal distribution, mean μ and finite, non-zero variance σ^2. Then, for any $\lambda > \sqrt{8/3} = 1.632\overline{9}$,

$$\Pr\left(|X - \mu| \geq \lambda\sigma\right) \leq \frac{4}{9\lambda^2}. \tag{1.2}$$

It is common in the construction of control charts, and other statistical heuristics, to set $\lambda = 3$, corresponding to an upper probability bound of 4/81 = 0.04938, and to construct *3-sigma* limits to bound nearly all (i.e. 95%) of the values of a process output.

- The empirical investigation of sundry probability distributions that at least 99% of observations occurred within three standard deviations of the mean.

Shewhart summarized the conclusions by saying:

... the fact that the criterion which we happen to use has a fine ancestry in highbrow statistical theorems does not justify its use. Such justification must come from empirical evidence that it works. As the practical engineer might say, the proof of the pudding is in the eating.

Though he initially experimented with limits based on probability distributions, Shewhart ultimately wrote:

Some of the earliest attempts to characterize a state of statistical control were inspired by the belief that there existed a special form of frequency function f *and it was early argued that the normal law characterized such a state. When the normal* *law was found to be inadequate, then generalized functional forms were tried. Today, however, all hopes of finding a unique functional form* f *are blasted.*

The control chart is intended as a heuristic. Deming insisted that it is not an hypothesis test and is not motivated by the Neyman-Pearson lemma. He contended that the disjoint nature of population and sampling frame in most industrial situations compromised the use of conventional statistical techniques. Deming's intention was to seek insights into the cause system of a process *...under a wide range of unknowable circumstances, future and past....* He claimed that, under such conditions, *3-sigma* limits provided *... a rational and economic guide to minimum economic loss...* from the two errors:

- Ascribe a variation or a mistake to a special cause when in fact the cause belongs to the system (common cause). In statistics this is a Type I error
- Ascribe a variation or a mistake to the system (common causes) when in fact the cause was special. In statistics this is a Type II error

Common cause variation plots as an irregular pattern, mostly within the control limits. Any observations outside the limits, or patterns within, suggest (*signal*) a special-cause. The run chart provides a context in which to interpret signals and can be beneficially annotated with events in the business.

1.3 From classical control charts to fuzzy control charts

It is not surprising that uncertainty exists in the human world. To survive in our world, we are engaged in making decisions, managing and analyzing information, as well as predicting future events. All of these activities utilize information that is available and help us try to cope with information that is not. A rational approach toward decision-making should take human subjectivity into account, rather than employing only objective probability measures. A research work incorporating uncertainty into decision analysis is basically done through the probability theory and/or the fuzzy set theory. The former represents the stochastic nature of decision analysis while the latter captures the subjectivity of human behavior. The fuzzy set theory is a perfect means for modeling uncertainty (or imprecision) arising from mental phenomena which is neither random nor stochastic.

When human subjectivity plays an important role in defining the quality characteristics, the classical control charts may not be applicable since they require certain information. Fuzzy control charts are inevitable to use when the statistical data in consideration are uncertain or vague; or available information about the process is incomplete, linguistic or includes human subjectivity. A general comparison of traditional Shewhart control charts and fuzzy control charts is given in Table 1.3.

Table 1.3: Comparison of Traditional Shewhart and Fuzzy Control Charts

Comparison issue	**Traditional Shewhart Control Charts**	**Fuzzy Control Charts**
Number of quality characteristics	Only one quality characteristic	Multiple quality characteristics
Availability and type of statistical data	Completely required and certain	Vague, uncertain, and incomplete information
Information used in base period	Historical data	Experts' experience rules
Judgment	in control or out of control	Further intermediate linguistic decisions
Advantages	1. Easier for considering one quality characteristic 2. More objective	1. Provide more accurate control standards for the process based on experts' experience expressed in degree of membership 2. More flexible for the definitions of the fuzzy inference rules
Disadvantages	1. Inflexible control limits 2. Sample size influences the width of control limits 3. Historical data are needed to obtain the formal control limits	1. Inference outcomes are based on the subjective experience rules 2. Supplemental rules (for systematic changes) of the traditional control charts cannot be used

1.4 Scope and aim of the thesis

This thesis aims at developing some models for the construction and interpretation of the fuzzy control charts with linguistic, uncertain, and vague data. Section 2 is a review of the statistical process control. Statistical process control charts are given in Section 3. Unnatural pattern analyses for the classical process control charts are explained in Section 4. Basics of the fuzzy sets theory required to construct fuzzy control charts are presented in Section 5. In Section 6, fuzzy control charts are developed. Fuzzy unnatural pattern analyses for the developed fuzzy control charts are proposed in Section 7. Numerical examples are presented in Section 8 and finally a conclusion is given at the end of the thesis.

2 STATISTICAL PROCESS CONTROL (SPC)

2.1 Introduction

Statistical process control was pioneered by Walter A. Shewhart and taken up by W. Edwards Deming with significant effect by the Americans during World War II to improve industrial production. Deming was also instrumental in introducing SPC methods to Japanese industry after that war. Dr. Shewhart created the basis for the control chart and the concept of a state of statistical control by carefully designed experiments. While Dr. Shewhart drew from pure mathematical statistical theories, he understood data from physical processes never produce a “normal distribution curve” (a Gaussian distribution, also commonly referred to as a “bell curve”). He discovered that observed variation in manufacturing data did not always behave the same way as data in nature (Brownian motion of particles). Dr. Shewhart concluded that while every process displays variation, some processes display controlled variation that is natural to the process, while others display uncontrolled variation that is not present in the process causal system at all times.

SPC encompasses the following basic ideas:

- Quality is conformance to specifications.
- Processes and products vary.
- Variation in processes and products can be measured.
- Variation follows identifiable patterns.
- Variation due to assignable causes distorts the bell shape.
- Variation is detected and controlled through SPC

Classical Quality control was achieved by observing important properties of the finished product and accept/reject the finished product. As opposed to this statistical process control uses statistical tools to observe the performance of the production line to predict significant deviations that may result in reject products.

The underlying assumption in the SPC method is that any production process will produce products whose properties vary slightly from their designed values, even when the production line is running normally, and these variances can be analyzed statistically to control the process. For example, a breakfast cereal packaging line may be designed to fill each cereal box with 500 grams of product, but some boxes will have slightly more than 500 grams, and some will have slightly less, producing a distribution of net weights. If the production process itself changes (for example, the machines doing the manufacture begin to wear) this distribution can shift or spread out. For example, as its cams and pulleys wear out, the cereal filling machine may start putting more cereal into each box than it was designed to. If this change is allowed to continue unchecked, product may be produced that fall outside the tolerances of the manufacturer or consumer, causing product to be rejected.

By using statistical tools, the operator of the production line can discover that a significant change has been made to the production line, by wear and tear or other means, and correct the problem - or even stop production - before producing product outside specifications. An example of such a statistical tool would be the Shewhart control chart, and the operator in the aforementioned example plotting the net weight in the Shewhart chart.

A production system is a process hierarchy, consisting of basic processes and their respective sub-processes and sub-subprocesses. Process control is a critical part of operations. Process control is a complex combination of measurement, comparison, and correction. Box *et. al.* [35] and Box and Luceno [36] cite two techniques for dealing with process control issues: techniques of process monitoring and techniques of process adjustment.

Process monitoring strategies focuses on process disruptions/special cause elimination, the detection, isolation, and removal of influences over and above common cause or natural variation that enters a process by virtue of controllable or uncontrollable variables.

Process adjustment strategies focuses on process regulation/adjustment, the manipulation of identified controllable input/transformation variables so as to influence the value of an output variable [37].

2.2 SPC Tools

SPC can be applied to any process. Its major tools are briefly explained in the following [38]:

- Histogram: The histogram is a graphical data summary tool which allows to group observed data into cells, or predefined categories, in order to discover data location and dispersion characteristics (without a sophisticated numerical analysis). The histogram is a very valuable and underrated data analysis tool. Two types of histograms are:
    1. a frequency count histogram
    2. a relative frequency or proportion histogram.
- Check Sheet: A check sheet is a simple tool used to record and classify observed data. Primarily, there are two types of check sheets [39]:
    1. Tabular check sheets
    2. Pictorial check sheets.
- Pareto Chart: In nineteenth-century Italy, the Italian economist Vilfredo Pareto observed that about 80 percent of the country's wealth was controlled by about 20 percent of the population. This observation lead to what is now known as Pareto Principle; it is also known as "80-20" rule. Juran [40] and Juran and Gryna [41] applied this concept to the causes of quality failures. They stated that 20 percent of the causes account for 80 percent of the failures. In general, Pareto principle, applied to quality, suggests that majority of the quality losses are distributed in such a way that a "vital few" quality defects or problems always constitute a high percent of the overall quality losses [38].

- Cause and Effect Diagram: A cause is a fundamental condition or stimulus of some sort that ultimately creates a result or effect. We may proceed from cause to effect, or conversely from effects to cause. Most analyses work in both directions in order to discover or document causes, effects, and cause-effect linkages. Cause-Effect analyses are usually summarized in a cause-effect diagram that is developed by Ishikawa for the purpose of representing the relationship between an effect and the potential or possible causes influencing it. The cause-effect diagram first helps us to discover possible root causes of defects and then helps us understand the failure mechanism involved, so that we can prevent or eliminate them by proactive-reactive actions [38].
- Strafication Analyses: Strafication is the process of breaking down or sorting a large database so that meaningful subset, classifications, or summaries can be developed. It allows us to effectively and efficiently navigate through huge volumes of data, seeking out the clues to quality improvement buried therein.
- Scatter Diagram: A scatter diagram provides the opportunity to view a data set in multiple dimensions in order to detect trends, spot best operating regions, explore cause-effect relationships, and so on [38].
- Control Charts: The seventh fundamental tool is the statistical process control (SPC) chart. These tools are based on the principles of probability and statistics. Control charts that are the main scope of this thesis are discussed in the next section.

3 STATISTICAL PROCESS CONTROL CHARTS (SPCC)

3.1 Introduction

Every process varies. If you write your name ten times, your signatures will all be similar, but no two signatures will be exactly alike. There is an inherent variation, but it varies between predictable limits. If, as you are signing your name, someone bumps your elbow, you get an unusual variation due to what is called a “special cause”. If you are cutting diamonds, and someone bumps your elbow, the special cause can be expensive. For many, many processes, it is important to notice special causes of variation as soon as they occur.

There is also “common cause” variation. Consider a baseball pitcher. If he has good control, most of his pitches are going to be where he wants them. There will be some variation, but not too much. If he is “wild”, his pitches are not going where he wants them; there is more variation. There may not be any special causes - no wind, no change in the ball - just more “common cause” variation. The result: more walks are issued, and there are unintended fat pitches out over the plate where batters can hit them. In baseball, control wins ballgames. Likewise, in most processes, reducing common cause variation saves money. [42]

Happily, there are easy-to-use charts which make it easy see both special and common cause variation in a process. They are called control charts, or sometimes Shewhart charts, after their inventor, Walter Shewhart, of Bell Labs. There are many different subspecies of control charts which can be applied to the different types of process data which are typically available.

All control charts have three basic components:

- a centerline, usually the mathematical average of all the samples plotted.
- upper and lower statistical control limits that define the constraints of common cause variations.

- performance data plotted over time.

Control charts are used to routinely monitor quality. Depending on the number of process characteristics to be monitored, there are two basic types of control charts. The first, referred to as a univariate control chart, is a graphical display (chart) of one quality characteristic. The second, referred to as a multivariate control chart, is a graphical display of a statistic that summarizes or represents more than one quality characteristic.

If a single quality characteristic has been measured or computed from a sample, the control chart shows the value of the quality characteristic versus the sample number or versus time. In general, the chart contains a center line that represents the mean value for the in-control process. Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL), are also shown on the chart. These control limits are chosen so that almost all of the data points will fall within these limits as long as the process remains in-control. The figure below illustrates this.

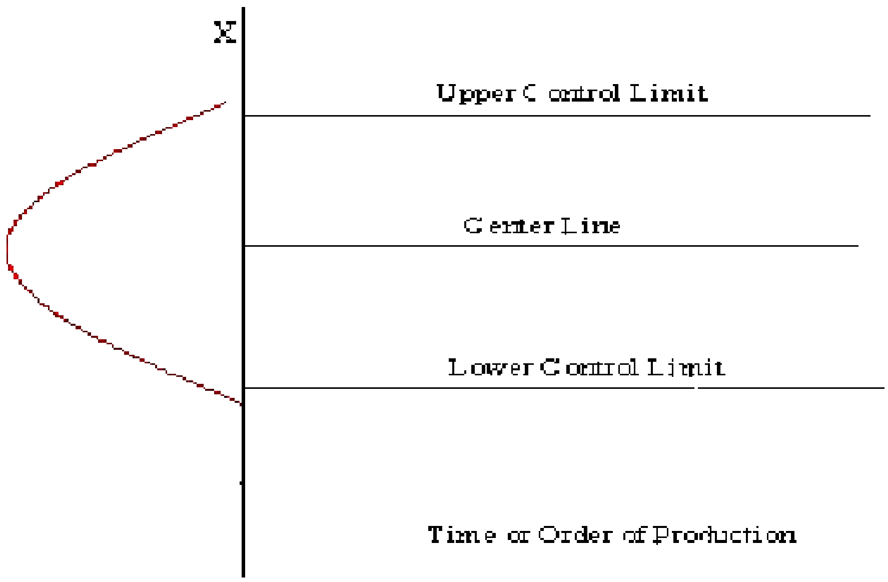

Figure 3.1: Illustration of control limits.

The control limits as pictured in the graph might be 0.001 *probability* limits. If so, and if chance causes alone were present, the probability of a point falling above the upper limit would be one out of a thousand, and similarly, a point falling below the lower limit would be one out of a thousand. We would be searching for an assignable cause if a point would fall outside these limits. Where we put these limits will determine the risk of undertaking such a search when in reality there is no assignable cause for variation.

Since two out of a thousand is a very small risk, the 0.001 limits may be said to give practical assurances that, if a point falls outside these limits, the variation was caused be an assignable cause. It must be noted that two out of one thousand is a purely arbitrary number. There is no reason why it could have been set to one out a hundred or even larger. The decision would depend on the amount of risk the management of the quality control program is willing to take. In general (in the world of quality control) it is customary to use limits that approximate the 0.002 standard.

Letting *X* denote the value of a process characteristic, if the system of chance causes generates a variation in *X* that follows the normal distribution, the 0.001 probability limits will be very close to the 3σ limits. From normal tables we glean that the $3\boldsymbol{\sigma}$ in one direction is 0.00135, or in both directions 0.0027. For normal distributions, therefore, the 3σ limits are the practical equivalent of 0.001 probability limits.

In the U.S., whether *X* is normally distributed or not, it is an acceptable practice to base the control limits upon a multiple of the standard deviation. Usually this multiple is 3 and thus the limits are called 3σ limits. This term is used whether the standard deviation is the universe or population parameter, or some estimate thereof, or simply a "standard value" for control chart purposes. It should be inferred from the context what standard deviation is involved. (Note that in the U.K., statisticians generally prefer to adhere to probability limits.)

If the underlying distribution is skewed, say in the positive direction, the 3-sigma limit will fall short of the upper 0.001 limit, while the lower 3-sigma limit will fall below the 0.001 limit. This situation means that the risk of looking for assignable causes of positive variation when none exists will be greater than one out of a thousand. But the risk of searching for an assignable cause of negative variation, when none exists, will be reduced. The net result, however, will be an increase in the risk of a chance variation beyond the control limits. How much this risk will be increased will depend on the degree of skewness.

If variation in quality follows a Poisson distribution, for example, for which *np* = 0.8, the risk of exceeding the upper limit by chance would be raised by the use of 3σ limits from 0.001 to 0.009 and the lower limit reduces from 0.001 to 0. For a Poisson distribution, the mean and variance both equal *np*. Hence the upper 3σ limit is 0.8 + 3 $\sqrt{0.8}$ = 3.48 and the lower limit = 0. For *np* = 0.8 the probability of getting more than 3 successes is 0.009.

If a data point falls outside the control limits, we assume that the process is probably out of control and that an investigation is warranted to find and eliminate the cause or causes.

Does this mean that when all points fall within the limits, the process is in control? Not necessarily. If the plot looks non-random, that is, if the points exhibit some form of systematic behavior, there is still something wrong. For example, if the first 25 of 30 points fall above the center line and the last 5 fall below the center line, we would wish to know why this is so. Statistical methods to detect sequences or nonrandom patterns can be applied to the interpretation of control charts. To be sure, "in control" implies that all points are between the control limits and they form a random pattern.

3.2 Statistical Basis of the Control Charts

A typical control chart is the graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time. Control limits are chosen so that if the process is in control, nearly all of the sample points fall between them. As long as the points plot within the control limits, the process is assumed to be in control, and no action is necessary. However, a point that plots outside of the control limits is interpreted as evidence that the process is out of control, and investigation and corrective action is required to find and eliminate the assignable cause or causes responsible for this behavior. It is customary to connect the sample points on the control chart with straight-line segments, so that it is easier to visualize how the sequence of points has evolved over time.

Even if all the points plot inside the control limits, if they behave in a systematic or nonrandom manner, then it is an indication that the process is out of control. For example, if 18 of the last 20 points plotted above the center line but below the upper control limit and only two of these points plotted below the center line but above the lower control limit, we would be very suspicious that something was wrong. If the process is in control, all the plotted points should have an essentially random pattern. Methods for looking for sequences or nonrandom patterns can be applied to control charts as an aid in detecting out-of-control conditions. Usually, there is a reason why a particular nonrandom pattern appears on a control chart, and if it can be found and eliminated, process performance can be improved [38].

There is a close connection between control charts and hypothesis testing. Essentially, the control chart is a test of the hypothesis that the process is in a state of statistical control. A point plotting within the control limits is equivalent to failing to reject the hypothesis of statistical control. Just as in hypothesis testing, we may think of the probability of type I error of the control chart (concluding the process is out of control when it is really in control) and the probability of type II error of the control chart (concluding the process is in control when it is really out of control). It is occasionally helpful to use the operating-characteristic curve of a control chart to display its probability of type II error. This would be an indication of the ability of the control chart to detect process shifts of different magnitudes [38].

The control chart is a device for describing in a precise manner exactly what is meant by statistical control. The most important use of a control chart is to improve the process. Most processes do not operate in a state of statistical control. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved. This process improvement activity using the control chart is illustrated in Figure 3.2.

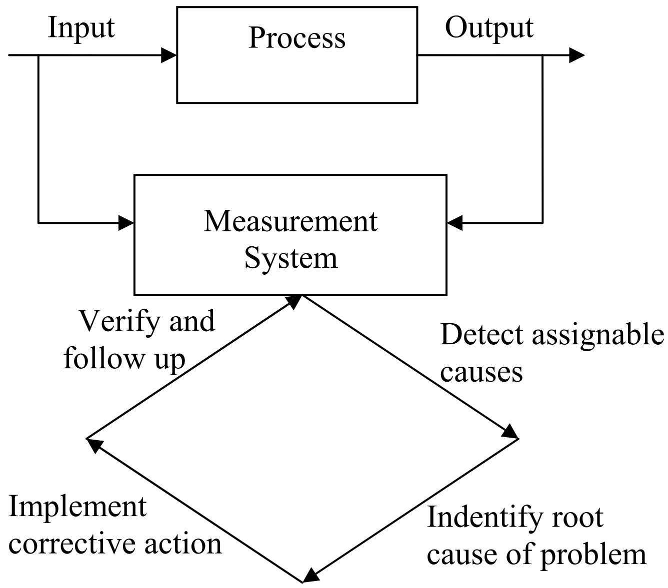

Figure 3.2: Process improvement using the control chart.

Some reasons for the popularity of the control charts are given below [38]:

- Control charts are proven a technique for improving productivity: A successful control chart program will reduce the scrap and rework, which are the primary productivity-killers in any operation. If you reduce scrap and rework, then productivity increases, cost decreases, and production capacity (measured in the number of good parts per hour) increases.

- Control charts are effective in defect prevention: The control helps keep the process in control, which is consistent with the "do it right the first time" philosophy. It is never cheaper to sort out "good" units from "bad" units later on than it is to build it right initially. If you do not have effective process control, you are paying someone to make a nonconforming product.

- Control chart prevent unnecessary process adjustments: A control chart can distinguish between background noise and abnormal variation; no other device including a human operator is as effective in making this distinction. If process operators adjust the process based on periodic tests unrelated to a control chart program, they will often overreact to the background noise and make unneeded adjustments. These unnecessary adjustments can actually result in a deterioration of process performance. In other words, the control chart is consistent with the "if it is not broken, do not fix it" philosophy.

- Control charts provide diagnostic information: frequently, the pattern of points on the control chart will contain information of diagnostic value to an experienced operator or engineer. This information allows the implementation of a change in the process that improves its performance.
- Control charts provide information about process capability: The control chart provides information about the value of important process parameters and their stability over time. This allows an estimate of process capability to be made. This information is of tremendous use to product and process designers.

3.3 Control Limits

Specifying the control limits is one of the critical decisions that must be made in designing a control chart. By moving the control limits further from the center line, the risk of a type I error is decreased. However, widening the control limits will also increase the risk of a type II error. If the control limits are moved closer to the center line, the opposite effect is obtained: The risk of type I error is increased, while the risk of type II error is decreased [38].

One of the salient characteristics of the distribution of the example data is the tendency to build up observations in the center of the distribution. This characteristic is known as central tendency. Central tendency is usually expressed in three ways: (a) the average value termed the arithmetic mean, (b) the middle value termed the median, and (c) the most frequently occurring value termed the mode [43]. The arithmetic mean, $\overline{X}$, is by far the most used measure of central tendency and is the basis of the definition of the center line of the control charts.

$$CL = \overline{X} = \frac{\sum_i X_i}{n}, \tag{3.1}$$

where X_i is the characteristic value of the observed data and n is the number of data initially available to construct the control chart.

