ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

COORDINATION IN A TWO-STAGE CAPACITATED SUPPLY CHAIN WITH MULTIPLE SUPPLIERS

Ph.D. Thesis by
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Department : Industrial Engineering

Programme : Industrial Engineering

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FOREWORD

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December 2009 Peral Toktaş Palut

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A : Interarrival time of the manufacturer

b_i : Backorder cost per unit backordered at supplier i per unit time, $i=1,\ldots,n$

B_i : Outstanding backorders at supplier i, $i=1,\ldots,n$

b_M : Backorder cost per unit backordered at the manufacturer per unit time

B_M : Outstanding backorders at the manufacturer

B_T : Total number of outstanding backorders

$\tilde{C}$: Approximate average total costs per unit time for supplier j and the manufacturer

C_M : Average cost per unit time for the manufacturer

$\tilde{C}_M$: Approximate average cost per unit time for the manufacturer

C_{S_i} : Average cost per unit time for supplier i, $i=1,\ldots,n$

C_T : Average total backorder and holding costs per unit time for the overall system

$\tilde{C}_T$: Approximate average total backorder and holding costs per unit time for the overall system

$C_{(.)}^2$: Squared coefficient of variation of the random variable (.)

$\tilde{C}_M^B$: Approximate average cost per unit time for the manufacturer after the backorder cost subsidy contract

$C_{S_j}^B$: Average cost per unit time for supplier j after the backorder cost subsidy contract

$\tilde{C}_M^C$: Approximate average cost per unit time for the manufacturer after the cost sharing contract

$\tilde{C}_{S_j}^C$: Approximate average cost per unit time for supplier j after the cost sharing contract

$\tilde{C}_M^P$: Approximate average cost per unit time for the manufacturer after the transfer payment contract based on Pareto improvement

$C_{S_j}^P$: Average cost per unit time for supplier j after the transfer payment contract based on Pareto improvement

$D_{S_j}^B$: Difference between the average backorder costs per unit time for supplier j before and after the transfer payment in the backorder cost subsidy contract

$E[.]$: Expected value of the random variable (.)

$f_{(.)}(t)$: Probability density function of the random variable (.)

h_i : Holding cost per unit inventory per unit time for supplier i, $i=1,\ldots,n$

I_i : Inventory level of supplier i, $i=1,\ldots,n$
L_i : Order fulfillment lead time for supplier i, $i=1,\ldots,n$
L_M : Order fulfillment lead time for the manufacturer
L_S : Order fulfillment lead time for the overall system
N_i : Number of jobs in supplier i's system, $i=1,\ldots,n$
N_{q_M} : Number of jobs in the manufacturer's queue
N_M : Number of jobs in the manufacturer's system
r_{ij} : Normalized value of alternative i in terms of criterion j, $i=1,\ldots,m$, $j=1,\ldots,n$
S_i : Base stock level of supplier i, $i=1,\ldots,n$
S_i^* : Optimal centralized solution for supplier i, $i=1,\ldots,n$
S_i^{o} : Optimal decentralized solution for supplier i, $i=1,\ldots,n$
T^C : Amount of transfer payment in the cost sharing contract
T^P : Amount of transfer payment in the transfer payment contract based on Pareto improvement
V_i : Value of alternative i, $i=1,\ldots,m$
v_{ij} : Value of alternative i in terms of criterion j, $i=1,\ldots,m$, $j=1,\ldots,n$
$Var(.)$: Variance of the random variable (.)
w_j : Weight of criterion j, $j=1,\ldots,n$
w_j^* : New weight of criterion j, $j=1,\ldots,n$
X : Time between demands
Y_i : Time until the next service completion for supplier i, $i=1,\ldots,n$
α_B : Parameter of the backorder cost subsidy contract
α_C : Parameter of the cost sharing contract
δ_j : The minimum change in the current weight w_j of criterion j so that the ranking of the alternatives is reversed, $j=1,\ldots,n$
λ : Customer demand rate
μ_i : Service rate of supplier i, $i=1,\ldots,n$
μ_M : Service rate of the manufacturer
$\pi_{(.)}$: Steady-state probability of state (.)
ρ_i : Traffic intensity of supplier i, $i=1,\ldots,n$
ρ_M : Traffic intensity of the manufacturer

COORDINATION IN A TWO-STAGE CAPACITATED SUPPLY CHAIN WITH MULTIPLE SUPPLIERS

SUMMARY

The aim of this thesis is to coordinate the inventory policies in a decentralized supply chain with stochastic demand by means of contracts. The system considered is a decentralized two-stage supply chain consisting of multiple independent suppliers and a manufacturer with limited production capacities. The suppliers operate on a make-to-stock basis and apply base stock policy to manage their inventories. On the other hand, the manufacturer employs a make-to-order strategy.

Since the suppliers are capacitated, each supplier is modeled as an $M/M/1$ make-to-stock queue under necessary assumptions. Furthermore, the average outstanding backorders and the average inventory level of each supplier are derived using the queuing model.

On the other hand, to model the manufacturer as a queuing system, first an approximate distribution is derived for the interarrival times of the manufacturer. The idea behind the approximation is the expectation that the supplier with the minimum base stock level affects the interarrival times of the manufacturer the most. Then, the manufacturer is modeled as a $GI/M/1$ queue under necessary assumptions. Moreover, the average number of jobs in the manufacturer's system and the average outstanding backorders at the manufacturer are obtained using the queuing model.

After the supply chain has been modeled as a queuing system, the centralized and decentralized models are developed. In the centralized model, the objective of the single decision maker is to minimize the average total backorder and holding costs per unit time for the overall system. The decision variables are the base stock levels of the suppliers. Therefore, in the decentralized model, the objective of each supplier is to minimize the average cost per unit time for his own system.

When the optimal solutions to the centralized and decentralized models are compared, it is concluded that only the supplier with the minimum base stock level needs coordination. Therefore, contracts are prepared between that supplier and the manufacturer.

Three different transfer payment contracts are studied in this thesis. These are the backorder cost subsidy contract, the transfer payment contract based on Pareto improvement, and the cost sharing contract. Each contract is evaluated according to its coordination ability and whether it is Pareto improving or not. The analyses of the contracts point out that all three contracts have the ability to coordinate the supply chain. However, when the Pareto improvement is taken into account, the cost sharing contract seems to be the one that will be preferred by both members.

In this thesis, also a numerical study is performed to compare the centralized and decentralized systems based on SCOR Model performance metrics, which are the total number of outstanding backorders, the order fulfillment lead time, the supply

chain response time, the total backorder and holding costs, and the inventory days of supply. The results denote that the decentralized system has a better performance than the centralized system according to the total number of outstanding backorders and the order fulfillment lead time, which are customer-facing metrics. On the other hand, the centralized system performs better according to the internal-facing metrics, which are the total backorder and holding costs and the inventory days of supply. Finally, according to the supply chain response time, which is also a customer-facing metric, it is found that the centralized system generally has a better performance than the decentralized system.

After the centralized and decentralized systems have been compared based on these performance metrics, the simple additive weighting method is used to decide which system is more preferable. When each criterion is taken as equally important, it is found that the decentralized system is preferred over the centralized system. Then, a sensitivity analysis is performed to determine the most sensitive criterion. The results indicate that the inventory days of supply is the most sensitive criterion; and it is followed by the total backorder and holding costs, and the supply chain response time, respectively. On the other hand, the total number of outstanding backorders and the order fulfillment lead time are insensitive to the ranking of the systems. The results obtained from the sensitivity analysis also point out that the decentralized system is more preferable than the centralized system.

KAPASİTESİ SINIRLI ÇOKLU TEDARİKÇİDEN OLUŞAN İKİ KADEMELİ BİR TEDARİK ZİNCİRİNİN KOORDİNASYONU

ÖZET

Bu tezin amacı, rassal talebe sahip merkezkaç bir tedarik zincirindeki envanter politikalarını kontratlar aracılığıyla koordine etmektir. Ele alınan sistem, sınırlı üretim kapasitesine sahip çoklu bağımsız tedarikçi ve bir üreticiden oluşan iki kademeli merkezkaç bir tedarik zinciridir. Tedarikçiler stok için üretim yapmakta ve envanter yönetiminde temel stok yöntemini kullanmaktadır. Üretici ise sipariş için üretim prensibine göre çalışmaktadır.

Tedarikçilerin kapasitesi sınırlı olduğu için, gerekli varsayımlar altında her tedarikçi bir $M/M/1$ stok-için-üretim kuyruk sistemi olarak modellenmiştir. Ayrıca, kuyruk modeli kullanılarak her tedarikçinin ortalama bekleyen sipariş miktarı ve ortalama envanter seviyesi elde edilmiştir.

Diğer yandan, üreticinin bir kuyruk sistemi olarak modellenebilmesi için, öncelikle gelişlerarası sürelerinin yaklaşık dağılımı bulunmuştur. Söz konusu dağılım, en düşük temel stok seviyesine sahip tedarikçinin üreticinin gelişlerarası sürelerini en çok etkileyeceği beklentisinden yola çıkarak elde edilmiştir. Daha sonra, gerekli varsayımlar altında üretici bir $GI/M/1$ kuyruk sistemi olarak modellenmiştir. Bunun yanı sıra, kuyruk modeli kullanılarak üreticinin sistemindeki ortalama iş sayısı ve ortalama bekleyen sipariş miktarı bulunmuştur.

Tedarik zincirinin bir kuyruk sistemi olarak modellenmesinden sonra, merkezi ve merkezkaç modeller geliştirilmiştir. Merkezi modelde karar vericinin amacı, sistemin tümü için birim zamandaki ortalama toplam bekleyen sipariş ve elde tutma maliyetlerini enküçüklemektir. Karar değişkenleri tedarikçilerin temel stok seviyeleridir. Bu nedenle merkezkaç modelde, her bir tedarikçi kendi sistemi için birim zamandaki ortalama maliyeti enküçüklemeye çalışır.

Merkezi ve merkezkaç modellerin eniyi çözümleri karşılaştırıldığında, sadece en düşük temel stok seviyesine sahip tedarikçinin koordine edilmesi gerektiği sonucuna varılmıştır. Bu nedenle, sadece bu tedarikçi ve üretici arasında kontratlar hazırlanmıştır.

Bu tezde, transfer ödemesine dayalı üç farklı kontrat üzerine çalışılmıştır. Bu kontratlar, bekleyen sipariş maliyetini destekleme kontratı, Pareto iyileştirmeye dayalı transfer ödemesi kontratı ve maliyet paylaşımı kontratıdır. Her kontrat, koordinasyon yeteneği ve Pareto iyileştiren olup olmaması yönünden değerlendirilmiştir. Sonuç olarak, üç kontratın da tedarik zincirinin koordinasyonunu sağladığı ispatlanmıştır. Pareto iyileştirme göz önüne alındığında ise, maliyet paylaşımı kontratının her iki üye tarafından da tercih edilmesi beklenebilir.

Bu tezde ayrıca, merkezi ve merkezkaç sistemlerin SCOR Model performans ölçütleri açısından karşılaştırılması için sayısal bir çalışma gerçekleştirilmiştir. Ele alınan performans ölçütleri, toplam bekleyen sipariş miktarı, sipariş karşılama süresi,

tedarik zinciri cevap süresi, toplam bekleyen sipariş ve elde tutma maliyetleri ve envanter gün sayısıdır. Sonuçlar, müşteriye-dönük ölçütler olan toplam bekleyen sipariş miktarı ve sipariş karşılama süresi açısından, merkezkaç sistemin merkezi sisteme nazaran daha iyi bir performansa sahip olduğunu göstermektedir. Diğer yandan, içe-dönük ölçütler olan toplam bekleyen sipariş ve elde tutma maliyetleri ve envanter gün sayısına göre ise, merkezi sistem daha iyi bir performansa sahiptir. Son olarak, yine müşteriye-dönük bir ölçüt olan tedarik zinciri cevap süresine bakıldığında, merkezi sistemin merkezkaç sisteme nazaran genellikle daha iyi bir performans gösterdiği bulunmuştur.

Merkezi ve merkezkaç sistemler söz konusu performans ölçütlerine göre karşılaştırıldıktan sonra, hangi sistemin daha tercih edilir olduğunu belirlemek için basit toplamlı ağırlıklandırma yöntemi kullanılmıştır. Her ölçütün eşit öneme sahip olması durumunda, merkezkaç sistemin merkezi sisteme nazaran tercih edildiği görülmektedir. Daha sonra, en duyarlı ölçütü belirlemek için duyarlılık analizi uygulanmıştır. Sonuçlar, envanter gün sayısının en duyarlı ölçüt olduğunu göstermektedir. Bunu sırasıyla, toplam bekleyen sipariş ve elde tutma maliyetleri ve tedarik zinciri cevap süresi takip etmektedir. Toplam bekleyen sipariş miktarı ve sipariş karşılama süresinin ise sistemlerin sıralamasına duyarsız olduğu bulunmuştur. Duyarlılık analizinden elde edilen sonuçlar, aynı zamanda merkezkaç sistemin merkezi sisteme nazaran daha tercih edilebilir olduğunu göstermektedir.

1. INTRODUCTION

Intensifying competition in today's business environment has brought the need of paying more attention to the design and management of supply chains. Starting from the effective product design, selection of the suppliers, facility location decisions, inventory management, distribution strategies, information technology, and finally the coordination and integration activities are critical factors for an effective supply chain.

Supply chain management can be defined as the integration of all the activities taking place beginning from the arrival of the demand, until the time the products are distributed to the end customer. According to Simchi-Levi et al. (2000), "supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements."

The origins of the supply chain management can be traced back to 1950s and 1960s, when traditional mass manufacturing was employed to reduce costs and improve productivity. In the 1960s and 1970s, the manufacturers noticed the importance of inventory management and storage costs. In the 1980s, the companies utilized new strategies such as just-in-time manufacturing, Kanban system, lean manufacturing, and total quality management to improve quality, manufacturing efficiency, and delivery times. In the 1990s, as the competition intensified further, the companies began to form supply chain partnerships to achieve specific objectives and benefits. In addition, they began to understand the necessity of integrating the activities through the supply chain. The improvement of information technology has aided the evolution of the integrated supply chain concept. Today, the companies continue to investigate the ways of effective supply chain management to stay competitive in the market (Wisner et al., 2005, pp. 10-12).

Supply chains generally consist of multiple agents, such as suppliers, manufacturers, warehouses, and distribution centers. In a supply chain, if there is a single decision maker who tries to optimize the overall system, such a structure is called centralized. However, generally the agents have conflicting objectives even they belong to the same entity. For instance, manufacturers would like to produce in large lot sizes in order to reduce setup costs. However, this would increase the inventory amounts, and hence the holding costs, which contradicts the objectives of the warehouses. On the other hand, a supply chain in which each agent tries to optimize its own system is referred to as decentralized.

A centralized system leads to global optimization, whereas, a decentralized system results in local optimization of the agents. Therefore, to achieve the global optimal solution in a decentralized supply chain, the conflicting objectives of the agents should be aligned through coordination issues.

Supply chain coordination can be accomplished via contracting on a set of transfer payments between the supply chain members. A contract is said to coordinate the supply chain, if each member acts rationally according to the supply chain optimal solution, i.e., the decentralized solution is equal to the centralized solution. There are also other initiatives to coordinate a supply chain, such as quick response, efficient consumer response, and vendor managed inventory. In quick response, by sharing information, supply chain members work together to respond more quickly to customer needs. This brings forth better customer service and fewer inventories. Efficient consumer response, in which real-time point-of-sale data can be viewed by all supply chain members, is another concept that concerns with speed and flexibility. Thus, safety stock inventories can be reduced (Wisner et al., 2005, pp. 208). Finally, in vendor-managed inventory, the vendor (supplier) takes on the responsibility of managing the buyer's (retailer's) inventory. Both agents can benefit from this arrangement. For example, the supplier takes the advantage of reduced forecast uncertainties, and hence safety stocks, while the retailer relieves from the responsibility of specifying, placing, and monitoring purchase orders and benefits from guaranteed service levels (Aviv and Federgruen, 1998).

The scope of this thesis is the coordination of the inventory policies in a decentralized supply chain with stochastic demand by means of contracts. The system considered is a decentralized two-stage supply chain consisting of multiple independent suppliers and a manufacturer. The system operates in a manufacture-to-order environment, i.e., the suppliers and the manufacturer employ make-to-stock and make-to-order strategies, respectively. The manufacturer orders each component from a particular supplier and production cannot start until all components arrive. The transfer times between the suppliers and the manufacturer are negligible. The inventory of each component at each supplier is controlled by an $(S-1, S)$ base stock policy. The suppliers and the manufacturer have a limited capacity of production. Backorders are allowed in the system and capacity of the backlog queue at each supplier is infinite. End customer demand arrives in single units and it is stochastic.

The aim of this thesis is to develop transfer payment contracts between the suppliers and the manufacturer, so that the suppliers choose the base stock levels that are optimal for the overall supply chain. In other words, the aim is to coordinate the inventory policies of the suppliers via contracts. To the best of our knowledge, the coordination of the inventory policies in a capacitated supply chain with multiple suppliers has not been explored yet.

Figure 1.1 depicts the flow chart of the methodology used in this thesis. In summary, first, the supply chain is modeled as a queuing system since the suppliers and the manufacturer have a limited capacity of production. Afterwards, using the principles of queuing theory, the performance measures of the suppliers and the manufacturer are obtained. Then, the centralized and decentralized models are developed based on these performance measures. Comparison of the optimal solutions to these models reveals that the supply chain needs to be coordinated. Therefore, different transfer payment contracts are examined for the coordination of the supply chain; and each contract is evaluated according to its coordination ability and whether it is Pareto improving or not. Finally, among these contracts, the one that can coordinate the supply chain and that is the most advantageous for all parties is suggested for the coordination of the supply chain in this thesis.

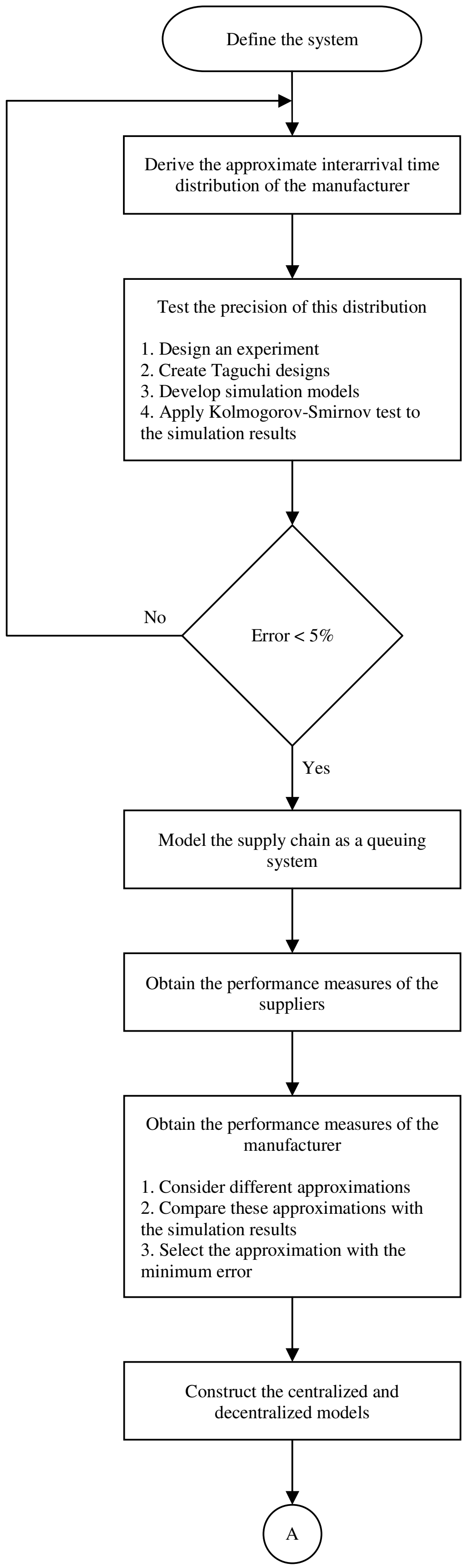

Figure 1.1: The flow chart of the methodology used in this thesis.

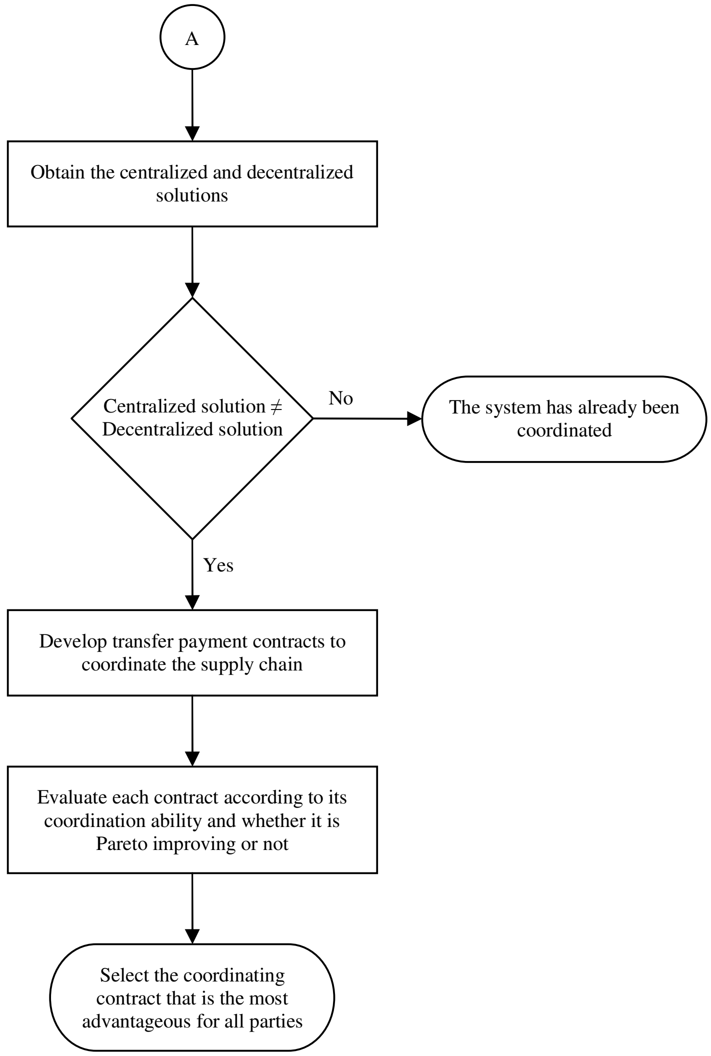

Figure 1.1 (continued): The flow chart of the methodology used in this thesis.

The thesis is organized as follows. The supply chain contracting literature related to the coordination in decentralized supply chains with stochastic demand is reviewed in the first section of chapter two. Since the suppliers operate on a make-to-stock basis in the system considered, this is followed by a brief review of the literature on make-to-stock system models.

In chapter three, the interarrival time distribution of the manufacturer in the case of one supplier and two suppliers are derived. Also, an approximate interarrival time distribution is developed for a system with two or more suppliers. Then, the supply chain is modeled as a queuing system and the following performance measures are obtained: The average outstanding backorders and the average inventory level of the suppliers, the average number of jobs in the manufacturer's system, and the average outstanding backorders at the manufacturer.

In chapter four, using these performance measures, the centralized and decentralized models are developed; and the optimal solutions to these models are derived.

Comparison of the centralized and decentralized solutions points out that the supply chain needs coordination. Therefore, in chapter five, three different transfer payment contracts are studied to coordinate the supply chain. These are the backorder cost subsidy contract, the transfer payment contract based on Pareto improvement, and the cost sharing contract. Then, each contract is evaluated whether it can coordinate the supply chain and whether it is Pareto improving or not.

Chapter six presents a numerical study. In this chapter, experimental designs are developed to compare the centralized and decentralized systems based on SCOR Model performance metrics, which are the total number of outstanding backorders, the order fulfillment lead time, the supply chain response time, the total backorder and holding costs, and the inventory days of supply. Then, the simple additive weighting method is used to decide which system is more preferable. Also, a sensitivity analysis is performed to determine the most sensitive criterion.

Finally, the concluding remarks and the future research directions are given in chapter seven.

2. LITERATURE REVIEW

The literature related to the scope of this thesis can be analyzed in two parts. In the first part, the contracting literature concerned with the coordination of decentralized supply chains with stochastic demand is reviewed. This is followed by a brief review of the literature on models of make-to-stock systems, according to which the suppliers operate in the system taken into consideration.

2.1 The Literature on Supply Chain Contracts

The contracting literature on supply chains with stochastic demand can be mainly divided into two categories. Most of the research is on the coordination of supply chains in a single-period setting, i.e., the newsvendor model, and also its extensions. In the newsvendor model, generally there exists only one replenishment opportunity for the retailer. There are also relatively fewer studies on the coordination of supply chains in an infinite horizon setting with many replenishment opportunities.

2.1.1 The newsvendor model and its extensions

In the classical newsvendor model, there is a single supplier and a retailer. The retailer sells a single product and faces stochastic demand. There is just one opportunity for the retailer to order inventory from the supplier before the selling season begins. The decision variable is the order quantity of the retailer. In a decentralized system, since the retailer tries to minimize his own costs and does not take the supplier’s profit into consideration, he orders less inventory than the supply chain optimal order amount. Thus, an incentive scheme is needed for the retailer to increase his order quantity.

In the literature, different contract types have been studied to coordinate this supply chain and its extensions. The most widely used ones are the wholesale-price contract, buyback contract, revenue-sharing contract, quantity-discount contract, quantity-flexibility contract, sales-rebate contract, and price-discount contract (Cachon, 2003). As the main scope of this thesis is the coordination of supply chains in an infinite horizon setting, the research on the newsvendor model and its extensions is briefly reviewed below, giving a few examples for each contract type.

2.1.1.1 Wholesale-price contract

In the wholesale-price contract, the supplier determines a wholesale price per unit purchased by the retailer. However, because of double marginalization, this contract fails to coordinate the supply chain. Double marginalization was first discussed by Spengler (1950). It occurs when the supplier determines a wholesale price greater than his marginal costs and this gives rise to a retail price greater than the supply chain optimal price. Since there are two margins in this scheme, the supply chain cannot be coordinated. Coordination can only be achieved if the supplier has a nonpositive profit.

Cachon (2004) examines three types of wholesale-price contracts for coordinating a supplier and a retailer. In the push contract, the retailer can submit an order before the selling season and there is a single wholesale price determined by the supplier. In the pull contract, the retailer can place an order during the selling season and again there is just one wholesale price. The third contract, which is the advance-purchase discount contract, has two wholesale prices. There is a discounted price for the orders given before the selling season starts and a regular price for the orders given during the season. It is shown that the advance-purchase discount contract may coordinate the supply chain and arbitrarily allocate the profits between the supplier and the retailer.

Debo and Sun (2005) study the coordination between a manufacturer and a retailer, where the retailer faces the repeated version of the single-period newsvendor model. In each period of an infinite horizon, before the demand is realized, the manufacturer and the retailer subsequently determine the wholesale price and the order quantity, respectively. Inventory carriage between the periods is not allowed. The authors point out that if the manufacturer and the retailer discount the future stream of profits with a sufficiently high factor, the coordination can be achieved using a wholesale-price contract.