Once the center line is determined, upper and lower control limits are set as 3σ above and below the center line. Some analysts suggest using two sets of limits on control charts. The 3σ limits are the usual action limits; that is, when a point plots outside of this limit, a search for an assignable cause is made and corrective action is taken if necessary. The second set of the control limits is the 2σ control limits known as warning limits. If one or more points fall between the warning limits and action limits, or very close to the warning limit, one should be suspicious that the process may not be operating properly. One possible action to take when this occurs is to increase sampling frequency and to use these additional data in conjunction with the suspicious points to investigate the state of control of the process. Warning limits increase the sensitivity of the control chart. Their disadvantage is that they do not have a precise interpretation and may be confusing to operating personnel. This is not a serious objection, however [38].

3.4 Classification of SPCC

SPCC's are usually classified in two ways: Classification based on the number of quality characteristics and type of the quality characteristics [44].

3.4.1 Classification Based on the Number of Variables

Based on the number of characteristic variables in consideration SPCC can be categorized into two categories as univariate control charts and multivariate control charts.

Univariate SPCC

In this type of SPCC, there is only one characteristic to be observed with the control charts. Classical Shewhart Chart uses one quality characteristic and so known as univariate control chart. Examples of univariate control charts can be given as *p*, *np*, and *c* charts.

Multivariate SPCC

SPCC in this category deals with more than one quality characteristic at the same time. As a special case, a control chart with two quality characteristics in consideration is called as a bivariate control chart.

3.4.2 Classification Based on the Quality Characteristics

Consider that you are evaluating the output from a process. Conceptually, you could evaluate the products in two basic ways. In the first way you could measure a key characteristic using a continuous scale. This produces variable (continuous) data. In the second way you would simply classify the products as "conforming" or "nonconforming". This produces attribute (discrete) data. A SPCC based on a continuous and measurable data is called variables SPCC, while that on a discrete and immeasurable data are called attributes SPCC.

Variables SPCC

Variables control charts are used to evaluate variation in a process where the measurement is a variable, i.e. the variable can be measured on a continuous scale (e.g. height, weight, length, concentration). There are two main types of variables control charts. One (e.g. x-bar chart, Delta chart) evaluates variation *between* samples. Non-random patterns (signals) in the data on these charts would indicate a possible change in central tendency from one sampling period to the next. One way of thinking about the use of a variables control chart is that you are testing the hypothesis that a particular sample mean came from the population of sample means represented by the control limits of the process. If the particular sample mean is within the control limits, your conclusion is that it does come from that population. If the particular sample mean is outside the control limits, you conclusion is that it may have come from some other distribution (i.e. a distribution with a mean that is higher or lower than this population mean.

The other type of variables control chart (e.g. R-chart, S-chart, Moving Range chart) evaluates variation *within* samples. Non-random patterns (signals) in the data on these charts would indicate a possible change in the variation within the samples.

Attributes SPCC

The Shewhart control chart plots quality characteristics that can be measured and expressed numerically. We measure weight, height, position, thickness, etc. If we cannot represent a particular quality characteristic numerically, or if it is impractical to do so, we then often resort to using a quality characteristic to sort or classify an item that is inspected into one of two "buckets".

An example of a common quality characteristic classification would be designating units as "conforming units" or "nonconforming units". Another quality characteristic criteria would be sorting units into "non defective" and "defective" categories. Quality characteristics of that type are called *attributes.*

Control charts dealing with the number of *defects* or *nonconformities* are called c charts (for count).

Control charts dealing with the proportion or fraction of defective product are called p charts (for proportion).

There is another chart which handles defects per unit, called the u chart (for unit). This applies when we wish to work with the average number of nonconformities per unit of product.

4 UNNATURAL PATTERN ANALYSES

When a process is in statistical control, a control chart will display the known patterns of variation. When the control chart points deviate from these known patterns, the process is considered to be out of control. The control chart distinguishes between normal and non-normal variation through the use of statistical tests and control limits. The control limits are calculated using the rules of probability so that when a point is determined to be out of control, it is due to an assignable cause and not due to a normal variation. Points outside the control limits are not the only criteria to determine out of control conditions. All points may be inside the limits and the process may still be out of control if it does not display a normal pattern of variation. Zone tests are, which are hypothesis tests in a modified form, used to determine out of control conditions. They are used to test if the plotted points are following a normal pattern of variation. For a control chart to be effective, some action must be taken as a result of the chart pattern. When the process average is centered where it is supposed to be, and the variability displays a normal pattern, the process is considered to be in control. A normal pattern means that the process is aligned with the probabilities of the normal distribution. Large abnormal variability and unnatural patterns indicate out of control conditions. Out of control conditions usually have assignable causes that must be investigated and resolved.

The control charts may indicate an out-of-control condition when either one or more points fall beyond the control limits or plotted points show some non-random patterns of behavior. Unnatural (non-random) patterns for classical control charts have been extensively studied. Over the years, many rules have been developed to detect non-random patterns within the control limits. Under the pattern-recognition approach, numerous researches have defined several types of out-of-control patterns (e.g. trends, cyclic pattern, mixture, etc.) with a specific set of possible causes. When a process exhibits any of these unnatural patterns, it implies that those patterns may provide valuable information for process improvement.

Zones of a control chart used in zone tests are bounded by the standard deviations of the data as illustrated in Figure 4.1. Probabilities of each zone based on the normal distribution are depicted in Figure 4.2.

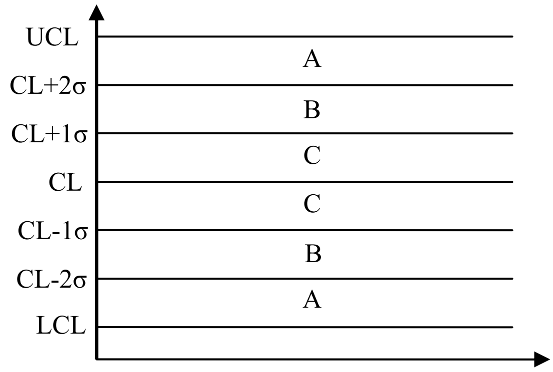

Figure 4.1: Zones of a control chart

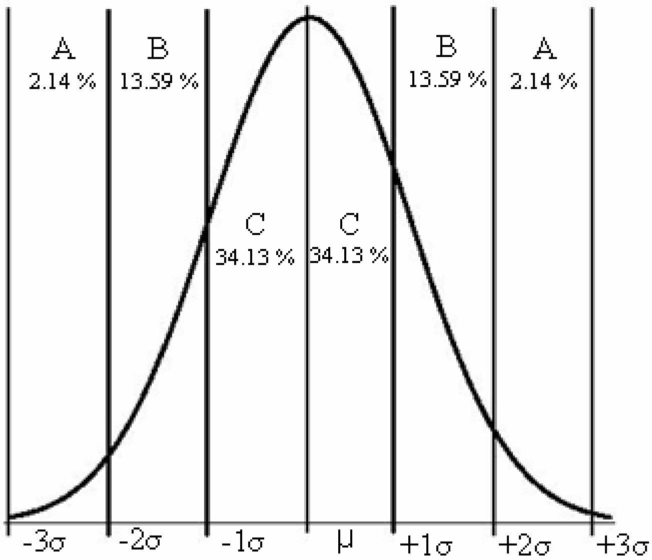

Figure 4.2: Zones and probabilities of normal distribution

The main idea behind defining a rule for an unnatural pattern is the probability of the occurrence: These rules are based on the premise that a specific run of data has a low probability of occurrence in a completely random stream of data. In general, probability of occurrence of an unnatural pattern is less than 1%. In the literature, there exist some unnatural patterns defined for the crisp cases. There is no certain rule about which unnatural patterns to use and the selection of a set of rules depends on the user preferences. Unnatural patterns are defined for the short runs, i.e., rules for a 15-20 consecutive points on the chart are investigated.

Numerous supplementary rules, like zone tests or run rules [39, 45-48] have been developed to assist quality practitioners in detection of unnatural patterns for the crisp control charts. Run rules are based on the premise that a specific run of data has a low probability of occurrence in a completely random stream of data. If a run occurs, then this must mean that something has changed in the process to produce a nonrandom or unnatural pattern. Based on the expected percentages in each zone, sensitive run tests can be developed for analyzing the patterns of variation in the various zones.

Western Electric suggested a set of decision rules for detecting unnatural patterns on control charts. Specifically, it suggested concluding that the process is out of control if any of the following conditions is satisfied [45].

Rule 1: A single point falls outside of the control limits (beyond $\pm 3\sigma$ limits) (Figure 4.3)

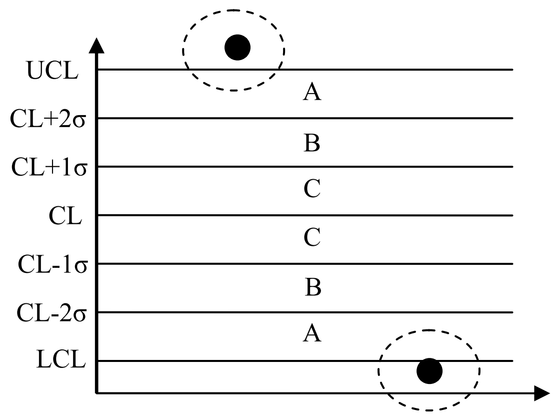

Figure 4.3: Representation of Rule 1 of Western Electric

Rule 2: Two out of three successive points fall in zone A or beyond (The odd point may be anywhere. Only two points count) (Figure 4.4).

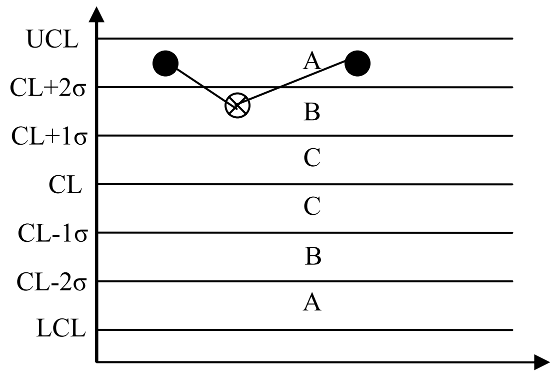

Figure 4.4: Representation of Rule 2 of Western Electric

Rule 3: Four out of five successive points fall in zone B or beyond (The odd point may be anywhere. Only four points count) (Figure 4.5).

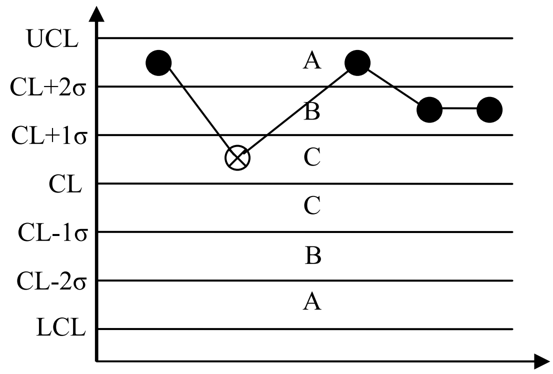

Figure 4.5: Representation of Rule 3 of Western Electric

Rule 4: Eight successive points fall in zone C or beyond (Figure 4.6).

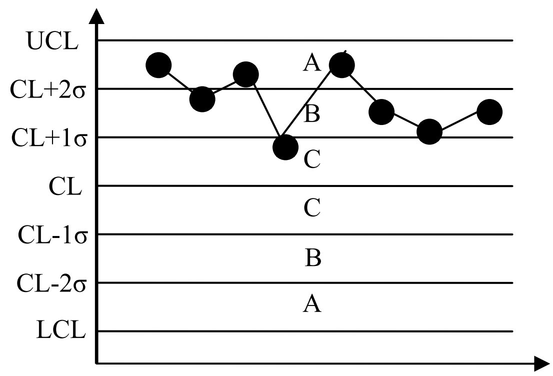

Figure 4.6: Representation of Rule 4 of Western Electric

One-sided probabilities of the rules above are calculated as 0.00135, 0.0015, 0.0027, and 0.0039, respectively.

Grant and Leavenworth recommended that nonrandom variations are likely to be presented if any one of the following sequences of points occurs in the control charts [39].

Rule 1: 7 consecutive points on the same side of the center line (Figure 4.7).

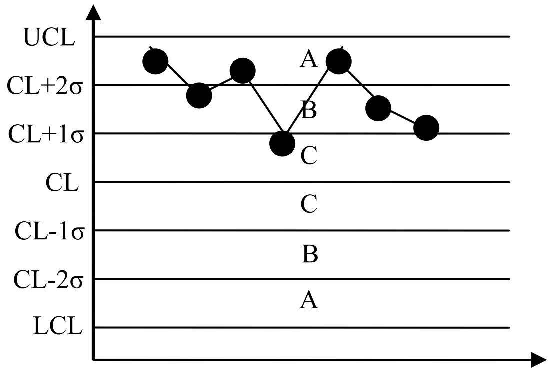

Figure 4.7: Representation of Rule 1 of Grant and Leavenworth

Rule 2: At least 10 of 11 consecutive points on the same side of the center line.

Rule 3: At least 12 of 14 consecutive points on the same side of the center line.

Rule 4: At least 14 of 17 consecutive points on the same side of the center line.

One-sided probabilities of the rules above are calculated as 0.00781, 0.00586, 0.00647, and 0.00636, respectively.

Nelson proposed the following rules for unnatural patterns [46, 47]:

Rule 1: One or more points outside of the control limits;

Rule 2: 9 consecutive points in the same side of center line

Rule 3: 6 points in a row steadily increasing or decreasing (Figure 4.8)

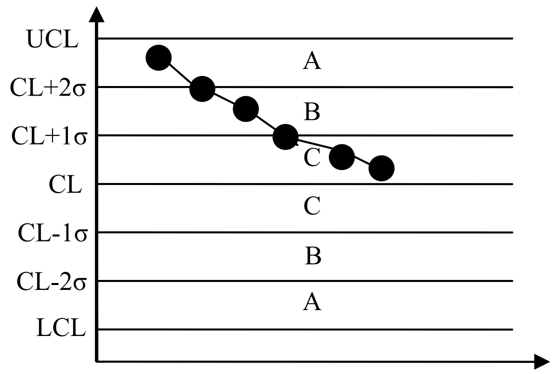

Figure 4.8: Representation of Rule 3 of Nelson

Rule 4: 14 points in a row altering up and down (Figure 4.9)

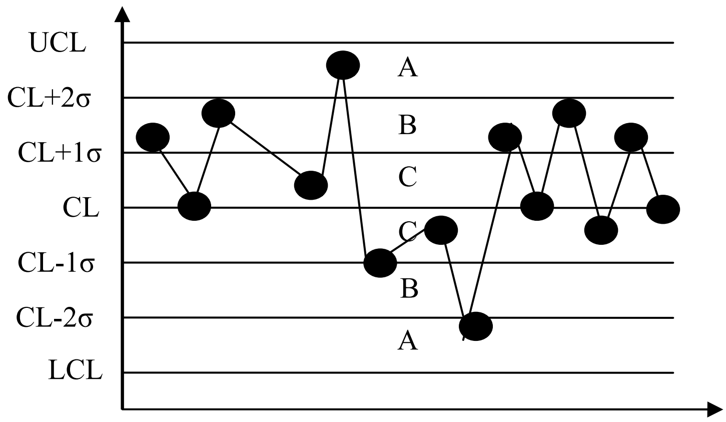

Figure 4.9: Representation of Rule 3 of Nelson

Rule 5: 2 out of 3 points in a row in zone A or beyond

Rule 6: 4 out of 5 points in zone B or beyond

Rule 7: 15 points in a row in zones C, above and below the centerline

Rule 8: 8 points in a row on both sides of the centerline with none in zone C

Unnatural patterns tend to fluctuate too wide or they fail to balance around the centerline. The portrayal of natural and unnatural patterns is what makes the control chart a very useful tool for statistical process and quality control. When a chart is interpreted, we look for special patterns such as cycles, trends, freaks, mixtures, groupings or bunching of measurements, and sudden shifts in levels [45].

5 FUZZY SET THEORY

5.1 Introduction

The boundaries of classical sets are required to be drawn precisely and, therefore, set membership is determined with complete certainty. An individual is either definitely a member of the set or definitely not a member of it. This sharp distinction is also reflected in classical logic, where each proposition is treated as either true or false. However, most sets and propositions are not so neatly characterized. It is not surprising that uncertainty exists in the human world. To survive in our world, we are engaged in making decisions, managing and analyzing information, as well as predicting future events. All of these activities utilize information that is available and help us try to cope with information that is not. Lack of information, of course, produces uncertainty, which is the condition where the possibility of error exists. This interplay of information and uncertainty is the hallmark of complexity. Research that attempts to model uncertainty into decision analysis is done basically through probability theory and/or fuzzy set theory. The former represents the stochastic nature of decision analysis while the latter captures the subjectivity of human behavior.

A classical (crisp) set is normally defined as a collection of elements or objects $x \in X$ which can be finite, countable, or overcountable. Each single element can either belong to or not belong to a set A, $A \subseteq X$. In the former case, the statement "x belongs to A" is true, whereas in the latter case it is false. It is possible to describe such a classical set in different ways: one can either enumerate the elements that belong to the set; describe the set analytically, i.e., by stating conditions for membership ($A = \{x \mid x \le 4\}$); or define the member elements by using the characteristic function, in which 1 indicates memberships and 0 nonmemberships. Fuzzy set theory is developed for solving problems in which descriptions of activities and observations are imprecise, vague, and uncertain. The term "fuzzy" refers to the situation in which there are no well-defined boundaries of the set of the activities or observations to which the descriptions apply. In fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set. [49]

5.2 Literature Survey

Professor Zadeh's paper [50] on fuzzy sets introduced the concept of a class with unsharp boundaries and marked the beginning of a new direction by providing a basis for a qualitative approach to the analysis of complex systems in which linguistic rather than numerical variables are employed to describe system behavior and performance. This approach centers on building better models of human reasoning and decision-making. His unorthodox ideas were initially met with some skepticism but they have since gained wide acceptance.

Fuzzy sets were introduced in 1965 by Lotfi Zadeh with a view to reconcile mathematical modeling and human knowledge in the engineering sciences. Since then, a considerable body of literature has blossomed around the concept of fuzzy sets in an incredible wide range of areas, from mathematics and logics to traditional and advanced engineering methodologies. Applications are found in many contexts, from medicine to finance, from human factors to consumer products, from vehicle control to computational linguistics, and so on… Fuzzy logic is now currently used in the industrial practice of advanced information technology [51].

Recent Applications of fuzzy sets in the last decades can be found in [52-103]. A literature survey about the fuzzy set theory applications in production management research can be referred to the study of Guiffrida and Nagi [104].

5.3 Basic Concepts and Definitions

5.3.1 Definition of a Fuzzy Set and Membership Function

Fuzzy set theory is composed of an organized body of mathematical tools particularly well-suited for handling incomplete information, the unhappiness of classes of objects or situations, or gradualness of preference profiles, in a flexible way. It offers a unifying framework for modeling various types of information ranging from precise numerical, interval-valued data, to symbolic and linguistic knowledge, with a stress on semantics rather than syntax (hence, some misunderstandings with logicians). [51]

Let X be classical (ordinary) set of objects, called the universe, whose generic elements are denoted by *x*, namely, $X = \{x\}$. A fuzzy set $\tilde{A}$ in *X* is characterized by a membership function $\mu_x(A)$ which associates with each element in X a real number in the interval [0, 1]. If *X* is a collection of objects denoted by *x*, the fuzzy set $\tilde{A}$ in *X* is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in X)\}$$

$\mu_A(x)$ is called the membership function or grade of membership (sometimes degree of compatibility or degree of truth) of x in $\tilde{A}$ which maps X to the membership space M. The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed in $\tilde{A}$. Membership function is not limited to values between 0 and 1. If $\sup(\mu_x(x)) = 1$ the fuzzy set $\tilde{A}$ is called as normal. A nonempty fuzzy set $\tilde{A}$ can be normalized by dividing $\mu_A(x)$ by $\sup(\mu_x(x))$. As a matter of convenience, otherwise stated, we will generally assume that fuzzy sets are normalized. In defining a membership function, the universal (crisp) set X is always assumed to be a classical set.

As an example, suppose that a university defines class levels according to the Table 5.1 and seeking to represent the concept of an experienced undergraduate student.

By contrast with the crisp sets based on the precisely defined class levels, the vague term experienced undergraduate student corresponds to a genuine fuzzy set. This fuzzy set consists of individuals whose degrees of membership in the set range from 0 to 1, and thus the graph of their membership degrees provides a transition from 0 to 1. Depending on our judgment of how many completed credit hours are required for an undergraduate student to be regarded as experienced, one might represent the transition from inexperienced to experienced as depicted in Figure 5.1.

Table 5.1: Undergraduate Class Levels

Class Level	Credit Hours
Freshman	0-32
Sophomore	33-62
Junior	63-94
Senior	95-126

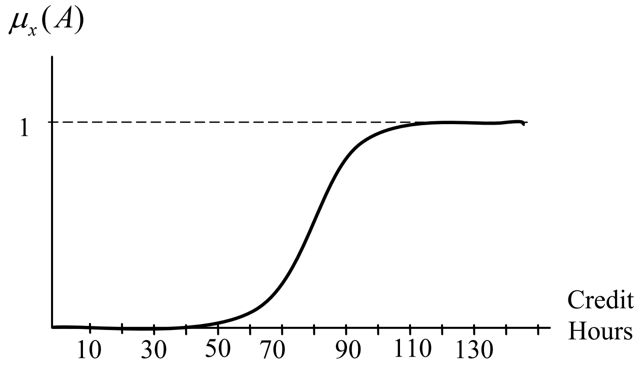

Figure 5.1: Representation of experienced undergraduate students.

The transition from nonmembership to full membership takes the form of a smooth curve which increases in height from left to right. Membership functions may exhibit other types of shapes depending upon the decision maker. In many fuzzy sets the exact shape of the transition from 0 to 1 is not critical. Indeed, sometimes we do not know for sure how to draw the transition from zero membership to total membership to capture the meaning of a linguistic term, such as medium, in a given context. The reason is that such a transitional shape must be based on empirical evidence of how the term in question is used in that context; many times this evidence is incomplete. However, most applications of fuzzy se theory do not show great sensitivity to the actual shapes of the membership functions involved. Hence, simple shapes are usually favored. [105]

Membership functions can be represented in the following ways:

- Graphical representation, as illustrated in Figure 5.1,

- Tabular and list representation: As the list of all ordered pairs consisting of each membership degree together with the label of individual. Some of the tabular and list representations used in the literature are given below.

$$\tilde{A} = \{x_1 / 0.8, x_2 / 0.3, x_3 / 0.9, x_4 / 1.0\} \qquad \textbf{(5.1)}$$

$$\tilde{A} = \{0.8 / x_1, 0.3 / x_2, 0.9 / x_3, 1.0 / x_4\} \qquad \textbf{(5.2)}$$

$$\tilde{A} = 0.8 / x_1 + 0.3 / x_2 + 0.9 / x_3 + 1.0 / x_4 \qquad \textbf{(5.3)}$$

Notice that the symbols / and + do not stand for division and summation; they are merely the correspondence between an element in the universal set and its membership degree, and connector between the elements, respectively.

Analytic representation: When a universal set is infinite, which is usually the case for a set of real numbers, it is impossible to list all the elements together with their membership grades. These kinds of fuzzy sets are represented by an analytic form, which describes the shapes of the membership function. An example of analytic representation of membership functions is given below.

$$A(x) = \begin{cases} x - 4 & \text{for} \quad 4 \le x \le 5, \\ 6 - x & \text{for} \quad 5 \le x \le 6, \\ 0 & \quad \text{otherwise.} \end{cases} \qquad \textbf{(5.4)}$$

5.3.2 Complement of a Fuzzy Set

The complement of $\tilde{A}$, denoted by $\bar{\tilde{A}}$, is defined as:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x), \forall x \in X \qquad \textbf{(5.5)}$$

Let $X = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$, the possible speed at which cars can cruise over a long distance. Then the fuzzy set, comfortable car speed for long distance travel, $\tilde{A}$ can subjectively be defined by the membership function as given below.

$\tilde{A} = \{30/0.7,\ 40/0.75,\ 50/0.80,\ 60/0.80,\ 70/1.0,\ 80/0.8,\ 90/0.30\}$

Then, the fuzzy set of uncomfortable car speed for long distance travel, $\tilde{\tilde{A}}$, can be written as:

$\tilde{\tilde{A}} = \{10/1.0,\ 20/1.0,\ 30/0.30,\ 40/0.25,\ 50/0.20,\ 60/0.20,\ 80/0.20,\ 90/0.70\}$

Notice that elements of the fuzzy sets with zero membership degrees are omitted from the fuzzy set. The membership functions of $\tilde{A}$ and $\tilde{\tilde{A}}$ are illustrated in Figure 5.2.

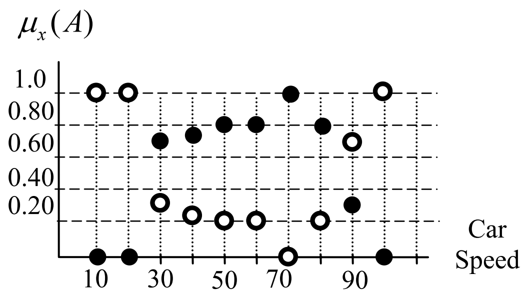

Figure 5.2: Illustration of $\tilde{A}$ (●) and $\tilde{\tilde{A}}$ (○).

5.3.3 Support of a Fuzzy Set

It is often necessary to consider such elements in a fuzzy set which have nonzero membership grades. These elements are called support of that fuzzy set. The support of a fuzzy set $\tilde{A}$, $S(\tilde{A})$, is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x) > 0$.

5.3.4 α-Cut of a Fuzzy Set

The (crisp) set of elements that belong to fuzzy set $\tilde{A}$ at least to the degree α is called the α-level set:

$$A_\alpha = \{x \in X \mid \mu_{\tilde{A}} \geq \alpha\} \quad \textbf{(5.6)}$$

$A_\alpha^{`} = \{x \in X \mid \mu_{\tilde{A}} > \alpha\}$ is called "strong α-level set" or "strong α-cut". The α-cut of a fuzzy set is a more general case of the support of a fuzzy set. When $\alpha = 0$, $A_\alpha = S(\tilde{A})$.

5.3.5 Convexity of a Fuzzy Set

The convexity of a fuzzy set is an important property from the point of view of the application aspect. A fuzzy set $\tilde{A}$ is convex if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_1)) \qquad \textbf{(5.7)}$$

where $x_1, x_2 \in X$ and $\lambda \in [0,1]$. Alternatively, a fuzzy set is convex if all α-level sets are convex. Figure 5.3 gives a convex fuzzy set and a nonconvex fuzzy set. Generally speaking, unless otherwise stated, the term fuzzy set will denote a convex fuzzy set.

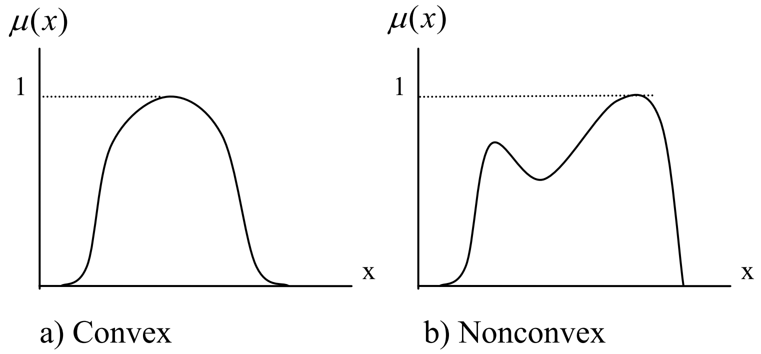

Figure 5.3: Example of convex and nonconvex fuzzy set.

5.3.6 Normality of a Fuzzy Set

A fuzzy set $\tilde{A}$ is normal if and only if there exists at least one x value such that $\mu_A(x) = 1$. This property guarantees that at least one element in a fuzzy set fully satisfies the phenomenon that the fuzzy set applies to. Unless otherwise stated, the term fuzzy set is also assumed to be a normal fuzzy set.

5.4 Fuzzy Numbers

The concept of fuzzy number arises from the fact that many quantifiable phenomena do not lend themselves to be characterized in terms of absolutely precise numbers.

Imprecise numerical quantities, such as "*close to 12*", "*about 15*", "*several*", "*near*", are represented with fuzzy numbers. A real fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line *R* with membership function f_A which possesses the following properties. [106]

- f_A is a continuous mapping from *R* to the closed interval [0, 1]
- $f_A(x) = 0$, for all $x \in (-\infty, a]$
- f_A is strictly increasing on $[a, b]$
- $f_A(x) = 1$, for all $x \in [b, c]$
- f_A is strictly decreasing on $[c, d]$
- $f_A(x) = 0$, for all $x \in [d, \infty)$;

where *a*, *b*, *c*, and *d* are real numbers. Different types of fuzzy numbers can be obtained by changing the positions of *a*, *b*, *c*, and *d* (i.e., $a = -\infty$, or $a = b$, or $b = c$, or $c = d$, or $d = \infty$), and/or defining different increasing functions for (a, b], and/or decreasing function for.[c, d) Unless specified, it is assumed that $\tilde{A}$ is convex, normal and bounded, i.e., $-\infty < a$, $d < \infty$.

The membership function μ_A of the fuzzy number *A* can be stated as

$$\mu_A(x) = \begin{cases} \mu_A^L(x), & \text{for} \quad a \le x \le b, \\ 1, & \text{for} \quad b \le x \le c, \\ \mu_A^R(x), & \text{for} \quad c \le x \le d, \\ 0, & \text{otherwise.} \end{cases} \tag{5.8}$$

where μ_A^L and μ_A^R are the left (increasing part) and right (decreasing part) membership functions of fuzzy number *A*, respectively.

A fuzzy number can be represented in discrete or continuous form. Although there are a great variety of shapes of membership functions, as exemplified in Figure 5.4, the most common are trapezoidal and triangular shapes. These types of fuzzy numbers are easy to construct and manipulate.

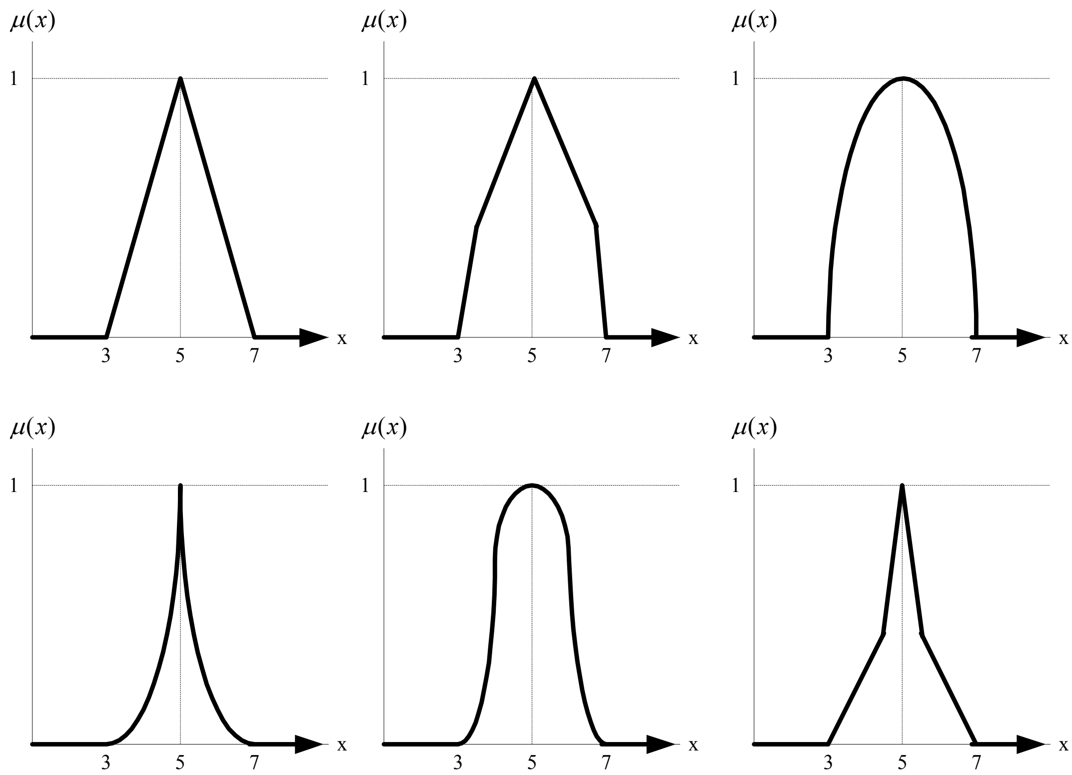

Figure 5.4: Possible fuzzy numbers to capture the concept of "around 5".

A trapezoidal fuzzy number (TraFN) illustrated in Figure 5.5 has the membership function as given in Eq. below.

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, & \text{for} \quad a\le x\le b,\\ 1, & \text{for} \quad b\le x\le c,\\ \dfrac{d-x}{d-c}, & \text{for} \quad c\le x\le d,\\ 0, & \text{otherwise.}\end{cases} \tag{5.9}$$

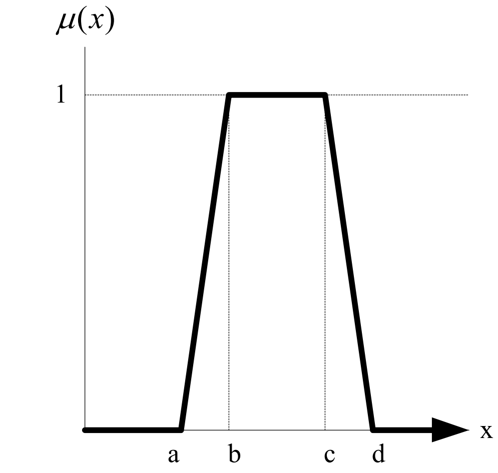

Figure 5.5: A trapezoidal fuzzy number (TraFN).

Trapezoidal fuzzy numbers are represented by four points: *a*, *b*, *c*, and *d* as illustrated in Figure 5.5. From this point forward, a TraFN will be denoted as (*a*, *b*, *c*, *d*).

A triangular fuzzy number (TriFN) as illustrated in Figure 5.6 is indeed a special case of the TraFN where *b*=*c*, and will be represented as (*a*, *b*, *d*) for the convenience to the TraFN. In this case, membership function of the TriFN becomes as follows:

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & \text{for} \quad a \le x \le b, \\ 1, & \text{for} \quad x = b, \\ \dfrac{d-x}{d-c}, & \text{for} \quad b \le x \le d, \\ 0, & \text{otherwise.} \end{cases} \tag{5.10}$$

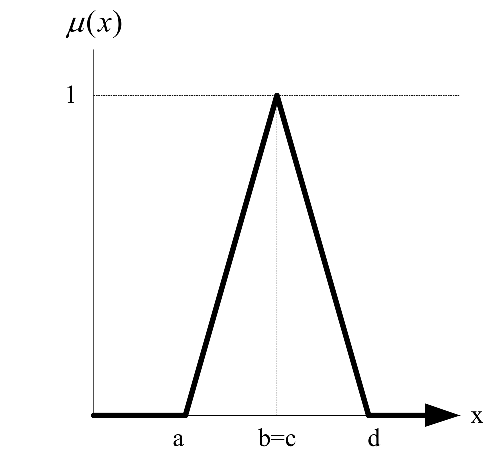

Figure 5.6: A triangular fuzzy number (TriFN).

5.5 Fuzzy Arithmetic

Basic operations for TriFNs and TraFNs used in this thesis are tabulated in Tables 5.2 and 5.3.

Table 5.2. Fuzzy operations for $M = (l, m, u)$, $N = (a, b, c)$

Image of N	$-N = (-c, -b, -a)$	**(5.11)**
Inverse of N	$N^{-1} = (\frac{1}{c}, \frac{1}{b}, \frac{1}{a})$	**(5.12)**
Addition	$M + N = (l + a, m + b, u + c)$	**(5.13)**
Subtraction	$M - N = (l - c, m - b, u - a)$	**(5.14)**
Scalar Multiplications		
$\forall k > 0, k \in R$	$kM = (kl, km, ku)$	**(5.15)**
$\forall k < 0, k \in R$	$kM = (ku, km, kl)$	**(5.16)**
Multiplications		
$M > 0, N > 0$	$MN = (la, mb, uc)$	**(5.17)**
$M < 0, N > 0$	$MN = (lc, mb, ua)$	**(5.18)**
$M < 0, N < 0$	$MN = (uc, mb, la)$	**(5.19)**
Division		
$M > 0, N > 0$	$\frac{M}{N} = (\frac{l}{c}, \frac{m}{b}, \frac{u}{a})$	**(5.20)**
$M < 0, N > 0$	$\frac{M}{N} = (\frac{u}{c}, \frac{m}{b}, \frac{l}{a})$	**(5.21)**
$M < 0, N < 0$	$\frac{M}{N} = (\frac{u}{a}, \frac{m}{b}, \frac{l}{c})$	**(5.22)**

Table 5.3: Fuzzy operations for $M = (a_1, b_1, c_1, d_1)$, $N = (a_2, b_2, c_2, d_2)$

Image of N	$-N = (-d_2, -c_2, -b_2, -a_2,)$	**(5.23)**
Inverse of N	$N^{-1} = (\frac{1}{d_2}, \frac{1}{c_2}, \frac{1}{b_2}, \frac{1}{a_2})$	**(5.24)**
Addition	$M + N = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$	**(5.25)**
Subtraction	$M - N = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$	**(5.26)**
Scalar Multiplications		
$\forall k > 0, k \in R$	$kM = (ka_1, kb_1, kc_1, kd_1)$	**(5.27)**
$\forall k < 0, k \in R$	$kM = (kd_1, kc_1, kb_1, ka_1)$	**(5.28)**
Multiplications		
$M > 0, N > 0$	$MN = (a_1a_2, b_1b_2, c_1c_2, d_1d_2)$	**(5.29)**
$M < 0, N > 0$	$MN = (a_2d_1, b_2c_1, c_2b_1, d_2a_1)$	**(5.30)**
$M < 0, N < 0$	$MN = (d_1d_2, c_1c_2, b_1b_2, a_1a_2)$	**(5.31)**
Division		
$M > 0, N > 0$	$\frac{M}{N} = (\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2})$	**(5.32)**
$M < 0, N > 0$	$\frac{M}{N} = (\frac{d_1}{d_2}, \frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2})$	**(5.33)**
$M < 0, N < 0$	$\frac{M}{N} = (\frac{d_1}{a_2}, \frac{c_1}{b_2}, \frac{b_1}{c_2}, \frac{a_1}{d_2})$	**(5.34)**

5.6 Comparison of Fuzzy Numbers

Fuzzy numbers cannot be easily compared to each other. So, in decision analysis it is very difficult to distinguish the best possible course of action among alternatives defined by means of fuzzy numbers. This is because fuzzy numbers do not provide a totally ordered set as real numbers do. Taxonomy of fuzzy ranking methods is given in Figure 5.7.

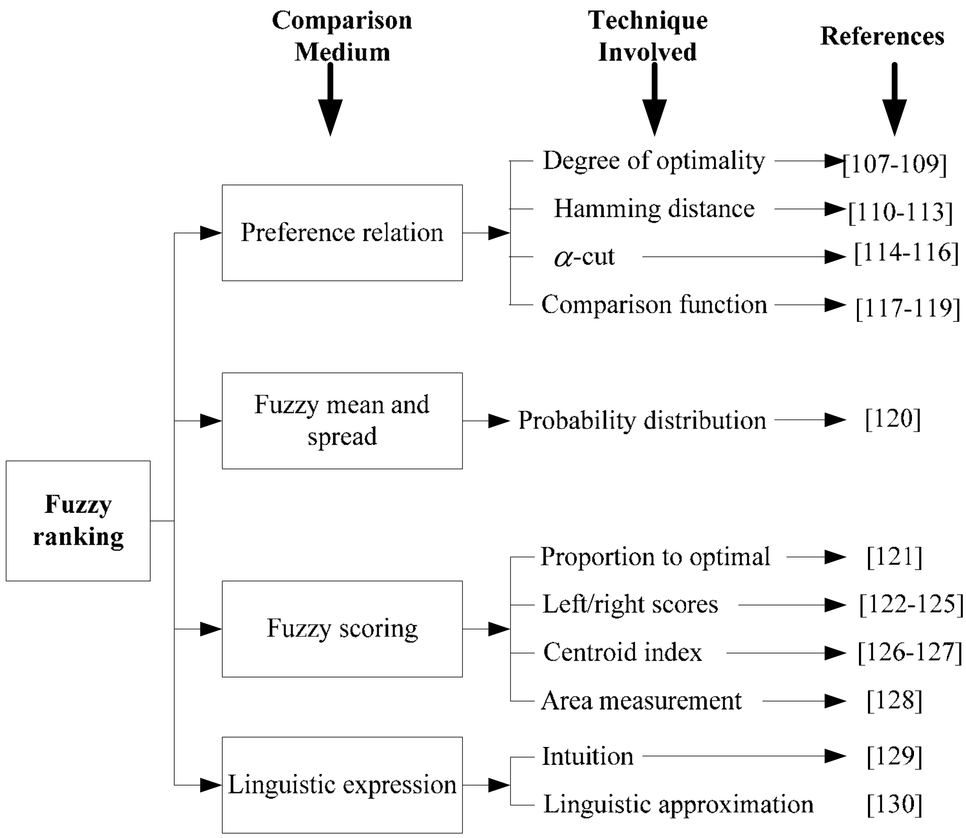

Figure 5.7: Taxonomy of fuzzy ranking methods.

A detailed description of the fuzzy ranking methods can be found from [131]. When dealing process control charts in fuzzy environment, as well as in classical way, we check fuzzy samples whether they are within the fuzzy control limits or not. In classical control charts, a sample is either within or beyond the control limits. While a sample is within the control limits, process is said to be in control with a membership degree of 1 with respect to be in control, a sample beyond the control limits is marked as an out of control situation with 0 membership degree of being in control. From this point of view, for a fuzzy sample, membership degree of being in control can possess any membership degree between 0 and 1. In order to reflect the concept of fuzziness, a fuzzy ranking with respect to the fuzzy scoring based on the area measurement for TriFN and TraFN's used in this thesis is explained below.

Let $\widetilde{LCL} = (a_1, b_1, c_1, d_1,)$ and $U\tilde{C}L = (a_2, b_2, c_2, d_2,)$ be two TraFN's corresponding to the lower and upper fuzzy control limits, respectively. A fuzzy sample, $\tilde{S} = (a_3, b_3, c_3, d_3,)$ needs to be compared to $\tilde{A}$ and $\tilde{B}$ in the following possibilities.

- $L\tilde{C}L \lesssim \tilde{S} \lesssim U\tilde{C}L$ process is in control with a membership degree of $0 \le \mu \le 1$,
- $\tilde{S} \lesssim L\tilde{C}L \lesssim U\tilde{C}L$ process is in control with a membership degree of $0 \le \mu \le 1$,
- $L\tilde{C}L \lesssim U\tilde{C}L \lesssim \tilde{S}$ process is in control with a membership degree of $0 \le \mu \le 1$,

where $\mu = 0$ and $\mu = 1$ refer to classical out of control and in control situations, respectively.

An example for each of these three conditions is illustrated through the Figures 5.8-10, respectively.