2.1.1.2 Buyback contract

In the buyback contract, which is also called return policy, the supplier charges the retailer a wholesale price per unit purchased, but pays back the retailer an amount per unit for the units unsold at the end of the season. Obviously, this amount should not be greater than the wholesale price.

Pasternack (1985) studies the buyback contracts in the newsvendor framework. He points out that the optimal solution cannot be obtained if the manufacturer offers the retailer full credit for all unsold units or refuses the return of unsold goods. He also shows that when the manufacturer offers a partial credit for unsold commodities, supply chain coordination can be achieved in a multi-retailer environment.

Donohue (2000) extends the basic newsvendor model such that production can be performed in two different modes and demand forecast updating is possible. The selling season is divided into two periods. In the first period, demand predictions are uncertain. Nevertheless, demand forecast can be updated in the second period. The manufacturer can produce in two different modes: slow and fast. If the manufacturer produces in the slow mode, he should start the production in the first period since its lead time is long. However, production can also start in the second period in the fast mode, which is more expensive than slow production. In this study, it is found that a buyback contract with three parameters, which are a different wholesale price for each period and a return price, can coordinate the manufacturer and the distributor in this supply chain.

2.1.1.3 Revenue-sharing contract

Under a revenue-sharing contract, the supplier charges the retailer a wholesale price for each unit purchased and the retailer shares a percentage of his revenue with the supplier.

Dana and Spier (2001) consider the revenue-sharing contracts in video rental industry with perfectly competitive multiple retailers. They demonstrate that a revenue-sharing contract, combined with a low purchasing price from the supplier, can coordinate the supply chain by softening the retail price competition and encouraging the retailers for holding inventory.

Cachon and Lariviere (2005) study the strengths and limitations of revenue-sharing contracts in a general supply chain model. They point out that if the retail price is fixed, the revenue-sharing contract is equivalent to the buyback contract. However, while the buyback contracts cannot coordinate the newsvendor model with price-dependent demand, the revenue-sharing contracts satisfy coordination. The authors also show that a supply chain with multiple retailers competing on quantities can be coordinated using revenue-sharing contracts. Nevertheless, if retailers compete both on price and quantity, the supply chain cannot be coordinated. Another limitation of revenue-sharing contracts is their failure to coordinate a supply chain with effort-dependent demand.

2.1.1.4 Quantity-discount contract

In the quantity-discount contract, the supplier reduces the wholesale price when the retailer's purchase amount exceeds some quantity threshold. Two types of quantity discounts are generally used: all-units discount and incremental-units discount. In the former, the discount is applied to all units, whereas in the latter, the discount is applied only to the units above the threshold.

In the newsvendor model with effort-dependent demand, the retailer takes some actions to increase the demand of customers. Cachon (2003) demonstrates that the quantity-discount contract can coordinate this supply chain since both the cost and benefit of the effort concern only the retailer. He also points out that a quantity-discount contract can coordinate the newsvendor with both price-dependent and effort-dependent demand. In this case, since the retailer earns all the revenue, he optimizes the price and the effort. As the quantity-discount schedule is contingent on the optimal price and effort, the quantity decision is not distorted and the supply chain is coordinated.

Weng (2004) studies the coordination of the generalized newsvendor model with the objective of maximizing the system's expected profit. He develops quantity-discount policies for encouraging the buyer to order the coordinated quantity. He shows that the most important result of coordination is the reduction of the operating costs. Due to this reduction, the expected profit of the system is increased through coordination.

2.1.1.5 Quantity-flexibility contract

Under the quantity-flexibility contract, the supplier charges a wholesale price per unit purchased and gives the retailer full refund for a specified amount of unsold units. Quantity-flexibility contract differs from the buyback contract in that the former gives full protection on a specified portion of the retailer's order, whereas since the buyback price is smaller than the wholesale price, the latter partially protects the retailer's entire order (Cachon, 2003).

Tsay (1999) considers a supply chain consisting of a manufacturer and a retailer. The retailer provides a planning forecast of his intended purchase, but does not have to comply with his plan. Thus, he has the incentive of over forecasting his purchase amount to increase the manufacturer's production quantity. This behavior can also be anticipated by the manufacturer. The author uses a quantity-flexibility contract to coordinate such an inefficient supply chain. In the contract, the retailer commits not to purchase less than a certain percentage below his forecast and the manufacturer guarantees to deliver up to a certain percentage above. The author shows that supply chain coordination can be achieved with this contract under certain conditions.

Wu (2005) studies the coordination of a supply chain consisting of a manufacturer and a retailer under a quantity-flexibility contract. In this model, the retailer shares his demand forecast with the manufacturer. Accordingly, the manufacturer decides the production capacity. Then, using the Bayesian procedure, the retailer updates the demand information and commits on the purchase amount, which is constrained by the negotiated flexibility and the manufacturer's production capacity. The results denote that the retailer prefers more quantity flexibility, whereas the manufacturer usually benefits from smaller flexibility. Under the quantity-flexibility contract with Bayesian updating procedure, the manufacturer and the retailer can share the benefits from information updating.

2.1.1.6 Sales-rebate contract

In the sales-rebate contract, the supplier charges a wholesale price for each unit purchased and pays the retailer a rebate per unit sold beyond a specified target level. This is called target rebate. There are also linear rebates, in which the rebate is paid for each unit sold.

Taylor (2002) studies supply chain coordination with sales-rebate contracts. He points out that when demand is not effort-dependent, a target sales-rebate contract can ensure coordination and both the manufacturer and the retailer can benefit. Nevertheless, coordination cannot be achieved with a linear rebate contract since the retailer can increase his marginal revenue but the manufacturer bears the entire financial burden. The author also examines coordination with effort-dependent demand and finds that the supply chain can be coordinated under a properly designed target sales-rebate contract and buyback contract. However, these contracts cannot ensure coordination alone. In addition, both members can benefit under the defined scheme.

Zhang et al. (2005) consider the coordination of a loss-averse newsvendor. They examine several contracts, one of which is the target sales-rebate contract. They point out that the allocation of the profits is influenced by the retailer's risk preference when target sales-rebate contract is used. If the retailer is loss-averse, selecting the parameters of the contract is burdensome. Furthermore, since the retailer's profit will decline quickly without an effort to increase the demand, he will exert more effort under this contract.

2.1.1.7 Price-discount contract

Similar to the buyback contract, the price-discount contract has a wholesale price and a buyback rate. These contracts differ in that the contract terms are conditional on the chosen retail price in the price-discount contract (Cachon and Lariviere, 2005).

Bernstein and Federgruen (2005) study the coordination of a supply chain with a single supplier and multiple retailers with price-dependent demand. The authors examine both the competing and noncompeting retailer cases. They show that with a linear price-discount contract, the supply chain can be coordinated when the retailers are noncompeting. In the case of competitive retailers, coordination can also be achieved using the price-discount scheme by adding a nonlinear component.

The discriminating and important features of the studies mentioned in this part are displayed in Table 2.1.

Table 2.1: Research on the newsvendor model and its extensions.

Reference	Number of upstream stage members	Number of downstream stage members	Contract type	Additional features
Cachon (2004)	1	1	Wholesale-price	Many replenishment opportunities in a season
Debo and Sun (2005)	1	1	Wholesale-price	Repeated version
Pasternack (1985)	1	Multiple	Buyback	-
Donohue (2000)	1	1	Buyback	Two-mode production Demand forecast updating
Dana and Spier (2001)	1	Multiple	Revenue-sharing	Perfectly competitive multiple retailers
Cachon and Lariviere (2005)	1	1/Multiple	Revenue-sharing	Price-dependent demand Multiple retailers competing on quantities
Cachon (2003)	1	1	Quantity-discount	Price-dependent and effort-dependent demand
Weng (2004)	1	1	Quantity-discount	-
Tsay (1999)	1	1	Quantity-flexibility	Demand forecast sharing
Wu (2005)	1	1	Quantity-flexibility	Demand forecast sharing Bayesian updating
Taylor (2002)	1	1	Sales-rebate	Effort-dependent demand
Zhang et al. (2005)	1	1	Sales-rebate	Loss-averse newsvendor
Bernstein and Federgruen (2005)	1	Multiple	Price-discount	Price-dependent demand Competing/Noncompeting multiple retailers

In summary, there are several studies on the newsvendor model and its extensions in the literature. All the studies mentioned in this part consider a two-stage supply chain with a single upstream stage member, whereas the numbers of downstream stage members differ in the studies. All the studies are based on the newsvendor model, but they also have different additional features as given in Table 2.1. Moreover, they consider different contract types for the coordination of the supply chain.

Recall that the newsvendor model and its extensions are not in the main scope of this thesis. Nevertheless, the studies belonging to this area have some similarities with this thesis such that they also investigate the coordination of the supply chain via contracts and they also consider stochastic demand.

2.1.2 Stochastic models in an infinite horizon setting

The literature on stochastic models in an infinite horizon setting that investigates the coordination of the inventory policies in a decentralized supply chain can be mainly analyzed in two groups. Some of the studies consider an uncapacitated supply system and some of them deal with capacitated supply chains.

2.1.2.1 Uncapacitated supply chain

Lee and Whang (1999) study the coordination of decentralized multi-echelon supply chains. For the centralized multi-echelon inventory problem, Clark and Scarf (1960) define the optimal policy for finite planning horizons. They show that for a series system with uncertain demand, the echelon inventory order-up-to policy applied at each installation is optimal. In this policy, each installation always orders up to bring its echelon inventory position to the order-up-to level. Extension of these results for infinite horizons is performed by Federgruen and Zipkin (1984). However, since these results are valid for a centralized system, it is not possible to use them directly for a decentralized multi-echelon supply chain. In the model of Lee and Whang (1999), the members of the supply chain use echelon inventory order-up-to policies. Only the last downstream member is charged a backorder cost for not filling a customer order on time. Thus, upstream members are reluctant to hold stocks and the last downstream member has to account for carrying extra inventories. Since the end products incur the highest inventory holding costs, such a system is inefficient. The authors develop a nonlinear transfer payment contract to align the incentives of the different members in the supply chain.

Chen (1999) considers a decentralized multi-echelon supply chain subject to material and information delays. Each member in the supply chain is charged an inventory holding cost, but the backorder cost is incurred only at the last downstream member as in the model of Lee and Whang (1999). Although the members are from the same firm, they can only access to local inventory information. The author finds that it is optimal for each member to follow an installation base stock policy, i.e., installation inventory order-up-to policy, in which each stage orders up to bring its installation inventory position to the order-up-to level. The author then defines a linear incentive alignment scheme based on accounting inventory levels such that the system optimal solution also optimizes each member's own system.

Cachon and Zipkin (1999) investigate a two-stage serial supply chain consisting of a supplier and a retailer. Both members are charged their own holding costs and they share the backorder cost for not filling a customer order on time. Base stock policy is applied at both stages. The authors use a game-theoretic approach and consider two non-cooperative games: echelon inventory game and local inventory game. In the former, the firms track echelon inventory, whereas in the latter, they track local inventory. In both games, the supplier and the retailer simultaneously choose their base stock levels. Since it is found that the optimal solution is not a Nash equilibrium, the authors prepare a set of linear contracts such that the Nash equilibrium is same as the optimal solution, thus eliminating each member's incentive to deviate from the optimal strategy. The authors also study two Stackelberg games, in one of which the supplier is the leader and in the other one, the retailer is the leader.

Finally, Cachon (2001) studies a two-stage supply chain with a single supplier and multiple retailers. Both the supplier and the retailers hold inventory managed by reorder point policy. Each member is charged a holding cost for his own inventory and also a backorder cost. The author uses a game-theoretic approach and considers a supermodular game. As it is proved that the optimal reorder points are frequently not a Nash equilibrium, a coordination mechanism is needed. The author studies different coordination strategies: a set of contracts to change the players' incentives so that the optimal solution is a Nash equilibrium; switching to the lowest cost equilibrium when there are multiple Nash equilibria; and giving all control to the supplier by letting him to choose all reorder points. Among these strategies, only the equilibrium change does not guarantee the optimal solution.

2.1.2.2 Capacitated supply chain

Cachon (1999) examines a two-echelon supply chain consisting of a capacitated supplier and a retailer. Both stages can hold inventory and have their own holding costs. The supplier and the retailer use a base stock policy to manage their inventories. The transfer times between the supplier and the retailer are negligible. Backorders are not allowed in the system. Thus, assuming independent, Poisson distributed demand and independent, exponentially distributed processing times, the system is modeled as an $M/M/1/c$ make-to-stock queue. To analyze the decentralized system, the author considers a non-cooperative game, in which both the supplier and the retailer choose their base stock levels simultaneously. Since the Nash equilibrium is not identical to the optimal solution, the author investigates several contracts to coordinate the supply chain. The contracts contain one or more of the following elements: a retailer holding cost subsidy; a lost sales transfer payment; and inventory holding cost sharing. It is found that the most effective contract includes both a lost sales transfer payment and inventory holding cost sharing.

Caldentey and Wein (2003) study the coordination of a decentralized supply chain consisting of a capacitated supplier and a retailer. The finished goods inventory is carried by the retailer. The retailer specifies his inventory policy and the supplier chooses the capacity of his manufacturing facility. The retailer is charged a holding cost; the supplier is charged a cost for building capacity; and backorder cost is shared between them. The order cost is negligible in the model, thus the retailer uses an $(S-1,S)$ base stock policy. Under necessary assumptions, the supplier's production facility is modeled as an $M/M/1$ make-to-stock queue with a continuous-state approximation. The main difference between this model and the model of Cachon and Zipkin (1999) is that the production process is an infinite-server queue in the former since the supplier is uncapacitated, whereas a single-server queue in the latter. Similar to the study of Cachon and Zipkin (1999), Caldentey and Wein (2003) also use a game-theoretic framework by considering a non-cooperative game between the supplier and the retailer, where the retailer chooses his base stock level and the supplier chooses his capacity, simultaneously. As the Nash equilibrium is not equal to the optimal solution, the authors develop linear transfer payment schemes to coordinate the supply chain and they also study Stackelberg games.

Jemaï and Karaesmen (2004) investigate a decentralized supply chain consisting of a capacitated manufacturer and a retailer. Both members may keep inventory managed by base stock policy. Each member is responsible for his own holding cost and backorder cost is shared between them. As in Cachon (1999), the transportation times between the manufacturer and the retailer are negligible. With this assumption, rather than inventory positioning, pure inventory ownership becomes the focus of this study. Then, the system can be modeled as an $M/M/1$ make-to-stock queue assuming that the necessary conditions are satisfied. In contrast to the study of Caldentey and Wein (2003), a discrete-state space model is employed. The authors use a game-theoretic approach in this study. They investigate a non-cooperative game in which both members choose their base stock levels and they also examine Stackelberg games. It is found that the system is not coordinated at the Nash equilibrium except under special cases and a set of simple linear contracts are studied to coordinate the system.

Finally, Gupta and Weerawat (2006) study a manufacture-to-order system consisting of a component supplier and a manufacturer, which are make-to-stock and make-to-order systems, respectively. Processing is required at both stages, distinguishing the manufacturer from a retailer. Both the supplier and the manufacturer have production capacities. Backorders are allowed in the model. The supplier employs a base stock policy and the only decision variable is the base stock level of the supplier. Under necessary conditions, the supplier is modeled as an $M/M/1$ make-to-stock queue. Although the arrival of components to the manufacturer is not a renewal process, the manufacturer is also approximated as an $M/M/1$ queue to incorporate the congestion effects at the manufacturer's production facility. In this study, three different revenue functions are defined. In the first function, revenue is a linear function of realized (or average) lead time. The second function models quoted lead time and the third one models lost sales. The authors develop three different contracts for the coordination of the decentralized model. These are fixed-markup contract, simple revenue-sharing contract, and two-part revenue-sharing contract. In the simple revenue-sharing contract, the manufacturer is the Stackelberg game leader. He chooses the revenue-fraction first, and then the supplier chooses the base stock level. The authors refer to this contract as the Stackelberg equilibrium contract and use it as a benchmark. The results denote that for each of the revenue functions, the two-part revenue-sharing contract can coordinate the supply chain.

The distinctive and important features of the studies mentioned in this part are given in Table 2.2.

In summary, there is a limited number of studies on stochastic models in an infinite horizon setting that investigate the coordination of the inventory policies in a decentralized supply chain. Some of these studies consider an uncapacitated supply system and some of them deal with capacitated supply chains.

Among the studies that consider an uncapacitated supply system, some of them deal with multi-echelon supply chains, whereas some of them are interested in two-stage systems. Backorders are allowed in the system in all studies. Therefore, a lost sales model seems to be missing in this area. Only the studies that consider a two-stage supply chain use a game theoretic framework. Thus, a further research area can be to incorporate game theory in a multi-echelon system. Finally, the studies use different inventory control policies and investigate different contracts to coordinate the supply chain.

All the studies that deal with a capacitated system consider a two-stage supply chain with a single member at each stage. The other similarities between these studies are given as follows: The base stock policy is selected as the inventory control policy; a game theoretic framework is used in the models; and the capacitated member or members are modeled using queuing theory. In some of the studies both members hold inventory, whereas in some of them only one of the members holds inventory. There are models that consider lost sales and/or allowed backorders. Finally, the studies investigate different contracts to coordinate the supply chain. Consequently, a system consisting of multiple members at one of the stages of the supply chain is missing in this area. This thesis fills that gap in the literature by considering multiple suppliers in the system as presented in Table 2.2.

Table 2.2: Research on stochastic models in an infinite horizon setting.

Reference	Number of upstream stage members	Number of downstream stage members	Capacity type	Inventory control policy	Game theoretic extension	Queuing model	Contract type	Additional features
Lee and Whang (1999)	Multi-echelon		Uncapacitated	Echelon inventory order-up-to policy	-	-	Nonlinear transfer payment	Backorders allowed
Chen (1999)	Multi-echelon		Uncapacitated	Installation inventory order-up-to policy	-	-	Linear incentive alignment scheme based on accounting inventory levels	Backorders allowed
Cachon and Zipkin (1999)	1	1	Uncapacitated	Base stock policy	Non-cooperative Stackelberg	-	Linear transfer payment	Backorders allowed Both members hold inventory Both members choose base stock levels
Cachon (2001)	1	Multiple	Uncapacitated	Reorder point policy	Supermodular	-	Three different coordination strategies	Backorders allowed All members hold inventory All members choose reorder points
Cachon (1999)	1	1	Capacitated	Base stock policy	Non-cooperative	$M/M/1/c$ make-to-stock queue	A contract including a lost sales transfer payment and inventory holding cost sharing	Lost sales Both members choose base stock levels
Caldentey and Wein (2003)	1	1	Capacitated	Base stock policy	Non-cooperative Stackelberg	$M/M/1$ make-to-stock queue (continuous-state approximation)	Linear transfer payment	Backorders allowed Only retailer holds inventory Retailer chooses base stock level Supplier chooses capacity
Jemaï and Karaesmen (2004)	1	1	Capacitated	Base stock policy	Non-cooperative Stackelberg	$M/M/1$ make-to-stock queue (discrete-state space)	Linear transfer payment	Backorders allowed Both members choose base stock levels
Gupta and Weerawat (2006)	1	1	Capacitated	Base stock policy	Stackelberg	Supplier: $M/M/1$ make-to-stock queue Manufacturer: $M/M/1$ queue	Two-part revenue-sharing contract	Backorders allowed / Lost sales Manufacture-to-order system Revenue is a function of lead time
This thesis	**Multiple**	**1**	**Capacitated**	**Base stock policy**	**-**	**Suppliers: $M/M/1$ make-to-stock queues** **Manufacturer: $GI/M/1$ queue**	**Three different transfer payment contracts**	**Backorders allowed** **Manufacture-to-order system**

In the system considered, the suppliers operate on a make-to-stock basis. Therefore, the literature on make-to-stock system models is briefly reviewed in the following section.

2.2 The Literature on Make-to-Stock System Models

The models of make-to-stock systems have been investigated in the literature especially in the last two decades. Most of the studies use approximations to model queuing networks consisting of make-to-stock queues.

Lee and Zipkin (1992) study a tandem queuing model, in which each stage holds its own inventory. In other words, their system is a tandem queuing network consisting of make-to-stock queues. Base stock policy is applied at each stage. Assuming that demands occur according to a Poisson process and unit production times are exponentially distributed, the authors approximate the point process describing the release of units from a stage by a Poisson process. Then, each stage behaves like an $M/M/1$ queue. They also define some performance measures such as average customer backorders outstanding, average work-in-process inventory, and average finished-goods inventory. Comparing the approximation estimates with the simulation results for two-stage and three-stage systems, they conclude that the approximation appears to be quite accurate.

Buzacott et al. (1992) investigate a manufacturing system consisting of a number of stages in series. Each stage holds inventory and has limited capacity. The authors consider both MRP and base stock policy to initiate the work release to each stage. Based on a sample path analysis, they develop bounds and approximations for shipment delays. Under the assumptions of Poisson demand process and exponential service times, they approximate the congestion at the second stage of a two-stage base stock system using an $M/M/1$ queue. The authors also derive the distribution of the time between releases to the second stage and they develop an alternative approximation using a $GI/M/1$ queuing model. Comparing the estimates of $M/M/1$ and $GI/M/1$ approximations with the simulation results denotes that the $GI/M/1$ queuing model improves the accuracy of the predictions.

Bai et al. (2004) derive the interdeparture time distributions for make-to-stock queues controlled via base stock policy, i.e., base stock inventory queues. Using

Palm probabilities, they relate the distribution of interdeparture times to residual arrival time of demands and residual time for a production completion. The main findings of their study are the interdeparture time probability distributions and squared coefficient of variations for the base stock inventory queues with birth and death production processes, such as $M/M/1$, $M/M/c$, and $M/M/\infty$ inventory queues.

Finally, Gupta and Selvaraju (2006) study capacitated serial supply systems, in which each stage holds inventory managed according to a base stock policy. The authors propose a modification to the approximations of Lee and Zipkin (1992) and Buzacott et al. (1992). Based on their approximation, they derive performance measures such as average number of units that need to be processed at the second stage, average inventory at each stage, and average number of backorders outstanding for a two-stage system. They also investigate systems with more than two stages. The authors then define a near-exact matrix-geometric procedure to compare their approximation with the others. Numerical tests denote that their approximation gives better results. They also study the optimization of the policy parameters.

To summarize, the supply chain contracting literature related to the coordination in decentralized supply chains with stochastic demand and the literature on make-to-stock system models have been reviewed in this chapter. After reviewing the literature, the study on the coordination of the decentralized supply chain begins by modeling each member as a queuing system. The queuing model is presented in the next chapter.

3. THE QUEUING MODEL

The supply chain considered in this thesis has two stages consisting of multiple independent suppliers and a manufacturer with limited production capacities. Let the number of suppliers be n, where $n \geq 2$. The suppliers operate on a make-to-stock basis and apply base stock policy to manage their inventories. Let S_i be the base stock level of supplier i for $i = 1,\ldots,n$. No inventory is held by the manufacturer, i.e., the manufacturer employs a make-to-order strategy.

In the system taken into consideration, the end customer demands occur according to a Poisson process with rate λ. The service times of supplier i are independent and identically distributed (i.i.d.) random variables having an exponential distribution with rate μ_i for $i = 1,\ldots,n$. The manufacturer has also i.i.d. and exponentially distributed service times with rate μ_M. Let ρ_i and ρ_M be the traffic intensity of supplier i and the manufacturer, respectively, where traffic intensity can be defined as the ratio of the arrival rate to the service rate. For the stability of the system, it is assumed that $0 < \rho_i < 1$ for all $i = 1,\ldots,n$ and $0 < \rho_M < 1$. See Appendix A for a complete list of all assumptions made in this thesis.

Under the conditions defined above, each supplier can be modeled as an $M/M/1$ make-to-stock queue. On the other hand, the interarrival time distribution of the manufacturer has to be derived to model the manufacturer as a queuing system. This distribution is obtained by Buzacott et al. (1992) in the case of a single supplier.

In the following part, the derivation of the manufacturer's interarrival time distribution for a system with one supplier is represented in a similar way to the study of Buzacott et al. (1992). In addition, the interarrival time distribution in the case of two suppliers is derived. Also, an approximate distribution is developed for a system with two or more suppliers.

3.1 Interarrival Time Distribution of the Manufacturer

3.1.1 Exact distribution in the case of one supplier

In the general case, let $N_i(t)$ be the number of jobs in supplier *i*'s system at time *t*; $I_i(t)$ be the inventory level of supplier *i* at time *t*; and $B_i(t)$ be the outstanding backorders at supplier *i* at time *t* for $i = 1, \ldots, n$. Under the previously defined conditions, the first stage of the supply chain behaves like an $M/M/1$ queue. Then, the number of jobs in supplier *i*'s system $\{N_i(t), t \geq 0\}$ forms a birth and death process. Solving the probability flow balance equations, we obtain

$$P_m^{(i)} = \lim_{t\to\infty} P\{N_i(t) = m\} = (1-\rho_i)\rho_i^m, \quad i = 1, \ldots, n, \quad m = 0, 1, \ldots, \qquad \textbf{(3.1)}$$

where $P_m^{(i)}$ denotes the steady-state probability that the number of jobs in supplier *i*'s system is equal to *m*.

In the case of one supplier, i.e., $i = 1$, the supplier can be in one of the three states immediately after a component has been released to the manufacturer: $0 < I_1(t) < S_1$; $I_1(t) = B_1(t) = 0$; and $B_1(t) > 0$. To find the distribution of the interdeparture times of the supplier, i.e., the interarrival times of the manufacturer, the steady-state probabilities of these three states have to be calculated first.

Lemma 3.1. In the case of one supplier, the states $0 < I_1(t) < S_1$, $I_1(t) = B_1(t) = 0$, and $B_1(t) > 0$ immediately after a component has been released to the manufacturer have the steady-state probabilities $\pi_{(.)}$ given by

$$\pi_{0<I_1(t)<S_1} = 1 - \rho_1^{S_1 - 1}, \qquad \textbf{(3.2)}$$

$$\pi_{I_1(t)=B_1(t)=0} = (1-\rho_1^2)\rho_1^{S_1 - 1}, \qquad \textbf{(3.3)}$$

and

$$\pi_{B_1(t)>0} = \rho_1^{S_1+1}, \qquad \textbf{(3.4)}$$

respectively.