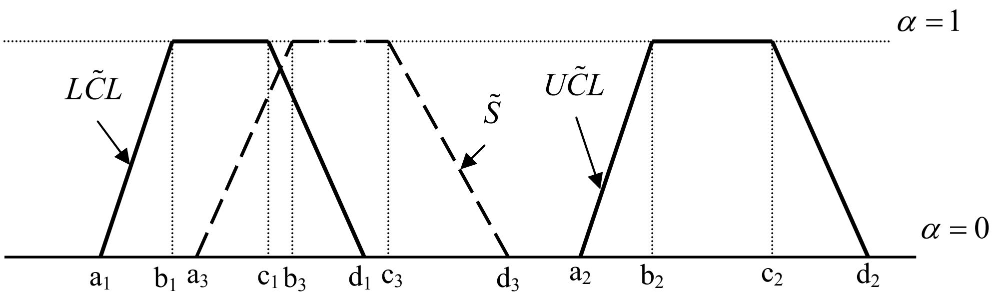

Figure 5.8: Illustration of $L\tilde{C}L \lesssim \tilde{S} \lesssim U\tilde{C}L$

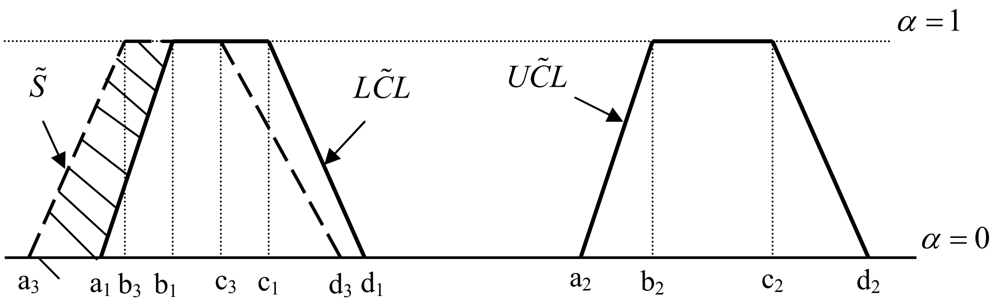

Figure 5.9. Illustration of $\tilde{S} \lesssim L\tilde{C}L \lesssim U\tilde{C}L$

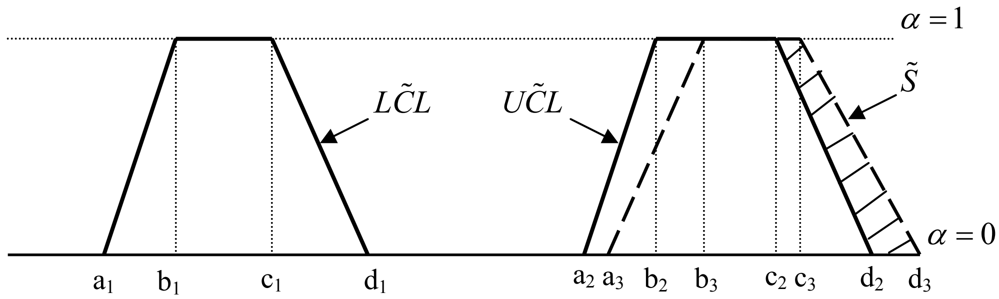

Figure 5.10: Illustration of $L\tilde{C}L \lesssim U\tilde{C}L \lesssim \tilde{S}$

$$\mu(L\tilde{C}L \le \tilde{S} \le U\tilde{C}L) = \begin{cases} 1 & if \quad a_3 \ge a_1 \wedge b_3 \ge b_1 \wedge d_3 \le d_2 \wedge c_3 \le c_2 \\ 0 & if \quad d_3 \le a_1 \vee a_3 \ge d_2 \\ 0 < \mu < 1 & otherwise \end{cases} \quad \textbf{(5.35)}$$

Membership degree of $L\tilde{C}L \lesssim \tilde{S} \lesssim U\tilde{C}L$ can be stated as

$$\mu = \beta = \frac{S_{in}}{S} = \frac{S - S_{out}}{S} \quad \textbf{(5.36)}$$

where S_{in} and S_{out} are the fuzzy sample areas falling between and beyond the fuzzy control limits, respectively; and S is the total fuzzy sample area. Therefore, $0 \le \mu = \beta \le 1$ is always satisfied and meaningful with respect to the definition of the membership degree.

5.7 Representative Values for Fuzzy Sets

The most commonly used four ways of representative (scalar) values for the fuzzy sets, which transforms fuzzy sets into crisp values are fuzzy mode, α-level fuzzy midrange, fuzzy median, and fuzzy average.

5.7.1 Fuzzy mode

The fuzzy mode of a fuzzy set F is the value of the base variable where the membership function equals to 1. This is stated as

$$f_{\text{mod}e} = \{ x \mid \mu_f(x) = 1 \}, \forall x \in F \quad \textbf{(5.37)}$$

If the membership function is unimodal, the fuzzy mode is unique. However fuzzy mode is easy to calculate, it may lead to a biased result when the membership function is extremely asymmetrical. The fuzzy mode for TriFN and TraFN are illustrated in Figure 5.11.

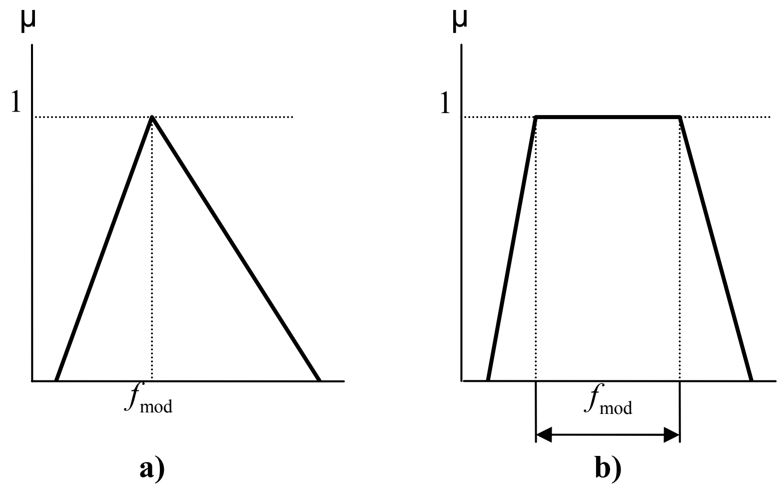

Figure 5.11: Illustration of fuzzy mode for **a)** TriFN, **b)** TraFN

5.7.2 *α*-Level Fuzzy Midrange

The α-level fuzzy midrange, $f_{mr}(\alpha)$, is defined as the midpoint of the ends of the α-level cut. An α-level cut, denoted by A_α, is a nonfuzzy set which comprises all elements whose membership is greater than or equal to α. If a_α and b_α are the end points of A_α, then

$$f_{mr}(\alpha) = \frac{1}{2}\left(a_\alpha + b_\alpha\right) \tag{5.38}$$

In fact, the fuzzy mode is a special case of α -level fuzzy midrange when $\alpha = 1$. α -level fuzzy midrange is also easy to calculate and more flexible since one can choose different levels of membership (α) of interest. The α-level fuzzy midrange for TriFN and TraFN are illustrated in Figure 5.12.

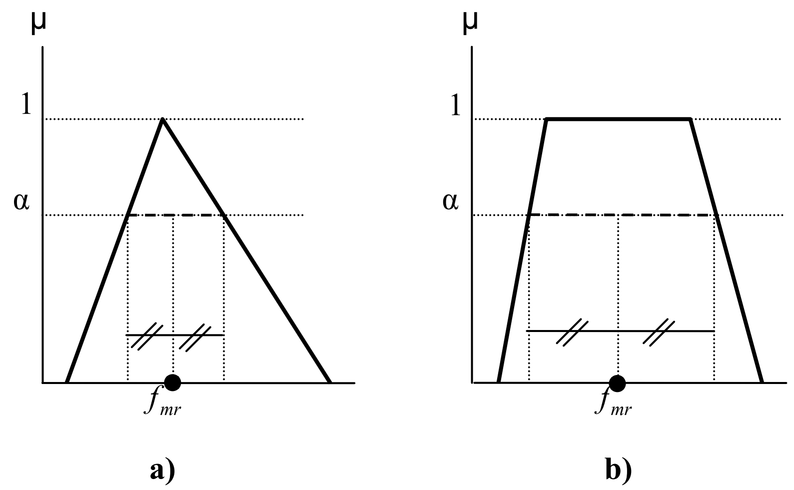

Figure 5.12: Illustration of α-level fuzzy midrange for **a)** TriFN, **b)** TraFN

5.7.3 Fuzzy Median

The fuzzy median, f_{med}, is the point which partitions the curve under the membership function of a fuzzy set into two equal regions satisfying the following equation:

$$\int_{a}^{f_{med}} \mu_f(x)dx = \int_{f_{med}}^{b} \mu_f(x)dx = \frac{1}{2}\int_{a}^{b} \mu_f(x)dx \quad \textbf{(5.39)}$$

where a and b are the end points in the base variable of the fuzzy set F such that $a < b$. If the area under the membership function is considered to be an appropriate measure of fuzziness, the fuzzy median may be thought to be suitable. The fuzzy median for TriFN and TraFN are illustrated in Figure 5.13.

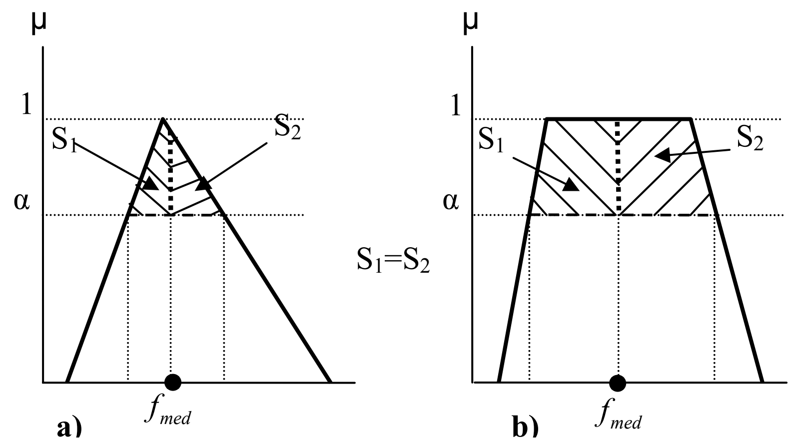

Figure 5.13: Illustration of fuzzy median for **a)** TriFN, **b)** TraFN

5.7.4 Fuzzy Average

The fuzzy average, f_{avg}, is defined as below:

$$f_{avg} = Av(x:F) = \frac{\int_{x=0}^{1} x\mu_f(x)dx}{\int_{x=0}^{1} \mu_f(x)dx} \quad \textbf{(5.40)}$$

When anyone wants to account for the shape of the membership function as well as its location, the fuzzy average will then be a better choice since it is derived from the extension principle and is basically a weighted average of the base variable.

6 FUZZY CONTROL CHARTS

Fuzzy set theory has been used to model systems that are hard to define precisely. As a methodology, fuzzy set theory incorporates imprecision and subjectivity into the model formulation and solution process. Fuzzy set theory represents an attractive tool to aid research in the development of the control charts.

6.1 Literature Survey

Bradshaw used fuzzy set theory as a basis for interpreting the representation of a graded degree of product conformance with a quality standard. When the costs resulting from substandard quality are related to the extent of nonconformance, a compatibility function exists which describes the grade of nonconformance associated with any given value of that quality characteristic. This compatibility function can then be used to construct fuzzy economic control charts on an acceptance control chart. The author stresses that fuzzy economic control chart limits are advantageous over traditional acceptance charts in that fuzzy economic control charts provide information on the severity as well as the frequency of product nonconformance [132].

Wang and Raz illustrated two approaches for constructing variable control charts based on linguistic data. When product quality can be classified using terms such as `perfect', `good', `poor', etc., membership functions can be used to quantify the linguistic quality descriptions. Representative (scalar) values for the fuzzy measures may be found using any one of four commonly used methods: (1) by using the fuzzy mode; (2) the alpha-level fuzzy midrange; (3) the fuzzy median; or (4) the fuzzy average. The representative values that result from any of these methods are then used to construct the control limits of the control chart. Wang and Raz illustrate the construction of an x-bar chart using the `probabilistic' control limits based on the estimate of the process mean, plus or minus three standard errors (in a fuzzy format), and by control limits expressed as membership functions [133].

Raz and Wang present a continuation of their 1990 work on the construction of control charts for linguistic data. Results based on simulated data suggest that, on the basis of sensitivity to process shifts, control charts for linguistic data outperform conventional percentage defective charts. The number of linguistic terms used to represent the observation was found to influence the sensitivity of the control chart [134].

Kanagawa *et al*. developed control charts for linguistic variables based on probability density functions which exist behind the linguistic data in order to control process average and process variability. This approach differs from the procedure of Wang and Raz in that the control charts are targeted at directly controlling the underlying probability distributions of the linguistic data [135].

Wang and Chen presented a fuzzy mathematical programming model and solution heuristic for the economic design of statistical control charts. The economic statistical design of an attribute *np*-chart is studied under the objective of minimizing the expected lost cost per hour of operation subject to satisfying constraints on the Type I and Type II errors. The authors argue that under the assumptions of the economic statistical model, the fuzzy set theory procedure presented improves the economic design of control charts by allowing more flexibility in the modeling of the imprecision that exist when satisfying Type I and Type II error constraints [136].

Gutierrez and Carmona noted that decisions regarding quality were inherently ambiguous and must be resolved based on multiple criteria. Hence, fuzzy multi-criteria decision theory provides a suitable framework for modeling quality decisions. The authors demonstrated the fuzzy multiple criteria framework in an automobile manufacturing example consisting of five decision alternatives (purchasing new machinery, workforce training, preventative maintenance, supplier quality and inspection) and four evaluation criteria (reduction of total cost, flexibility, leadtime and cost of quality) [137].

Khoo and Ho presented a framework for a fuzzy quality function deployment (FQFD) system in which the voice of the customer' could be expressed as both linguistic and crisp variables. The FQFD system was used to facilitate the documentation process and consists of four modules (planning, deployment, quality control and operation) and five supporting databases linked via a coordinating control mechanism. The

FQFD system was demonstrated for determining the basic design requirements of a flexible manufacturing system [138].

Glushkovsky and Florescu described how fuzzy set theory can be applied to quality improvement tools when linguistic data are available. The authors identified three general steps for formalizing linguistic quality characteristics: (1) universal set choosing; (2) definition and adequate formalization of terms; and (3) relevant linguistic description of the observation. Examples of the application of fuzzy set theory using linguistic characteristics to Pareto analysis, cause-and-effect diagrams, design of experiments, statistical control charts and process capability studies were demonstrated [139].

Yongting identified that failure to deal with quality as a fuzzy concept was a fundamental shortcoming of conformance with a quality standard. When the costs resulting from substandard quality are related to the extent of nonconformance, a compatibility function exists which describes the grade of nonconformance associated with any given value of that quality characteristic. This compatibility function can then be used to construct fuzzy economic control charts on an acceptance control chart. The author stresses that fuzzy economic control chart limits are advantageous over traditional acceptance charts in that fuzzy economic control charts provide information on the severity as well as the frequency of product nonconformance [140].

Grzegorzewski proposed a fuzzy control chart consisting of two complementary graphs for fuzzy observations represented by fuzzy numbers. The first graph incorporated a centre line which took the representative value of the fuzzy grand mean of all the samples as an estimate of the process level. Each sample was transformed to an interval symbolizing the fuzzy set of the sample mean. These intervals were plotted on the graph, and the failure of an interval to intersect with the centre line was taken as an indication of an out-of-control situation. Each interval also corresponded to a value plotted on the second graph, in which each value was interpreted as a degree of conviction that the process was out of control [141].

The aforementioned approaches for constructing fuzzy control charts established the centre line by calculating the representative value of the grand sample mean. However, these approaches all have the drawback that the fuzziness of the process level tends to be lost to a certain extent, and hence the resultant control charts lose some of the information associated with the original data. To retain the fuzziness of vague data, Grzegorzewski and Hryniewicz [142] proposed the use of a fuzzy control chart based on a statistical test to verify a fuzzy hypothesis with vague data. Their approach utilized the necessity index of strict dominance (NSD) proposed by Dubois and Prade [143] to test the fuzzy hypothesis and required the user to specify a value for the necessity index, ξ. A centre area, rather than a centre line, was determined by the $(1-\xi)$-level set of the grand sample mean, and was used to estimate the process level. The upper and lower control limits were determined using the method presented by Grzegorzewski [141]. Each sample was represented by the interval of the $(1-\xi)$-level set of the sample mean. The presence of an interval outside the control limits was taken as an indication that the process was no longer in control. In general, the aforementioned approaches for fuzzy process control employed the linguistic terms to assess product quality. However, the membership functions of these linguistic terms may not accurately reflect the expert's judgment since they are assigned arbitrarily along the scale without regard to the fuzziness of the expert's judgment. Kanagawa *et al*. [144] also commented that the membership functions of the linguistic terms used in their study and in that of Wang and Raz [133] were problematic.

Cheng presented the construction of fuzzy control charts for a process with fuzzy outcomes derived from the subjective quality ratings provided by a group of experts. The proposed fuzzy process control methodology comprises an off-line stage and an on-line stage. In the off-line stage, experts assign quality ratings to products based on a numerical scale. The individual numerical ratings are then aggregated to form collective opinions expressed in the form of fuzzy numbers. The collective knowledge applied by the experts when conducting the quality rating process is acquired through a process of fuzzy regression analysis performed by a neural network. In the on-line stage, the product dimensions are measured, and the fuzzy regression model is employed to automate the experts' judgments by mapping the measured dimensions to appropriate fuzzy quality ratings. The fuzzy quality ratings are then plotted on fuzzy control charts, whose construction and out-of-control conditions are developed using possibility theory. The developed control charts not only monitor the central tendency of the process, but also indicate its degree of fuzziness [145].

6.2 Fuzzy *p* Control Charts

In classical p charts, products are distinctly classified as "*conformed*" or "*nonconformed*" when determining fraction rejected. In fuzzy p control charts, when categorizing products, several linguistic terms can be used to denote the degree of being a nonconformed product such as "*standard*", "*second choice*", "*third choice*", "*chipped*", and so on… A membership degree of being a nonconformed product is assigned to each linguistic term. Then, sample means for each sample group, M_j, are calculated using Eq. 6.1 [88]:

$$M_j = \frac{\sum_{i=1}^{t} k_{ij} r_i}{m_j} \tag{6.1}$$

where t is the total number of linguistic terms, k_{ij} is the number of products categorized with the linguistic term i in the sample j, r_i is the membership degree of the linguistic term i, and m_j is the total number of products in sample j. Center line, CL, is the average of the means of the n sample groups and can be determined by Eq. 6.2.

$$CL = \overline{M_j} = \frac{\sum_{j=1}^{n} M_j}{n} \tag{6.2}$$

where n is the number of sample groups initially available. k_{ij} and r_i in Eq. 6.1, and so in Eq. 6.2, are the uncertain values and depend on the human subjective judgment. In another word, a sample can be belonged to the second choice category by a quality controller, while it may be included in the standard or third choice by another quality controller. In the same way, defining a membership degree for a category may depend on the quality controller preferences. Therefore, the value of M_j may lie between 0 and 1, as a result of these human judgments. It is clear that CL in Eq. 6.2 has a range between 0 and 1 too. To overcome the uncertainty in the determination of the CL, fuzzy set theory can successfully be adopted by defining CL as a triangular fuzzy number (TFN) whose fuzzy mode is CL, as shown in Figure 6.1. Then, for each sample mean, $L_j(\alpha)$ and $R_j(\alpha)$ can be calculated using Eqs. 6.3 and 6.4, respectively [88].

$$L_j(\alpha) = M_j\alpha \tag{6.3}$$

$$R_j(\alpha) = 1 - \lfloor (1 - M_j)\alpha \rfloor \tag{6.4}$$

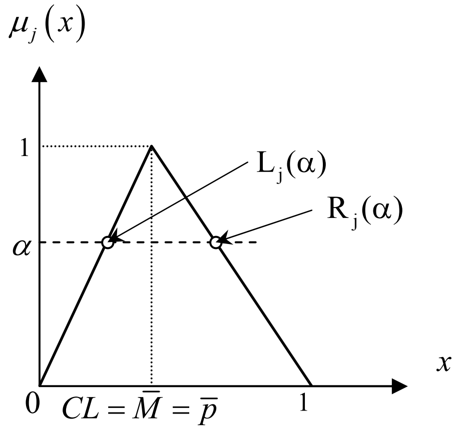

Figure 6.1: TFN representation of $\overline{M}$ and M_j of the sample j

Membership function of the $\overline{M}$, or CL , can be written as:

$$\mu_{M_j}(x) = \begin{cases} 0, & if\ x \le 0 \\ \dfrac{x}{\overline{M}}, & if\ 0 \le x \le \overline{M} \\ \dfrac{1-x}{1-\overline{M}}, & if\ \overline{M} \le x \le 1 \\ 0, & if\ x \ge 1 \end{cases} \tag{6.5}$$

Control limits for α-cut can also be represented by TFNs. Since the membership function of CL is divided into two components, then, each component will have its own CL , LCL and UCL . The membership function of the control limits depending upon the value of α is given in Eq. 6.6 and illustrated in Figure 6.2 [88].

$$
ControlLimits(\alpha) = \begin{cases}
\left\{\begin{array}{l}
CL^L = \bar{M}\alpha \\
LCL^L = \max\left\{CL^L - 3\sqrt{\dfrac{\left(CL^L\right)\left(1-CL^L\right)}{\bar{n}}}, 0\right\} \\
UCL^L = \min\left\{CL^L + 3\sqrt{\dfrac{\left(CL^L\right)\left(1-CL^L\right)}{\bar{n}}}, 1\right\}
\end{array}\right\}, & if\ 0 \le M_j \le \bar{M} \\
\left\{\begin{array}{l}
CL^R = 1 - \left[\left(1-\bar{M}\alpha\right)\alpha\right] \\
LCL^R = \max\left\{CL^R - 3\sqrt{\dfrac{\left(CL^R\right)\left(1-CL^R\right)}{\bar{n}}}, 0\right\} \\
UCL^R = \min\left\{CL^R + 3\sqrt{\dfrac{\left(CL^R\right)\left(1-CL^R\right)}{\bar{n}}}, 1\right\}
\end{array}\right\}, & if\ \bar{M} \le M_j \le 1
\end{cases} \quad \textbf{(6.6)}
$$

where $\bar{n}$ is the average sample size (*ASS*). When the *ASS* is used, the control limits do not change with the sample size. Hence, the control limits for all samples are the same.

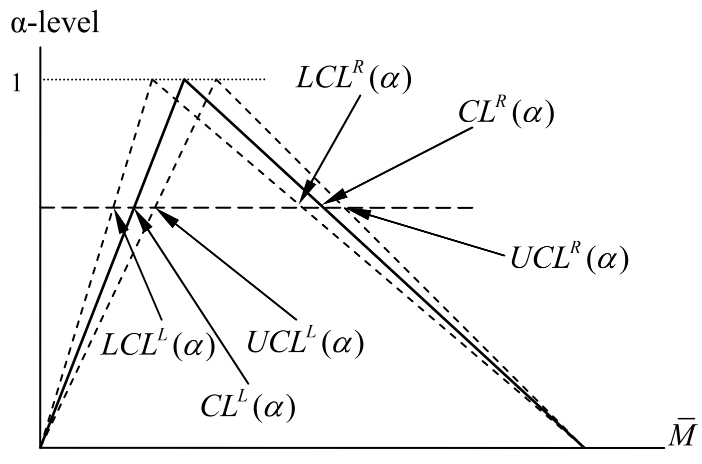

Figure 6.2: Illustration of the α-cut control limits.

For the variable sample size (*VSS*), $\bar{n}$ should be replaced by the size of the j th sample, n_j. Hence, control limits change for each sample depending upon the size of the sample. Therefore, each sample has its own control limits. The decision that whether process is in control (1) or out of control (0) for both *ASS* and *VSS* is as follows:

$$\text{Process Control} = \begin{cases} 1, & \text{if } LCL^L(\alpha) \le L_j(\alpha) \le UCL^L(\alpha) \wedge LCL^R(\alpha) \le R_j(\alpha) \le UCL^R(\alpha), \\ 0, & \text{otherwise.} \end{cases} \quad \textbf{(6.7)}$$

The value of α-cut is decided with respect to the tightness of inspection such that for a tight inspection, α values close to 1 may be used. As can be seen from Figure 6.2, while α reduces to 0 (decreasing the tightness of inspection), the range where the process is in control (difference between UCL and LCL) increases [88].

6.3 Fuzzy *c* Control Charts: A Direct Fuzzy Approach

In the crisp case, control limits for number of nonconformities are calculated by the Eqs. 6.8-10.

$$CL = \bar{c} \quad \textbf{(6.8)}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} \quad \textbf{(6.9)}$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} \quad \textbf{(6.10)}$$

where $\bar{c}$ is the mean of the nonconformities In the fuzzy case, where number of nonconformity includes human subjectivity or uncertainty, uncertain values such as "*between 10 and 14*" or "*approximately 12*" can be used to define number of nonconformities in a sample. Then number of nonconformity in each sample, or subgroup, can be represented by a trapezoidal fuzzy number (*a*, *b*, *c*, *d*) or a triangular fuzzy number (*a*, *b*, *d*) as shown in Figure 6.3**.** Note that a trapezoidal fuzzy number becomes triangular when *b*=*c*. For the ease of representation and calculation, a triangular fuzzy number is also represented as trapezoidal by (*a*, *b*, *b*, *d*) or (*a*, *c*, *c*, *d*).

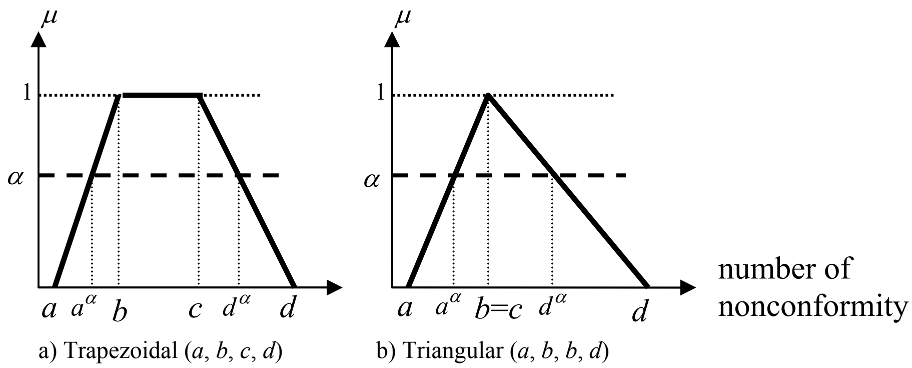

Figure 6.3: Representation of number of nonconformities by fuzzy numbers

Here, we propose a direct fuzzy approach (DFA) to deal with the vague data for the control charts. Transforming the vague data by representing them with their representative values may result in biased decisions for particular data especially when they are represented by asymmetrical fuzzy numbers. Center line, CL, given in Eq. 8, is the mean of the samples. For fuzzy case, where the numbers of nonconformities are represented by trapezoidal fuzzy numbers, fuzzy center line, $\widetilde{CL}$, can be determined using the arithmetic mean of the fuzzy numbers and written as in Eq. 6.11 (See Chen and Hwang (1992) for the fuzzy arithmetics performed in this paper).

$$\widetilde{CL} = \left(\frac{\sum_{j=1}^{n} a_j}{n}, \frac{\sum_{j=1}^{n} b_j}{n}, \frac{\sum_{j=1}^{n} c_j}{n}, \frac{\sum_{j=1}^{n} d_j}{n} \right) = \left(\bar{a}, \bar{b}, \bar{c}, \bar{d} \right) \quad \textbf{(6.11)}$$

where n is the number of fuzzy samples and $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ are the arithmetic means of the a, b, c, and d, respectively. $\widetilde{CL}$ can be rewritten as in Eq. 6.12. Then $\widetilde{LCL}$ and $\widetilde{UCL}$ are calculated using fuzzy arithmetics as given in Eqs. 6.13 and 6.14, respectively.

$$\widetilde{CL} = \left(\bar{a}, \bar{b}, \bar{c}, \bar{d} \right) = \left(CL_1, CL_2, CL_3, CL_4 \right) \quad \textbf{(6.12)}$$

$$\begin{aligned} \widetilde{LCL} &= \widetilde{CL} - 3\sqrt{\widetilde{CL}} \\ &= \left(CL_1 - 3\sqrt{CL_4}, CL_2 - 3\sqrt{CL_3}, CL_3 - 3\sqrt{CL_2}, CL_4 - 3\sqrt{CL_1} \right) \\ &= \left(LCL_1, LCL_2, LCL_3, LCL_4 \right) \end{aligned} \quad \textbf{(6.13)}$$

$$\begin{aligned}\widetilde{UCL} &= \widetilde{CL} + 3\sqrt{\widetilde{CL}} \\ &= \left(CL_1 + 3\sqrt{CL_1}, CL_2 + 3\sqrt{CL_2}, CL_3 + 3\sqrt{CL_3}, CL_4 + 3\sqrt{CL_4}\right) \\ &= \left(UCL_1, UCL_2, UCL_3, UCL_4\right)\end{aligned} \quad \textbf{(6.14)}$$

An α-cut is a nonfuzzy set which comprises all elements whose membership is greater than or equal to α. Applying α-cuts of fuzzy sets (Figure 6.3), values of a^{α} and d^{α} for samples and CL_1^{α} and CL_4^{α} (start and end points of the α-cut of CL) for center line are determined by Eqs. 6.15 and 6.16, respectively.

$$\begin{aligned} a^{\alpha} &= a + \alpha(b-a) \\ CL_1^{\alpha} &= CL_1 + \alpha\left(CL_2 - CL_1\right)\end{aligned} \quad \textbf{(6.15)}$$

$$\begin{aligned} d^{\alpha} &= d - \alpha(d-c) \\ CL_4^{\alpha} &= CL_4 - \alpha\left(CL_4 - CL_3\right)\end{aligned} \quad \textbf{(6.16)}$$

Using α-cut representations, fuzzy control limits can be rewritten as given in Eqs. 6.17-19.

$$\widetilde{CL}^{\alpha} = (CL_1^{\alpha}, CL_2, CL_3, CL_4^{\alpha}) \quad \textbf{(6.17)}$$

$$\widetilde{LCL}^{\alpha} = (LCL_1^{\alpha}, LCL_2, LCL_3, LCL_4^{\alpha}) \quad \textbf{(6.18)}$$

$$\widetilde{UCL}^{\alpha} = (UCL_1^{\alpha}, UCL_2, UCL_3, UCL_4^{\alpha}) \quad \textbf{(6.19)}$$

The results of these equations can be illustrated as in Figure 6.4. To retain the standard format of control charts and to facilitate the plotting of observations on the chart, it is necessary to convert the fuzzy sets associated with linguistic values into scalars referred to as *representative values*. This conversion may be performed in a number of ways as long as the result is intuitively representative of the range of the base variable included in the fuzzy set. Four ways, which are similar in principle to the measures of central tendency used in descriptive statistics, are *fuzzy mode*, *α-level fuzzy midrange*, *fuzzy median*, and *fuzzy average*. It should be pointed out that there is no theoretical basis supporting any one specifically and the selection between them should be mainly based on the ease of computation or preference of the user [133]. Conversion of fuzzy sets into crisp values results in loss of information in linguistic data. To retain the information of the linguistic data, we prefer to keep fuzzy sets as themselves and to compare fuzzy samples with the fuzzy control limits. For this reason, a direct fuzzy approach (DFA) based on the area measurement is proposed for the fuzzy control charts [54].

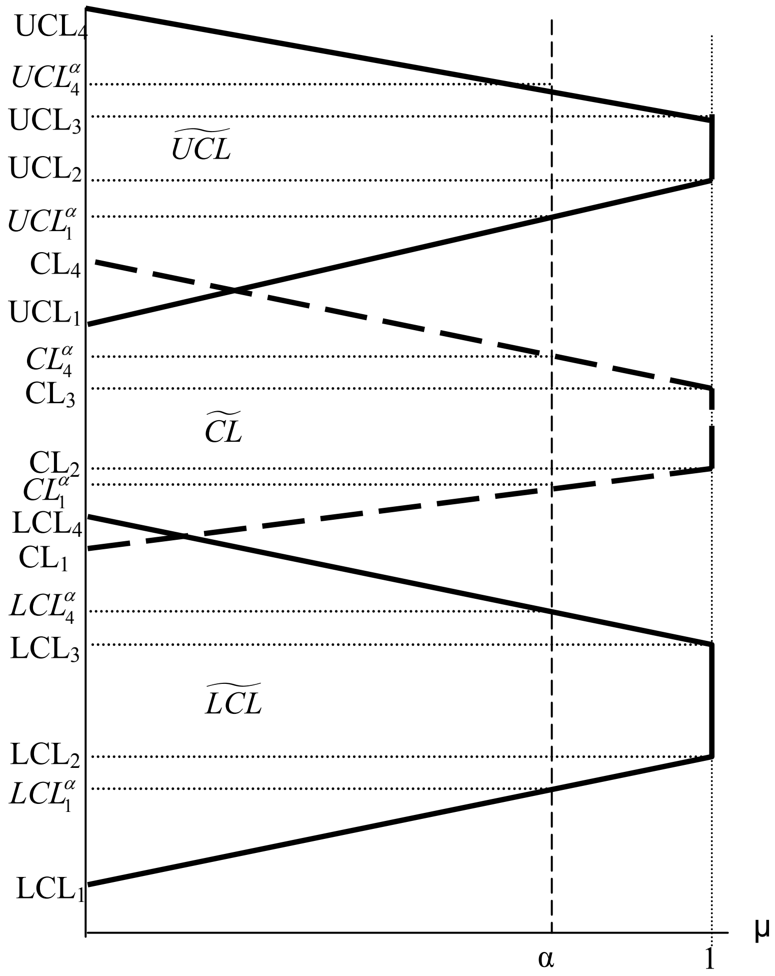

Figure 6.4: Representation of fuzzy control limits.

Decision about whether the process is in control can be made according to the percentage area of the sample which remains inside the $\widetilde{UCL}$ and/or $\widetilde{LCL}$ defined as fuzzy sets. When the fuzzy sample is completely involved by the fuzzy control limits, the process is said to be "*in-control*". If a fuzzy sample is totally excluded by the fuzzy control limits, the process is said to be "out of control". Otherwise, a sample is partially included by the fuzzy control limits. In this case, if the percentage area (β_j) which remains inside the fuzzy control limits is equal or greater than a predefined acceptable percentage (β), then the process can be accepted as "*rather in control*"; otherwise it can be stated as "*rather out of control*". Possible decisions resulting from DFA are illustrated in Figure 6.5. The parameters for determination of the sample area outside the control limits for α-level fuzzy cut are LCL_1, LCL_2, UCL_3, UCL_4, a, b, c, d, and α. The shape of the control limits and fuzzy sample are formed by the lines of $\overline{LCL_1\,LCL_2}$, $\overline{UCL_3\,UCL_4}$, $\overline{a\,b}$, and $\overline{c\,d}$. A flowchart to calculate area of the fuzzy sample outside the control limits is given in Figure 6.6. Sample area above the upper control limits, A_{out}^{U}, and sample area falling below the lower control limits, A_{out}^{L}, are calculated. Equations to compute A_{out}^{U} and A_{out}^{L} are given in Appendix A. Then, total sample area outside the fuzzy control limits, A_{out}, is the sum of the areas below fuzzy lower control limit and above fuzzy upper control limit. Percentage sample area within the control limits is calculated as given in Eq. 6.20.

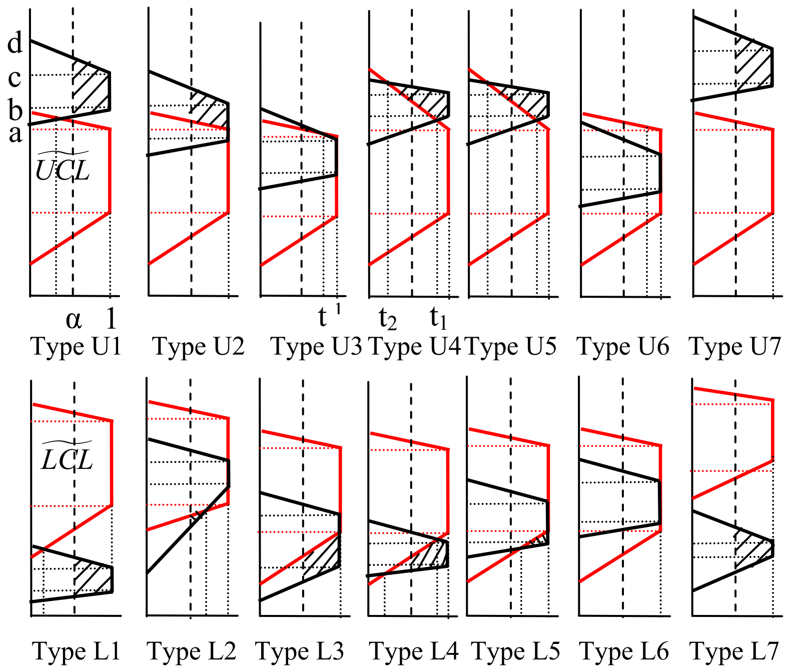

Figure 6.5: Illustration of all possible sample areas outside the fuzzy control limits at α-level cut.

$$\beta_j^{\alpha} = \frac{S_j^{\alpha} - A_{out,j}^{\alpha}}{S_j^{\alpha}} \tag{6.20}$$

where S_j^{α} is the sample area at α-level cut.

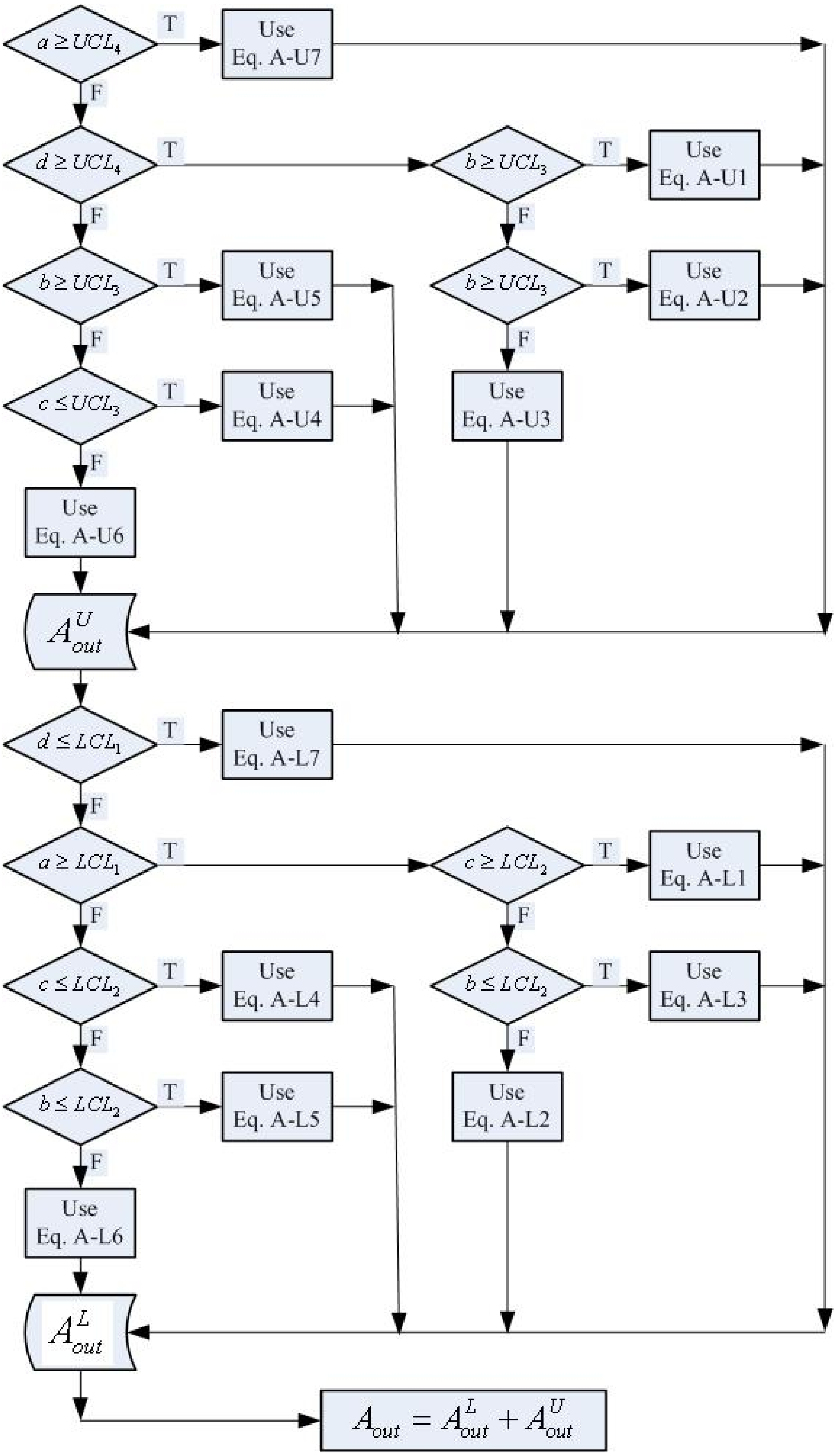

Figure 6.6: Flowchart to compute the area outside the fuzzy control limits

DFA provides the possibility of obtaining linguistic decisions like “rather in control” or “rather out of control”. Further intermediate levels of process control decisions are also possible by defining β in stages. For instance, it may be defined as given below which is more distinguished.

$$\text{Process Control}=\begin{cases}\text{in control,} & \text{for } 0.85 \le \beta_j \le 1, \\ \text{rather in control,} & \text{for } 0.60 \le \beta_j < 0.85, \\ \text{rather out of control,} & \text{for } 0.10 \le \beta_j < 0.60, \\ \text{out of control,} & \text{for } 0 \le \beta_j < 0.10. \end{cases} \quad \mathbf{(6.21)}$$

Intermediate levels of process control decisions are subjectively defined by the quality expert. In binary classification (crisp case), the quality expert may only know if the process is in control or out of control. These predefined levels refer the strengthens of the out of control. It can be used as a tracking and may give valuable information before the process is out of control. However intermediate levels are subjectively defined, it should refer to the depth of information the quality expert needs to take some preventive actions [54].

7 FUZZY UNNATURAL PATTERN ANALYSIS

Analysis of fuzzy unnatural patterns for fuzzy control charts is necessary to develop. In this section a model for the fuzzy control charts developed in the previous section is proposed.

The formula for calculating the probability of a fuzzy event A is a generalization of the probability theory: In the case which a sample space X is a continuum or discrete, the probability of a fuzzy event P(A) is given by Yen and Langari as:

$$P(A) = \begin{cases} \int \mu_A(x) P_X(x)\,dx, & \text{if } X \text{ is continuous,} \\ \sum_i \mu_A(x_i) P_X(x_i), & \text{if } X \text{ is discrete.} \end{cases} \quad \textbf{(7.1)}$$

where P_X denotes a classical probability distribution function of X for continuous sample space and probability function for discrete sample space, and μ_A is a membership function of the event A [146].

The membership degree of a fuzzy sample to belong to a region is directly related to its percentage area falling in that region, and therefore, it is continuous. For example, a fuzzy sample may be in zone B with a membership degree of 0.4 and in zone C with a membership degree of 0.6. While counting fuzzy samples in zone B, that sample is counted as 0.4.

Run rules are based on the premise that a specific run of data has a low probability of occurrence in a completely random stream of data. If a run occurs, then this must mean that something has changed in the process to produce a nonrandom or unnatural pattern. Based on the expected percentages in each zone, sensitive run tests can be developed for analyzing the patterns of variation in the various zones.