Proof. Let event D denote the departure of a component from the supplier. Then,

$$\begin{aligned}
\pi_{0<I_1(t)<S_1} &= \lim_{t\to\infty} P\left\{0 < I_1(t) < S_1 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{0 < N_1(t) < S_1 \mid D\right\} \\
&= \lim_{t\to\infty} \frac{P\left\{0 < N_1(t) < S_1, D\right\}}{P\{D\}} \\
&= \sum_{m=1}^{S_1-1} \frac{\lambda P_{m-1}^{(1)}}{\lambda} \\
&= 1-\rho_1^{S_1-1},
\end{aligned}$$

proving equation (3.2).

$$\begin{aligned}
\pi_{I_1(t)=B_1(t)=0} &= \lim_{t\to\infty} P\left\{I_1(t) = B_1(t) = 0 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = S_1 \mid D\right\} \\
&= \lim_{t\to\infty} \frac{P\left\{N_1(t) = S_1, D\right\}}{P\{D\}} \\
&= \frac{\lambda P_{S_1-1}^{(1)} + \mu_1 P_{S_1+1}^{(1)}}{\lambda} \\
&= \left(1-\rho_1^2\right)\rho_1^{S_1-1},
\end{aligned}$$

proving equation (3.3). Finally,

$$\begin{aligned}
\pi_{B_1(t)>0} &= \lim_{t\to\infty} P\left\{B_1(t) > 0 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) \geq S_1 + 1 \mid D\right\} \\
&= \lim_{t\to\infty} \frac{P\left\{N_1(t) \geq S_1 + 1, D\right\}}{P\{D\}} \\
&= \sum_{m=S_1+1}^{\infty} \frac{\mu_1 P_{m+1}^{(1)}}{\lambda} \\
&= \rho_1^{S_1+1},
\end{aligned}$$

proving equation (3.4). Thus, the proof of Lemma 3.1 is completed. □

Theorem 3.1. In the case of one supplier, the probability density function $f_{(.)}(t)$ of the interdeparture times of the supplier, i.e., the interarrival times of the manufacturer, is given by

$$f_A(t)=\lambda e^{-\lambda t}\left(1-\rho_1^{S_1+1}\right)+\mu_1 e^{-\mu_1 t}\rho_1^{S_1-1}-\left(\lambda+\mu_1\right)e^{-(\lambda+\mu_1)t}\left(1-\rho_1^2\right)\rho_1^{S_1-1}, \quad \textbf{(3.5)}$$

where A denotes the interarrival time of the manufacturer.

Proof. Recall that the single supplier can be in one of the three states just after a component has been released to the manufacturer. If $0<I_1(t)<S_1$ immediately after a release, the time to the next release equals to the time between demands. In the $I_1(t)=B_1(t)=0$ case, the next release occurs after the maximum of time between demands and time until the next service completion. Finally, if $B_1(t)>0,$ the time to the next release is equal to the time until the next service finishes.

Now, let X denote the time between demands and Y_i denote the time until the next service completion for supplier i, where $i=1,\ldots,n.$ Then, the probability density functions of X and Y_i are

$$f_X(t)=\lambda e^{-\lambda t} \quad \textbf{(3.6)}$$

and

$$f_{Y_i}(t)=\mu_i e^{-\mu_i t}, \quad i=1,\ldots,n, \quad \textbf{(3.7)}$$

respectively. Assuming independence between X and $Y_i,$ the probability density function of their maximum can be calculated as

$$f_{\max(X,Y_i)}(t)=\lambda e^{-\lambda t}+\mu_i e^{-\mu_i t}-\left(\lambda+\mu_i\right)e^{-(\lambda+\mu_i)t}, \quad i=1,\ldots,n. \quad \textbf{(3.8)}$$

Accordingly, in the case of one supplier, the interarrival times of the manufacturer have the probability density function given by

$$f_A(t)=f_X(t)\pi_{0<I_1(t)<S_1}+f_{\max(X,Y_1)}(t)\pi_{I_1(t)=B_1(t)=0}+f_{Y_1}(t)\pi_{B_1(t)>0}. \quad \textbf{(3.9)}$$

Substituting equations (3.2)-(3.4) and (3.6)-(3.8) into equation (3.9) completes the proof of Theorem 3.1. □

From equation (3.5), the expected value of the interarrival times of the manufacturer is calculated as

$$E[A]=\frac{1}{\lambda}, \quad \textbf{(3.10)}$$

and the variance is

$$\operatorname{Var}(A) = \frac{1}{\lambda^2}\left(1 - 2\rho_1^{S_1+1}\frac{1-\rho_1}{1+\rho_1}\right). \tag{3.11}$$

Hence, the squared coefficient of variation, which is the ratio of the variance to the square of the expected value, is given by

$$C_A^2 = 1 - 2\rho_1^{S_1+1}\frac{1-\rho_1}{1+\rho_1}. \tag{3.12}$$

3.1.2 Exact distribution in the case of two suppliers

In the case of two suppliers, the suppliers can be in one of the thirteen states immediately after both components have been released to the manufacturer. These states, their steady-state probabilities, and the time to the next release in each case are given below. While calculating the steady-state probabilities, the states of the suppliers are assumed to be conditionally independent from each other given both components have been departed from the suppliers.

i. For state $I_1(t) = S_1, B_2(t) > 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{I_1(t)=S_1, B_2(t)>0} &= \lim_{t\to\infty} P\left\{I_1(t) = S_1, B_2(t) > 0 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = 0, N_2(t) \geq S_2 + 1 \mid D\right\} \\
&= P_0^{(1)} \sum_{m=S_2+1}^{\infty} \frac{\mu_2 P_{m+1}^{(2)}}{\lambda} \\
&= (1-\rho_1)\rho_2^{S_2+1},
\end{aligned} \tag{3.13}$$

and the time to the next release is equal to Y_2.

ii. For state $I_1(t) = S_1, I_2(t) = B_2(t) = 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{I_1(t)=S_1, I_2(t)=B_2(t)=0} &= \lim_{t\to\infty} P\left\{I_1(t) = S_1, I_2(t) = B_2(t) = 0 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = 0, N_2(t) = S_2 \mid D\right\} \\
&= P_0^{(1)} \frac{\mu_2 P_{S_2+1}^{(2)}}{\lambda} \\
&= (1-\rho_1)(1-\rho_2)\rho_2^{S_2},
\end{aligned} \tag{3.14}$$

and the time to the next release is equal to the maximum of *X* and Y_2.

iii. For state $0 < I_1(t) < S_1, 0 < I_2(t) < S_2,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{0<I_1(t)<S_1,0<I_2(t)<S_2} &= \lim_{t\to\infty} P\{0 < I_1(t) < S_1, 0 < I_2(t) < S_2 \mid D\} \\
&= \lim_{t\to\infty} P\{0 < N_1(t) < S_1, 0 < N_2(t) < S_2 \mid D\} \\
&= \sum_{m=1}^{S_1-1} \frac{\lambda P_{m-1}^{(1)}}{\lambda} \sum_{m=1}^{S_2-1} \frac{\lambda P_{m-1}^{(2)}}{\lambda} \\
&= \left(1-\rho_1^{S_1-1}\right)\left(1-\rho_2^{S_2-1}\right),
\end{aligned} \tag{3.15}$$

and the time to the next release is equal to *X*.

iv. For state $0 < I_1(t) < S_1, I_2(t) = B_2(t) = 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{0<I_1(t)<S_1,I_2(t)=B_2(t)=0} &= \lim_{t\to\infty} P\{0 < I_1(t) < S_1, I_2(t) = B_2(t) = 0 \mid D\} \\
&= \lim_{t\to\infty} P\{0 < N_1(t) < S_1, N_2(t) = S_2 \mid D\} \\
&= \sum_{m=1}^{S_1-1} \frac{\lambda P_{m-1}^{(1)}}{\lambda} \frac{\lambda P_{S_2-1}^{(2)}}{\lambda} + \sum_{m=1}^{S_1-1} P_m^{(1)} \frac{\mu_2 P_{S_2+1}^{(2)}}{\lambda} \\
&= \left(1-\rho_1^{S_1-1}\right)(1-\rho_2)\rho_2^{S_2-1} + \rho_1\left(1-\rho_1^{S_1-1}\right)(1-\rho_2)\rho_2^{S_2},
\end{aligned} \tag{3.16}$$

and the time to the next release is equal to the maximum of *X* and Y_2.

v. For state $0 < I_1(t) < S_1, B_2(t) > 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{0<I_1(t)<S_1,B_2(t)>0} &= \lim_{t\to\infty} P\{0 < I_1(t) < S_1, B_2(t) > 0 \mid D\} \\
&= \lim_{t\to\infty} P\{0 < N_1(t) < S_1, N_2(t) \geq S_2 + 1 \mid D\} \\
&= \sum_{m=1}^{S_1-1} P_m^{(1)} \sum_{m=S_2+1}^{\infty} \frac{\mu_2 P_{m+1}^{(2)}}{\lambda} \\
&= \rho_1\left(1-\rho_1^{S_1-1}\right)\rho_2^{S_2+1},
\end{aligned} \tag{3.17}$$

and the time to the next release is equal to Y_2.

vi. For state $I_1(t) = B_1(t) = 0, 0 < I_2(t) < S_2$, the steady-state probability is given by

$$\begin{aligned}
\pi_{I_1(t)=B_1(t)=0,0<I_2(t)<S_2} &= \lim_{t\to\infty} P\left\{I_1(t) = B_1(t) = 0, 0 < I_2(t) < S_2 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = S_1, 0 < N_2(t) < S_2 \mid D\right\} \\
&= \frac{\lambda P_{S_1-1}^{(1)}}{\lambda} \sum_{m=1}^{S_2-1} \frac{\lambda P_{m-1}^{(2)}}{\lambda} + \frac{\mu_1 P_{S_1+1}^{(1)}}{\lambda} \sum_{m=1}^{S_2-1} P_m^{(2)} \\
&= (1-\rho_1)\rho_1^{S_1-1}\left(1-\rho_2^{S_2-1}\right) + (1-\rho_1)\rho_1^{S_1}\rho_2\left(1-\rho_2^{S_2-1}\right),
\end{aligned} \tag{3.18}$$

and the time to the next release is equal to the maximum of *X* and Y_1.

vii. For state $I_1(t) = B_1(t) = 0, I_2(t) = S_2$, the steady-state probability is given by

$$\begin{aligned}
\pi_{I_1(t)=B_1(t)=0,I_2(t)=S_2} &= \lim_{t\to\infty} P\left\{I_1(t) = B_1(t) = 0, I_2(t) = S_2 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = S_1, N_2(t) = 0 \mid D\right\} \\
&= \frac{\mu_1 P_{S_1+1}^{(1)}}{\lambda} P_0^{(2)} \\
&= (1-\rho_1)\rho_1^{S_1}(1-\rho_2),
\end{aligned} \tag{3.19}$$

and the time to the next release is equal to the maximum of *X* and Y_1.

viii. For state $I_1(t) = B_1(t) = 0, I_2(t) = B_2(t) = 0$, the steady-state probability is given by

$$\begin{aligned}
\pi_{I_1(t)=B_1(t)=0,I_2(t)=B_2(t)=0} &= \lim_{t\to\infty} P\left\{I_1(t) = B_1(t) = 0, I_2(t) = B_2(t) = 0 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = S_1, N_2(t) = S_2 \mid D\right\} \\
&= \frac{\lambda P_{S_1-1}^{(1)}}{\lambda}\frac{\lambda P_{S_2-1}^{(2)}}{\lambda} + \frac{\mu_1 P_{S_1+1}^{(1)}}{\lambda}\frac{\mu_2 P_{S_2+1}^{(2)}}{\lambda} \\
&= (1-\rho_1)\rho_1^{S_1-1}(1-\rho_2)\rho_2^{S_2-1} + (1-\rho_1)\rho_1^{S_1}(1-\rho_2)\rho_2^{S_2},
\end{aligned} \tag{3.20}$$

and the time to the next release is equal to the maximum of *X*, Y_1, and Y_2.

ix. For state $I_1(t) = B_1(t) = 0, B_2(t) > 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{I_1(t)=B_1(t)=0,B_2(t)>0} &= \lim_{t\to\infty} P\left\{I_1(t) = B_1(t) = 0, B_2(t) > 0 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) = S_1, N_2(t) \geq S_2 + 1 \mid D\right\} \\
&= \frac{\mu_1 P_{S_1+1}^{(1)}}{\lambda} \sum_{m=S_2+1}^{\infty} \frac{\mu_2 P_{m+1}^{(2)}}{\lambda} \\
&= (1-\rho_1)\rho_1^{S_1}\rho_2^{S_2+1},
\end{aligned} \tag{3.21}$$

and the time to the next release is equal to Y_2.

x. For state $B_1(t) > 0, 0 < I_2(t) < S_2,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{B_1(t)>0,0<I_2(t)<S_2} &= \lim_{t\to\infty} P\left\{B_1(t) > 0, 0 < I_2(t) < S_2 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) \geq S_1 + 1, 0 < N_2(t) < S_2 \mid D\right\} \\
&= \sum_{m=S_1+1}^{\infty} \frac{\mu_1 P_{m+1}^{(1)}}{\lambda} \sum_{m=1}^{S_2-1} P_m^{(2)} \\
&= \rho_1^{S_1+1}\rho_2\left(1-\rho_2^{S_2-1}\right),
\end{aligned} \tag{3.22}$$

and the time to the next release is equal to Y_1.

xi. For state $B_1(t) > 0, I_2(t) = S_2,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{B_1(t)>0,I_2(t)=S_2} &= \lim_{t\to\infty} P\left\{B_1(t) > 0, I_2(t) = S_2 \mid D\right\} \\
&= \lim_{t\to\infty} P\left\{N_1(t) \geq S_1 + 1, N_2(t) = 0 \mid D\right\} \\
&= \sum_{m=S_1+1}^{\infty} \frac{\mu_1 P_{m+1}^{(1)}}{\lambda} P_0^{(2)} \\
&= \rho_1^{S_1+1}(1-\rho_2),
\end{aligned} \tag{3.23}$$

and the time to the next release is equal to Y_1.

xii. For state $B_1(t) > 0, I_2(t) = B_2(t) = 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{B_1(t)>0, I_2(t)=B_2(t)=0} &= \lim_{t\to\infty} P\{B_1(t) > 0, I_2(t) = B_2(t) = 0 \mid D\} \\
&= \lim_{t\to\infty} P\{N_1(t) \geq S_1 + 1, N_2(t) = S_2 \mid D\} \\
&= \sum_{m=S_1+1}^{\infty} \frac{\mu_1 P_{m+1}^{(1)}}{\lambda} \frac{\mu_2 P_{S_2+1}^{(2)}}{\lambda} \\
&= \rho_1^{S_1+1}(1-\rho_2)\rho_2^{S_2},
\end{aligned} \tag{3.24}$$

and the time to the next release is equal to Y_1.

xiii. For state $B_1(t) > 0, B_2(t) > 0,$ the steady-state probability is given by

$$\begin{aligned}
\pi_{B_1(t)>0, B_2(t)>0} &= \lim_{t\to\infty} P\{B_1(t) > 0, B_2(t) > 0 \mid D\} \\
&= \lim_{t\to\infty} P\{N_1(t) \geq S_1 + 1, N_2(t) \geq S_2 + 1 \mid D\} \\
&= \sum_{m=S_1+1}^{\infty} \frac{\mu_1 P_{m+1}^{(1)}}{\lambda} \sum_{m=S_2+1}^{\infty} \frac{\mu_2 P_{m+1}^{(2)}}{\lambda} \\
&= \rho_1^{S_1+1}\rho_2^{S_2+1},
\end{aligned} \tag{3.25}$$

and the time to the next release is equal to the maximum of Y_1 and Y_2.

Theorem 3.2. In the case of two suppliers, the probability density function of the interarrival times of the manufacturer is given by

$$\begin{aligned}
f_A(t) &= \lambda e^{-\lambda t}\left(\rho_1^{S_1+1} - 1\right)\left(\rho_2^{S_2+1} - 1\right) \\
&\quad + \mu_1 e^{-\mu_1 t}\rho_1^{S_1-1}\left(1 + (\rho_1 - 1)\rho_2^{S_2}(1 + \rho_1\rho_2)\right) \\
&\quad + \mu_2 e^{-\mu_2 t}\rho_2^{S_2-1}\left(1 + (\rho_2 - 1)\rho_1^{S_1}(1 + \rho_1\rho_2)\right) \\
&\quad - (\lambda + \mu_1)e^{-(\lambda+\mu_1)t}\rho_1^{S_1-1}\left(1 - \rho_1^2 + \rho_2^{S_2}(\rho_1 - 1)(1 + \rho_1\rho_2)\right) \\
&\quad - (\lambda + \mu_2)e^{-(\lambda+\mu_2)t}\rho_2^{S_2-1}\left(1 - \rho_2^2 + \rho_1^{S_1}(\rho_2 - 1)(1 + \rho_1\rho_2)\right) \\
&\quad - (\mu_1 + \mu_2)e^{-(\mu_1+\mu_2)t}\rho_1^{S_1-1}\rho_2^{S_2-1}\left((\rho_1 - 1)(\rho_2 - 1)(1 + \rho_1\rho_2) + \rho_1^2\rho_2^2\right) \\
&\quad + (\lambda + \mu_1 + \mu_2)e^{-(\lambda+\mu_1+\mu_2)t}\rho_1^{S_1-1}\rho_2^{S_2-1}(\rho_1 - 1)(\rho_2 - 1)(1 + \rho_1\rho_2).
\end{aligned} \tag{3.26}$$

Proof. The probability density functions of X, Y_i, and their maximum are as given in equations (3.6)-(3.8), where $i = 1, 2$ in the case of two suppliers. In addition, the probability density function of the maximum of Y_1 and Y_2 is calculated as

$$f_{\max(Y_1,Y_2)}(t) = \mu_1 e^{-\mu_1 t} + \mu_2 e^{-\mu_2 t} - (\mu_1 + \mu_2)e^{-(\mu_1+\mu_2)t}, \tag{3.27}$$

and the probability density function of the maximum of X, Y_1, and Y_2 is given by

$$f_{\max(X,Y_1,Y_2)}(t) = \lambda e^{-\lambda t} + \mu_1 e^{-\mu_1 t} + \mu_2 e^{-\mu_2 t} - (\lambda+\mu_1) e^{-(\lambda+\mu_1)t} - (\lambda+\mu_2) e^{-(\lambda+\mu_2)t} \\ - (\mu_1+\mu_2) e^{-(\mu_1+\mu_2)t} + (\lambda+\mu_1+\mu_2) e^{-(\lambda+\mu_1+\mu_2)t}, \quad \textbf{(3.28)}$$

assuming that X and Y_i's are independent from each other. Then, similar to the proof of Theorem 3.1, using equations (3.13)-(3.25) and the probability density functions of the time to the next release for each case, it is not difficult to show that the interarrival times of the manufacturer have the probability density function as given in equation (3.26). □

3.1.3 The approximate distribution

Deriving the distribution of the interarrival times of the manufacturer becomes mathematically intractable as the number of suppliers gets larger. This brings forth the need of an approximate distribution.

The manufacturer cannot start production until all components arrive. Hence, the supplier with the minimum base stock level is expected to affect the interarrival times of the manufacturer the most. Thus, inspired by Theorem 3.1, an appropriate approximation for the probability density function of the interarrival times of the manufacturer can be given by

$$f_A(t) \simeq \lambda e^{-\lambda t}\left(1-\rho_j^{S_j+1}\right) + \mu_j e^{-\mu_j t} \rho_j^{S_j-1} - \left(\lambda+\mu_j\right) e^{-(\lambda+\mu_j)t}\left(1-\rho_j^2\right)\rho_j^{S_j-1}, \quad \textbf{(3.29)}$$

where supplier j is the one with the minimum base stock level among all suppliers[1], i.e., $j = \underset{i=1,\dots,n}{\arg\min}\, S_i$; ρ_j is the traffic intensity of supplier j; and μ_j is the service rate of supplier j.

Then, from equation (3.29), the approximate squared coefficient of variation is calculated as

$$C_A^2 \simeq 1 - 2\rho_j^{S_j+1}\frac{1-\rho_j}{1+\rho_j}. \quad \textbf{(3.30)}$$

[1] If more than one supplier has the minimum base stock level, then supplier j is the one having the highest traffic intensity among these suppliers.

For testing the precision of the approximate interarrival time distribution of the manufacturer presented in equation (3.29), simulation models are developed in the case of two, three, and four suppliers as described in Appendix B. The results (see Tables B.1-B.3) denote that the approximate distribution fits the interarrival time data of the manufacturer in 79 of the 81 cases, giving an error of just 2.47%. Since the error of the approximate distribution is reasonable, it is concluded that the interarrival time distribution of the manufacturer can be approximated as given in equation (3.29).

3.2 The Model and the Performance Measures

Recall that in the system taken into consideration, the end customer demands occur according to a Poisson process and the service times of the suppliers are i.i.d. and exponentially distributed random variables. Under these conditions, each supplier can be modeled as an $M/M/1$ make-to-stock queue. Furthermore, the performance measures of interest are the average outstanding backorders and the average inventory level of each supplier.

The probability distributions of the outstanding backorders and the inventory level of supplier *i* for $i = 1, \ldots, n$ are given by

$$P\{B_i = k\} = \lim_{t \to \infty} P\{B_i(t) = k\} = \begin{cases} \sum_{m=0}^{S_i} P_m^{(i)} = 1 - \rho_i^{S_i+1}, & k = 0 \\ P_{S_i+k}^{(i)} = (1-\rho_i)\rho_i^{S_i+k}, & k = 1, 2, \ldots \end{cases} \qquad \textbf{(3.31)}$$

and

$$P\{I_i = k\} = \lim_{t \to \infty} P\{I_i(t) = k\} = \begin{cases} \sum_{m=S_i}^{\infty} P_m^{(i)} = \rho_i^{S_i}, & k = 0 \\ P_{S_i-k}^{(i)} = (1-\rho_i)\rho_i^{S_i-k}, & k = 1, 2, \ldots, S_i, \end{cases} \qquad \textbf{(3.32)}$$

respectively.

Hence, the average outstanding backorders at supplier *i* can be calculated as

$$E[B_i] = \frac{\rho_i^{S_i+1}}{1-\rho_i}, \quad i = 1, \ldots, n, \qquad \textbf{(3.33)}$$

and the average inventory level of supplier i is given by

$$E[I_i] = S_i - \frac{\rho_i\left(1-\rho_i^{S_i}\right)}{1-\rho_i}, \quad i = 1,\ldots,n. \tag{3.34}$$

On the other hand, under the assumption that arrivals to the manufacturer form a renewal process, the manufacturer can be modeled as a $GI/M/1$ queue with the interarrival time distribution given in equation (3.29). Moreover, the performance measures of interest are the average number of jobs in the manufacturer's system and the average outstanding backorders at the manufacturer.

Shanthikumar and Buzacott (1980) investigate approximations for the mean number of jobs in $GI/G/1$ queuing systems. These approximations require only the squared coefficient of variations of the interarrival and service times, denoted by C_A^2 and C_S^2, respectively. The authors recommend different approximations for the various values of C_A^2 and C_S^2 as given below:

i. The approximation of Krämer and Langenbach-Belz (1976):

$$E[N_M] \simeq \rho_M + \frac{\rho_M^2\left(C_A^2 + C_S^2\right)}{2\left(1-\rho_M\right)} g\left(C_A^2, C_S^2, \rho_M\right), \tag{3.35}$$

where N_M denotes the number of jobs in the manufacturer's system and

$$g\left(C_A^2, C_S^2, \rho_M\right) = \begin{cases} \exp\left(\dfrac{-2\left(1-\rho_M\right)\left(1-C_A^2\right)^2}{3\rho_M\left(C_A^2+C_S^2\right)}\right), & C_A^2 \le 1 \\ \exp\left(-\left(1-\rho_M\right)\dfrac{\left(C_A^2-1\right)}{C_A^2+4C_S^2}\right), & C_A^2 \ge 1. \end{cases}$$

ii. The approximation of Marchal (1976):

$$E[N_M] \simeq \rho_M + \left(\frac{\rho_M^2\left(1+C_S^2\right)}{1+\rho_M^2 C_S^2}\right)\left(\frac{C_A^2 + \rho_M^2 C_S^2}{2\left(1-\rho_M\right)}\right). \tag{3.36}$$

iii. The approximation of Page (1972) by adding a slight modification to the original formula:

$$E[N_M] \simeq \rho_M + \frac{\rho_M^2}{2(1-\rho_M)}\left(C_A^2\left(1+C_S^2\right)+C_S^2\left(1-C_A^2\right)\exp\left(\frac{-2(1-\rho_M)}{3\rho_M}\right)\right). \qquad \textbf{(3.37)}$$

In addition, there are two other approximations for the mean number of jobs in a $GI/G/1$ queuing system presented by Buzacott and Shanthikumar (1993). These approximations are given by

$$E[N_M] \simeq \rho_M + \left(\frac{\rho_M\left(1+C_S^2\right)}{2-\rho_M+\rho_M C_S^2}\right)\left(\frac{\rho_M(2-\rho_M)C_A^2+\rho_M^2 C_S^2}{2(1-\rho_M)}\right) \qquad \textbf{(3.38)}$$

and

$$E[N_M] \simeq \rho_M + \frac{\rho_M^2\left(C_A^2+C_S^2\right)}{2(1-\rho_M)} + \frac{\left(1-C_A^2\right)C_A^2\rho_M}{2}. \qquad \textbf{(3.39)}$$

For selecting the best-fit approximation for the average number of jobs in the manufacturer's system among the approximations given in equations (3.35)-(3.39), simulation models are developed in the case of two, three, and four suppliers as described in Appendix C. Afterwards, the errors between the approximations and the simulation results are calculated. The results (see Tables C.1-C.3) denote that in the case of two, three, and four suppliers, Marchal (1976)'s approximation given in equation (3.36) has the minimum average errors of 2.74%, 3.28%, and 4.09%, respectively. Since these errors are in acceptable ranges, the approximation of Marchal (1976) is selected for the average number of jobs in the manufacturer's system.

As the service times of the manufacturer are exponentially distributed, C_S^2 is equal to one. By substituting C_A^2 calculated from equation (3.30) and $C_S^2 = 1$ into equation (3.36), the average number of jobs in the manufacturer's system is calculated as

$$E[N_M] \simeq \rho_M + \left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)(1-\rho_M)}\right), \qquad \textbf{(3.40)}$$

where $j = \arg\min_{i=1,\dots,n} S_i$.

As stated before, another performance measure of interest is the average outstanding backorders at the manufacturer. Since the manufacturer holds no inventory, the average outstanding backorders is equal to the average number of jobs in the manufacturer's queue, the derivation of which is given below.

From equation (3.29), the mean interarrival time of the manufacturer is calculated as $1/\lambda$. In addition, since the service times of the manufacturer are exponentially distributed, the mean service time is $1/\mu_M$. Then, using Little's formula, it is easy to prove that

$$E\left[N_{q_M}\right] = E\left[N_M\right] - \rho_M, \tag{3.41}$$

where N_{q_M} denotes the number of jobs in the manufacturer's queue.

Substituting equation (3.40) into equation (3.41) yields that the average number of jobs in the manufacturer's queue, i.e., the average outstanding backorders at the manufacturer, can be expressed as

$$E\left[N_{q_M}\right] = E\left[B_M\right] \simeq \left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right) - 2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right), \tag{3.42}$$

where B_M is the outstanding backorders at the manufacturer and $j = \arg\min_{i=1,\ldots,n} S_i$.

To summarize, the supply chain has been modeled as a queuing system in this chapter. The next chapter presents the centralized and decentralized models that are developed using the performance measures obtained from the queuing model.