For fuzzy control charts, based on the Western Electric rules [45], the following fuzzy unnatural pattern rules can be defined. Probabilities of these fuzzy events are calculated using normal approach to binomial distribution. The probability of each fuzzy rule (event) below depends on the definition of the membership function which is subjectively defined so that the probability of each of the fuzzy rules is as close as possible to the corresponding classical rule for unnatural patterns. The idea behind this approach may justify the following rules [54].

Rule 1: Any fuzzy data falling outside the three-sigma control limits with a ratio of more than predefined percentage (β) of sample area at desired α-level. Membership function for this rule can subjectively be defined as below:

$$\mu_1(x) = \begin{cases} 0, & \text{for } 0.85 \le x \le 1, \\ (x-0.60)/0.25, & \text{for } 0.60 \le x \le 0.85, \\ (x-0.10)/0.50, & \text{for } 0.10 \le x \le 0.60, \\ 1, & \text{for } 0 \le x \le 0.10, \end{cases} \tag{7.2}$$

Rule 2: A total membership degree *around* 2 from 3 consecutive points in zone A or beyond.

Probability of a sample being in zone A (0.0214) or beyond (0.00135) is 0.02275. Let membership function for this rule be defined as follows:

$$\mu_2(x) = \begin{cases} 0, & \text{for } 0 \le x \le 0.59, \\ (x-0.59)/1.41, & \text{for } 0.59 \le x \le 2, \\ 1, & \text{for } 2 \le x \le 3. \end{cases} \tag{7.3}$$

Using the membership function above, fuzzy probability given in Eq. 7.1 can be determined by Eq. 7.4.

$$\begin{aligned} \int_0^3 \mu_2(x)P_2(x)dx &= \int_0^{x_1} \mu_2(x)P_2(x)dx + \int_{x_1}^{x_2} \mu_2(x)P_2(x)dx + \int_{x_2}^{3} \mu_2(x)P_2(x)dx \\ &= \int_{x_1}^{x_2} \mu_2(x)P_2(x)dx + \int_{x_2}^{3} \mu_2(x)P_2(x)dx \end{aligned} \tag{7.4}$$

where,

$$P(X \ge x) = P\left(z \ge \frac{x-np}{\sqrt{npq}} \right) \tag{7.5}$$

To integrate the equation above, membership function is divided into sections each with a 0.05 width and $\mu_2(x)P_x(x)$ values for each section are added. For $x_1 = 0.59$ and $x_2 = 2$, the probability of the fuzzy event, rule 2, is determined as 0.0015, which corresponds to the crisp case of this rule.

In the following rules, the membership functions are set in the same way.

<u>Rule 3</u>: A total membership degree *around* 4 from 5 consecutive points in zone C or beyond:

$$\mu_3(x) = \begin{cases} 0, & \text{for } 0 \le x \le 2.42, \\ (x-2.42)/1.58, & \text{for } 2.42 \le x \le 4, \\ 1, & \text{for } 4 \le x \le 5. \end{cases} \qquad \textbf{(7.6)}$$

Fuzzy probability for this rule is calculated as 0.0027.

<u>Rule 4</u>: A total membership degree *around* 8 from 8 consecutive points on the same side of the centerline with the membership function below:

$$\mu_4(x) = \begin{cases} 0, & \text{for } 0 \le x \le 2.54, \\ (x-2.54)/5.46, & \text{for } 2.54 \le x \le 8. \end{cases} \qquad \textbf{(7.7)}$$

The fuzzy probability for the rule above is then determined as 0.0039

Based on Grant and Leavenworth's rules (1988), the following fuzzy unnatural pattern rules can be defined.

<u>Rule 1</u>: A total membership degree *around* 7 from 7 consecutive points on the same side of the center line. Fuzzy probability of this rule is 0.0079 when membership function is defined as below:

$$\mu_1(x) = \begin{cases} 0, & \text{for } 0 \le x \le 2.48, \\ (x-2.48)/4.52, & \text{for } 2.48 \le x \le 7. \end{cases} \qquad \textbf{(7.8)}$$

<u>Rule 2</u>: At least a total membership degree around 10 from 11 consecutive points on the same side of the center line. Fuzzy probability of this rule is 0.0058 when membership function is defined as below:

$$\mu_2(x) = \begin{cases} 0, & \text{for } 0 \le x \le 9.33, \\ (x - 9.33)/0.77, & \text{for } 9.33 \le x \le 10, \\ 1, & \text{for } 10 \le x \le 11. \end{cases} \quad \textbf{(7.9)}$$

Rule 3: At least a total membership degree around 12 from 14 consecutive points on the same side of the center line. If membership function is set as given below, then fuzzy probability of the rule is equal to 0.0065.

$$\mu_3(x) = \begin{cases} 0, & \text{for } 0 \le x \le 11.33, \\ (x - 11.33)/0.67, & \text{for } 11.33 \le x \le 12, \\ 1, & \text{for } 12 \le x \le 14. \end{cases} \quad \textbf{(7.10)}$$

Rule 4: At least a total membership degree around 14 from 17 consecutive points on the same side of the center line. Probability of this fuzzy event with the membership function below is 0.0062.

$$\mu_4(x) = \begin{cases} 0, & \text{for } 0 \le x \le 13.34, \\ (x - 13.34)/0.66, & \text{for } 13.34 \le x \le 14, \\ 1, & \text{for } 14 \le x \le 17. \end{cases} \quad \textbf{(7.11)}$$

Fuzzy unnatural pattern rules based on Nelson's Rules (1985) can be defined in the same way. Some of Nelson's rules (Rules 3 and 4) are different from the Western Electric Rules and Grant and Leavenworth's rules. In order to apply these rules to fuzzy control charts, fuzzy samples can be defuzzified using α-level fuzzy midranges of the samples. Remember that the α-level fuzzy midrange, f_{mr}^{α}, is defined as the midpoint of the ends of the α-cut. If a^{α} and d^{α} are the end points of α-cut, then,

$$f_{mr}^{\alpha} = \frac{1}{2}\left(a^{\alpha} + d^{\alpha}\right) \quad \textbf{(7.12)}$$

Then Nelson's 3rd and 4th rules are fuzzified as follows:

Rule 3: 6 points in a row steadily increasing or decreasing with respect to the desired α-level fuzzy midranges.

Rule 4: 14 points in a row altering up and down with respect to the desired α-level fuzzy midranges.

8 APPLICATIONS

8.1 α-Level Fuzzy Control Charts for Fraction Rejected

In order to compare our approach, a numerical example of Tunisie Porcelaine problem stated by Wang and Raz (1990) and Taleb and Limam (2002) will be handled. In the example presented, Taleb and Limam (2002) classified porcelain products into four categories with respect to the quality. When a product represents no default, or an invisible minor default, it is classified as a standard product (*S*). If it presents a visible minor default that does not affect the use of the product, then it is classified as second choice (*SC*). When there is a visible major default that does not affect the product use, it is called as third choice (*TC*). Finally, when the use is affected, the item is considered as chipped (*C*). Data for 30 samples of different sizes taken every half an hour is shown in Table 8.1.

Wang and Raz's Approaches

a) Probablistic Approach: We will use the fuzzy mode as the representative value of the fuzzy subset. For each sample j, sample mean M_j and the standard deviation SD_j, are determined. The results of these values, their means, and the corresponding control limits, are shown in Table 8.2. The control limits change when the sample size changes. Only on two occasions is the process deemed to be out of control: samples 8 and 29 as shown in Figure 8.1.

Table 8.1: Data of the Porcelain Process

Sample *j*	Standard	Second Choice	Third Choice	Chipped	Size
1	144	46	12	5	207
2	142	50	9	5	206
3	142	35	16	6	199
4	130	70	19	10	229
5	126	60	15	10	211
6	112	47	9	8	176
7	151	28	22	9	210
8	127	43	45	30	245
9	102	79	20	3	204
10	137	64	24	5	230
11	147	59	16	6	228
12	146	30	6	6	188
13	135	51	16	8	210
14	186	82	23	7	298
15	183	53	11	9	256
16	137	65	26	4	232
17	140	70	10	3	223
18	135	48	15	9	207
19	122	52	23	10	207
20	109	42	28	9	188
21	140	31	9	4	184
22	130	22	3	8	163
23	126	29	11	8	174
24	90	23	16	2	131
25	80	29	19	8	136
26	138	55	12	12	217
27	121	35	18	10	184
28	140	35	15	6	196
29	110	15	9	1	135
30	112	37	28	11	188

Table 8.2: Determined values of M_j, SD_j, UCL_j, LCL_j for 30 subgroups.

j	M_j	SD_j	UCL_j	UCL_j	***j***	M_j	SD_j	UCL_j	LCL_j
1	0.109	0.20	0.184	0.088	16	0.143	0.21	0.170	0.091
2	0.107	0.20	0.164	0.088	17	0.114	0.18	0.174	0.090
3	0.114	0.22	0.183	0.085	18	0.138	0.24	0.177	0.088
4	0.162	0.24	0.178	0.091	19	0.167	0.25	0.178	0.088
5	0.154	0.24	0.182	0.089	20	0.178	0.26	0.182	0.086
6	0.138	0.24	0.181	0.084	21	0.088	0.19	0.182	0.085
7	0.129	0.25	0.179	0.089	22	0.092	0.23	0.188	0.082
8	0.258	0.34	0.179	0.092	23	0.119	0.24	0.186	0.084
9	0.161	0.19	0.180	0.088	24	0.120	0.21	0.198	0.076
10	0.143	0.21	0.174	0.091	25	0.182	0.27	0.195	0.077
11	0.126	0.21	0.178	0.090	26	0.146	0.25	0.190	0.089
12	0.088	0.21	0.179	0.086	27	0.151	0.26	0.193	0.085
13	0.137	0.23	0.177	0.089	28	0.114	0.22	0.180	0.087
14	0.131	0.21	0.174	0.096	29	0.069	0.16	0.193	0.077
15	0.108	0.22	0.175	0.093	30	0.182	0.27	0.183	0.086
					Average	**0.136**	**0.229**		

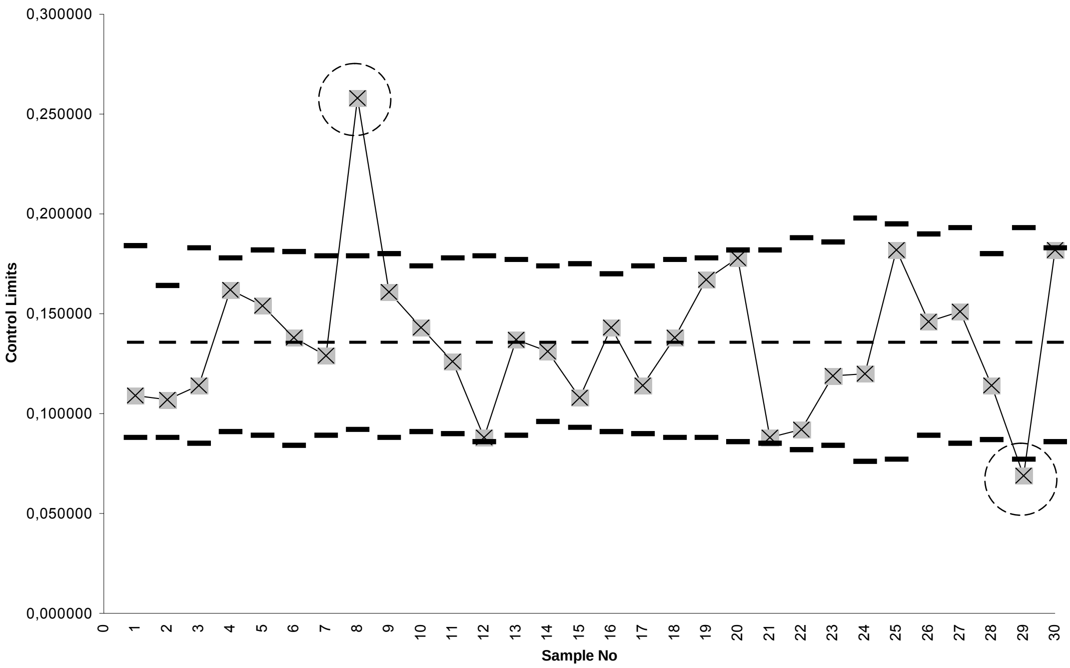

Figure 8.1: Fuzzy probabilistic control chart with fuzzy mode.

b) Membership Approach: For each sample, the membership function of the fuzzy subset corresponding to the sample observations is determined. Membership function for the porcelain process is as follows:

$$\mu_S(x) = \begin{cases} 0, & \text{for } x \le 0, \\ -x+1, & \text{for } 0 \le x \le 1, \\ 0, & \text{for } x \ge 1. \end{cases} \qquad \mu_{SC}(x) = \begin{cases} 0, & \text{for } x \le 0, \\ 4x, & \text{for } 0 \le x \le \frac{1}{4}, \\ -\frac{4}{3}x + \frac{4}{3}, & \text{for } \frac{1}{4} \le x \le 1, \\ 0, & \text{for } x \ge 1. \end{cases}$$

$$\mu_{TC}(x) = \begin{cases} 0, & \text{for } x \le 0, \\ 2x, & \text{for } 0 \le x \le \frac{1}{2}, \\ 2-2x, & \text{for } \frac{1}{2} \le x \le 1, \\ 0, & \text{for } x \ge 1. \end{cases} \qquad \mu_C(x) = \begin{cases} 0, & \text{for } x \le 0, \\ x, & \text{for } 0 \le x \le 1, \\ 0, & \text{for } x \ge 1. \end{cases} \tag{8.1}$$

By the use of the fuzzy mode transformation, the representative values for fuzzy subsets shown in Table 8.3 are determined.

Table 8.3: Representative values of linguistic terms

Linguistic Term	Representative Value
S	0
SC	0.25
TC	0.5
C	1

The membership functions for the porcelain data are also illustrated in Figure 8.2.

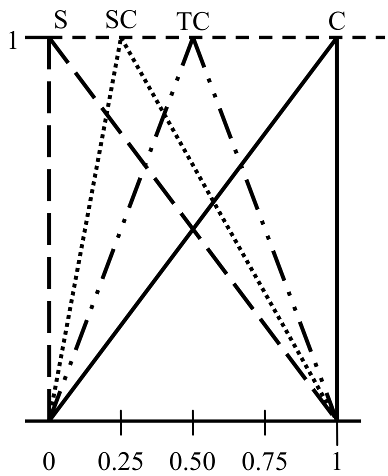

Figure 8.2: Membership functions for the porcelain data.

By applying membership approach to the porcelain data, fuzzy membership control chart is obtained as in Figure 8.3. As can be seen from Figure 8.3, only samples 8 and 29 are an out of control state. Note that fuzzy control limits, here, are calculated as follows.

$$LCL = \max\{0, [CL - k\sigma]\}$$
$$UCL = \min\{1, [CL + k\sigma]\} \qquad \textbf{(8.2)}$$

where

$$\sigma = \frac{1}{m}\sum_{j=1}^{m} SD_j \tag{8.3}$$

and

$$SD_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{t} k_{ij}(r_j - M_j)^2} \tag{8.4}$$

The value of k is calculated by the use of *Monte Carlo* simulation so that a pre-specified type I error probability yields. In this example, the value of k, used here, is approximately 0.2795, *CL* is 0.136, and σ is 0.229. Then UCL and LCL are calculated as 0.200 and 0.072, respectively.

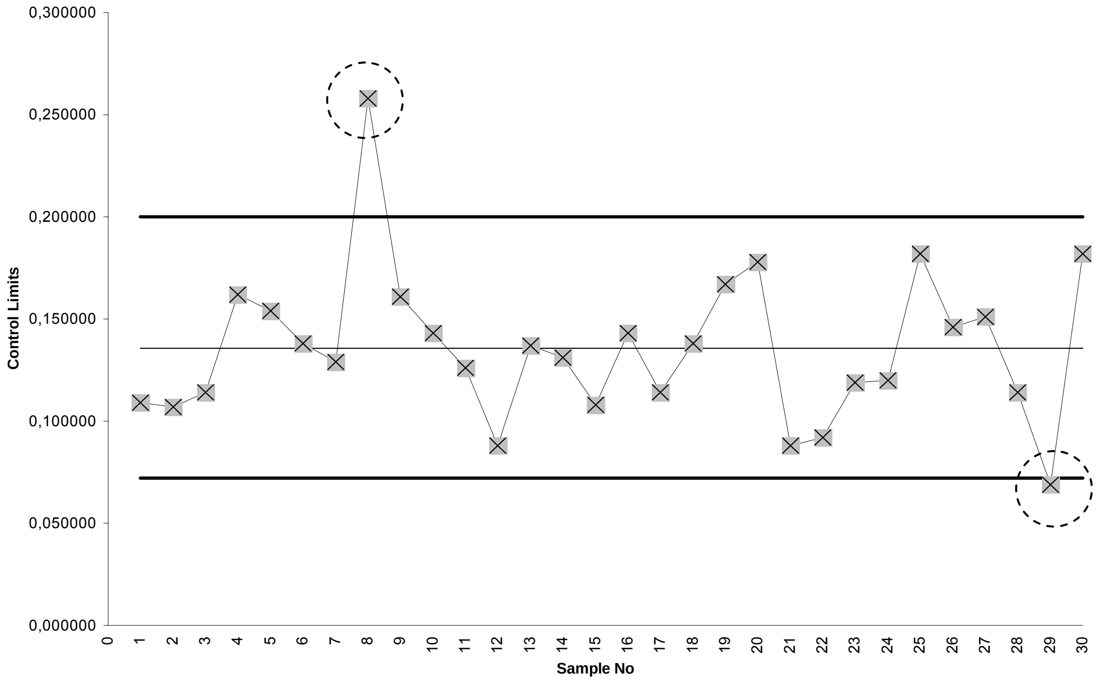

Figure 8.3: Fuzzy membership control chart with fuzzy mode transformation.

Assume that quality control expert decides for reduced inspection, say α=0.30. If we apply the ASS approach, then center lines and control limits for α=0.30 can be determined using Eq. (6.6), as:

$CL^L(\alpha = 0.30) = 0.040800$ $\quad$ $CL^R(\alpha = 0.30) = 0.740800$

$LCL^L(\alpha = 0.30) = 0.0$ $\quad$ $LCL^R(\alpha = 0.30) = 0.648321$

$UCL^L(\alpha = 0.30) = 0.082550$ $\quad$ $UCL^R(\alpha = 0.30) = 0.833279$

As can be seen from Figure 8.4, corresponding control chart for α=0.30, all the samples are in control.

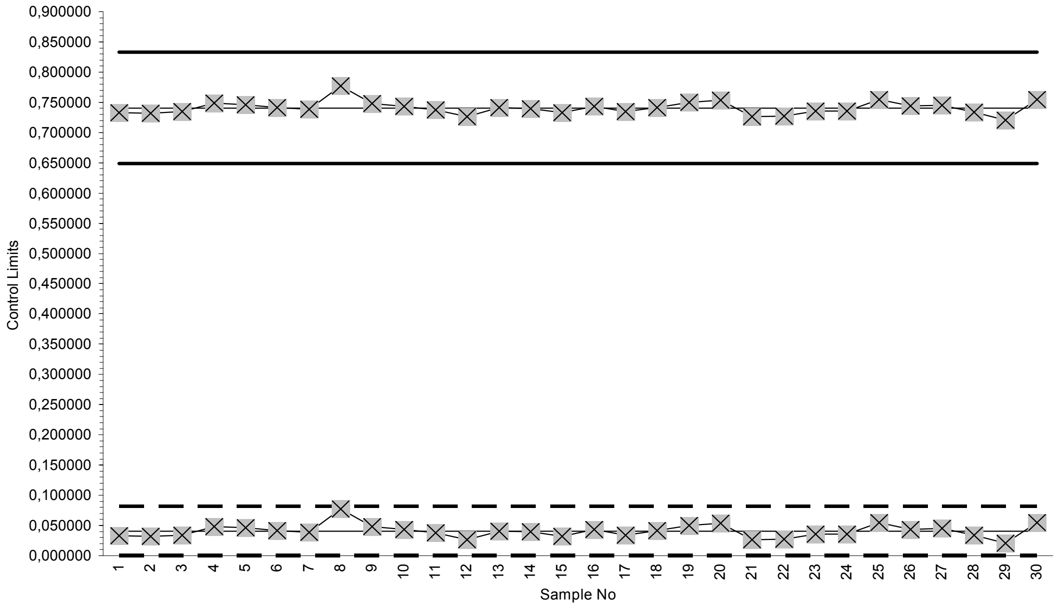

Figure 8.4: α-cut fuzzy control chart for α=0.30 (ASS approach)

For a tighter inspection with α=0.50, control chart is obtained as shown in Figure 8.5. Note that sample 8 begins to be out of control while α is chosen as 0.39 or greater.

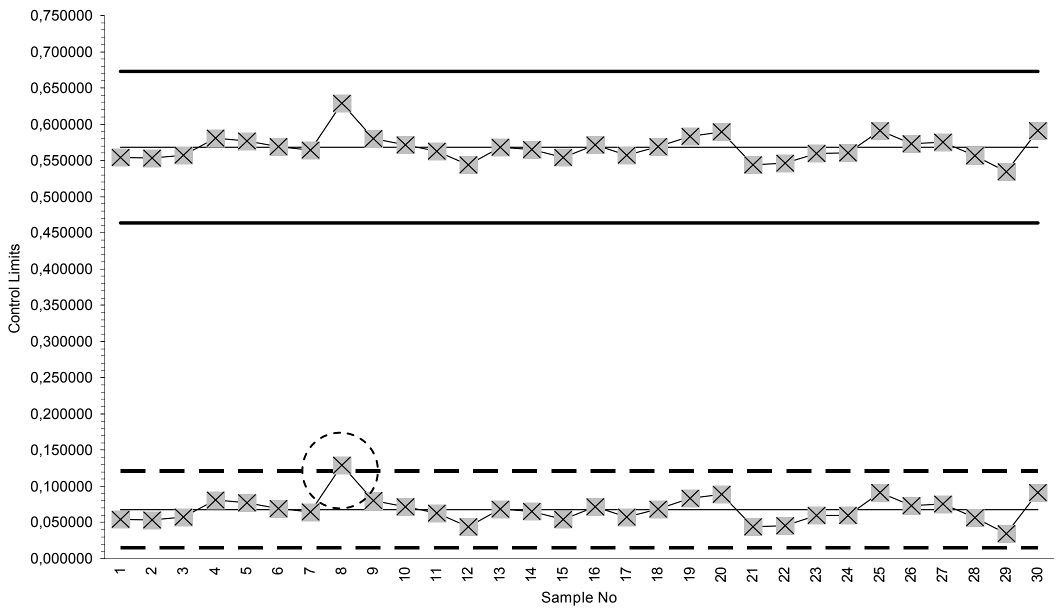

Figure 8.5: α-cut fuzzy control chart for α=0.50 (ASS approach)

When α=1.0, the crisp control limits are obtained as in Figure 8.6.

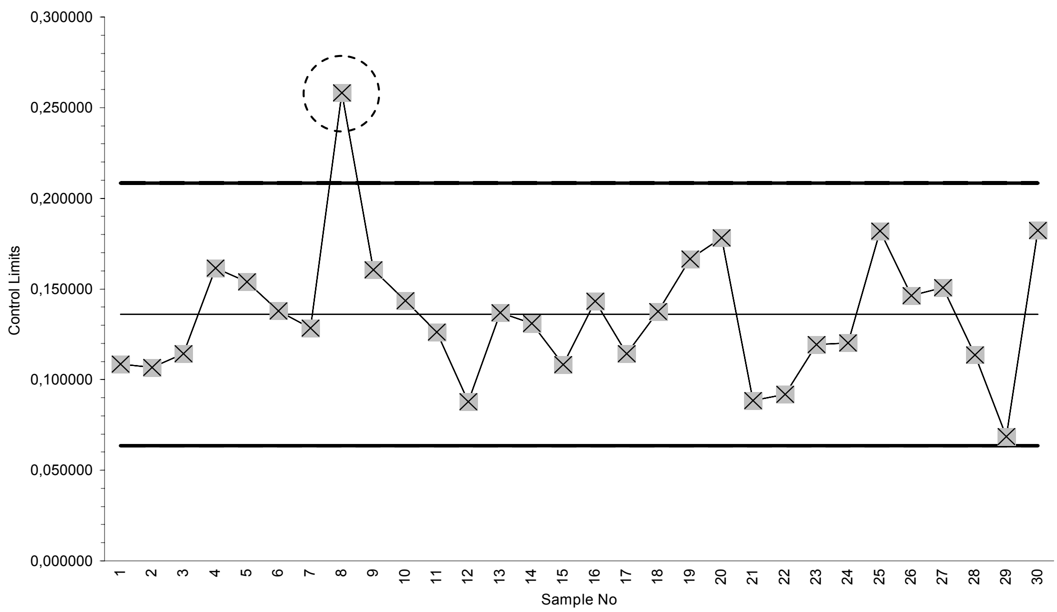

Figure 8.6: α-Cut fuzzy control chart for α=1.0 (Crisp Case, ASS approach)

If we use the VSS approach for the same example, control charts for α=0.30, α=0.50, and α=1.0 are obtained as in Figures 8.7, 8.8 and 8.9. While increasing α-cut, namely tightening the inspection, sample 8 starts to be out of control while $\alpha \geq 0.33$.

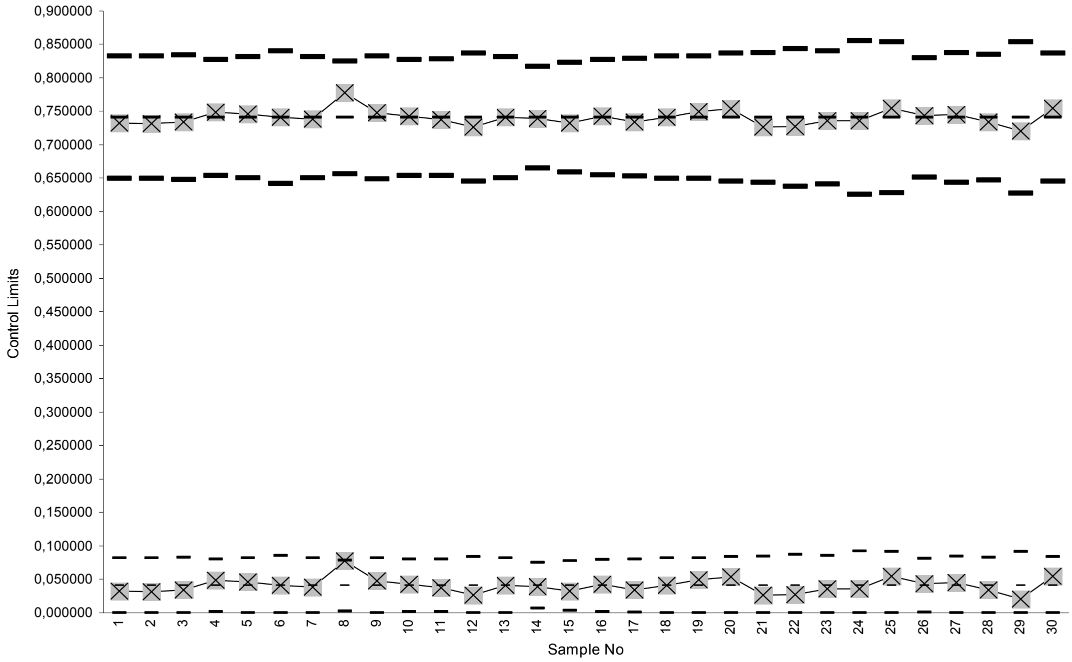

Figure 8.7: α-cut fuzzy control chart for α=0.30 (VSS approach)

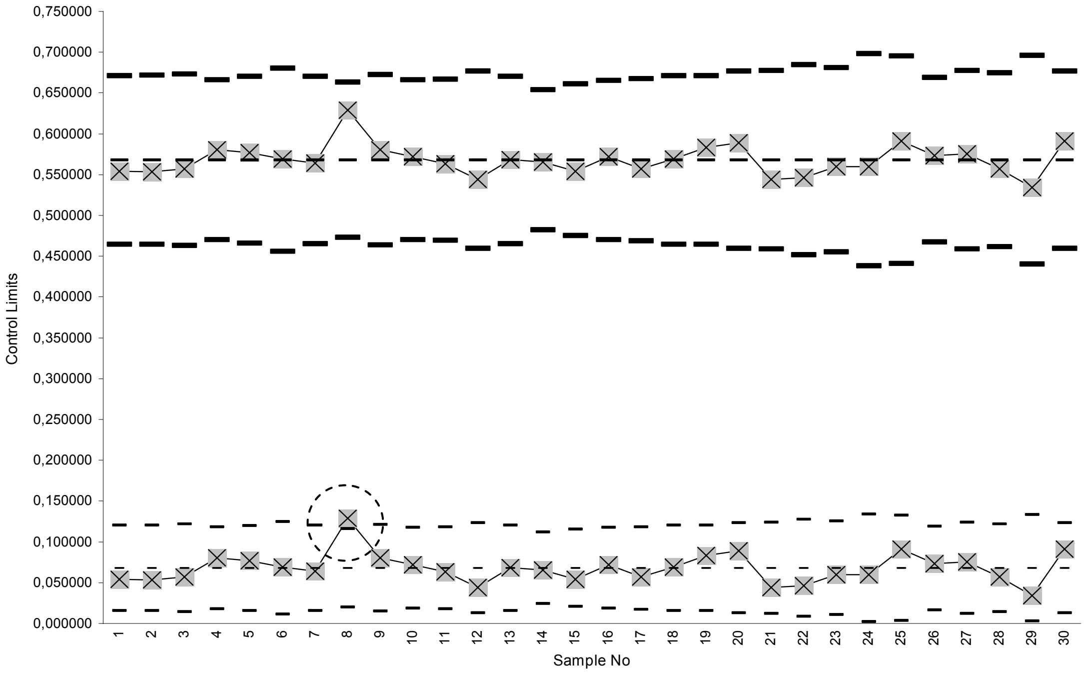

Figure 8.8: α-Cut fuzzy control chart for α=0.50 (VSS approach)

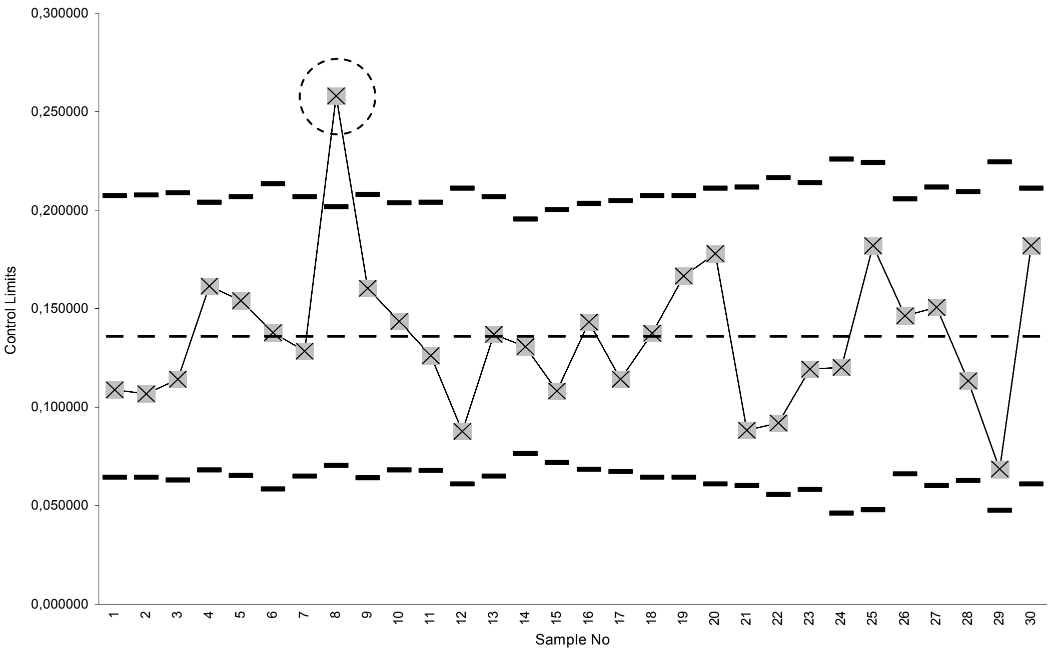

Figure 8.9: α-Cut fuzzy control chart for α=1.0 (Crisp Case, VSS approach)

Wang and Raz (1990) attempted to extend the use of control charts to linguistic variables by presenting several ways for determining the center line and the control limits. Kanagawa and Ohta (1993) proposed control charts for linguistic data with the above values of *UCL*, *LCL*, and *CL*, from a standpoint different to that of Wang and Raz in order not only to control the process average but also to control the process variability.

Our approach differs from previous studies from the point of view of inspection tightness. The quality controller is able to define the tightness of the inspection depending on the nature of the products and manufacturing processes. This is possible by selecting the value of α-cut freely. Quality controller may decide using higher values of α-cut for products that require a tighter inspection. Our approach is very easy in computation and similar to the crisp control charts [88].

8.2 α-Level Fuzzy Control Charts for number of nonconformıtıes

The samples from a toy company producing large-sized toys are taken every 4 h to control number of nonconformities. Because of the large dimensions of the toys, the number of nonconformities may also be large. The data collected from 30 subgroups are linguistic as shown in Table 8.4.

Table 8.4: Number of nonconformities for 30 subgroups.

Sample No	*Approximately*	*Between*	*Sample No*	*Approximately*	*Between*
1	30		*16*	40	
2		20-30	*17*		32-50
3		5-12	*18*	39	
4	6		*19*		15-21
5	38		*20*	28	
6		20-24	*21*		32-35
7		4-8	*22*		10-25
8		36-44	*23*	30	
9		11-15	*24*	25	
10		10-13	*25*		31-41
11	6		*26*		10-25
12	32		*27*		5-14
13	13		*28*		28-35
14		50-52	*29*		20-25
15		38-41	*30*	8	

The linguistic expressions in Table 8.4 are represented by fuzzy numbers as shown in Table 8.5. These numbers are subjectively identified by the quality control expert who also sets a = 0.60 and minimum acceptable ratio as b = 0.70.

Table 8.5: Fuzzy number (a,b,c,d) representation of 30 subgroups.

No	*a*	*b*	*c*	*d*	*No*	*a*	*b*	*c*	*d*
1	25	30	30	35	*16*	33	40	40	44
2	15	20	30	35	*17*	28	32	50	60
3	4	5	12	15	*18*	33	39	39	43
4	3	6	6	8	*19*	12	15	21	38
5	32	38	38	45	*20*	23	28	28	36
6	16	20	24	28	*21*	28	32	35	42
7	3	4	8	12	*22*	14	18	28	33
8	27	36	44	50	*23*	24	30	30	34
9	9	11	15	20	*24*	20	25	25	31
10	7	10	13	15	*25*	25	31	41	46
11	3	6	6	10	*26*	7	10	25	28
12	27	32	32	37	*27*	3	5	14	20
13	11	13	13	15	*28*	23	28	35	38
14	39	50	52	55	*29*	17	20	25	29
15	28	38	41	45	*30*	5	8	8	15
Average of 30 subgroups						18.13	22.67	26.93	32.07

Using Eqs. 6.11-14, $\tilde{CL}$, $L\tilde{CL}$, and $U\tilde{CL}$ are determined as follows:

$\tilde{CL} = (18.13, 22.67, 26.93, 32.07)$

$L\tilde{CL} = (1.15, 7.10, 12.65, 19.29)$

$U\tilde{CL} = (30.91, 36.95, 42.50, 49.05)$

Applying an α-cut of 0.60, values of $\tilde{CL}^{\alpha=0.60}$, $L\tilde{CL}^{\alpha=0.60}$, and $U\tilde{CL}^{\alpha=0.60}$ are calculated as follows. (See Eqs. 6.17-19)

$\tilde{CL}^{\alpha=0.60} = (20.85, 22.67, 26.93, 28.99)$

$L\tilde{C}L^{\alpha=0.60} = (4.72, 7.10, 12.65, 15.31)$

$U\tilde{C}L^{\alpha=0.60} = (36.95, 36.95, 42.50, 45.12)$

The fuzzy modes, α-level fuzzy midranges, and α-level fuzzy medians of the fuzzy control limits above are summarized in Table 8.6.

Table 8.6: Control limits and their representative values based on fuzzy mode, fuzzy midrange, and fuzzy median

	Fuzzy number				Fuzzy Transformation Method		
	a	b	c	d	Mode	Midrange (α=0.60)	Median (α=0.60)
CL	18.13	22.67	26.93	32.07	[22.67,26.9	24.95	24.88
LCL	1.15	7.10	12.65	19.29	[7.10,12.65]	10.05	9.96
UCL	30.91	36.95	42.5	49.05	[36.95,42.5	38.95	39.79

The decisions about the process control resulted from each sample based on the fuzzy mode, α-level fuzzy midrange, and α-level fuzzy median are given in Table 8.7.

Table 8.7: Decisions based on fuzzy mode, fuzzy midrange, and fuzzy median (α=0.60, β=0.70)

S_j	$f_{\text{mod},j}$		β_j	$f_{\text{mod},j}$ Decision	$f_{mr,j}^{\alpha=0.60}$	$f_{mr,j}^{\alpha=0.60}$ Decision	$f_{med,j}^{\alpha=0.60}$	$f_{med,j}^{\alpha=0.60}$ Decision
1	30	30	100.00	In Control	30.00	In Control	30.00	In Control
2	20	30	100.00	In Control	25.00	In Control	25.00	In Control
3	5	12	70.04	Rather In Control	8.90	Out of Control	8.70	Out of Control
4	6	6	0.00	Out of Control	5.80	Out of Control	5.90	Out of Control
5	38	38	100.00	In Control	38.20	In Control	38.10	In Control
6	20	24	100.00	In Control	22.00	In Control	22.00	In Control
7	4	8	22.56	Rather Out of Control	6.60	Out of Control	6.30	Out of Control
8	36	44	81.28	Rather In Control	39.40	In Control	39.70	In Control
9	11	15	100.00	In Control	13.60	In Control	13.30	In Control
10	10	13	100.00	In Control	11.30	In Control	11.40	In Control
11	6	6	0.00	Out of Control	6.20	Out of Control	6.10	Out of Control
12	32	32	100.00	In Control	32.00	In Control	32.00	In Control
13	13	13	100.00	In Control	13.00	In Control	13.00	In Control
14	50	52	0.00	Out of Control	49.40	Out of Control	50.20	Out of Control
15	38	41	100.00	In Control	38.30	In Control	38.90	In Control
16	40	40	100.00	In Control	39.40	In Control	39.70	In Control
17	32	50	58.35	Rather Out of Control	42.20	Out of Control	41.60	Out of Control
18	39	39	100.00	In Control	38.60	In Control	38.80	In Control
19	15	21	100.00	In Control	20.80	In Control	19.40	In Control
20	28	28	100.00	In Control	28.60	In Control	28.30	In Control
21	32	35	100.00	In Control	34.10	In Control	33.80	In Control
22	18	28	100.00	In Control	23.20	In Control	23.10	In Control
23	30	30	100.00	In Control	29.60	In Control	29.80	In Control
24	25	25	100.00	In Control	25.20	In Control	25.10	In Control
25	31	41	100.00	In Control	35.80	In Control	35.90	In Control
26	10	25	100.00	In Control	17.50	In Control	17.50	In Control
27	5	14	76.69	Rather In Control	10.30	In Control	9.90	Out of Control
28	28	35	100.00	In Control	31.10	In Control	31.30	In Control
29	20	25	100.00	In Control	22.70	In Control	22.60	In Control
30	8	8	100.00	In Control	8.80	Out of Control	8.40	Out of Control

The overall results of these approaches are summarized in Table 8.8. As it is clearly seen, some different decisions are obtained. For example, sample 3 indicates "Rather in control" when fuzzy mode transformation or DFA (85.81 percent of the sample is inside the control limits) is used, but it also indicates "out of control" when fuzzy midrange or fuzzy median is used. On the other hand, while sample 11 indicates an "out of control" situation when fuzzy mode, fuzzy midrange, or fuzzy median is used, DFA results in "Rather in control" since 74.38 percent of the fuzzy sample is inside the fuzzy control limits. Another typical result is sample 27's, which reveals 3 different process control decisions. According to the fuzzy mode transformation and DFA, this sample indicates "Rather in Control", while fuzzy midrange transformation results in "In Control" and fuzzy median results in "Out of Control". DFA shows that 87.67 percent of this sample is within the fuzzy control limits and it is strongly "Rather in Control" for β=0.70. Sample 30 is another example that reveals different decisions.

Table 8.8: Comparison of alternative approaches: Fuzzy mode, fuzzy midrange, fuzzy median, and DFA (α = 0.60 and β=0.70)

j	$f_{\mathrm{mod},j}$ Decision	$f_{mr,j}^{\alpha=0.60}$ Decision	$f_{med,j}^{\alpha=0.60}$ Decision	DFA (α=0.60) Decision	*j*	$f_{\mathrm{mod},j}$ Decision	$f_{mr,j}^{\alpha=0.60}$ Decision	$f_{med,j}^{\alpha=0.60}$ Decision	DFA (α=0.60) Decision
1	In Control	In Control	In Control	In Control	16	In Control	In Control	In Control	In Control
2	In Control	In Control	In Control	In Control	17	Rather Out of Control	Out of Control	Out of Control	Rather Out of Control
3	Rather In Control	Out of Control	Out of Control	Rather In Control	18	In Control	In Control	In Control	In Control
4	Out of Control	Out of Control	Out of Control	Rather Out of Control	19	In Control	In Control	In Control	In Control
5	In Control	In Control	In Control	In Control	20	In Control	In Control	In Control	In Control
6	In Control	In Control	In Control	In Control	21	In Control	In Control	In Control	In Control
7	Rather Out of Control	Out of Control	Out of Control	Rather Out of Control	22	In Control	In Control	In Control	In Control
8	Rather In Control	In Control	In Control	Rather In Control	23	In Control	In Control	In Control	In Control
9	In Control	In Control	In Control	In Control	24	In Control	In Control	In Control	In Control
10	In Control	In Control	In Control	In Control	25	In Control	In Control	In Control	In Control
11	Out of Control	Out of Control	Out of Control	Rather In Control	26	In Control	In Control	In Control	In Control
12	In Control	In Control	In Control	In Control	**27**	Rather In Control	In Control	Out of Control	Rather In Control
13	In Control	In Control	In Control	In Control	28	In Control	In Control	In Control	In Control
14	Out of Control	Out of Control	Out of Control	Out of Control	29	In Control	In Control	In Control	In Control
15	In Control	In Control	In Control	In Control	**30**	In Control	Out of Control	Out of Control	In Control

DFA provides the possibility of making linguistic decisions like "rather in control" or "rather out of control". Further intermediate levels of process control decisions are also possible by defining different intervals for β. For instance, it may be defined as in Eq. 8.5.

$$\text{Process Control}=\begin{cases}\text{in control,} & \text{for } 0.85 \le \beta_j \le 1,\\ \text{rather in control,} & \text{for } 0.60 \le \beta_j < 0.85,\\ \text{rather out of control,} & \text{for } 0.10 \le \beta_j < 0.60,\\ \text{out of control,} & \text{for } 0 \le \beta_j < 0.10.\end{cases} \qquad \textbf{(8.5)}$$

More intervals for the process control decisions can be subjectively defined by the decision-maker [61].