4. THE CENTRALIZED AND DECENTRALIZED MODELS

In the centralized system, there is a single decision maker who tries to optimize the overall supply chain. On the other hand, in the decentralized system, each supplier tries to optimize his own entity and the manufacturer does likewise. In this chapter, the centralized and decentralized models are developed; and the solutions to these models are given. Notice that the centralized system is also considered in this thesis since the centralized solution is used as a reference point for the performance of the decentralized system.

4.1 The Centralized Model

In the centralized model, the objective of the single decision maker is to minimize the average total backorder and holding costs per unit time for the overall system. The decision variables are the base stock levels of the suppliers. Other cost terms, such as the unit production cost or the order processing cost could also be included in the objective function. However, since these cost terms do not include the decision variables, adding them would not affect the optimal solution.

Now, let b_i be the backorder cost per unit backordered at supplier *i* per unit time; b_M be the backorder cost per unit backordered at the manufacturer per unit time; and h_i be the holding cost per unit inventory per unit time for supplier *i*, where $i = 1, \ldots, n.$ It is assumed that $b_i > 0$ and $h_i > 0$ for all $i = 1, \ldots, n,$ and $b_M > 0.$

In addition to the notation given above, let C_{S_i} denote the average cost per unit time for supplier *i*, where $i = 1, \ldots, n,$ and C_M denote the average cost per unit time for the manufacturer. Then, C_{S_i} and C_M can be expressed as

$$C_{S_i} = b_i E[B_i] + h_i E[I_i], \quad i = 1, \ldots, n \qquad \textbf{(4.1)}$$

and

$$C_M = b_M E\left[B_M\right], \tag{4.2}$$

respectively. Notice that the average cost of the manufacturer is only equal to his average backorder cost since no inventory is held by the manufacturer.

By substituting equations (3.33) and (3.34) into equation (4.1), C_{S_i} can be written as a function of S_i given by

$$C_{S_i}\left(S_i\right) = b_i\left(\frac{\rho_i^{S_i+1}}{1-\rho_i}\right) + h_i\left(S_i - \frac{\rho_i\left(1-\rho_i^{S_i}\right)}{1-\rho_i}\right), \quad i = 1,\ldots,n. \tag{4.3}$$

On the other hand, by substituting equation (3.42) into equation (4.2), C_M can be expressed as a function of S_j as given below:

$$C_M\left(S_j\right) \simeq \tilde{C}_M\left(S_j\right) = b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right) - 2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right), \tag{4.4}$$

where $j = \underset{i=1,\ldots,n}{\arg\min}\, S_i$.

Finally, let C_T be the average total backorder and holding costs per unit time for the overall system. Then, using equations (4.3) and (4.4), C_T can be written as a function of $S_1,\ldots,S_n$ given by

$$\begin{aligned} C_T\left(S_1,\ldots,S_n\right) &= \sum_{i=1}^{n} C_{S_i}\left(S_i\right) + C_M\left(S_j\right) \\ &\simeq \sum_{i=1}^{n} b_i\left(\frac{\rho_i^{S_i+1}}{1-\rho_i}\right) + \sum_{i=1}^{n} h_i\left(S_i - \frac{\rho_i\left(1-\rho_i^{S_i}\right)}{1-\rho_i}\right) \\ &\quad + b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right) - 2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right), \end{aligned} \tag{4.5}$$

where $j = \underset{i=1,\ldots,n}{\arg\min}\, S_i$.

Recall that in the centralized model, the decision maker tries to minimize the average total backorder and holding costs per unit time for the overall system. Then, from equation (4.5), the centralized system leads to the following nonlinear optimization problem:

$$
\begin{aligned}
\text{minimize} \quad \tilde{C}_T\left(S_1,\ldots,S_n\right) &= \sum_{i=1}^{n} C_{S_i}\left(S_i\right) + \tilde{C}_M\left(S_j\right) \\
&= \sum_{i=1}^{n} b_i \left(\frac{\rho_i^{S_i+1}}{1-\rho_i}\right) + \sum_{i=1}^{n} h_i \left(S_i - \frac{\rho_i\left(1-\rho_i^{S_i}\right)}{1-\rho_i}\right) \\
&\quad + b_M \left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right) - 2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right)
\end{aligned} \qquad \textbf{(4.6)}
$$

subject to $S_i \geq 0, \quad i = 1,\ldots,n,$

where $j = \arg\min_{i=1,\ldots,n} S_i.$

To find the optimal solution to the centralized model, the condition $j = \arg\min_{i=1,\ldots,n} S_i$ is not taken into consideration at the beginning. Then, for the nonlinear optimization problem presented in equation (4.6), the Karush-Kuhn-Tucker (KKT) conditions are given by

$$
\begin{aligned}
& h_j + \left(b_j + h_j\right)\left(\frac{\ln \rho_j}{1-\rho_j}\right)\rho_j^{S_j+1} \\
& \quad - b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)\left(\ln \rho_j\right)}{1+\rho_j}\right)\rho_j^{S_j+1} - u_j = 0,
\end{aligned} \qquad \textbf{(4.7a)}
$$

$$
h_i + \left(b_i + h_i\right)\left(\frac{\ln \rho_i}{1-\rho_i}\right)\rho_i^{S_i+1} - u_i = 0, \quad i = 1,\ldots,n, \quad i \neq j, \qquad \textbf{(4.7b)}
$$

$$
S_i \geq 0, \quad i = 1,\ldots,n, \qquad \textbf{(4.7c)}
$$

$$
u_i S_i = 0, \quad i = 1,\ldots,n, \qquad \textbf{(4.7d)}
$$

$$
u_i \geq 0, \quad i = 1,\ldots,n. \qquad \textbf{(4.7e)}
$$

Recall that the suppliers considered in this thesis apply base stock policy to manage their inventories. Therefore, similar to Cachon (1999) and Gupta and Weerawat (2006), it is assumed that the optimal base stock levels of the suppliers are not equal to zero.

According to equation (4.7e), $u_j \geq 0$ since $j \in \{1,\ldots,n\}$. Then, if $\partial\tilde{C}_T(0)/\partial S_j < 0$, equation (4.7a) does not hold giving that $S_j = 0$ is not optimal. Again using equation

(4.7e), if $\partial \tilde{C}_T(0)/\partial S_i < 0$ for all $i \neq j$, equation (4.7b) does not hold resulting in that $S_i = 0$ is not optimal. Consequently, throughout the thesis it is assumed that

$$h_j + (b_j + h_j)\left(\frac{\ln \rho_j}{1-\rho_j}\right)\rho_j - b_M\left(\frac{2\rho_M^2}{(1+\rho_M^2)(1-\rho_M)}\right)\left(\frac{(1-\rho_j)(\ln \rho_j)}{1+\rho_j}\right)\rho_j < 0 \qquad \textbf{(4.8)}$$

and

$$h_i + (b_i + h_i)\left(\frac{\ln \rho_i}{1-\rho_i}\right)\rho_i < 0, \quad i = 1,\dots,n, \quad i \neq j. \qquad \textbf{(4.9)}$$

Lemma 4.1. $\tilde{C}_T(S_1,\dots,S_n)$ is a strictly convex function on $\mathbb{R}^n$ for given *j*.

Proof. The Hessian of $\tilde{C}_T(S_1,\dots,S_n)$ is given by

$$H(S_1,\dots,S_n) = \begin{bmatrix} H_{11} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & H_{22} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & H_{jj} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & H_{nn} \end{bmatrix},$$

where

$$\begin{aligned} H_{jj} = {} & (b_j + h_j)\left(\frac{(\ln \rho_j)^2}{1-\rho_j}\right)\rho_j^{S_j+1} \\ & - b_M\left(\frac{2\rho_M^2}{(1+\rho_M^2)(1-\rho_M)}\right)\left(\frac{(1-\rho_j)(\ln \rho_j)^2}{1+\rho_j}\right)\rho_j^{S_j+1}, \end{aligned} \qquad \textbf{(4.10)}$$

$$H_{ii} = (b_i + h_i)\left(\frac{(\ln \rho_i)^2}{1-\rho_i}\right)\rho_i^{S_i+1}, \quad i = 1,\dots,n, \quad i \neq j, \qquad \textbf{(4.11)}$$

and all other entries are zero.

Using the assumption given in equation (4.8), it can be shown that

$$(\ln \rho_j)\rho_j\left(\frac{b_j + h_j}{1-\rho_j} - b_M\left(\frac{2\rho_M^2}{(1+\rho_M^2)(1-\rho_M)}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\right) < 0 \qquad \textbf{(4.12)}$$

because $h_j > 0$. Notice that the assumptions made for all $i = 1,\dots,n$ are also valid for *j* since $j \in \{1,\dots,n\}$.

Then, as $0 < \rho_j < 1$, equation (4.12) leads to

$$\frac{b_j + h_j}{1-\rho_j} - b_M \left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right) > 0, \tag{4.13}$$

giving that $H_{jj} > 0$ for $S_j \in \mathbb{R}$ (see equation (4.10)).

It is also obvious that since $b_i > 0$, $h_i > 0$, and $0 < \rho_i < 1$ for all $i = 1,\dots,n$, $H_{ii} > 0$ for $S_i \in \mathbb{R}$ for all $i \neq j$ (see equation (4.11)).

Consequently, all the *k*th order leading principal minors of the Hessian are positive. Therefore, the Hessian is positive definite for all $S_i \in \mathbb{R}$ for $i = 1,\dots,n$, which is a sufficient condition for $\tilde{C}_T\left(S_1,\dots,S_n\right)$ to be a strictly convex function on $\mathbb{R}^n$. □

Theorem 4.1. The global optimal solution to the centralized model presented in equation (4.6) is given by

$$S_j^* = \frac{\ln\left(\dfrac{-h_j}{\left(\dfrac{b_j+h_j}{1-\rho_j} - b_M\left(\dfrac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\dfrac{1-\rho_j}{1+\rho_j}\right)\right)\left(\ln\rho_j\right)\rho_j}\right)}{\ln\rho_j}, \quad j = j^*, \tag{4.14}$$

$$S_i^* = \frac{\ln\left(\dfrac{-h_i\left(1-\rho_i\right)}{\left(b_i+h_i\right)\left(\ln\rho_i\right)\rho_i}\right)}{\ln\rho_i}, \quad i = 1,\dots,n, \quad i \neq j^*, \tag{4.15}$$

where $j^* = \arg\min_{j\in J} \tilde{C}_T\left(S_1,\dots,S_j,\dots,S_n\right)$ and $j \in J$ iff $S_j = \min_{i=1,\dots,n} S_i$ for $j = 1,\dots,n$.

Proof. To calculate the optimal solution to the centralized model, the model has to be solved n times, each time setting j equal to $1,\dots,n$, respectively. Recall that the condition $j = \arg\min_{i=1,\dots,n} S_i$ is ignored while solving the model each time.

For each $j = 1,\ldots,n,$ when u_j is set to zero in KKT condition (4.7a), S_j is calculated as given in equation (4.14).

In addition, from the assumption presented in equation (4.8), it is easy to prove that

$$0 < \frac{-h_j}{\left(\frac{b_j + h_j}{1-\rho_j} - b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\right)\left(\ln \rho_j\right)\rho_j} < 1. \qquad \textbf{(4.16)}$$

Then, using equation (4.16) and $0 < \rho_j < 1,$ equation (4.14) results in $S_j > 0,$ satisfying the KKT conditions.

Likewise, for each $j = 1,\ldots,n,$ when u_i is set to zero in KKT condition (4.7b) for all $i \neq j,$ S_i is found as given in equation (4.15).

Furthermore, using the assumption presented in equation (4.9), it can be shown that

$$0 < \frac{-h_i\left(1-\rho_i\right)}{\left(b_i + h_i\right)\left(\ln \rho_i\right)\rho_i} < 1, \quad i = 1,\ldots,n, \quad i \neq j. \qquad \textbf{(4.17)}$$

Then, from equation (4.17), and since $0 < \rho_i < 1$ for all $i = 1,\ldots,n,$ equation (4.15) gives $S_i > 0$ for all $i \neq j,$ satisfying the KKT conditions.

As the centralized model presented in equation (4.6) is a minimization problem having linear constraints and an objective function that is strictly convex on $\mathbb{R}^n$ for given *j* (see Lemma 4.1), and as the solution for each $j = 1,\ldots,n$ satisfies the KKT conditions, each solution is the unique global optimal solution to the model for the *j* given. However, since the condition $j = \arg\min_{i=1,\ldots,n} S_i$ is ignored while solving the model each time, some of the solutions may not be feasible for the centralized model. Nevertheless, it is easy to prove that at least one of the solutions is feasible. Briefly, if all the base stock levels were calculated using equation (4.15), it is obvious that one of them would be the minimum; for instance, say S_1. It is also easy to see that the base stock level calculated from equation (4.14) is always smaller than the one calculated from equation (4.15) for the same supplier. Therefore, when *j* is set to one, S_1 calculated from equation (4.14) will also be the minimum, proving that at least one of the solutions is feasible.

After finding separate solutions for each $j = 1,\ldots,n,$ to find the optimal solution to the centralized model, the condition $j = \arg\min_{i=1,\ldots,n} S_i$ has to be taken into account. Consequently, among the solutions in which $S_j = \min_{i=1,\ldots,n} S_i,$ the one that minimizes the objective function given in equation (4.6) is the global optimal solution to the centralized model, concluding the proof. □

Remark 4.1. The optimal solution given in equations (4.14) and (4.15) ignores the integrality of the base stock levels of the suppliers. However, the optimal integer solution to the centralized model can also be found as follows. First, the model has to be solved *n* times, each time setting *j* equal to $1,\ldots,n,$ respectively. Then each time, S_j is rounded to $\lfloor S_j \rfloor$ and $\lceil S_j \rceil;$ S_i is rounded to $\lfloor S_i \rfloor$ and $\lceil S_i \rceil$ for all $i \neq j;$ and the objective function value is calculated for all feasible combinations, where $\operatorname{int} S_j = \min_{i=1,\ldots,n} \operatorname{int} S_i.$ Here, $\lfloor x \rfloor$ stands for the largest integer less than or equal to *x*; $\lceil x \rceil$ stands for the smallest integer greater than or equal to *x*; and $\operatorname{int} x$ denotes the rounded integer value of *x*. Notice that if $\operatorname{int} S_j = \operatorname{int} S_i$ for some $i \neq j,$ then ρ_j must be greater than or equal to ρ_i for a combination to be feasible. Among all feasible combinations, the one that minimizes the objective function presented in equation (4.6) gives the optimal integer solution to the centralized model.

4.2 The Decentralized Model

In the decentralized model, the objective of each member of the supply chain is to minimize the average cost per unit time for his own system. Therefore, supplier *i* for $i = 1,\ldots,n$ tries to minimize his average backorder and holding costs per unit time, which is presented in equation (4.3). Since the decision variables are the base stock levels of the suppliers, this leads to the following decentralized model for supplier *i*, where $i = 1,\ldots,n$:

$$\text{minimize} \quad C_{S_i}\left(S_i\right) = b_i\left(\frac{\rho_i^{S_i+1}}{1-\rho_i}\right) + h_i\left(S_i - \frac{\rho_i\left(1-\rho_i^{S_i}\right)}{1-\rho_i}\right) \qquad \textbf{(4.18)}$$

$$\text{subject to} \quad S_i \geq 0, \quad i \in \{1,\ldots,n\}.$$

From the manufacturer's point of view, as the manufacturer holds no inventory, he only wants to minimize his average backorder cost per unit time, which is given in equation (4.4). However, since the decision variables are the base stock levels of the suppliers, the manufacturer is not included in the decentralized model.

From equation (4.4), notice that the average backorder cost per unit time for the manufacturer depends on S_j, where $j = \arg\min_{i=1,\ldots,n} S_i$. Therefore, even though the manufacturer is not included in the decentralized model, he is affected by supplier *j*'s decision.

Again using equation (4.4), it can be seen that $\tilde{C}_M\left(S_j\right)$ is minimized for $S_j = 0$, which can be interpreted as follows: As S_j approaches zero, the arrival of component *j* to the manufacturer takes longer time on average. Hence, the average number of jobs in the manufacturer's queue, i.e., the average number of outstanding backorders arisen from the manufacturer's own system, decreases, reducing the average backorder cost per unit time for the manufacturer.

Theorem 4.2. The unique global optimal solution to the decentralized model for supplier *i* presented in equation (4.18) is given by

$$S_i^{\text{o}} = \frac{\ln\left(\dfrac{-h_i\left(1-\rho_i\right)}{\left(b_i+h_i\right)\left(\ln\rho_i\right)\rho_i}\right)}{\ln\rho_i}, \quad i \in \{1,\ldots,n\}. \qquad \textbf{(4.19)}$$

Proof. For the nonlinear optimization problem given in equation (4.18), the KKT conditions can be written as

$$h_i + \left(b_i + h_i\right)\left(\frac{\ln\rho_i}{1-\rho_i}\right)\rho_i^{S_i+1} - u_i = 0, \quad i \in \{1,\ldots,n\}, \qquad \textbf{(4.20a)}$$

$$S_i \geq 0, \quad i \in \{1,\ldots,n\}, \qquad \textbf{(4.20b)}$$

$$u_i S_i = 0, \quad i \in \{1,\ldots,n\}, \qquad \textbf{(4.20c)}$$

$$u_i \geq 0, \quad i \in \{1,\ldots,n\}. \qquad \textbf{(4.20d)}$$

When u_i is set to zero in KKT condition (4.20a) for $i \in \{1,\ldots,n\}$, S_i is found as given in equation (4.19).

On the other hand, recall the assumption that the optimal base stock levels of the suppliers are not equal to zero. Therefore, the assumption given in equation (4.9) and accordingly equation (4.17) can be extended to all $i = 1, \ldots, n$ in the decentralized case. Then, using equation (4.17) and $0 < \rho_i < 1$ for all $i = 1, \ldots, n,$ equation (4.19) gives $S_i > 0$ for $i \in \{1, \ldots, n\},$ satisfying the KKT conditions.

Finally, since $b_i > 0,$ $h_i > 0,$ and $0 < \rho_i < 1$ for all $i = 1, \ldots, n,$

$$\frac{\partial^2 C_{S_i}(S_i)}{\partial S_i^2} = (b_i + h_i)\left(\frac{(\ln \rho_i)^2}{1-\rho_i}\right)\rho_i^{S_i+1} > 0$$

for $S_i \in \mathbb{R},$ indicating that $C_{S_i}(S_i)$ given in equation (4.18) is a strictly convex function on $\mathbb{R}.$

As a result, since the decentralized model for supplier *i* presented in equation (4.18) is a minimization problem having a linear constraint and an objective function that is strictly convex on $\mathbb{R},$ and since the solution S_i^{o} given in equation (4.19) satisfies the KKT conditions, it is also the unique global optimal solution to the decentralized model for supplier *i*, where $i = 1, \ldots, n,$ concluding the proof. □

Remark 4.2. The optimal solution S_i^{o} given in equation (4.19) ignores the integrality of the base stock level of supplier *i*, where $i = 1, \ldots, n.$ However, the optimal integer solution to the decentralized model can easily be found, such that the optimal integer value of S_i is the one among $\lfloor S_i^{\text{o}} \rfloor$ and $\lceil S_i^{\text{o}} \rceil,$ which minimizes the objective function given in equation (4.18).

As a summary, the centralized and decentralized models are developed and the optimal solutions to these models are derived in this chapter. The next chapter continues with the comparison of these optimal solutions and accordingly the development of the transfer payment contracts for the coordination of the decentralized system.

5. COORDINATION OF THE DECENTRALIZED SYSTEM

A supply chain is coordinated if each member acts rationally according to the supply chain optimal solution, i.e., the decentralized solution is equal to the centralized solution. Notice that the optimal solution to the centralized model given in equation (4.6) is indeed the approximate centralized solution to the system, since the model has an approximate objective function. However, from now on, it will be referred to as the "centralized solution" for simplicity. Similarly, the "decentralized solution" refers to the optimal solution to the decentralized model given in equation (4.18).

Comparing the centralized solution given in equations (4.14) and (4.15) with the decentralized solution given in equation (4.19)[2], it is found that $S_i^* = S_i^{\mathrm{o}}$ for all $i \neq j$ and $S_j^* \neq S_j^{\mathrm{o}}$, where $j = j^*$ as defined in Theorem 4.1. Therefore, a coordination mechanism should be investigated between supplier *j* and the manufacturer.

Proposition 5.1. The centralized solution for supplier *j* given in equation (4.14) is smaller than the decentralized solution given in equation (4.19), i.e., $S_j^* < S_j^{\mathrm{o}}$.

Proof. Using the assumption given in equation (4.8) and $b_j > 0,\ h_j > 0,\ 0 < \rho_j < 1,$ $b_M > 0,$ and $0 < \rho_M < 1,$ it is easy to show that

$$\frac{-h_j}{\left(\dfrac{b_j + h_j}{1-\rho_j} - b_M\left(\dfrac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\dfrac{1-\rho_j}{1+\rho_j}\right)\right)\left(\ln \rho_j\right)\rho_j} > \frac{-h_j}{\left(\dfrac{b_j + h_j}{1-\rho_j}\right)\left(\ln \rho_j\right)\rho_j}. \quad \textbf{(5.1)}$$

By taking the natural logarithm of both sides and dividing by $\ln \rho_j$, the proof of $S_j^* < S_j^{\mathrm{o}}$ is completed. □

As explained in the previous chapter, the average backorder cost per unit time for the manufacturer decreases as S_j approaches zero. Therefore, one can already anticipate

[2] Notice that equation (4.19) is valid for all suppliers, including supplier *j*.

the fact $S_j^* < S_j^o$, which means that a coordinating contract has to decrease the base stock level of supplier *j*. Decreasing S_j yields lower average holding cost, whereas higher average backorder cost per unit time for this supplier. Hence, the manufacturer has to prepare a contract to induce supplier *j* to choose a smaller base stock level than his decentralized solution. Remark that since the average cost per unit time for each supplier depends only on his base stock level (see equation (4.3)), a change in the average cost function per unit time for supplier *j* does not affect the optimal strategies of the other suppliers. Therefore, the fact $S_i^* = S_i^o$ for all $i \neq j$ remains the same after the contract.

In the following sections of this chapter, three different transfer payment contracts are studied to coordinate the supply chain. These are the backorder cost subsidy contract, the transfer payment contract based on Pareto improvement, and the cost sharing contract. These contracts are examined in this thesis since they are expected to encourage supplier *j* to select a smaller base stock level than his decentralized solution, which is necessary for the coordination process.

Besides its ability to coordinate the supply chain, a contract should also be Pareto improving, i.e., at least one of the supply chain members should be strictly better off without making any other member worse off after the transfer payment. Therefore, each contract is evaluated according to its coordination ability and whether it is Pareto improving or not.

In this chapter, the average cost functions per unit time for the manufacturer (and also for supplier *j* in the cost sharing contract) after the transfer payments depend on $\tilde{C}_M\left(S_j\right)$ given in equation (4.4), yielding that these functions are approximate. In addition, recall that the centralized model given in equation (4.6) is also developed using $\tilde{C}_M\left(S_j\right)$. Therefore, the contracts are based on the average cost functions per unit time for supplier *j* and the approximate average cost functions per unit time for the manufacturer (and also for supplier *j* in the cost sharing contract). The contracts are also evaluated whether they are Pareto improving or not according to these cost functions.

5.1 The Backorder Cost Subsidy Contract

As stated before, a coordinating contract has to induce supplier *j* to choose a smaller base stock level than his decentralized solution. Hence, a backorder cost subsidy contract, in which the manufacturer covers some part of supplier *j*'s backorder costs, seems to be able to coordinate the supply chain.

In the backorder cost subsidy contract, the manufacturer pays supplier *j* $\alpha_B b_j$ per unit backordered at supplier *j* per unit time, where $0<\alpha_B<1$. Then, after the transfer payment, the average cost function per unit time for supplier *j* and the approximate average cost function per unit time for the manufacturer that are given in equations (4.3)[3] and (4.4) modify to

$$C_{S_j}^B\left(S_j\right)=\left(1-\alpha_B\right)b_j\left(\frac{\rho_j^{S_j+1}}{1-\rho_j}\right)+h_j\left(S_j-\frac{\rho_j\left(1-\rho_j^{S_j}\right)}{1-\rho_j}\right) \quad \textbf{(5.2)}$$

and

$$\tilde{C}_M^B\left(S_j\right)=b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right)+\alpha_B b_j\left(\frac{\rho_j^{S_j+1}}{1-\rho_j}\right), \quad \textbf{(5.3)}$$

respectively.

Theorem 5.1. The backorder cost subsidy contract coordinates the supply chain for

$$\alpha_B=b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{b_j\left(1+\rho_j\right)}\right). \quad \textbf{(5.4)}$$

Proof. Let us start the proof by showing that α_B given in equation (5.4) is feasible, i.e., $0<\alpha_B<1$. Since $b_j>0$, $0<\rho_j<1$, $b_M>0$, and $0<\rho_M<1$, it is obvious that $\alpha_B>0$. The proof of $\alpha_B<1$ is not that simple, but a step by step procedure brings forth the proof. First, it is not hard to show that

$$\frac{\ln\left(\frac{\rho_j-1}{\left(\ln\rho_j\right)\rho_j}\right)}{\ln\rho_j}<0. \quad \textbf{(5.5)}$$

[3] Notice that equation (4.3) is valid for all suppliers, including supplier *j*.

Since $S_j^* > 0$ (see the proof of Theorem 4.1), equation (5.5) can be rewritten as

$$\frac{\ln\left(\dfrac{\rho_j - 1}{(\ln \rho_j)\rho_j}\right)}{\ln \rho_j} < S_j^*. \tag{5.6}$$

Then, substituting equation (4.14) into equation (5.6) leads to

$$\frac{h_j}{\dfrac{b_j + h_j}{1-\rho_j} - b_M\left(\dfrac{2\rho_M^2}{(1+\rho_M^2)(1-\rho_M)}\right)\left(\dfrac{1-\rho_j}{1+\rho_j}\right)} < 1 - \rho_j. \tag{5.7}$$

Finally, using equation (4.13), equation (5.7) yields $\alpha_B < 1$.

Now, let us prove that the backorder cost subsidy contract coordinates the supply chain for α_B given in equation (5.4). As $b_j > 0$, $h_j > 0$, $0 < \rho_j < 1$, and $0 < \alpha_B < 1$,

$$\frac{\partial^2 C_{S_j}^B(S_j)}{\partial S_j^2} = \left((1-\alpha_B)b_j + h_j\right)\left(\frac{(\ln \rho_j)^2}{1-\rho_j}\right)\rho_j^{S_j+1} > 0$$

for $S_j \in \mathbb{R}$, indicating that $C_{S_j}^B(S_j)$ given in equation (5.2) is a strictly convex function on $\mathbb{R}$. Then, for α_B given in equation (5.4), solving the first order condition

$$h_j + \left((1-\alpha_B)b_j + h_j\right)\left(\frac{\ln \rho_j}{1-\rho_j}\right)\rho_j^{S_j+1} = 0$$

results in S_j^* as presented in equation (4.14).

Consequently, since the decentralized model for supplier *j* after the transfer payment is a minimization problem with a strictly convex objective function $C_{S_j}^B(S_j)$ over a convex set $S_j \geq 0$, S_j^* given in equation (4.14) is the unique global optimal solution to the decentralized model for supplier *j* after the transfer payment. As supplier *j*'s decentralized solution is equal to S_j^*, it is proved that the backorder cost subsidy contract coordinates the supply chain for α_B given in equation (5.4). □

Theorem 5.2. The backorder cost subsidy contract is not Pareto improving.

Proof. Using equations (4.3), (5.2), and (5.4), the average costs per unit time for supplier j before and after the transfer payment are given by

$$C_{S_j}\left(S_j^{\mathrm{o}}\right)=b_j\left(\frac{\rho_j^{S_j^{\mathrm{o}}+1}}{1-\rho_j}\right)+h_j\left(S_j^{\mathrm{o}}-\frac{\rho_j\left(1-\rho_j^{S_j^{\mathrm{o}}}\right)}{1-\rho_j}\right) \tag{5.8}$$

and

$$\begin{aligned}C_{S_j}^{B}\left(S_j^{*}\right)=&\left(1-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{b_j\left(1+\rho_j\right)}\right)\right)b_j\left(\frac{\rho_j^{S_j^{*}+1}}{1-\rho_j}\right)\\&+h_j\left(S_j^{*}-\frac{\rho_j\left(1-\rho_j^{S_j^{*}}\right)}{1-\rho_j}\right),\end{aligned} \tag{5.9}$$

respectively.

From equations (5.8) and (5.9), the difference $D_{S_j}^{B}$ between the average backorder costs per unit time for supplier j before and after the transfer payment is given by

$$D_{S_j}^{B}=b_j\left(\frac{\rho_j^{S_j^{\mathrm{o}}+1}}{1-\rho_j}\right)-\left(1-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{b_j\left(1+\rho_j\right)}\right)\right)b_j\left(\frac{\rho_j^{S_j^{*}+1}}{1-\rho_j}\right). \tag{5.10}$$

Substituting equations (4.14) and (4.19) into equation (5.10) yields

$$D_{S_j}^{B}=\frac{1}{b_j+h_j}-\frac{b_j\left(1+\rho_j\right)-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(1-\rho_j\right)^2}{b_j\left(\left(b_j+h_j\right)\left(1+\rho_j\right)-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(1-\rho_j\right)^2\right)}. \tag{5.11}$$

Then, $\alpha_B<1$ gives that the numerator of the second term in equation (5.11) is positive and equation (4.13) yields that its denominator is also positive. Afterwards, it is easily proved that $D_{S_j}^{B}>0$, i.e., the average backorder cost per unit time for supplier j decreases after the transfer payment.

On the other hand, it is easy to prove that the average holding cost function per unit time for supplier *j*, which is the second term in equation (4.3), is strictly convex on $\mathbb{R}$ and takes its minimum value at

$$S_j = \frac{\ln\left(\dfrac{\rho_j - 1}{(\ln \rho_j)\rho_j}\right)}{\ln \rho_j}.$$

Therefore, for

$$S_j > \frac{\ln\left(\dfrac{\rho_j - 1}{(\ln \rho_j)\rho_j}\right)}{\ln \rho_j},$$

the function is increasing in S_j.

From Proposition 5.1 and equation (5.6), it is known that

$$S_j^{\text{o}} > S_j^{*} > \frac{\ln\left(\dfrac{\rho_j - 1}{(\ln \rho_j)\rho_j}\right)}{\ln \rho_j},$$

which proves the decrease of the average holding cost per unit time for supplier *j* after the transfer payment.

Consequently, it is shown that the average cost per unit time for supplier *j* decreases after the contract, i.e., $C_{S_j}^{B}\left(S_j^{*}\right) < C_{S_j}\left(S_j^{\text{o}}\right)$.

Now, let us examine the manufacturer. Using equations (4.4), (5.3), and (5.4), the approximate average costs per unit time for the manufacturer before and after the transfer payment are given by

$$\tilde{C}_M\left(S_j^{\text{o}}\right) = b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{(1+\rho_j)(1+\rho_M^2) - 2\rho_j^{S_j^{\text{o}}+1}(1-\rho_j)}{2(1+\rho_j)(1-\rho_M)}\right) \qquad \textbf{(5.12)}$$

and

$$\tilde{C}_M^B\left(S_j^*\right)=b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j^*+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right)$$

$$+b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{b_j\left(1+\rho_j\right)}\right)b_j\left(\frac{\rho_j^{S_j^*+1}}{1-\rho_j}\right) \tag{5.13}$$

$$=b_M\left(\frac{\rho_M^2}{1-\rho_M}\right),$$

respectively.

From equations (5.12) and (5.13), since $S_j^{\text{o}}>0$ (see the proof of Theorem 4.2), $0<\rho_j<1,$ $b_M>0,$ and $0<\rho_M<1,$ it is easily proved that $\tilde{C}_M^B\left(S_j^*\right)>\tilde{C}_M\left(S_j^{\text{o}}\right).$

As a result, the average cost per unit time for supplier *j* decreases after the transfer payment, whereas the approximate average cost per unit time for the manufacturer increases. Therefore, the backorder cost subsidy contract is not Pareto improving. □

5.2 The Transfer Payment Contract Based on Pareto Improvement

In the transfer payment contract based on Pareto improvement, the manufacturer pays supplier *j* an amount that makes the manufacturer as well off after the transfer payment as before. The transfer payment satisfying this condition is given by

$$T^P\left(S_j\right)=b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\left(\rho_j^{S_j+1}+\frac{h_j\left(1-\rho_j\right)}{\left(b_j+h_j\right)\left(\ln\rho_j\right)}\right). \tag{5.14}$$

Then, after the transfer payment, the average cost function per unit time for supplier *j* and the approximate average cost function per unit time for the manufacturer that are given in equations (4.3) and (4.4) become

$$C_{S_j}^P\left(S_j\right)=b_j\left(\frac{\rho_j^{S_j+1}}{1-\rho_j}\right)+h_j\left(S_j-\frac{\rho_j\left(1-\rho_j^{S_j}\right)}{1-\rho_j}\right)-T^P\left(S_j\right) \tag{5.15}$$

and

$$\tilde{C}_M^P\left(S_j\right)=b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right)+T^P\left(S_j\right), \qquad \textbf{(5.16)}$$

respectively, where $T^P\left(S_j\right)$ is as given in equation (5.14).

Theorem 5.3. The transfer payment contract based on Pareto improvement coordinates the supply chain.

Proof. Using equation (4.13) and $0<\rho_j<1$,

$$\frac{\partial^2 C_{S_j}^P\left(S_j\right)}{\partial S_j^2}=\left(b_j+h_j\right)\left(\frac{\left(\ln\rho_j\right)^2}{1-\rho_j}\right)\rho_j^{S_j+1}$$
$$-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)\left(\ln\rho_j\right)^2}{1+\rho_j}\right)\rho_j^{S_j+1}>0$$

for $S_j\in\mathbb{R}$, pointing out that $C_{S_j}^P\left(S_j\right)$ given in equation (5.15) is a strictly convex function on $\mathbb{R}$. Then, the solution to the first order condition

$$h_j+\left(b_j+h_j\right)\left(\frac{\ln\rho_j}{1-\rho_j}\right)\rho_j^{S_j+1}-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)\left(\ln\rho_j\right)}{1+\rho_j}\right)\rho_j^{S_j+1}=0$$

is S_j^* as given in equation (4.14).

Notice that the decentralized model for supplier *j* after the transfer payment is a minimization problem having a strictly convex objective function $C_{S_j}^P\left(S_j\right)$ over a convex set $S_j\geq 0.$ Hence, S_j^* given in equation (4.14) is the unique global optimal solution to the decentralized model for supplier *j* after the transfer payment. Since supplier *j*'s decentralized solution is equal to $S_j^*,$ it is proved that the transfer payment contract based on Pareto improvement coordinates the supply chain. □

Theorem 5.4. The transfer payment contract based on Pareto improvement is Pareto improving.

Proof. The average cost per unit time for supplier *j* before the transfer payment is as given in equation (5.8). On the other hand, using equation (5.15), the average cost per unit time for supplier *j* after the transfer payment is given by

$$C_{S_j}^P\left(S_j^*\right)=b_j\left(\frac{\rho_j^{S_j^*+1}}{1-\rho_j}\right)+h_j\left(S_j^*-\frac{\rho_j\left(1-\rho_j^{S_j^*}\right)}{1-\rho_j}\right)$$
$$-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\left(\rho_j^{S_j^*+1}+\frac{h_j\left(1-\rho_j\right)}{\left(b_j+h_j\right)\left(\ln\rho_j\right)}\right). \quad \textbf{(5.17)}$$

By substituting equation (4.19) into equation (5.8), and equation (4.14) into equation (5.17), the difference between the average costs per unit time for supplier j before and after the transfer payment can be expressed as

$$C_{S_j}\left(S_j^{\circ}\right)-C_{S_j}^P\left(S_j^*\right)=-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{\left(b_j+h_j\right)\left(1+\rho_j\right)}\right)$$
$$-\ln\left(1-b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{\left(b_j+h_j\right)\left(1+\rho_j\right)}\right)\right). \quad \textbf{(5.18)}$$

From equation (4.13), and since $b_j>0,\ h_j>0,\ 0<\rho_j<1,\ b_M>0,$ and $0<\rho_M<1,$ it can be shown that

$$0<b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{\left(b_j+h_j\right)\left(1+\rho_j\right)}\right)<1. \quad \textbf{(5.19)}$$

For simplicity, let

$$z=b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)^2}{\left(b_j+h_j\right)\left(1+\rho_j\right)}\right). \quad \textbf{(5.20)}$$

Then, equation (5.18) can be rewritten as

$$C_{S_j}\left(S_j^{\circ}\right)-C_{S_j}^P\left(S_j^*\right)=-z-\ln\left(1-z\right). \quad \textbf{(5.21)}$$

Afterwards, it can be easily proved that the function given in equation (5.21) is strictly convex on $\mathbb{R}$ and takes its minimum value zero at $z=0.$ Then, since $0<z<1$ (see equations (5.19) and (5.20)), the function given in equation (5.21) is always positive, i.e., $C_{S_j}^P\left(S_j^*\right)<C_{S_j}\left(S_j^{\circ}\right).$

Now, recall that the transfer payment $T^P\left(S_j\right)$ given in equation (5.14) satisfies that the manufacturer is as well off after the transfer payment as before. To prove this statement, let us examine the manufacturer. The approximate average cost per unit time for the manufacturer before the transfer payment is as given in equation (5.12). On the other hand, using equation (5.16), the approximate average cost per unit time for the manufacturer after the transfer payment is given by

$$\tilde{C}_M^P\left(S_j^*\right)=b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j^*+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right)$$
$$+b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\left(\rho_j^{S_j^*+1}+\frac{h_j\left(1-\rho_j\right)}{\left(b_j+h_j\right)\left(\ln\rho_j\right)}\right). \quad \textbf{(5.22)}$$

Notice that the terms including $\rho_j^{S_j^*+1}$ cancel each other in equation (5.22). Then, substituting equation (4.19) into equation (5.12) proves that $\tilde{C}_M^P\left(S_j^*\right)=\tilde{C}_M\left(S_j^{\mathrm{o}}\right)$.

Consequently, the average cost per unit time for supplier *j* decreases after the transfer payment, and the approximate average cost per unit time for the manufacturer remains the same. Therefore, the contract is Pareto improving. □

5.3 The Cost Sharing Contract

In the cost sharing contract, similar to the study of Caldentey and Wein (2003), the manufacturer pays supplier *j* an amount such that supplier *j* covers α_C of their approximate average total costs per unit time after the transfer payment, i.e.,

$$\tilde{C}_{S_j}^C\left(S_j\right)=\alpha_C\tilde{C}\left(S_j\right), \quad \textbf{(5.23)}$$

and the manufacturer covers $\left(1-\alpha_C\right)$ of their approximate average total costs per unit time given by

$$\tilde{C}_M^C\left(S_j\right)=\left(1-\alpha_C\right)\tilde{C}\left(S_j\right), \quad \textbf{(5.24)}$$

where $0<\alpha_C<1$ and

$$
\begin{aligned}
\tilde{C}\left(S_j\right) &= C_{S_j}\left(S_j\right)+\tilde{C}_M\left(S_j\right) \\
&= b_j\left(\frac{\rho_j^{S_j+1}}{1-\rho_j}\right)+h_j\left(S_j-\frac{\rho_j\left(1-\rho_j^{S_j}\right)}{1-\rho_j}\right) \\
&\quad +b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right).
\end{aligned}
\tag{5.25}
$$

Then, the transfer payment satisfying equations (5.23) and (5.24) is given by

$$T^C\left(S_j\right)=\left(1-\alpha_C\right)C_{S_j}\left(S_j\right)-\alpha_C\tilde{C}_M\left(S_j\right),$$

where $C_{S_j}\left(S_j\right)$ and $\tilde{C}_M\left(S_j\right)$ are as given in equations (4.3) and (4.4), respectively.

Theorem 5.5. The cost sharing contract coordinates the supply chain.

Proof. Using equation (4.13), $0<\rho_j<1,$ and $\alpha_C>0,$

$$
\begin{aligned}
\frac{\partial^2\tilde{C}_{S_j}^C\left(S_j\right)}{\partial S_j^2} &= \alpha_C\left(b_j+h_j\right)\left(\frac{\left(\ln\rho_j\right)^2}{1-\rho_j}\right)\rho_j^{S_j+1} \\
&\quad -\alpha_C b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)\left(\ln\rho_j\right)^2}{1+\rho_j}\right)\rho_j^{S_j+1}>0
\end{aligned}
$$

for $S_j\in\mathbb{R},$ indicating that $\tilde{C}_{S_j}^C\left(S_j\right)$ given in equation (5.23) is a strictly convex function on $\mathbb{R}.$ Then, solving the first order condition

$$
\begin{aligned}
&\alpha_C h_j+\alpha_C\left(b_j+h_j\right)\left(\frac{\ln\rho_j}{1-\rho_j}\right)\rho_j^{S_j+1} \\
&\quad -\alpha_C b_M\left(\frac{2\rho_M^2}{\left(1+\rho_M^2\right)\left(1-\rho_M\right)}\right)\left(\frac{\left(1-\rho_j\right)\left(\ln\rho_j\right)}{1+\rho_j}\right)\rho_j^{S_j+1}=0
\end{aligned}
$$

yields S_j^* as given in equation (4.14).

As a result, since the decentralized model for supplier *j* after the transfer payment is a minimization problem having a strictly convex objective function $\tilde{C}_{S_j}^C\left(S_j\right)$ over a convex set $S_j\geq 0,$ S_j^* given in equation (4.14) is the unique global optimal solution to the decentralized model for supplier *j* after the transfer payment. As supplier *j*'s decentralized solution is equal to S_j^*, it is proved that the cost sharing contract coordinates the supply chain. □

Theorem 5.6. The cost sharing contract is Pareto improving for any α_C satisfying

$$\alpha_C \in \left[1-\frac{\tilde{C}_M\left(S_j^{\mathrm{o}}\right)}{\tilde{C}\left(S_j^*\right)}, \frac{C_{S_j}\left(S_j^{\mathrm{o}}\right)}{\tilde{C}\left(S_j^*\right)}\right] \cap (0,1), \tag{5.26}$$

where $C_{S_j}\left(S_j^{\mathrm{o}}\right)$ is as given in equation (5.8), $\tilde{C}_M\left(S_j^{\mathrm{o}}\right)$ is as given in equation (5.12), and from equation (5.25),

$$\begin{aligned} \tilde{C}\left(S_j^*\right) &= C_{S_j}\left(S_j^*\right)+\tilde{C}_M\left(S_j^*\right) \\ &= b_j\left(\frac{\rho_j^{S_j^*+1}}{1-\rho_j}\right)+h_j\left(S_j^*-\frac{\rho_j\left(1-\rho_j^{S_j^*}\right)}{1-\rho_j}\right) \\ &\quad +b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right)-2\rho_j^{S_j^*+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right). \end{aligned} \tag{5.27}$$

Furthermore, there is always an α_C satisfying equation (5.26).

Proof. The average cost per unit time for supplier *j* before the transfer payment is $C_{S_j}\left(S_j^{\mathrm{o}}\right)$ as given in equation (5.8). On the other hand, using equation (5.23), the approximate average cost per unit time for supplier *j* after the transfer payment can be expressed as

$$\tilde{C}_{S_j}^C\left(S_j^*\right)=\alpha_C\tilde{C}\left(S_j^*\right), \tag{5.28}$$

where $\tilde{C}\left(S_j^*\right)$ is as given in equation (5.27).

Then, supplier *j* is at least as well off after the transfer payment as before if and only if $C_{S_j}\left(S_j^{\mathrm{o}}\right) \ge \tilde{C}_{S_j}^C\left(S_j^*\right)$, leading to

$$\alpha_C \le \frac{C_{S_j}\left(S_j^{\mathrm{o}}\right)}{\tilde{C}\left(S_j^*\right)} \tag{5.29}$$

from equation (5.28).

In the same manner, the approximate average cost per unit time for the manufacturer before the transfer payment is $\tilde{C}_M\left(S_j^{\text{o}}\right)$ as given in equation (5.12); and from equation (5.24), the approximate average cost per unit time for the manufacturer after the transfer payment is given by

$$\tilde{C}_M^C\left(S_j^*\right) = \left(1-\alpha_C\right)\tilde{C}\left(S_j^*\right), \tag{5.30}$$

where $\tilde{C}\left(S_j^*\right)$ is as presented in equation (5.27).

Then, the manufacturer is at least as well off after the transfer payment as before if and only if $\tilde{C}_M\left(S_j^{\text{o}}\right) \geq \tilde{C}_M^C\left(S_j^*\right)$, which gives

$$\alpha_C \geq 1-\frac{\tilde{C}_M\left(S_j^{\text{o}}\right)}{\tilde{C}\left(S_j^*\right)} \tag{5.31}$$

from equation (5.30).

Consequently, as presented in equation (5.26), if α_C satisfies both conditions given in equations (5.29) and (5.31), and also if $0 < \alpha_C < 1$, then the cost sharing contract is Pareto improving. Thus, the proof of the first part of the theorem is completed.

On the other hand, to prove that there is always an α_C satisfying equation (5.26), notice that the following three conditions should be fulfilled:

$$1-\frac{\tilde{C}_M\left(S_j^{\text{o}}\right)}{\tilde{C}\left(S_j^*\right)} < \frac{C_{S_j}\left(S_j^{\text{o}}\right)}{\tilde{C}\left(S_j^*\right)}, \tag{5.32}$$

$$1-\frac{\tilde{C}_M\left(S_j^{\text{o}}\right)}{\tilde{C}\left(S_j^*\right)} < 1, \tag{5.33}$$

and

$$\frac{C_{S_j}\left(S_j^{\text{o}}\right)}{\tilde{C}\left(S_j^*\right)} > 0. \tag{5.34}$$

Since S_j^* given in equation (4.14) is the optimal solution when the approximate average total costs per unit time for the overall system is tried to be minimized, then it is obvious that $\tilde{C}\left(S_j^*\right) < \tilde{C}\left(S_j^{\text{o}}\right)$, resulting in that the condition given in equation (5.32) is met.

To prove equation (5.33), let us first verify that $\tilde{C}_M\left(S_j^{\text{o}}\right) > 0$ and $\tilde{C}\left(S_j^*\right) > 0.$

Notice that equation (5.12) is equivalent to

$$\tilde{C}_M\left(S_j^{\text{o}}\right) = b_M\left(\frac{\rho_M^2}{1-\rho_M}\right)\left(1-\left(\frac{2}{1+\rho_M^2}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\rho_j^{S_j^{\text{o}}+1}\right), \tag{5.35}$$

and since $S_j^{\text{o}} > 0$ (see the proof of Theorem 4.2), $0 < \rho_j < 1,$ and $0 < \rho_M < 1,$ it is easy to confirm that

$$\left(\frac{2}{1+\rho_M^2}\right)\left(\frac{1-\rho_j}{1+\rho_j}\right)\rho_j^{S_j^{\text{o}}+1} < 2\left(\frac{1-\rho_j}{1+\rho_j}\right)\rho_j < 1. \tag{5.36}$$

Then, using equations (5.35) and (5.36), $b_M > 0,$ and $0 < \rho_M < 1,$ it is proved that

$$\tilde{C}_M\left(S_j^{\text{o}}\right) > 0. \tag{5.37}$$

Now, let us show that $\tilde{C}\left(S_j^*\right) > 0,$ where $\tilde{C}\left(S_j^*\right)$ is as defined in equation (5.27).

First, since $b_j > 0$ and $0 < \rho_j < 1,$ it is obvious that

$$b_j\left(\frac{\rho_j^{S_j^*+1}}{1-\rho_j}\right) > 0. \tag{5.38}$$

Second, to prove

$$S_j^* - \frac{\rho_j\left(1-\rho_j^{S_j^*}\right)}{1-\rho_j} > 0,$$

both sides are multiplied by $\left(1-\rho_j\right)$ giving

$$S_j^*\left(1-\rho_j\right) - \rho_j\left(1-\rho_j^{S_j^*}\right) > 0. \tag{5.39}$$

Afterwards, it can be easily proved that the function given in equation (5.39) is decreasing in ρ_j and takes its minimum value zero at $\rho_j = 1.$ Then, since $0 < \rho_j < 1,$ the function given in equation (5.39) is always positive, leading to

$$S_j^* - \frac{\rho_j\left(1-\rho_j^{S_j^*}\right)}{1-\rho_j} > 0. \tag{5.40}$$

Thereupon, using equation (5.40) and $h_j > 0,$ it is proved that

$$h_j\left(S_j^* - \frac{\rho_j\left(1-\rho_j^{S_j^*}\right)}{1-\rho_j}\right) > 0. \tag{5.41}$$

Finally, equations (5.38) and (5.41) yield that

$$C_{S_j}\left(S_j^*\right) > 0. \tag{5.42}$$

On the other hand, similar to $\tilde{C}_M\left(S_j^{\text{o}}\right) > 0$ given in equation (5.37),

$$\tilde{C}_M\left(S_j^*\right) = b_M\left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right) - 2\rho_j^{S_j^*+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)}\right) > 0 \tag{5.43}$$

can also be proved easily.

Then, from equations (5.42) and (5.43), it is verified that

$$\tilde{C}\left(S_j^*\right) > 0. \tag{5.44}$$

Consequently, using equations (5.37) and (5.44), it is proved that the condition given in equation (5.33) is satisfied.

Finally, to prove equation (5.34), let us first verify that $C_{S_j}\left(S_j^{\text{o}}\right) > 0.$

Similar to $C_{S_j}\left(S_j^*\right) > 0$ given in equation (5.42),

$$C_{S_j}\left(S_j^{\text{o}}\right) > 0 \tag{5.45}$$

can also be shown easily.

Then, from equations (5.44) and (5.45), it is proved that the condition given in equation (5.34) is satisfied.

As a result, since all the conditions given in equations (5.32)-(5.34) are met, there is always an α_C satisfying equation (5.26), completing the proof. □

5.4 Comparison of the Contracts

Three different transfer payment contracts are studied in this thesis to coordinate the decentralized system. These are the backorder cost subsidy contract, the transfer payment contract based on Pareto improvement, and the cost sharing contract. All the contracts have the ability to coordinate the supply chain as given in Theorem 5.1, Theorem 5.3, and Theorem 5.5.

However, besides its ability to coordinate the supply chain, a contract should also be Pareto improving. Otherwise, at least one of the members of the supply chain will not be desirous to participate in the contract. When the contracts are evaluated from this perspective, the backorder cost subsidy contract fails as presented in Theorem 5.2. Conversely, the other two contracts are Pareto improving as given in Theorem 5.4 and Theorem 5.6. Among the other two contracts, in the transfer payment contract based on Pareto improvement, only the supplier is better off after the transfer payment, but the manufacturer is just as well off after the contract as before. On the other hand, in the cost sharing contract, both members can be better off after the transfer payment for an appropriately selected contract parameter. The comparison of the contracts is summarized in Table 5.1.

Table 5.1: Comparison of the contracts.

Contract	Coordination ability	Pareto improvement	Supplier's cost after the contract	Manufacturer's cost after the contract
Backorder cost subsidy contract	Yes	No	Decreases	Increases
Transfer payment contract based on Pareto improvement	Yes	Yes	Decreases	Remains same
Cost sharing contract	Yes	Yes	Decreases	Decreases

Consequently, while all three contracts have the ability to coordinate the supply chain, the cost sharing contract has a dominance over the other contracts when Pareto improvement is taken into account. Therefore, both members of the supply chain will be more advantageous under this contract. As a result, the cost sharing contract is suggested for the coordination of the decentralized supply chain.

Recall that in this thesis, the centralized and decentralized models are developed based on the average backorder and holding costs per unit time. Therefore, according to this cost metric, the centralized system performs better than the decentralized system. Then, three different transfer payment contracts are studied in this chapter for the coordination of the supply chain, so that the average total costs per unit time for the decentralized system become equal to that of the centralized system after the transfer payment. However, when different performance metrics are taken into account, the decentralized system may have a better performance than the centralized system. Therefore, the next chapter presents a numerical study to compare these systems also based on other performance metrics.

6. NUMERICAL STUDY

In this chapter, the centralized and decentralized systems are compared based on SCOR Model performance metrics. The SCOR Model, i.e., The Supply-Chain Operations Reference-model, is developed by the Supply-Chain Council; and it provides a framework and standardized terminology to help organizations integrating a number of management tools. To our knowledge, it is the most widely accepted supply chain reference model in use (Cohen and Roussel, 2005).

SCOR Model defines five key performance attributes, which are reliability, responsiveness, flexibility, cost, and assets. Among these attributes, reliability, responsiveness, and flexibility directly affect the customer, i.e., they are customer-facing, whereas cost and assets have a direct impact on the company, i.e., internal-facing (Presutti and Mawhinney, 2007).

In the light of these attributes, SCOR Model associates several performance metrics with each attribute. The metrics are defined in three levels such that level one metrics are designed to provide a view of the overall supply chain effectiveness. These metrics are then decomposed into a group of more detailed level two and level three metrics (Cohen and Roussel, 2005).

To be able to compare the centralized and decentralized systems unbiasedly, the performance metrics are chosen such that one metric is associated with each SCOR Model performance attribute as given in Table 6.1. While selecting the performance metrics, their calculability using the models presented in this thesis is taken into account. The complete list of SCOR Model performance metrics can be found in Cohen and Roussel (2005).

Table 6.1: The performance metrics and corresponding performance attributes.