8.3 A Numerical Example for Fuzzy Unnatural Pattern Analysis

Fuzzy unnatural pattern analysis for the numerical example given in Section 8.2 is carried out in this section. Fuzzy control limits were determined as:

$$\widetilde{CL}^{\alpha=0.60} = (20.85, 22.67, 26.93, 28.99)$$

$$\widetilde{LCL}^{\alpha=0.60} = (4.72, 7.10, 12.65, 15.31)$$
$$\widetilde{UCL}^{\alpha=0.60} = (34.53, 36.95, 42.50, 45.12)$$

Fuzzy zones are calculated and tabulated in the Table 8.9.

Table 8.9: Fuzzy zones calculated for the example

Zone	a	b	c	d
UCL^{α}	34.53	36.95	42.50	45.12
+2 σ	29.97	32.19	37.31	39.74
+1 σ	25.41	27.43	32.12	34.37
CL^{α}	20.85	22.67	26.93	28.99
-1 σ	15.47	17.48	22.17	24.43
-2 σ	10.10	12.29	17.41	19.87
LCL^{α}	4.72	7.10	12.65	15.31

Based on the Western Electric Rules 1-4, membership functions in Eqs. 6.2, 6.3, 6.6, and 6.7 are used. These membership functions set the degree of unnaturalness for each rule. As an example, when a total membership degree of 1.90 is calculated for the rule 2, its degree of unnaturalness is determined from $\mu_2(x)$ as 0.9291.

In order to make calculations easy and mine our sample database for unnaturalness a computer program is coded using Fortran 90 programming language. Table 8.10 gives total membership degrees of the fuzzy samples in various zones.

Total membership degrees of the fuzzy samples (and degree of unnaturalness) for the fuzzified Western Electric Rules are given in Table 8.11. Sample 14 shows an out of control situation with respect to Rule 1, while samples 10 and 16 indicate an unnatural pattern with respect to the Rule 2. Their degrees of unnaturalness are determined from the membership function of the Rule 2. As an example, considering Rule 2 of the fuzzified Western Electric Rules, membership degrees of samples 15 and 16 become 2 and it corresponds to a degree of unnaturalness of 1.

Table 8.10: Membership degrees of fuzzy samples for different zones (A: Above, B: Below)

	A +3σ	in +3σ	B +3σ	A +2σ	in +2σ	B +2σ	A +1σ	in +1σ	B +1σ	A CL	in CL	B CL	A -1σ	in -1σ	B -1σ	A -2σ	in -2σ	B -2σ	A -3σ	in -3σ	B -3σ
1	0	0	1	0	0.24	0.76	0	1	0	0.94	0.06	0	1	0	0	1	0	0	1	0	0
2	0	0	1	0	0.04	0.96	0	0.38	0.62	0.25	0.52	0.23	0.64	0.36	0	0.97	0.03	0	1	0	0
3	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0.18	0.82	0	0.86	0.14
4	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0.68	0.32
5	0	1	0	0.32	0.68	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
6	0	0	1	0	0	1	0	0	1	0	0.54	0.46	0.27	0.73	0	0.95	0.05	0	1	0	0
7	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0.58	0.42
8	0.13	0.73	0.14	0.61	0.39	0	0.97	0.03	0	1	0	0	1	0	0	1	0	0	1	0	0
9	0	0	1	0	0	1	0	0	1	0	0	1	0	0.05	0.95	0	0.89	0.11	0.37	0.63	0
10	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0.55	0.45	0.01	0.99	0
11	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0.74	0.26
12	0	0	1	0	0.96	0.04	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0
13	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	1	0	0.02	0.98	0
14	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
15	0	0.98	0.02	0.56	0.44	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
16	0	1	0	0.72	0.28	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
17	0.39	0.39	0.22	0.65	0.35	0	0.9	0.1	0	1	0	0	1	0	0	1	0	0	1	0	0
18	0	1	0	0.49	0.51	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
19	0	0	1	0	0	1	0	0.03	0.97	0	0.28	0.72	0.13	0.67	0.21	0.58	0.42	0	0.97	0.03	0
20	0	0	1	0	0.05	0.95	0	1	0	0.58	0.42	0	1	0	0	1	0	0	1	0	0
21	0	0.2	0.8	0	0.99	0.01	0.61	0.39	0	1	0	0	1	0	0	1	0	0	1	0	0
22	0	0	1	0	0	1	0	0.22	0.78	0.09	0.53	0.39	0.48	0.52	0	0.87	0.13	0	1	0	0
23	0	0	1	0	0.17	0.83	0	1	0	0.89	0.11	0	1	0	0	1	0	0	1	0	0
24	0	0	1	0	0	1	0	0.2	0.8	0	1	0	0.89	0.11	0	1	0	0	1	0	0
25	0	0.51	0.49	0.28	0.61	0.1	0.72	0.28	0	1	0	0	1	0	0	1	0	0	1	0	0
26	0	0	1	0	0	1	0	0.01	0.99	0	0.24	0.76	0.14	0.42	0.44	0.43	0.46	0.11	0.72	0.28	0
27	0	0	1	0	0	1	0	0	1	0	0	1	0	0.01	0.99	0	0.38	0.62	0.12	0.76	0.12
28	0	0.04	0.96	0	0.53	0.47	0.27	0.73	0	0.87	0.13	0	1	0	0	1	0	0	1	0	0
29	0	0	1	0	0	1	0	0.03	0.97	0	0.63	0.37	0.39	0.61	0	0.98	0.02	0	1	0	0
30	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0.02	0.98	0	1	0

Table 8.11: Total membership degrees of the fuzzy samples in zones for the fuzzified Western Electric Rules.

Sample No	Beyond $\pm 3\sigma$	In or Above Fuzzy CL			In or Below Fuzzy CL		
		Rule 2	Rule 3	Rule 4	Rule 2	Rule 3	Rule 4
1	0.00	0.24	1	1	0	0	0.06
2	0.00	0.04	0.38	0.77	0.03	0.36	0.75
3	0.14	0	0	0	0.86	0.86	0.86
4	0.32	0	0	0	0.68	0.68	0.68
5	0.00	1	1	1	0	0	0
6	0.00	0	0	0.54	0.05	0.73	1
7	0.42	0	0	0	0.58	0.58	0.58
8	0.13	0.87	0.87	0.87	0	0	0
9	0.00	0	0	0	1	1	1
10	0.00	0	0	0	**1 (μ^*=1)**	1	1
11	0.26	0	0	0	0.74	0.74	0.74
12	0.00	0.96	1	1	0	0	0
13	0.00	0	0	0	1	1	1
14	**1.00**	0	0	0	0	0	0
15	0.00	1	1	1	0	0	0
16	0.00	**1 (μ^*=1)**	1	1	0	0	0
17	0.39	0.61	0.61	0.61	0	0	0
18	0.00	1	1	1	0	0	0
19	0.00	0	0.03	0.28	0.42	0.87	1
20	0.00	0.05	1	1	0	0	0.42
21	0.00	0.99	1	1	0	0	0
22	0.00	0	0.22	0.61	0.13	0.52	0.91
23	0.00	0.17	1	1	0	0	0.11
24	0.00	0	0.2	1	0	0.11	1
25	0.00	0.9	1	1	0	0	0
26	0.00	0	0.01	0.24	0.57	0.86	1
27	0.12	0	0	0	0.88	0.88	0.88
28	0.00	0.53	1	1	0	0	0.13
29	0.00	0	0.03	0.63	0.02	0.61	1
30	0.00	0	0	0	1	1	1

* unnatural sample with the corresponding degree of unnaturalness defined by the membersip functions for each rule.

Total membership degrees of the fuzzy samples in zones for fuzzified Grant and Leavenworth's rules are represented in Table 8.12.

As can be seen from Table 8.12, no samples indicate an unnatural pattern with respect to the fuzzified Grant and Leavenworth's rules.

Rules 1, 2, 5, 6, 7, and 8 among Nelson's rules can be examined in the same way. For Nelson's Rules 3 and 4, fuzzy samples are defuzzified by using α-level fuzzy midranges (given in Table 8.7) in order to check whether next sample shows an increment or decrement or alternating. α-level fuzzy midranges for α=0.60 are illustrated in Figure 8.10, which refers no unnaturalness with respect to the Nelson's Rules 3 and 4.

Table 8.12: Total membership degrees of the fuzzy samples in zones for fuzzified Grant and Leavenworth's rules

Sample No	In or Above CL	In or Below CL	In or Above CL				In or Below CL			
			Rule 1	Rule 2	Rule 3	Rule 4	Rule 1	Rule 2	Rule 3	Rule 4
1	1.00	0.06	-	-	-	-	-	-	-	-
2	0.77	0.75	-	-	-	-	-	-	-	-
3	0.00	0.86	-	-	-	-	-	-	-	-
4	0.00	0.68	-	-	-	-	-	-	-	-
5	1.00	0.00	-	-	-	-	-	-	-	-
6	0.54	1.00	-	-	-	-	-	-	-	-
7	0.00	0.58	3.31	-	-	-	3.93	-	-	-
8	0.87	0.00	3.18	-	-	-	3.87	-	-	-
9	0.00	1.00	2.41	-	-	-	4.12	-	-	-
10	0.00	1.00	2.41	-	-	-	4.26	-	-	-
11	0.00	0.74	2.41	4.18	-	-	4.32	6.67	-	-
12	1.00	0.00	2.41	4.18	-	-	4.32	6.61	-	-
13	0.00	1.00	1.87	3.41	-	-	4.32	6.86	-	-
14	0.00	0.00	1.87	3.41	5.18	-	3.74	6.00	7.67	-
15	1.00	0.00	2.00	4.41	5.18	-	3.74	5.32	7.61	-
16	1.00	0.00	3.00	4.41	5.41	-	2.74	5.32	6.86	-
17	0.61	0.00	3.61	4.48	6.02	7.79	1.74	4.32	6.00	7.67
18	1.00	0.00	4.61	5.48	7.02	7.79	1.00	3.74	5.32	7.61
19	0.28	1.00	3.89	4.89	6.30	7.30	2.00	4.74	6.32	7.86
20	1.00	0.42	4.89	5.89	6.76	8.30	1.42	4.16	5.74	7.42
21	1.00	0.00	5.89	6.89	7.76	9.30	1.42	3.16	5.16	6.74
22	0.61	0.91	5.51	7.51	7.51	8.92	2.34	3.34	6.08	7.66
23	1.00	0.11	5.51	7.51	8.51	9.38	2.45	3.45	5.19	6.77
24	1.00	1.00	5.90	8.51	9.51	10.38	3.45	3.45	5.19	7.19
25	1.00	0.00	5.90	9.51	10.51	10.51	3.45	3.45	4.45	7.19
26	0.24	1.00	5.86	8.75	9.75	10.75	3.45	4.45	5.45	7.19
27	0.00	0.88	4.86	7.75	9.75	10.75	3.91	5.33	5.33	7.07
28	1.00	0.13	4.86	8.14	10.75	11.75	4.04	5.46	5.46	6.46
29	0.63	1.00	4.87	7.77	10.38	11.38	4.12	6.46	6.46	7.46
30	0.00	1.00	3.87	7.49	9.38	11.38	5.01	6.46	7.46	7.46

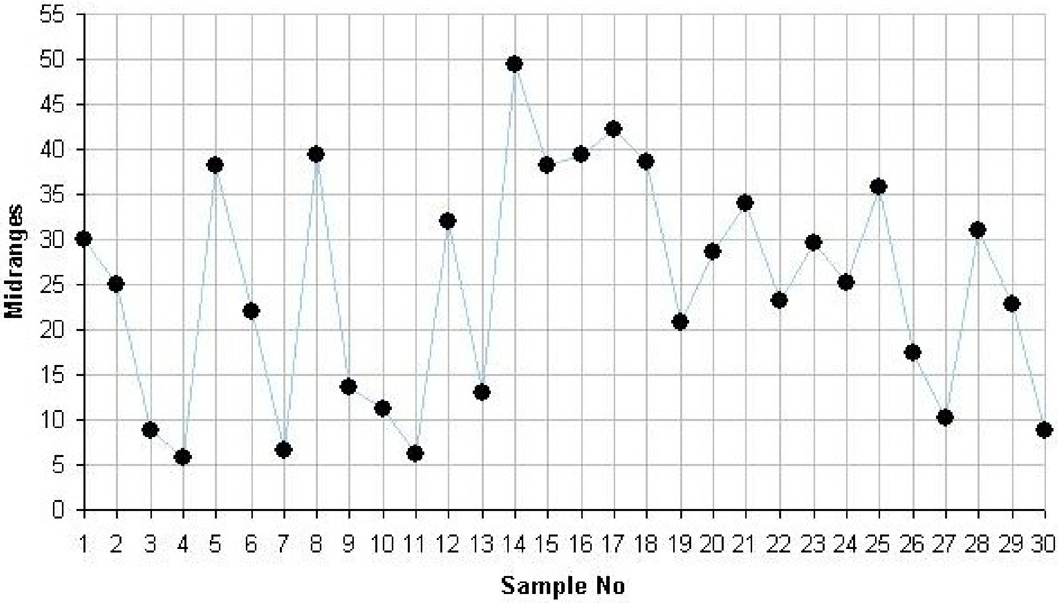

Figure 8.10: α-level (α =0.60) fuzzy midranges of the fuzzy samples

CONCLUSIONS

Fuzzy sets in the study of control charts were first used by Wang and Raz (1990) by means of fuzzy transformation. The aim was to represent the uncertainty in the available data. Followed Wand and Raz some similar models were proposed for the fuzzy control charts. Their models were based on the transformation to the crisp case using one of the representative values of fuzzy sets. This transformation is problematic since the characteristics of data were lost upon the transformation at early stages of the model. For example, when the uncertainty is represented by extremely asymmetrical fuzzy numbers, none of the representative values are successful to handle the information provided by the data. Furthermore, their study simply investigated the usual means of the control charts that resulted with the state of "in control" or "out of control". To make such a strict decision under uncertainty was another failure, i.e. there were no further intermediate levels of decisions available. On the other hand, the most meaningful part of the control charts was not studied: unnatural pattern analyses for fuzzy control charts.

In this study, the elimination of the gap in the construction and interpretation of fuzzy control charts is aimed. First, the existing models of fuzzy control charts were improved. α-cut of fuzzy sets is successfully introduced in order to reflect the tightness of the inspection that can subjectively be set by the quality controller according to the importance of the inspection. Using this approach, α-cut fuzzy p charts for linguistic data were developed. The proposed approach is effective in detecting process shifts.

In construction of the fuzzy c charts, loss of the properties of the fuzzy data was taken into consideration. For these purposes, a new approach based on the area measurement is developed and named as "Direct Fuzzy Approach (DFA)". In this model, linguistic or uncertain data are represented by triangular and trapezoidal fuzzy numbers. Based on the central tendency method, center line for fuzzy control chart is determined using fuzzy arithmetic operations. Then, lower and upper control limits of the chart are also determined as fuzzy numbers rather than transforming them by using representative values. As a result, both the sample data and control limits were obtained as fuzzy numbers and their comparison to judge the state of process control is carried out by means of a method based on the area measurement. At this stage, several intermediate decisions are able to set by the quality controller so that the strength of the signal can be defined. DFA provides the possibility of making linguistic decisions like “rather in control” or “rather out of control”.

If all of the points on the control chart lie between the defined control limits, the process is simply said to be in control. Does this mean that when all points fall within the limits, the process is in control? Not necessarily. If the plot looks non-random, that is, if the points exhibit some form of systematic behavior, there is still something wrong. Statistical methods to detect sequences or nonrandom patterns were extensively applied to the interpretation of classical control charts. To be sure, “in control” implies that all points are between the control limits and they form a random pattern. In this study, unnatural pattern analyses are also developed for the proposed direct fuzzy approach. Some fuzzy rules for the DFA are defined and their probability of occurrence is calculated using the probabilities of fuzzy events. It is, of course, possible that one can set further different fuzzy unnatural pattern rules in the light of the proposed model. In fuzzy unnatural pattern analyses, the degree of being an unnatural pattern is also defined so that the quality controller can be aware of the membership degree to say that an unnatural pattern exists. This model is also illustrated with a numerical example.

For a further study, construction and interpretation of multivariate fuzzy control charts is suggested. It is clear that the study of multivariate fuzzy control charts is going to open new research areas.

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APPENDIX A

Equations to compute A_{out}^{U} and A_{out}^{L} given in Figure 6.6

$$A_{out}^{U} = \frac{1}{2}\left[\left(d^{\alpha} - UCL_4^{\alpha}\right) + \left(d^{t} - UCL_4^{t}\right)\right]\left(\max\left(t-\alpha,0\right)\right) +$$
$$\frac{1}{2}\left[\left(d^{z} - a^{z}\right) + \left(c-b\right)\right]\left(\min\left(1-t,1-\alpha\right)\right)$$

(A-U1)

where,

$$t = \frac{UCL_4 - a}{\left(b-a\right)+\left(c-b\right)} \text{ and } z = \max\left(t,\alpha\right)$$

$$A_{out}^{U} = \frac{1}{2}\left[\left(d^{\alpha} - UCL_4^{\alpha}\right) + \left(c - UCL_3\right)\right]\left(1-\alpha\right) \quad \textbf{(A-U2)}$$

$$A_{out}^{U} = \frac{1}{2}\left(d^{\alpha} - UCL_4^{\alpha}\right)\left(\max\left(t-\alpha,0\right)\right)$$

where, **(A-U3)**

$$t = \frac{UCL_4 - d}{\left(UCL_4 - UCL_3\right)-\left(d-c\right)}$$

$$A_{out}^{U} = \frac{1}{2}\left[\left(c - UCL_3\right) + \left(d^{z} - UCL_4^{z}\right)\right]\left(\min\left(1-t,1-\alpha\right)\right)$$

where, **(A-U4)**

$$t = \frac{UCL_4 - d}{\left(UCL_4 - UCL_3\right)-\left(d-c\right)} \text{ and } z = \max\left(t,\alpha\right)$$

$$A_{out}^{U} = \frac{1}{2}\left[\left(d^{z_2} - UCL_4^{z_2}\right) + \left(d^{t_1} - UCL_4^{t_1}\right)\right]\left(\min\left(\max\left(t_1-\alpha,0\right),t_1-t_2\right)\right) +$$
$$\frac{1}{2}\left[\left(d^{z_1} - a^{z_1}\right) + \left(c-b\right)\right]\left(\min\left(1-t_1,1-\alpha\right)\right) \quad \textbf{(A-U5)}$$

where,

$$t_1 = \frac{UCL_4 - a}{(b-a)+(UCL_4 - UCL_3)} \ , \ t_2 = \frac{UCL_4 - d}{(UCL_4 - UCL_3)-(d-c)} \ ,$$

$z_1 = \max(\alpha, t_1)$, and $z_2 = \max(\alpha, t_2)$

$$A_{out}^{U} = 0 \quad \textbf{(A-U6)}$$

$$A_{out}^{U} = \frac{1}{2}\left[\left(d^{\alpha} - a^{\alpha}\right) + (c-b)\right](1-\alpha) \quad \textbf{(A-U7)}$$

$$A_{out}^{L} = \frac{1}{2}\left[\left(LCL_1^{\alpha} - a^{\alpha}\right) + \left(LCL_1^{t} - a^{t}\right)\right]\left(\max(t-\alpha, 0)\right) + \frac{1}{2}\left[\left(d^{z} - a^{z}\right) + (c-b)\right]\left(\min(1-t, 1-\alpha)\right) \quad \textbf{(A-L1)}$$

where,

$$t = \frac{d - LCL_1}{(LCL_2 - LCL_1)+(d-c)} \text{ and } z = \max(\alpha, t)$$

$$A_{out}^{L} = \frac{1}{2}\left[\left(d^{\alpha} - a^{\alpha}\right) + (c-b)\right](1-\alpha) \quad \textbf{(A-L2)}$$

$$A_{out}^{L} = \frac{1}{2}\left[\left(LCL_1^{\alpha} - a^{\alpha}\right) + (LCL_2 - b)\right](1-\alpha) \quad \textbf{(A-L3)}$$

$$A_{out}^{L} = \frac{1}{2}\left[\left(LCL_1^{z_2} - a^{z_2}\right) + \left(LCL_1^{t_1} - a^{t_1}\right)\right]\left(\min\left(\max(t_1-\alpha, 0), t_1 - t_2\right)\right) + \frac{1}{2}\left[\left(d^{z_1} - a^{z_1}\right) + (c-b)\right]\left(\min(1-t, 1-\alpha)\right) \quad \textbf{(A-L4)}$$

where,

$$t_1 = \frac{d - LCL_1}{(LCL_2 - LCL_1)+(d-c)} \ , \ t_2 = \frac{a - LCL_1}{(LCL_2 - LCL_1)-(b-a)}$$

$z_1 = \max(\alpha, t_1)$, and $z_2 = \max(\alpha, t_2)$

$$A_{out}^{L} = \frac{1}{2}\left[\left(LCL_1^{z} - a^{z}\right) + (LCL_2 - b)\right]\left(\min(1-t, 1-\alpha)\right) \quad \textbf{(A-L5)}$$

where,

$$t = \frac{a - LCL_1}{\left(LCL_2 - LCL_1\right) - \left(b - a\right)} \text{ , and } z = \max\left(\alpha, t\right)$$

$$A_{out}^{L} = 0 \qquad \textbf{(A-L6)}$$

$$A_{out}^{L} = \frac{1}{2}\Big[\left(d^{\alpha} - a^{\alpha}\right) + \left(c - b\right)\Big]\left(1 - \alpha\right) \qquad \textbf{(A-L7)}$$

CURRICULUM VITAE

Murat GÜLBAY was born in İskenderun-Hatay in 1973. Having finished İskenderun Technical School with a first degree award in 1991, he has started his bachelor science at mechanical engineering department of the University of Gaziantep in 1991. Including a year in English prep class, he has graduated from the mechanical engineering department in 1996. In the same year, he has started to work as a research assistant in Industrial Engineering Department of University of Gaziantep, which was newly established. In 1998, he has finished his master of science in Industrial Engineering Program of the Institute of Science and Technology of the University of Gaziantep. He has started his PhD at Istanbul Technical University with the research assistantship in 2000.

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