Performance metrics	Performance attributes				
	Customer-facing			Internal-facing	
	Reliability	Responsiveness	Flexibility	Cost	Assets
Total number of outstanding backorders	X				
Order fulfillment lead time		X			
Supply chain response time			X		
Total backorder and holding costs				X	
Inventory days of supply					X

6.1 Design of Experiment

To compare the performance of the centralized and decentralized supply chains, three different experiments are designed for a system with four independent suppliers and a manufacturer. The reason of performing different experiments is to consider several cases in which the traffic intensities of the supply chain members can be classified as low, medium, and high. In all experiments, $\lambda = 1$, $b_i \in \{25, 50, 100\}$, $h_i \in \{1, 10, 20\}$, and $b_M \in \{400, 600, 800\}$ for $i = 1, \ldots, 4$. The experiments differ in the values that ρ_i and ρ_M can take on: $\rho_i \in \{0.10, 0.40, 0.55\}$ and $\rho_M \in \{0.28, 0.30\}$ in the first experiment; $\rho_i \in \{0.35, 0.50, 0.67\}$ and $\rho_M \in \{0.38, 0.40\}$ in the second experiment; and lastly, $\rho_i \in \{0.45, 0.60, 0.90\}$ and $\rho_M \in \{0.48, 0.50\}$ in the third experiment, where $i = 1, \ldots, 4$. Then, for each experiment, Taguchi designs[4] are created using Minitab 15, each with 54 runs. The final data set satisfying the assumptions given in equations (4.8) and (4.9) is given in Table 6.2.

Table 6.2: The final data set of the experiments.

No.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_M	b_1	b_2	b_3	b_4	b_M	h_1	h_2	h_3	h_4
1	0.10	0.55	0.55	0.55	0.28	100	100	100	25	400	1	1	1	1
2	0.55	0.55	0.55	0.40	0.28	50	25	25	100	800	10	10	1	1
3	0.55	0.55	0.55	0.40	0.28	50	25	25	50	600	1	1	20	20
4	0.40	0.10	0.55	0.10	0.28	50	100	50	100	600	20	1	10	1
5	0.40	0.40	0.55	0.10	0.30	100	50	25	100	400	10	1	20	10
6	0.35	0.67	0.67	0.67	0.38	100	100	100	25	400	1	1	1	1
7	0.35	0.67	0.67	0.67	0.38	100	100	100	50	600	10	10	10	10
8	0.67	0.67	0.67	0.50	0.38	50	25	25	100	800	10	10	1	1
9	0.67	0.67	0.67	0.50	0.38	50	25	25	50	600	1	1	20	20
10	0.50	0.35	0.67	0.35	0.38	50	100	50	100	600	20	1	10	1
11	0.67	0.67	0.50	0.35	0.40	100	25	50	100	600	1	20	1	10
12	0.50	0.50	0.67	0.35	0.40	100	50	25	100	400	10	1	20	10
13	0.45	0.90	0.90	0.90	0.48	100	100	100	25	400	1	1	1	1
14	0.45	0.90	0.90	0.90	0.48	100	100	100	50	600	10	10	10	10
15	0.90	0.90	0.90	0.60	0.48	50	25	25	100	800	10	10	1	1
16	0.90	0.90	0.90	0.60	0.48	50	25	25	50	600	1	1	20	20
17	0.60	0.45	0.90	0.45	0.48	50	100	50	100	600	20	1	10	1
18	0.60	0.60	0.90	0.45	0.50	100	50	25	100	400	10	1	20	10

[4] While creating Taguchi designs, the values are entered in the order of minimum, maximum, and median values.

6.2 The Centralized and Decentralized Solutions

After the final experimental design has been determined, for the 18 systems given in Table 6.2, the integer centralized and decentralized solutions are calculated as explained in Remark 4.1 and Remark 4.2, respectively.

Table 6.3 presents the integer centralized and decentralized solutions. The numbers in bold denote the base stock levels of supplier *j*, where *j* is determined as explained in Remark 4.1.

Notice that in each system, only the base stock level of supplier *j* differs in the centralized and decentralized solutions as stated before and $\text{int}\, S_j^* < \text{int}\, S_j^{\text{o}}$ as proved in Proposition 5.1 for the continuous case. Also, observe that in the systems where $\text{int}\, S_j^{\text{o}} = \text{int}\, S_i^{\text{o}}$ for some $i \neq j$, $\rho_j \geq \rho_i$ as stated previously.

Table 6.3: The centralized and decentralized solutions (in integer).

No.	$\text{int}\, S_1^*$	$\text{int}\, S_2^*$	$\text{int}\, S_3^*$	$\text{int}\, S_4^*$	$\text{int}\, S_1^{\text{o}}$	$\text{int}\, S_2^{\text{o}}$	$\text{int}\, S_3^{\text{o}}$	$\text{int}\, S_4^{\text{o}}$
1	**1**	7	7	5	**2**	7	7	5
2	2	**0**	5	5	2	**2**	5	5
3	6	5	**0**	1	6	5	**1**	1
4	**0**	2	2	2	**1**	2	2	2
5	2	4	**0**	1	2	4	**1**	1
6	**3**	11	11	8	**4**	11	11	8
7	**1**	5	5	4	**2**	5	5	4
8	4	**0**	8	6	4	**3**	8	6
9	9	8	2	**0**	9	8	2	**1**
10	**0**	4	4	4	**1**	4	4	4
11	11	**0**	5	2	11	**2**	5	2
12	3	5	**1**	2	3	5	**2**	2
13	**4**	43	43	30	**5**	43	43	30
14	**0**	22	22	17	**3**	22	22	17
15	17	11	30	**7**	17	11	30	**9**
16	37	30	7	**0**	37	30	7	**2**
17	**0**	5	17	5	**2**	5	17	5
18	4	7	7	**1**	4	7	7	**3**

6.3 Comparison of the Centralized and Decentralized Systems

In this section, the centralized and decentralized systems are compared based on the performance metrics presented in Table 6.1. The definitions and calculations of the metrics are given below. Notice that the expected values of the metrics are used to compare the two systems.

6.3.1 Total number of outstanding backorders

Total number of outstanding backorders, which is denoted by B_T, is the sum of the outstanding backorders at the suppliers and the manufacturer. From equations (3.33) and (3.42), the expected value of this metric is calculated as

$$\begin{aligned} E[B_T] &= \sum_{i=1}^{4} E[B_i] + E[B_M] \\ &\simeq \sum_{i=1}^{4} \frac{\rho_i^{S_i+1}}{1-\rho_i} + \left(\frac{2\rho_M^2}{1+\rho_M^2}\right)\left(\frac{(1+\rho_j)(1+\rho_M^2) - 2\rho_j^{S_j+1}(1-\rho_j)}{2(1+\rho_j)(1-\rho_M)}\right), \end{aligned} \tag{6.1}$$

where $j = \arg\min_{i=1,\ldots,n} S_i$.

The comparison of the centralized and decentralized systems according to the average total number of outstanding backorders is presented in Table 6.4, where "Increase (%)" denotes the percentage increase of the decentralized system over the centralized system according to the relevant metric.

Table 6.4: Comparison of the systems according to the average total number of outstanding backorders.

No.	Average total number of outstanding backorders		
	Centralized system	Decentralized system	Increase (%)
1	0.2171	0.2086	–3.92
2	1.7369	0.9069	–47.79
3	1.6609	1.1254	–32.24
4	1.1129	0.7337	–34.08
5	1.4480	0.9149	–36.81
6	0.3851	0.3720	–3.40
7	1.3547	1.2478	–7.89
8	2.7165	1.3345	–50.87
9	2.2141	1.7481	–21.05
10	1.5904	1.1243	–29.31
11	2.3581	1.2728	–46.03
12	1.8084	1.3730	–24.08
13	1.0471	1.0314	–1.50
14	4.4119	3.7800	–14.32
15	5.1888	5.1639	–0.48
16	6.7037	5.8129	–13.29
17	3.3662	2.4753	–26.46
18	5.3478	5.1032	–4.57
Min	0.2171	0.2086	–50.87
Max	6.7037	5.8129	–0.48
Average	2.4816	1.9849	–22.12

Table 6.4 indicates that the average total number of outstanding backorders is higher in the centralized system than in the decentralized system for each case. Notice that in the centralized system, the base stock level of supplier *j* decreases (see Proposition 5.1), increasing his average outstanding backorders, while reducing that of the manufacturer's. On the other hand, the average numbers of outstanding backorders at the other suppliers remain the same. Consequently, as also depicted in Figure 6.1, the sum of all these terms increases in the centralized system. The results denote that according to the relevant metric, the minimum percentage increase of the decentralized system over the centralized system is calculated as −0.48%, the average increase as −22.12%, and the maximum increase as −50.87%.

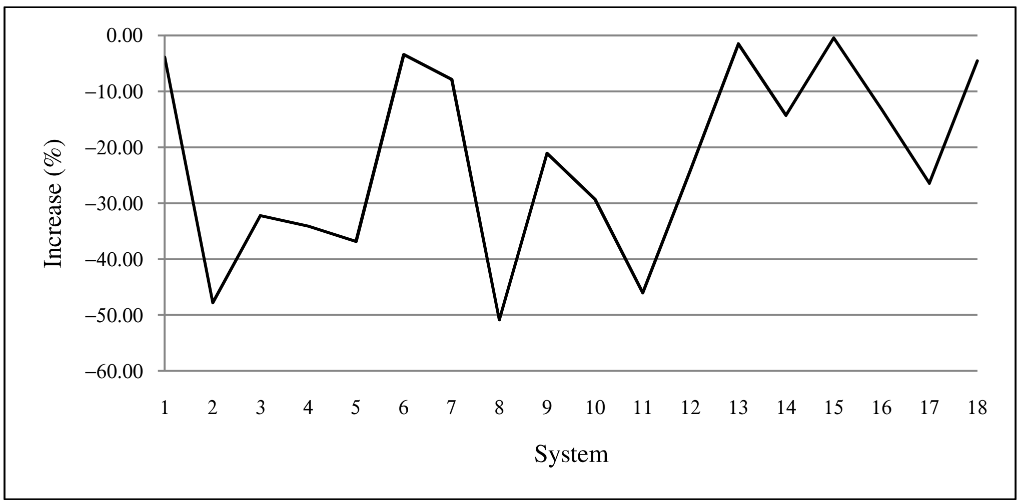

Figure 6.1: The percentage increase of the decentralized system over the centralized system according to the average total number of outstanding backorders.

6.3.2 Order fulfillment lead time

Order fulfillment lead time is the number of time units from the customer order authorization to the customer order receipt. Under the queuing theory concept, Buzacott and Shanthikumar (1993) determine the distribution of the time to fill a demand in an $M/M/1$ make-to-stock queue under stationary conditions.

Recall that each supplier is modeled as an $M/M/1$ make-to-stock queue in this thesis and let L_i denote the order fulfillment lead time for supplier *i*, where $i = 1,\ldots,4$. Then, from Buzacott and Shanthikumar (1993),

$$P\{L_i > x\} = \rho_i^{S_i} e^{-(\mu_i - \lambda)x}, \quad x \geq 0, \qquad \textbf{(6.2)}$$

giving the expected value

$$E[L_i] = \frac{\rho_i^{S_i}}{\mu_i - \lambda}, \quad i = 1, \ldots, 4. \tag{6.3}$$

Also recall that the manufacturer is modeled as a $GI/M/1$ queue and N_{q_M} denotes the number of jobs in the manufacturer's queue, i.e., the outstanding backorders at the manufacturer. Then, if L_M denotes the order fulfillment lead time for the manufacturer, using Little's formula it is easy to prove that

$$E[L_M] = \frac{E[N_{q_M}]}{\lambda} + \frac{1}{\mu_M}. \tag{6.4}$$

Finally, the expected value of the order fulfillment lead time for the overall system, denoted by L_S, can be expressed as

$$E[L_S] = E\left[\max_{i=1,\ldots,4} L_i\right] + E[L_M]. \tag{6.5}$$

In this thesis, two different approximations are considered to calculate the expected value of the maximum of suppliers' order fulfillment lead times, which is needed to calculate $E[L_S]$ from equation (6.5). In the first approximation, it is assumed that the order fulfillment lead times of the suppliers are independent from each other. If L denotes their maximum, then from

$$P\{L < x\} = \prod_{i=1}^{4} P\{L_i < x\}, \quad x \geq 0, \tag{6.6}$$

the expected value of L can be calculated, where $P\{L_i < x\}$ is found using equation (6.2). However, since the suppliers are triggered by the same arrival process, actually they are not independent. According to Zhao and Simchi-Levi (2006), the order fulfillment lead time in a single product assembly system with dependent component delays is stochastically smaller than the order fulfillment lead time in an analogous system with independent component delays. Consequently, this approximation overestimates the maximum of suppliers' order fulfillment lead times.

Therefore, a second approximation is also used such that the expected value of the maximum of suppliers' order fulfillment lead times is equal to the maximum of their expected values given in equation (6.3). However, such an approximation is known to be underestimating.

Since the first approximation overestimates and the second one underestimates, taking their average is expected to give a better result. Therefore, their average is used to find the expected value of the maximum of suppliers' order fulfillment lead times. Then, by substituting this value and $E[L_M]$ found from equation (6.4) into equation (6.5), the expected value of the order fulfillment lead time for the overall system is calculated.

The comparison of the centralized and decentralized systems according to the average order fulfillment lead time is presented in Table 6.5.

Table 6.5: Comparison of the systems according to the average order fulfillment lead time.

No.	Average order fulfillment lead time		
	Centralized system	Decentralized system	Increase (%)
1	0.4720	0.4688	–0.67
2	1.6861	0.9303	–44.83
3	1.6479	1.1596	–29.63
4	1.1407	0.8525	–25.26
5	1.6352	1.1219	–31.39
6	0.7276	0.7224	–0.72
7	1.2999	1.2743	–1.97
8	2.7131	1.4181	–47.73
9	1.8885	1.7108	–9.41
10	1.6846	1.2544	–25.54
11	2.6542	1.5919	–40.02
12	2.0429	1.6240	–20.51
13	1.4100	1.4041	–0.42
14	3.3810	3.2320	–4.41
15	4.5380	4.5318	–0.14
16	5.7523	5.5531	–3.46
17	2.9633	2.6263	–11.37
18	5.4055	5.3801	–0.47
Min	0.4720	0.4688	–47.73
Max	5.7523	5.5531	–0.14
Average	2.3913	2.0476	–16.55

Table 6.5 points out that the average order fulfillment lead time is higher in the centralized system than in the decentralized system for each case. This situation can also be interpreted in a similar way as the increase of the total number of outstanding backorders in the centralized system. According to the average order fulfillment lead time, the minimum percentage increase of the decentralized system over the centralized system is calculated as −0.14%, the average increase as −16.55%, and the

maximum increase as −47.73%. The percentage increases are also represented in Figure 6.2.

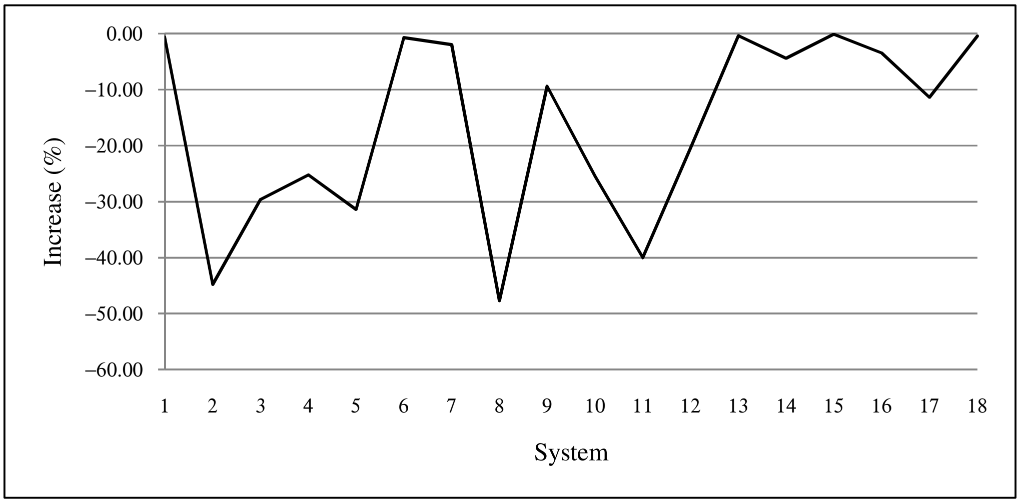

Figure 6.2: The percentage increase of the decentralized system over the centralized system according to the average order fulfillment lead time.

6.3.3 Supply chain response time

Supply chain response time is the amount of time it takes a supply chain to respond to an unplanned significant increase or decrease in demand without cost penalty (Bolstorff and Rosenbaum, 2003). In this thesis, the increase in demand is taken as 10%. Accordingly, this metric calculates the amount of increase in the order fulfillment lead time when λ rises to 1.10. Therefore, first the expected values of the new order fulfillment lead times are calculated as explained in section 6.3.2. Then, the differences between the new values and the ones given in Table 6.5 are calculated, giving the average supply chain response times.

The comparison of the centralized and decentralized systems according to the average supply chain response time is given in Table 6.6.

Table 6.6 denotes that the average supply chain response times are generally (in 13 of the 18 cases) lower in the centralized system than in the decentralized system, i.e., the centralized system responds to a 10% increase in demand more quickly. Notice that the response times dramatically rise for systems 13-18, since at least one of the suppliers has a traffic intensity of 0.99 when the demand increases by 10% in these systems.

Table 6.6: Comparison of the systems according to the average supply chain response time.

No.	Average supply chain response time		
	Centralized system	Decentralized system	Increase (%)
1	0.0881	0.0884	0.34
2	0.2308	0.2213	–4.10
3	0.2100	0.2163	3.01
4	0.1078	0.1643	52.35
5	0.1934	0.1953	1.02
6	0.2410	0.2402	–0.31
7	0.5786	0.5883	1.66
8	0.6771	0.6195	–8.51
9	0.5242	0.5948	13.47
10	0.2703	0.3582	32.52
11	0.5867	0.5456	–7.00
12	0.5780	0.5360	–7.26
13	98.1802	98.1899	0.01
14	108.7690	109.0387	0.25
15	110.7346	110.7441	0.01
16	109.0396	109.2991	0.24
17	73.9594	74.2758	0.43
18	79.5465	79.6196	0.09
Min	0.0881	0.0884	–8.51
Max	110.7346	110.7441	52.35
Average	32.4731	32.5297	4.35

The percentage increase of the decentralized system over the centralized system according to the average supply chain response time is also depicted in Figure 6.3. According to this metric, the minimum increase is calculated as −8.51%, the average increase as 4.35%, and the maximum increase as 52.35%.

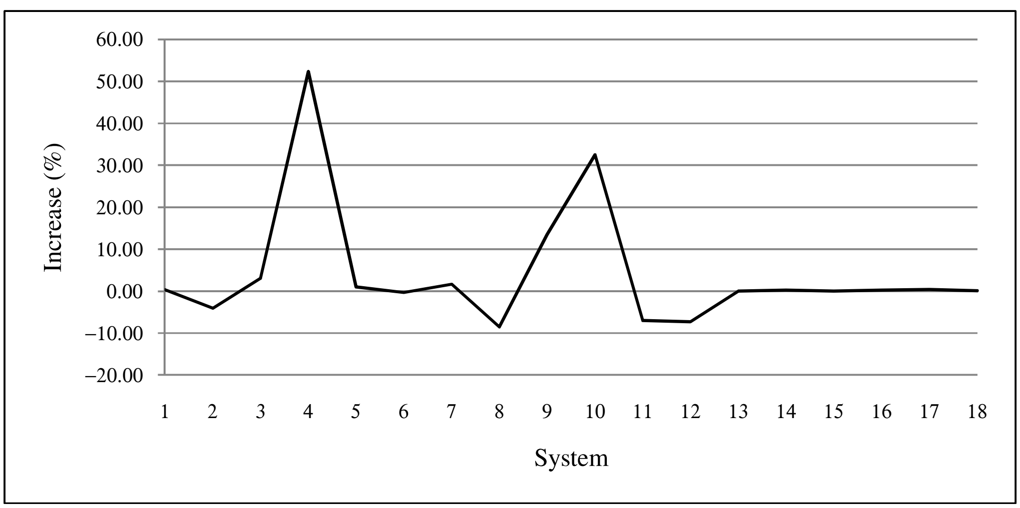

Figure 6.3: The percentage increase of the decentralized system over the centralized system according to the average supply chain response time.

6.3.4 Total backorder and holding costs

From equations (3.33), (3.34), and (3.42), the expected value of the total backorder and holding costs is given by

$$\begin{aligned} E[C_T] &= \sum_{i=1}^{4} b_i E[B_i] + \sum_{i=1}^{4} h_i E[I_i] + b_M E[B_M] \\ &\simeq \sum_{i=1}^{4} b_i \left(\frac{\rho_i^{S_i+1}}{1-\rho_i} \right) + \sum_{i=1}^{4} h_i \left(S_i - \frac{\rho_i \left(1-\rho_i^{S_i}\right)}{1-\rho_i} \right) \\ &\quad + b_M \left(\frac{2\rho_M^2}{1+\rho_M^2} \right) \left(\frac{\left(1+\rho_j\right)\left(1+\rho_M^2\right) - 2\rho_j^{S_j+1}\left(1-\rho_j\right)}{2\left(1+\rho_j\right)\left(1-\rho_M\right)} \right), \end{aligned} \quad \textbf{(6.7)}$$

where $j = \arg\min_{i=1,\ldots,n} S_i$.

Table 6.7 presents the comparison of the centralized and decentralized systems according to the average total backorder and holding costs.

Table 6.7: Comparison of the systems according to the average total backorder and holding costs.

No.	Average total backorder and holding costs		
	Centralized system	Decentralized system	Increase (%)
1	65.5971	66.1820	0.89
2	132.2308	140.3867	6.17
3	113.7549	117.7113	3.48
4	111.8585	116.3214	3.99
5	106.2978	108.3283	1.91
6	127.8397	128.0887	0.19
7	314.6279	320.5158	1.87
8	252.9050	263.3618	4.13
9	207.3459	212.6975	2.58
10	201.8388	207.1904	2.65
11	213.1738	222.9510	4.59
12	192.1756	197.3586	2.70
13	300.2937	300.5390	0.08
14	898.9457	914.1856	1.70
15	681.6098	682.4440	0.12
16	497.9630	512.2474	2.87
17	457.4969	471.7813	3.12
18	425.4746	432.7779	1.72
Min	65.5971	66.1820	0.08
Max	898.9457	914.1856	6.17
Average	294.5239	300.8371	2.49

Table 6.7 indicates that the average total backorder and holding costs is lower in the centralized system than in the decentralized system for each case. This result is

predictable since the models are based on cost minimization. According to the relevant metric, the minimum percentage increase of the decentralized system over the centralized system is calculated as 0.08%, the average increase as 2.49%, and the maximum increase as 6.17%. The percentage increase in the cost of the decentralized system over the centralized system is generally referred to as "competition penalty" in the literature, which is also represented in Figure 6.4.

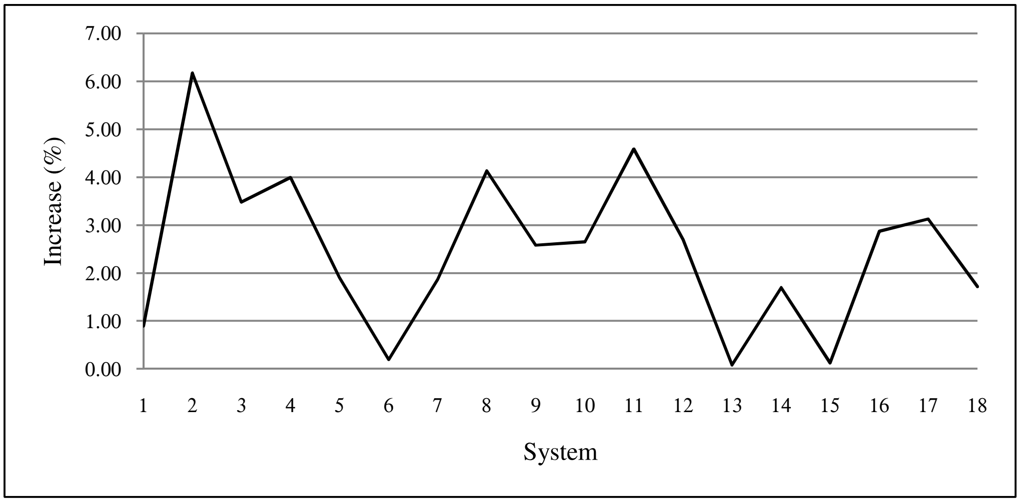

Figure 6.4: The percentage increase of the decentralized system over the centralized system according to the average total backorder and holding costs.

6.3.5 Inventory days of supply

Inventory days of supply is the number of days that cash is tied up as inventory (Bolstorff and Rosenbaum, 2003). This metric is calculated as

$$\text{Average inventory days of supply} = \frac{\text{Average aggregate value of inventory}}{\text{Annual cost of goods sold}/365 \text{ days}}. \qquad \textbf{(6.8)}$$

In the computation of average inventory days of supply, some values are needed that are not included in the models; and these values are derived as follows, where $i = 1, \ldots, 4$:

i. The unit cost of supplier *i* is determined such that h_i is 10%–40% of the unit cost.

ii. The selling price of supplier *i*, i.e., the direct material cost of the manufacturer based on supplier *i*, is determined such that b_i is 25%–80% of the selling price.

iii. The value of unit inventory of supplier *i* is the direct material cost of the manufacturer based on supplier *i*. Then, the average value of inventory for a supplier is calculated by multiplying his average inventory level with the corresponding value of unit inventory.

iv. Cost of goods sold (COGS) is taken as 1.40 times of the total direct material cost of the manufacturer. Since $0 < \rho_i < 1$ for $i = 1, \ldots, 4$, $0 < \rho_M < 1$, and $\lambda = 1$, assuming that one demand arrives per hour, it is expected that also one unit is sold per hour. Then, if there are 2080 working hours in a year, the expected number of units sold per year is also 2080. Therefore, annual COGS is calculated by multiplying COGS with 2080.

The data used to calculate the average inventory days of supply is given in Table 6.8.

Table 6.8: Data used to calculate the average inventory days of supply.

No.	Unit costs of the suppliers				Direct material cost of the manufacturer based on				Total direct material cost	COGS
	Sup. 1	Sup. 2	Sup. 3	Sup. 4	Sup. 1	Sup. 2	Sup. 3	Sup. 4		
1	10	10	10	10	125	125	125	31	406	569
2	75	50	10	10	125	63	63	250	500	700
3	10	10	50	75	125	63	63	125	375	525
4	50	10	25	10	63	125	63	125	375	525
5	100	10	50	100	250	125	63	250	688	963
6	10	10	10	10	125	125	125	31	406	569
7	75	75	75	50	125	125	125	63	438	613
8	75	50	10	10	125	63	63	250	500	700
9	10	10	50	75	125	63	63	125	375	525
10	50	10	25	10	63	125	63	125	375	525
11	10	50	10	100	250	63	125	250	688	963
12	100	10	50	100	250	125	63	250	688	963
13	10	10	10	10	125	125	125	31	406	569
14	75	75	75	50	125	125	125	63	438	613
15	75	50	10	10	125	63	63	250	500	700
16	10	10	50	75	125	63	63	125	375	525
17	50	10	25	10	63	125	63	125	375	525
18	100	10	50	100	250	125	63	250	688	963

Table 6.9 presents the comparison of the centralized and decentralized systems according to the average inventory days of supply.

Table 6.9 points out that the average inventory days of supply is lower in the centralized system than in the decentralized system for each case. Recall that in the centralized system, the base stock level of supplier j decreases, whereas the base stock levels of the other suppliers remain the same. Therefore, this result is predictable.

Table 6.9: Comparison of the systems according to the average inventory days of supply.

No.	Average inventory days of supply		
	Centralized system	Decentralized system	Increase (%)
1	0.5188	0.5570	7.36
2	0.3681	0.3861	4.88
3	0.3063	0.3157	3.07
4	0.1819	0.1944	6.89
5	0.1830	0.1881	2.80
6	0.8480	0.8860	4.48
7	0.2982	0.3296	10.54
8	0.4837	0.5085	5.12
9	0.4383	0.4592	4.77
10	0.3396	0.3501	3.08
11	0.5715	0.5815	1.76
12	0.2621	0.2684	2.40
13	2.9602	2.9981	1.28
14	1.1647	1.2455	6.94
15	1.0556	1.1793	11.71
16	1.6723	1.7158	2.60
17	0.5492	0.5709	3.96
18	0.3004	0.3782	25.89
Min	0.1819	0.1881	1.28
Max	2.9602	2.9981	25.89
Average	0.6946	0.7285	6.08

The percentage increase of the decentralized system over the centralized system according to the average inventory days of supply is also depicted in Figure 6.5. According to this metric, the minimum increase is calculated as 1.28%, the average increase as 6.08%, and the maximum increase as 25.89%.

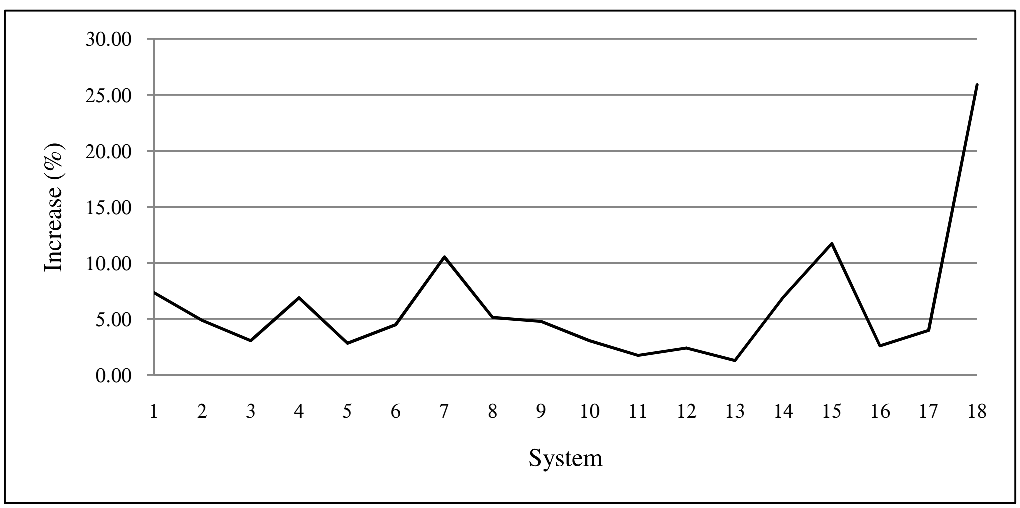

Figure 6.5: The percentage increase of the decentralized system over the centralized system according to the average inventory days of supply.

Notice that for all the performance metrics considered in this thesis, the less the better. The results denote that the decentralized system has a better performance than the centralized system according to the total number of outstanding backorders and the order fulfillment lead time, which are customer-facing metrics. On the other hand, according to the internal-facing metrics, which are the total backorder and holding costs and the inventory days of supply, the centralized system has a better performance in all cases. Finally, if there is a 10% increase in demand, the centralized system generally responds to this increase more quickly. Therefore, according to the supply chain response time, which is also a customer-facing metric, the centralized system generally has a better performance than the decentralized system.

6.4 Selection Among the Centralized and Decentralized Systems

In section 6.3, the centralized and decentralized systems are compared based on five performance metrics presented in Table 6.1; and in this section, a multi-criteria decision making method is used to decide which system is more preferable. For this purpose, the simple additive weighting (SAW) method is selected.

According to Yoon and Hwang (1995), among the multi-criteria decision making methods such as SAW, weighted product method, TOPSIS or ELECTRE, no one has a significant dominance over another. Therefore, and also since its implementation is simple, the SAW method is preferred in this thesis.

To select among the centralized and decentralized systems, the average of the data given in Tables 6.4-6.7 and 6.9 is used as presented in Table 6.10.

Table 6.10: The average values of the performance metrics for the centralized and decentralized systems.

Performance metrics	Centralized system	Decentralized system
Total number of outstanding backorders	2.4816	1.9849
Order fulfillment lead time	2.3913	2.0476
Supply chain response time	32.4731	32.5297
Total backorder and holding costs	294.5239	300.8371
Inventory days of supply	0.6946	0.7285

To apply the SAW method, first the data has to be normalized. In this thesis, the Manhattan distance based normalization method (Ginevičius and Podvezko, 2008;

Triantaphyllou, 2000, pp. 138-139) is preferred to normalize the data since it is simple to interpret and it conserves proportionality.

In the SAW method, the value of alternative *i* can be expressed as

$$V_i = \sum_{j=1}^{n} w_j r_{ij}, \quad i = 1, \ldots, m, \tag{6.9}$$

where V_i is the value of alternative *i*; r_{ij} is the normalized value of alternative *i* in terms of criterion *j*; and w_j is the weight of criterion *j*. Notice that the weights of the criteria should add up to one.

Additionally, in the Manhattan distance based normalization, the normalized values are calculated by

$$r_{ij} = \frac{v_{ij}}{\sum_{i=1}^{m} |v_{ij}|}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \tag{6.10}$$

where v_{ij} is the value of alternative *i* in terms of criterion *j*.

Table 6.11 presents the normalized values of the alternatives in terms of each criterion. In Table 6.11, A_i denotes alternative *i*, where $i = 1$ for the centralized system and $i = 2$ for the decentralized system. Besides, C_j denotes criterion *j*, where $j = 1$ for the total number of outstanding backorders; $j = 2$ for the order fulfillment lead time; $j = 3$ for the supply chain response time; $j = 4$ for the total backorder and holding costs; and $j = 5$ for the inventory days of supply.

Table 6.11: The normalized values of the alternatives in terms of each criterion.

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	0.5556	0.5387	0.4996	0.4947	0.4881
A_2	0.4444	0.4613	0.5004	0.5053	0.5119

Since each performance metric is equally important in the SCOR model, the weight of each criterion is taken equal to each other, i.e., $w_j = 0.20$ for all $j = 1, \ldots, 5,$ while applying the SAW method. Then, using equation (6.9) and the data given in Table 6.11, the values of the alternatives are calculated as $V_1 = 0.5153$ and $V_2 = 0.4847$. Recall that the less the better for all criteria, giving that the alternative with the minimum value has to be selected. Then, since $V_2 < V_1$, it is concluded that the decentralized system is preferred over the centralized system.

6.5 Sensitivity Analysis

In the previous section, when each criterion is taken as equally important, it is found that the decentralized system is preferred over the centralized system according to the five criteria given in Table 6.11. In addition, this section presents a sensitivity analysis performed in a similar way as explained in Triantaphyllou (2000).

The objective of the sensitivity analysis is to determine the most sensitive criterion. First, from equations (6.11a) and (6.11b), the minimum change δ_j in the current weight w_j of criterion *j* is calculated for each $j = 1,\ldots,5$ so that the ranking of the alternatives A_1 and A_2 is reversed:

$$\delta_j < \frac{V_1 - V_2}{r_{1j} - r_{2j}}, \quad \text{if } r_{1j} < r_{2j}, \quad j = 1,\ldots,5, \qquad \textbf{(6.11a)}$$

$$\delta_j > \frac{V_1 - V_2}{r_{1j} - r_{2j}}, \quad \text{if } r_{1j} > r_{2j}, \quad j = 1,\ldots,5, \qquad \textbf{(6.11b)}$$

where V_1 and V_2 are calculated from equation (6.9); r_{1j} and r_{2j} are calculated from equation (6.10). Notice that only the weight of criterion *j* is modified each time, while the weights of the other criteria remain the same. Also note that $V_2 < V_1$ in the current situation as given in the previous section.

After calculating δ_j for each $j = 1,\ldots,5,$ the new weight w_j^* of each criterion *j* is calculated by

$$w_j^* = w_j - \delta_j, \quad j = 1,\ldots,5, \qquad \textbf{(6.12)}$$

giving the following results. Notice that $w_j^* \notin [0,1]$ for $j = 1,\ldots,5$ since these are the values obtained before normalization:

i. $w_1^* < -0.0758,$ indicating that the ranking of the alternatives A_1 and A_2 is reversed for w_1 smaller than -0.0758. However, since w_1 is infeasible in this interval, it is not possible to change the ranking of the alternatives by changing w_1. This situation can also be seen in Figure 6.6. Notice that for all feasible values of w_1, the decentralized system is preferred over the centralized system, i.e., the ranking of the alternatives does not change. Also note that in Figure 6.6, the weights after normalization are considered; and the values of the alternatives are calculated using the normalized weights[5].

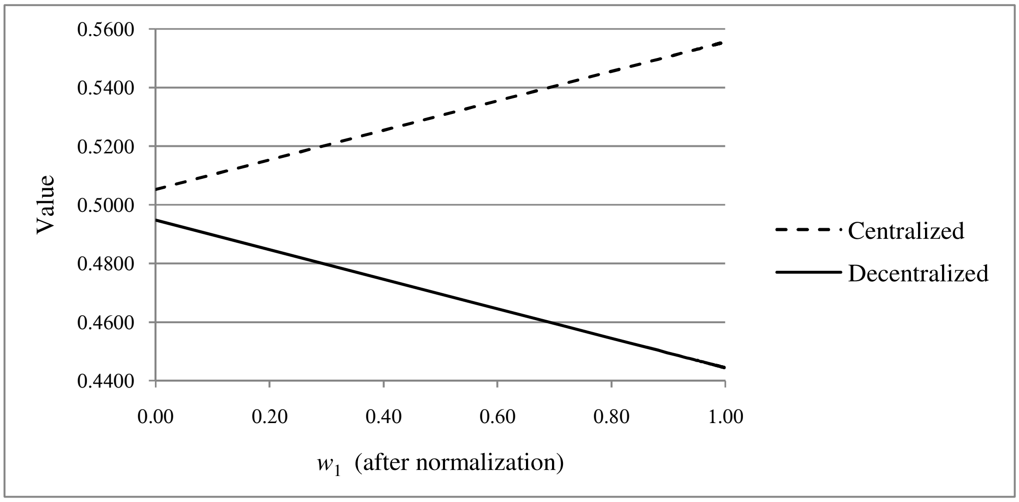

Figure 6.6: The values of the alternatives as a function of w_1 after normalization.

ii. $w_2^* < -0.1960$, pointing out that it is also impossible to alter the ranking of the alternatives by changing w_2 as depicted in Figure 6.7.

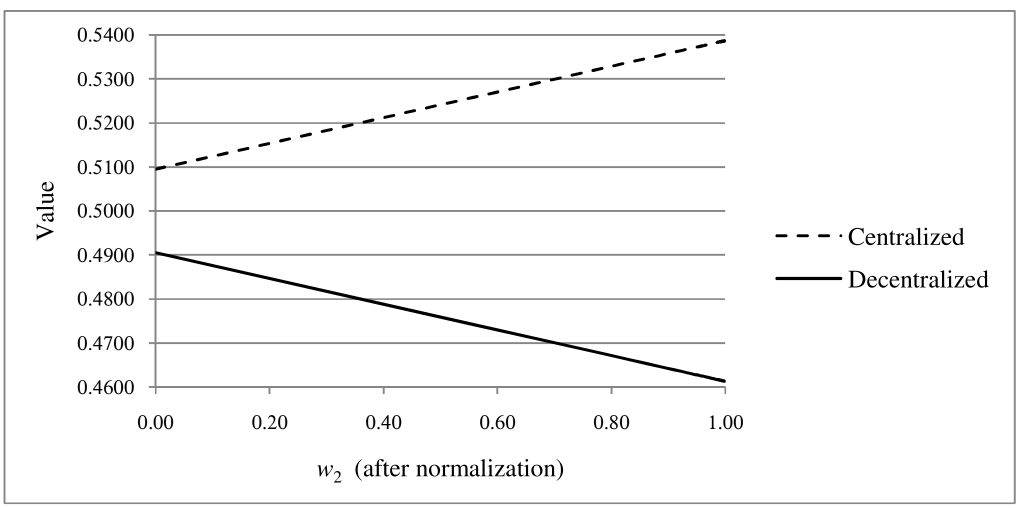

Figure 6.7: The values of the alternatives as a function of w_2 after normalization.

[5] This is also valid for Figures 6.7-6.10.

iii. $w_3^* > 35.3690$ (corresponds to greater than 0.9779 after normalization), meaning that for w_3 belonging to this interval, the ranking of the alternatives A_1 and A_2 is reversed as represented in Figure 6.8.

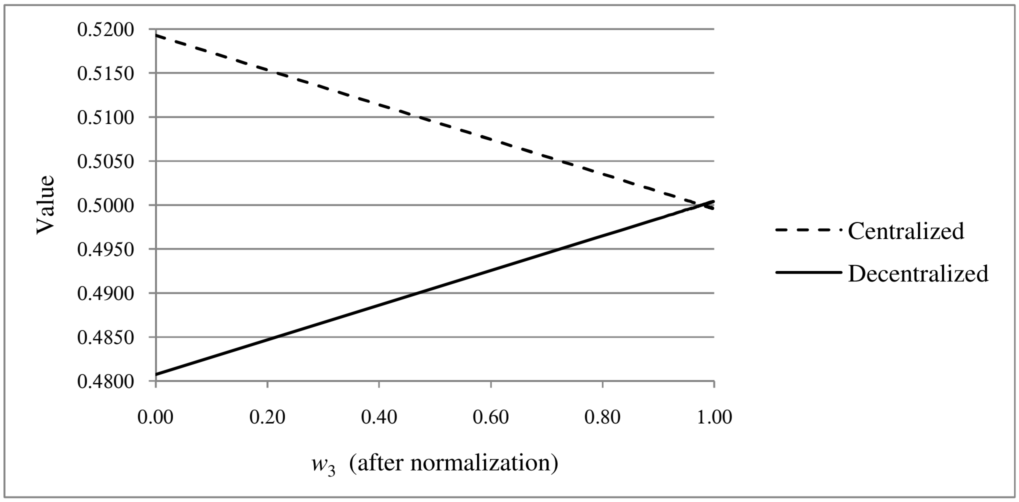

Figure 6.8: The values of the alternatives as a function of w_3 after normalization.

iv. $w_4^* > 3.0918$ (corresponds to greater than 0.7944 after normalization), indicating that when w_4 takes a value in this interval, the ranking of the alternatives A_1 and A_2 changes as depicted in Figure 6.9.

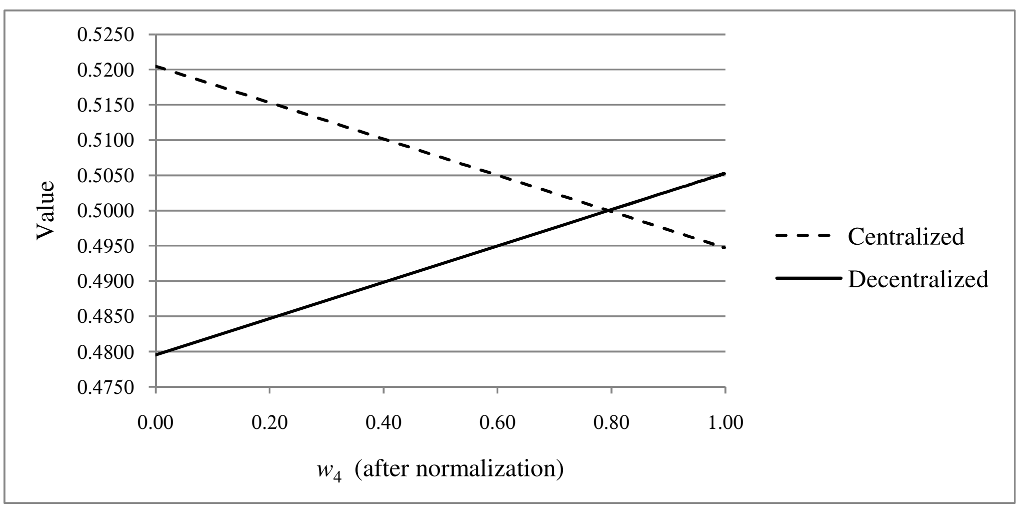

Figure 6.9: The values of the alternatives as a function of w_4 after normalization.

v. $w_5^* > 1.4870$ (corresponds to greater than 0.6502 after normalization), meaning that the ranking of the alternatives A_1 and A_2 is reversed for w_5 belonging to this interval. This result can also be seen in Figure 6.10.

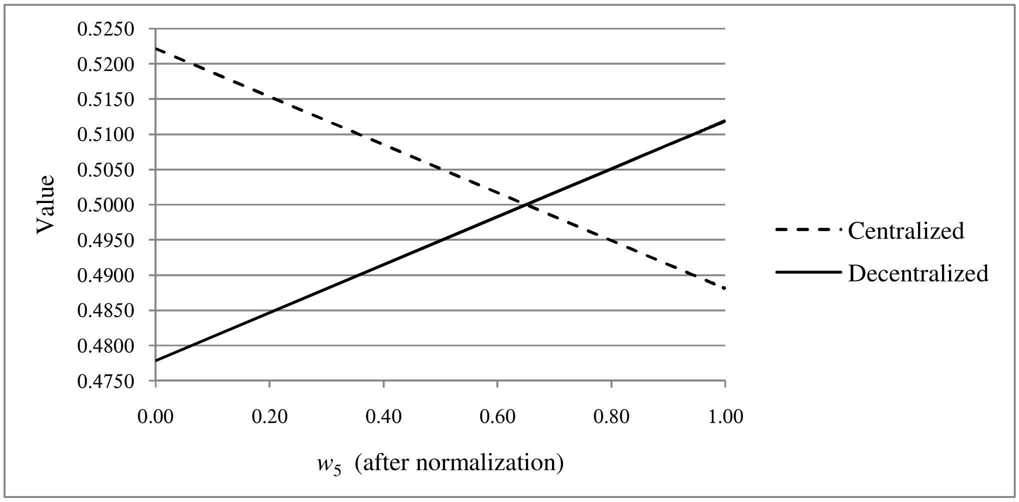

Figure 6.10: The values of the alternatives as a function of w_5 after normalization.

In summary, if each criterion is taken as equally important, it is found that the decentralized system is preferred over the centralized system. The results denote that when the weight of criterion j is modified and the others remain the same for $j = 1, \ldots, 5,$ changing the values of w_1 or w_2 cannot alter this ranking. However, for $w_3^* > 35.3690,$ $w_4^* > 3.0918,$ or $w_5^* > 1.4870,$ the ranking of the alternatives changes and the centralized system becomes more preferred over the decentralized system.

Calculating the minimum percentage increases between the current and new weights of the criteria gives 17584.49%, 1445.89%, and 643.48% for $j = 3, 4, 5,$ respectively. Notice that since w_1^* and w_2^* are not feasible, they are not included in this calculation. Then, the most sensitive criterion is the one for which the minimum percentage increase between its current and new weights is the smallest among the others.

Consequently, the inventory days of supply is the most sensitive criterion; and it is followed by the total backorder and holding costs, and the supply chain response time, respectively. On the other hand, the total number of outstanding backorders and the order fulfillment lead time are insensitive to the ranking of the alternatives.

Now, let us design an experiment, where $w_j \in \{1, 2, 5, 10, 20\}$ before normalization for $j = 1, \ldots, 5.$ For this purpose, a full factorial design is created using Minitab 15 with 3125 runs. First, the weights of the criteria are normalized so that they add up to one. Then, using the normalized weights and the data given in Table 6.11, the value of each alternative is calculated. The results denote that in 2901 of the 3125 cases, the decentralized system is preferred over the centralized system, giving a percentage of 92.83%.

Finally, the results of the numerical study can be summarized as follows:

i. The decentralized system has a better performance than the centralized system according to the total number of outstanding backorders and the order fulfillment lead time, which are customer-facing metrics. On the other hand, the centralized system performs better according to the internal-facing metrics, which are the total backorder and holding costs and the inventory days of supply. Finally, according to the supply chain response time, which is also a customer-facing metric, it is found that the centralized system generally has a better performance than the decentralized system.

ii. When each criterion is taken as equally important, the decentralized system is preferred over the centralized system.

iii. The sensitivity analysis denotes that by altering the weights of the total number of outstanding backorders and the order fulfillment lead time, the ranking of the alternatives cannot be changed, i.e., the decentralized system is always preferred over the centralized system. On the other hand, the centralized system becomes more preferred than the decentralized system if and only if a decision maker assigns a weight greater than 0.9779 to the supply chain response time; a weight greater than 0.7944 to the total backorder and holding costs; or a weight greater than 0.6502 to the inventory days of supply.

iv. According to the experiment designed in this chapter, it is found that the decentralized system is preferred over the centralized system in 92.83% of all cases.

Consequently, in this thesis it is concluded that the decentralized system is more preferable than the centralized system due to the following reasons: (i) By altering the weights of two criteria, the centralized system cannot be more preferred than the decentralized system; (ii) the minimum weights needed to make the centralized system more preferable are high for the other three criteria; and (iii) the experiment gives a very high percentage for the cases where the decentralized system is preferred over the centralized system.

7. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This thesis investigates a decentralized two-stage supply chain consisting of multiple independent suppliers and a manufacturer with limited production capacities. The suppliers operate on a make-to-stock basis and apply base stock policy to manage their inventories. On the other hand, the manufacturer employs a make-to-order strategy. The aim of this thesis is to coordinate the inventory policies of the suppliers in the supply chain.

Assuming that the end customer demands occur according to a Poisson process, and the service times of the suppliers and the manufacturer are i.i.d. and exponentially distributed random variables, each supplier is modeled as an $M/M/1$ make-to-stock queue. Furthermore, the average outstanding backorders and the average inventory level of each supplier are derived using the queuing model.

On the other hand, the interarrival time distribution of the manufacturer has to be derived to model the manufacturer as a queuing system. Therefore, first the exact distributions in the case of one supplier and two suppliers are derived. However, deriving the distribution of the interarrival times of the manufacturer becomes mathematically intractable as the number of suppliers increases. Thus, an approximate distribution is developed for a system with two or more suppliers. The idea behind the approximation is the expectation that the supplier with the minimum base stock level affects the interarrival times of the manufacturer the most.

For testing the precision of the approximate interarrival time distribution of the manufacturer, simulation models are developed in the case of two, three, and four suppliers. The results denote that the approximate distribution produces an error of 2.47%, denoting that it can be reasonably used as the interarrival time distribution of the manufacturer. Then, the manufacturer is modeled as a $GI/M/1$ queue under the assumption that the arrivals to the manufacturer form a renewal process. Moreover, the average number of jobs in the manufacturer's system and the average outstanding backorders at the manufacturer are obtained using the queuing model.

After the supply chain has been modeled as a queuing network, the centralized and decentralized systems are taken into account. Notice that the centralized system is also considered in this thesis since the centralized solution is used as a reference point for the performance of the decentralized system.

In the centralized model, the objective of the single decision maker is to minimize the average total backorder and holding costs per unit time for the overall system. The decision variables are the base stock levels of the suppliers. Since the suppliers apply base stock policy to manage their inventories, throughout the thesis it is assumed that the optimal base stock levels of the suppliers are not equal to zero.

After constructing the centralized model, the unique global optimal solution to the model is found. Although this solution ignores the integrality of the base stock levels of the suppliers, the way of finding the optimal integer solution to the centralized model is also defined.

On the other hand, in the decentralized model, the objective of each member of the supply chain is to minimize the average cost per unit time for his own system. Therefore, each supplier tries to minimize his average backorder and holding costs per unit time. However, since the decision variables are the base stock levels of the suppliers, the manufacturer is not included in the decentralized model. Nevertheless, the decision of the supplier with the minimum base stock level also affects the manufacturer.

After the decentralized model has been developed, the unique global optimal solution to the decentralized model for each supplier is derived. Similar to the centralized case, the optimal integer solution to the decentralized model can also be found easily.

When the centralized and decentralized solutions are compared, it is concluded that only the supplier with the minimum base stock level needs coordination. Therefore, contracts are prepared between that supplier and the manufacturer.

Three different transfer payment contracts are studied in this thesis. These are the backorder cost subsidy contract, the transfer payment contract based on Pareto improvement, and the cost sharing contract. Each contract is evaluated according to its coordination ability and whether it is Pareto improving or not. If a contract is not Pareto improving even it coordinates the supply chain, then at least one of the members of the supply chain will not desire to participate in the contract.

It is proved that all three contracts have the ability to coordinate the supply chain. However, they differ in whether they are Pareto improving or not. It is found that only the backorder cost subsidy contract is not Pareto improving. Among the other two contracts, in the transfer payment contract based on Pareto improvement, only the supplier is better off after the contract and the manufacturer remains the same. On the other hand, in the cost sharing contract, both the supplier and the manufacturer can be better off after the transfer payment for an appropriately selected contract parameter. Therefore, the cost sharing contract seems to be more advantageous to both members.

In this thesis, also a numerical study is performed to compare the centralized and decentralized systems based on SCOR Model performance metrics. The performance metrics are chosen such that exactly one metric is associated with exactly one of the SCOR Model performance attributes, which are reliability, responsiveness, flexibility, cost, and assets. Then, the corresponding performance metrics are determined as the total number of outstanding backorders, the order fulfillment lead time, the supply chain response time, the total backorder and holding costs, and the inventory days of supply, respectively.

The results of the numerical study point out that the decentralized system has a better performance than the centralized system according to the total number of outstanding backorders and the order fulfillment lead time, which are customer-facing metrics. On the other hand, the centralized system performs better according to the internal-facing metrics, which are the total backorder and holding costs and the inventory days of supply. Finally, according to the supply chain response time, which is also a customer-facing metric, it is found that the centralized system generally has a better performance than the decentralized system.

After the centralized and decentralized systems have been compared based on five performance metrics, a multi-criteria decision making method is used to decide which system is more preferable. For this purpose, the simple additive weighting method is selected. When each criterion is taken as equally important, it is found that the decentralized system is preferred over the centralized system. Then, a sensitivity analysis is performed to determine the most sensitive criterion. The results indicate that the inventory days of supply is the most sensitive criterion; and it is followed by the total backorder and holding costs, and the supply chain response time, respectively. On the other hand, the total number of outstanding backorders and the order fulfillment lead time are insensitive to the ranking of the alternatives. The results obtained from the sensitivity analysis also point out that the decentralized system is more preferable than the centralized system.

To summarize, the main contributions of this thesis are

i. The derivation of the approximate interarrival time distribution of the manufacturer in the presence of two or more suppliers, and accordingly finding the approximate performance measures of the manufacturer such as the average number of jobs in the manufacturer's system and the average outstanding backorders at the manufacturer;

ii. The development of the transfer payment contracts to coordinate the inventory policies in a capacitated supply chain with multiple suppliers.

Finally, the future research directions can be given as follows:

i. Considering competing suppliers and using a game-theoretic framework to examine the coordination issues. Notice that the suppliers considered in this thesis are independent and noncompeting. As a further study, games within the suppliers and also between the suppliers and the manufacturer can be incorporated into the models.

ii. Studying other types of incentives that may coordinate the supply chain.

iii. Developing a lost sales model.

iv. Incorporating also other performance metrics into the models so that the optimal base stock levels of the suppliers are calculated by taking different performance metrics into consideration.

v. Relaxing the assumptions that the end customer demands occur according to a Poisson process, or the service times of the suppliers and the manufacturer are i.i.d. and exponentially distributed random variables. Although the exact solution for such an extension cannot be found in the case of a capacitated supplier, approximations and simulation methods can be used as also adopted in this thesis.

vi. Relaxing the assumption that the transfer times between the suppliers and the manufacturer are negligible.

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APPENDICES

APPENDIX A **:** List of Assumptions

APPENDIX B **:** Testing the Precision of the Approximate Interarrival Time Distribution of the Manufacturer

APPENDIX C **:** Selecting the Best-Fit Approximation for the Average Number of Jobs in the Manufacturer's System

APPENDIX A

i. Transfer times between the suppliers and the manufacturer are negligible.

ii. End customer demands arrive in single units according to a Poisson process with rate λ.

iii. Supplier *i* and the manufacturer have i.i.d. and exponentially distributed service times with rate μ_i for $i = 1, \ldots, n$ and μ_M, respectively.

iv. For the stability of the system, $0 < \rho_i < 1$ for $i = 1, \ldots, n$ and $0 < \rho_M < 1$, where ρ_i and ρ_M denote the traffic intensities of supplier *i* and the manufacturer, respectively.

v. *X* and Y_i for $i = 1, \ldots, n$ are independent from each other, where *X* denotes the time between demands and Y_i denotes the time until the next service completion for supplier *i*.

vi. In the case of two suppliers, their states are conditionally independent from each other given both components have been departed from the suppliers.

vii. Arrivals to the manufacturer form a renewal process.

viii. $b_i > 0$ and $h_i > 0$ for $i = 1, \ldots, n,$ and $b_M > 0,$ where b_i denotes the backorder cost per unit backordered at supplier *i* per unit time; b_M denotes the backorder cost per unit backordered at the manufacturer per unit time; and h_i denotes the holding cost per unit inventory per unit time for supplier *i*.

ix. The optimal base stock levels of the suppliers are not equal to zero.

APPENDIX B

For testing the precision of the approximate interarrival time distribution of the manufacturer presented in equation (3.29), simulation models are developed in the case of two, three, and four suppliers.

In each case, the end customer demand rate is set to one. Three different values are used for the traffic intensities of the suppliers and the manufacturer: 0.50, 0.67, and 0.80. The base stock levels of the suppliers can also take on three values: 3, 5, and 7. Since considering all the combinations is too time consuming, Taguchi designs are created using Minitab 15, each with 27 runs.

The simulation models are developed using Arena 9.0. The replication length of each run is 10,000 time units and the number of replications is set to 10.

Kolmogorov-Smirnov (K-S) test is used to test the precision of the approximate interarrival time distribution of the manufacturer. Tables B.1-B.3 present the K-S test statistics and *p*-values for the approximate distribution in the case of two, three, and four suppliers, respectively. If more than one supplier has the minimum base stock level, the supplier with the highest traffic intensity is taken into consideration among these suppliers. The tables also include the results for the exponential distribution since most of the studies in the literature use the exponential distribution to approximate the interarrival times of the manufacturer in the presence of one supplier.

In Tables B.1-B.3, the *p*-values that are not significant at the 0.01 level are marked in bold. The results denote that the exponential distribution fits the interarrival time data of the manufacturer in just 23 of the 81 cases, giving an error of 71.60%. On the other hand, the approximate distribution fits the data in 79 of the 81 cases, producing an error of just 2.47%. Also, the distribution of the errors among the models for two, three, and four suppliers are balanced. Consequently, the results denote that the interarrival time distribution of the manufacturer can be approximated as given in equation (3.29).

Table B.1: K-S test statistics and *p*-values in the case of two suppliers.

No	ρ_M	ρ_1	ρ_2	S_1	S_2	Exponential dis.		Approximate dis.	
						p	K-S	p	K-S
1	0.50	0.50	0.50	3	3	**0.0001**	0.0225	0.5115	0.0082
2	0.50	0.50	0.50	3	5	0.0329	0.0143	0.1278	0.0117
3	0.50	0.50	0.50	3	7	0.0584	0.0133	0.0946	0.0123
4	0.50	0.67	0.67	5	3	**0.0003**	0.0209	0.5030	0.0082
5	0.50	0.67	0.67	5	5	0.0178	0.0153	0.7198	0.0069
6	0.50	0.67	0.67	5	7	0.2628	0.0100	0.4888	0.0083
7	0.50	0.80	0.80	7	3	**0.0025**	0.0182	0.1105	0.0120
8	0.50	0.80	0.80	7	5	**0.0047**	0.0173	0.4339	0.0087
9	0.50	0.80	0.80	7	7	0.1100	0.0120	0.7364	0.0068
10	0.67	0.50	0.80	5	3	**0.0000**	0.0233	0.6792	0.0072
11	0.67	0.50	0.80	5	5	0.2016	0.0106	0.0984	0.0122
12	0.67	0.50	0.80	5	7	0.3616	0.0093	0.4803	0.0084
13	0.67	0.67	0.50	7	3	0.0113	0.0160	0.1582	0.0112
14	0.67	0.67	0.50	7	5	0.1153	0.0119	0.0783	0.0127
15	0.67	0.67	0.50	7	7	0.2876	0.0098	0.1358	0.0116
16	0.67	0.80	0.67	3	3	**0.0017**	0.0186	0.1097	0.0119
17	0.67	0.80	0.67	3	5	**0.0064**	0.0168	0.1902	0.0108
18	0.67	0.80	0.67	3	7	0.0801	0.0126	0.0685	0.0129
19	0.80	0.50	0.67	7	3	0.0105	0.0160	0.0423	0.0137
20	0.80	0.50	0.67	7	5	0.0595	0.0131	0.1438	0.0114
21	0.80	0.50	0.67	7	7	0.0348	0.0141	0.0595	0.0131
22	0.80	0.67	0.80	3	3	**0.0000**	0.0236	0.3593	0.0092
23	0.80	0.67	0.80	3	5	**0.0005**	0.0203	0.4518	0.0085
24	0.80	0.67	0.80	3	7	**0.0009**	0.0196	0.6457	0.0074
25	0.80	0.80	0.50	5	3	**0.0016**	0.0188	0.7671	0.0066
26	0.80	0.80	0.50	5	5	0.1069	0.0120	0.4293	0.0087
27	0.80	0.80	0.50	5	7	0.1702	0.0110	0.2311	0.0103

Table B.2: K-S test statistics and p-values in the case of three suppliers.

No	ρ_M	ρ_1	ρ_2	ρ_3	S_1	S_2	S_3	Exponential dis.		Approximate dis.	
								p	K-S	p	K-S
1	0.50	0.50	0.50	0.50	3	3	3	**0.0003**	0.0209	0.0521	0.0133
2	0.50	0.50	0.50	0.67	3	5	5	**0.0004**	0.0205	0.6993	0.0070
3	0.50	0.50	0.50	0.80	3	7	7	**0.0037**	0.0176	0.8235	0.0063
4	0.50	0.67	0.67	0.50	5	3	3	**0.0000**	0.0350	0.0304	0.0144
5	0.50	0.67	0.67	0.67	5	5	5	**0.0002**	0.0211	0.0249	0.0146
6	0.50	0.67	0.67	0.80	5	7	7	**0.0095**	0.0163	0.3130	0.0096
7	0.50	0.80	0.80	0.50	7	3	3	**0.0005**	0.0201	0.1658	0.0110
8	0.50	0.80	0.80	0.67	7	5	5	**0.0000**	0.0271	0.0287	0.0146
9	0.50	0.80	0.80	0.80	7	7	7	0.0418	0.0138	0.5892	0.0077
10	0.67	0.50	0.80	0.67	5	3	7	**0.0000**	0.0248	0.7105	0.0070
11	0.67	0.50	0.80	0.80	5	5	3	**0.0000**	0.0233	0.0566	0.0132
12	0.67	0.50	0.80	0.50	5	7	5	0.1086	0.0120	0.1468	0.0113
13	0.67	0.67	0.50	0.67	7	3	7	**0.0015**	0.0190	0.9513	0.0052
14	0.67	0.67	0.50	0.80	7	5	3	**0.0092**	0.0163	0.2703	0.0099
15	0.67	0.67	0.50	0.50	7	7	5	0.3066	0.0096	0.5562	0.0079
16	0.67	0.80	0.67	0.67	3	3	7	**0.0000**	0.0246	0.7983	0.0064
17	0.67	0.80	0.67	0.80	3	5	3	**0.0007**	0.0198	0.1346	0.0115
18	0.67	0.80	0.67	0.50	3	7	5	**0.0000**	0.0232	0.9924	0.0043
19	0.80	0.50	0.67	0.80	7	3	5	**0.0000**	0.0259	0.3296	0.0095
20	0.80	0.50	0.67	0.50	7	5	7	0.1636	0.0111	0.2247	0.0103
21	0.80	0.50	0.67	0.67	7	7	3	**0.0013**	0.0189	0.2449	0.0101
22	0.80	0.67	0.80	0.80	3	3	5	**0.0000**	0.0347	**0.0041**	0.0176
23	0.80	0.67	0.80	0.50	3	5	7	**0.0000**	0.0248	0.6392	0.0074
24	0.80	0.67	0.80	0.67	3	7	3	**0.0000**	0.0284	0.1397	0.0115
25	0.80	0.80	0.50	0.80	5	3	5	**0.0000**	0.0235	0.4045	0.0089
26	0.80	0.80	0.50	0.50	5	5	7	0.1703	0.0110	0.3815	0.0090
27	0.80	0.80	0.50	0.67	5	7	3	**0.0000**	0.0233	0.5183	0.0081

Table B.3: K-S test statistics and *p*-values in the case of four suppliers.

No	ρ_M	ρ_1	ρ_2	ρ_3	ρ_4	S_1	S_2	S_3	S_4	Exponential dis.		Approximate dis.	
										p	K-S	p	K-S
1	0.50	0.50	0.50	0.50	0.50	3	3	3	3	**0.0000**	0.0346	**0.0003**	0.0210
2	0.50	0.50	0.50	0.67	0.67	3	5	5	5	**0.0000**	0.0239	0.0105	0.0160
3	0.50	0.50	0.50	0.80	0.80	3	7	7	7	**0.0012**	0.0191	0.4313	0.0086
4	0.50	0.67	0.67	0.50	0.67	5	3	3	5	**0.0000**	0.0338	0.0457	0.0137
5	0.50	0.67	0.67	0.67	0.80	5	5	5	7	**0.0005**	0.0202	0.1405	0.0115
6	0.50	0.67	0.67	0.80	0.50	5	7	7	3	**0.0000**	0.0234	0.1520	0.0113
7	0.50	0.80	0.80	0.50	0.80	7	3	3	7	**0.0007**	0.0196	0.0218	0.0148
8	0.50	0.80	0.80	0.67	0.50	7	5	5	3	**0.0000**	0.0246	0.1354	0.0116
9	0.50	0.80	0.80	0.80	0.67	7	7	7	5	**0.0009**	0.0197	0.1446	0.0115
10	0.67	0.50	0.80	0.67	0.50	5	3	7	5	**0.0000**	0.0254	0.8136	0.0064
11	0.67	0.50	0.80	0.80	0.67	5	5	3	7	**0.0000**	0.0230	0.3424	0.0093
12	0.67	0.50	0.80	0.50	0.80	5	7	5	3	**0.0000**	0.0231	0.0728	0.0128
13	0.67	0.67	0.50	0.67	0.67	7	3	7	7	**0.0003**	0.0206	0.0956	0.0122
14	0.67	0.67	0.50	0.80	0.80	7	5	3	3	**0.0000**	0.0289	0.2581	0.0101
15	0.67	0.67	0.50	0.50	0.50	7	7	5	5	0.0927	0.0123	0.3462	0.0093
16	0.67	0.80	0.67	0.67	0.80	3	3	7	3	**0.0000**	0.0342	0.0150	0.0157
17	0.67	0.80	0.67	0.80	0.50	3	5	3	5	**0.0000**	0.0229	0.1438	0.0113
18	0.67	0.80	0.67	0.50	0.67	3	7	5	7	**0.0015**	0.0189	0.6753	0.0072
19	0.80	0.50	0.67	0.80	0.50	7	3	5	7	**0.0000**	0.0330	0.0970	0.0123
20	0.80	0.50	0.67	0.50	0.67	7	5	7	3	**0.0000**	0.0234	0.6290	0.0074
21	0.80	0.50	0.67	0.67	0.80	7	7	3	5	**0.0002**	0.0213	0.2752	0.0099
22	0.80	0.67	0.80	0.80	0.67	3	3	5	3	**0.0000**	0.0259	0.2974	0.0097
23	0.80	0.67	0.80	0.50	0.80	3	5	7	5	**0.0000**	0.0324	0.0635	0.0132
24	0.80	0.67	0.80	0.67	0.50	3	7	3	7	**0.0000**	0.0325	0.1090	0.0121
25	0.80	0.80	0.50	0.80	0.80	5	3	5	5	**0.0001**	0.0228	0.3287	0.0094
26	0.80	0.80	0.50	0.50	0.50	5	5	7	7	0.1523	0.0112	0.0486	0.0135
27	0.80	0.80	0.50	0.67	0.67	5	7	3	3	**0.0000**	0.0297	0.2943	0.0097

APPENDIX C

For selecting the best-fit approximation for the average number of jobs in the manufacturer's system among the approximations given in equations (3.35)-(3.39), simulation models are developed in the case of two, three, and four suppliers as explained in Appendix B.

Recall that the end customer demand rate is set to one in each case. Three different values are used for the traffic intensities of the suppliers and the manufacturer: 0.50, 0.67, and 0.80. The base stock levels of the suppliers can also take on three values: 3, 5, and 7. Since considering all the combinations is too time consuming, Taguchi designs are created using Minitab 15, each with 27 runs.

The simulation models are developed using Arena 9.0. The replication length of each run is 10,000 time units and the number of replications is set to 10.

In Tables C.1-C.3, the average number of jobs in the manufacturer's system obtained from the simulation results is compared with the approximations given in equations (3.35)-(3.39) in the case of two, three, and four suppliers, respectively. While calculating the approximate average number of jobs in the manufacturer's system, the squared coefficient of variation of the interarrival times of the manufacturer is taken as given in equation (3.30). In addition, if more than one supplier has the minimum base stock level, the supplier with the highest traffic intensity is taken into consideration among these suppliers.

Tables C.1-C.3 also present the errors between the approximations and the simulation results. The errors are absolute percentage values; and in the case of two, three, and four suppliers, the approximation of Marchal (1976) given in equation (3.36) has the minimum average errors of 2.74%, 3.28%, and 4.09%, respectively. The approximation presented in equation (3.38) follows with corresponding average errors of 2.81%, 3.36%, and 4.20%. There is a slight increase in the average errors as the number of suppliers gets larger, but this increase is acceptable. As a result, Marchal (1976)'s approximation is selected for the average number of jobs in the manufacturer's system.

Table C.1: Average number of jobs in the manufacturer's system in the case of two suppliers.

No	Simul. results	(3.35)	Error (3.35)	(3.36)	Error (3.36)	(3.37)	Error (3.37)	(3.38)	Error (3.38)	(3.39)	Error (3.39)
1	0.9126	0.9893	8.41	0.9833	7.75	0.9845	7.88	0.9844	7.87	0.9996	9.53
2	0.9302	0.9893	6.35	0.9833	5.71	0.9845	5.84	0.9844	5.82	0.9996	7.46
3	0.9358	0.9893	5.71	0.9833	5.08	0.9845	5.20	0.9844	5.19	0.9996	6.81
4	0.9284	0.9790	5.46	0.9681	4.29	0.9704	4.53	0.9701	4.50	0.9984	7.55
5	0.9402	0.9908	5.39	0.9857	4.84	0.9867	4.95	0.9866	4.94	0.9997	6.33
6	0.9398	0.9908	5.43	0.9857	4.88	0.9867	4.99	0.9866	4.98	0.9997	6.37
7	0.9545	0.9759	2.24	0.9636	0.96	0.9662	1.23	0.9659	1.19	0.9979	4.55
8	0.9513	0.9849	3.53	0.9767	2.67	0.9784	2.84	0.9782	2.82	0.9992	5.03
9	0.9521	0.9904	4.03	0.9851	3.47	0.9861	3.58	0.9860	3.56	0.9997	5.00
10	1.9395	1.9665	1.40	1.9448	0.28	1.9511	0.60	1.9480	0.44	1.9961	2.92
11	1.9648	1.9899	1.28	1.9756	0.55	1.9796	0.75	1.9776	0.65	2.0091	2.25
12	1.9526	2.0232	3.61	2.0205	3.48	2.0212	3.51	2.0209	3.50	2.0267	3.79
13	1.9800	2.0016	1.09	1.9912	0.56	1.9940	0.71	1.9926	0.64	2.0153	1.78
14	2.0043	2.0232	0.94	2.0205	0.81	2.0212	0.84	2.0209	0.83	2.0267	1.12
15	2.0172	2.0193	0.11	2.0152	0.10	2.0163	0.04	2.0158	0.07	2.0247	0.37
16	1.8791	1.9665	4.65	1.9448	3.50	1.9511	3.83	1.9480	3.66	1.9961	6.22
17	1.8774	1.9665	4.75	1.9448	3.59	1.9511	3.92	1.9480	3.76	1.9961	6.32
18	1.9482	1.9665	0.94	1.9448	0.17	1.9511	0.14	1.9480	0.01	1.9961	2.46
19	3.9516	3.8709	2.04	3.8446	2.71	3.8530	2.50	3.8471	2.65	3.9019	1.26
20	3.7253	3.9425	5.83	3.9302	5.50	3.9340	5.60	3.9314	5.53	3.9566	6.21
21	4.0059	3.9743	0.79	3.9687	0.93	3.9704	0.89	3.9692	0.92	3.9806	0.63
22	3.7261	3.8522	3.38	3.8224	2.58	3.8320	2.84	3.8252	2.66	3.8875	4.33
23	3.7382	3.8709	3.55	3.8446	2.85	3.8530	3.07	3.8471	2.91	3.9019	4.38
24	3.7697	3.8709	2.68	3.8446	1.99	3.8530	2.21	3.8471	2.05	3.9019	3.51
25	3.8162	3.9329	3.06	3.9187	2.69	3.9231	2.80	3.9200	2.72	3.9493	3.49
26	3.9338	3.9059	0.71	3.8863	1.21	3.8925	1.05	3.8882	1.16	3.9287	0.13
27	3.9189	3.9059	0.33	3.8863	0.83	3.8925	0.67	3.8882	0.78	3.9287	0.25
Average error (%)			3.25	-	**2.74**	-	2.85	-	2.81	-	4.08

Table C.2: Average number of jobs in the manufacturer's system in the case of three suppliers.

No	Simul. results	(3.35)	Error (3.35)	(3.36)	Error (3.36)	(3.37)	Error (3.37)	(3.38)	Error (3.38)	(3.39)	Error (3.39)
	0.9354	0.9893	5.76	0.9833	5.13	0.9845	5.25	0.9844	5.24	0.9996	6.86
2	0.9380	0.9893	5.46	0.9833	4.83	0.9845	4.95	0.9844	4.94	0.9996	6.56
3	0.9377	0.9893	5.51	0.9833	4.87	0.9845	5.00	0.9844	4.98	0.9996	6.60
4	0.9467	0.9790	3.42	0.9681	2.26	0.9704	2.50	0.9701	2.48	0.9984	5.46
5	0.9285	0.9908	6.72	0.9857	6.16	0.9867	6.27	0.9866	6.26	0.9997	7.67
6	0.9388	0.9908	5.54	0.9857	4.99	0.9867	5.10	0.9866	5.09	0.9997	6.48
7	0.9618	0.9759	1.46	0.9636	0.18	0.9662	0.45	0.9659	0.42	0.9979	3.75
8	0.9447	0.9849	4.25	0.9767	3.39	0.9784	3.56	0.9782	3.54	0.9992	5.76
9	0.9476	0.9904	4.53	0.9851	3.96	0.9861	4.07	0.9860	4.06	0.9997	5.50
10	1.9614	1.9665	0.26	1.9448	0.84	1.9511	0.53	1.9480	0.68	1.9961	1.77
11	1.8994	1.9665	3.53	1.9448	2.39	1.9511	2.72	1.9480	2.55	1.9961	5.09
12	1.9496	2.0232	3.78	2.0205	3.64	2.0212	3.68	2.0209	3.66	2.0267	3.95
13	1.9039	2.0016	5.13	1.9912	4.59	1.9940	4.74	1.9926	4.66	2.0153	5.85
14	1.9260	1.9665	2.11	1.9448	0.98	1.9511	1.30	1.9480	1.14	1.9961	3.64
15	2.0272	2.0232	0.20	2.0205	0.33	2.0212	0.29	2.0209	0.31	2.0267	0.03
16	1.8309	1.9665	7.41	1.9448	6.22	1.9511	6.56	1.9480	6.39	1.9961	9.02
17	1.8788	1.9665	4.67	1.9448	3.52	1.9511	3.85	1.9480	3.68	1.9961	6.25
18	1.9588	1.9665	0.39	1.9448	0.71	1.9511	0.40	1.9480	0.56	1.9961	1.90
19	3.6205	3.8709	6.92	3.8446	6.19	3.8530	6.42	3.8471	6.26	3.9019	7.77
20	3.8611	3.9425	2.11	3.9302	1.79	3.9340	1.89	3.9314	1.82	3.9566	2.47
21	3.7635	3.8709	2.85	3.8446	2.16	3.8530	2.38	3.8471	2.22	3.9019	3.68
22	3.6737	3.8522	4.86	3.8224	4.05	3.8320	4.31	3.8252	4.13	3.8875	5.82
23	3.8166	3.8709	1.42	3.8446	0.73	3.8530	0.95	3.8471	0.80	3.9019	2.23
24	3.8059	3.8709	1.71	3.8446	1.02	3.8530	1.24	3.8471	1.08	3.9019	2.52
25	3.7479	3.9329	4.93	3.9187	4.56	3.9231	4.67	3.9200	4.59	3.9493	5.37
26	3.7242	3.9059	4.88	3.8863	4.35	3.8925	4.52	3.8882	4.40	3.9287	5.49
27	3.6674	3.8709	5.55	3.8446	4.83	3.8530	5.06	3.8471	4.90	3.9019	6.39
Average error (%)			3.90	-	**3.28**	-	3.43	-	3.36	-	4.96

Table C.3: Average number of jobs in the manufacturer's system in the case of four suppliers.

o	Simul. results	(3.35)	Error (3.35)	(3.36)	Error (3.36)	(3.37)	Error (3.37)	(3.38)	Error (3.38)	(3.39)	Error (3.39)
1	0.9441	0.9893	4.79	0.9833	4.16	0.9845	4.28	0.9844	4.27	0.9996	5.88
2	0.9293	0.9893	6.45	0.9833	5.81	0.9845	5.94	0.9844	5.92	0.9996	7.56
3	0.9423	0.9893	4.98	0.9833	4.35	0.9845	4.48	0.9844	4.46	0.9996	6.07
4	0.9167	0.9790	6.80	0.9681	5.61	0.9704	5.86	0.9701	5.83	0.9984	8.92
5	0.9427	0.9908	5.11	0.9857	4.57	0.9867	4.67	0.9866	4.66	0.9997	6.05
6	0.9229	0.9893	7.19	0.9833	6.54	0.9845	6.67	0.9844	6.66	0.9996	8.30
7	0.9439	0.9759	3.38	0.9636	2.08	0.9662	2.36	0.9659	2.32	0.9979	5.72
8	0.9400	0.9893	5.24	0.9833	4.61	0.9845	4.73	0.9844	4.72	0.9996	6.33
9	0.9299	0.9908	6.55	0.9857	6.00	0.9867	6.11	0.9866	6.09	0.9997	7.50
10	1.8761	1.9665	4.82	1.9448	3.66	1.9511	3.99	1.9480	3.83	1.9961	6.39
11	1.9222	1.9665	2.31	1.9448	1.18	1.9511	1.50	1.9480	1.34	1.9961	3.85
12	1.9255	1.9665	2.13	1.9448	1.00	1.9511	1.33	1.9480	1.16	1.9961	3.66
13	1.9258	2.0016	3.93	1.9912	3.40	1.9940	3.54	1.9926	3.47	2.0153	4.65
14	1.9120	1.9665	2.85	1.9448	1.72	1.9511	2.04	1.9480	1.88	1.9961	4.40
15	1.9794	2.0232	2.21	2.0205	2.08	2.0212	2.11	2.0209	2.09	2.0267	2.39
16	1.9203	1.9665	2.41	1.9448	1.28	1.9511	1.60	1.9480	1.44	1.9961	3.95
17	1.8795	1.9665	4.63	1.9448	3.47	1.9511	3.81	1.9480	3.64	1.9961	6.20
18	1.8851	1.9665	4.32	1.9448	3.17	1.9511	3.50	1.9480	3.33	1.9961	5.89
19	3.6575	3.8709	5.83	3.8446	5.12	3.8530	5.35	3.8471	5.18	3.9019	6.68
20	3.7974	3.8709	1.93	3.8446	1.24	3.8530	1.46	3.8471	1.31	3.9019	2.75
21	3.5624	3.8709	8.66	3.8446	7.92	3.8530	8.16	3.8471	7.99	3.9019	9.53
22	3.5790	3.8522	7.63	3.8224	6.80	3.8320	7.07	3.8252	6.88	3.8875	8.62
23	3.5707	3.8709	8.41	3.8446	7.67	3.8530	7.91	3.8471	7.74	3.9019	9.28
24	3.6380	3.8709	6.40	3.8446	5.68	3.8530	5.91	3.8471	5.75	3.9019	7.25
25	3.8650	3.9329	1.76	3.9187	1.39	3.9231	1.50	3.9200	1.42	3.9493	2.18
26	3.6656	3.9059	6.55	3.8863	6.02	3.8925	6.19	3.8882	6.07	3.9287	7.18
27	3.7002	3.8709	4.61	3.8446	3.90	3.8530	4.13	3.8471	3.97	3.9019	5.45
Average error (%)			4.89	-	**4.09**	-	4.30	-	4.20	-	6.02

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