ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

DYNAMICAL ANALYSIS OF A PASSIVE DYNAMIC WALKING BIPED ROBOT

M.Sc. THESIS

Burak YÜKSEL

Department of Control and Automation Engineering

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

İKİ BACAKLI PASİF DİNAMİK YÜRÜYEN BİR ROBOTUN DİNAMİK ANALİZİ

YÜKSEK LİSANS TEZİ

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Burak Yüksel, a **M.Sc.** student of ITU **Graduate School of Science Engineering and Technology** student ID **504101103**, successfully defended the **thesis** entitled "**DYNAMICAL ANALYSIS OF A PASSIVE DYNAMIC WALKING BIPED ROBOT**", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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To my beloved family,

FOREWORD

Pursuing an academic career has never been a simple task for the incipient scientists. I always believed that with motivation and enthusiasm, there is no task to be failed. I started to study on my M.Sc. Thesis by keeping these thoughts on mind and enjoying the world of Mathematics, Physics and Engineering. Therefore, motivation and enjoying the science were always my guides on my path. I hope I always keep this attitude in my entire academic career and make many contributions to science.

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June 2012 | Burak YÜKSEL
M.Sc. Control and Automation Engineer

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ABBREVIATIONS

BIPED : Bi-Pedal
DOF : Degree of Freedom
O(n) : Order of n
SOA : Spatial Operator Algebra
VRML : Virtual Reality Modeling Language

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DYNAMICAL ANALYSIS OF A PASSIVE DYNAMIC WALKING ROBOT

SUMMARY

The researches on Biped robots are promising a significant development both in robotic and medical fields. The dynamics of bipeds, or two-legged systems, is an attractive topic for researchers for many decades, since it reveals the mathematics and physics behind the human-walking. This yields many important applications such as improvement in design of prosthetics.

Passive dynamic walking, which is a challenging design problem, is a case, where a biped mechanism walks only by the gravitational force on a shallow slope ground. Two-legged system repeats full walking cycle under the influence of the gravitational force. This idea is mostly being used for efficiency problem of bipeds, since researches on this area introduced improved design techniques.

This thesis aims to study the dynamical analysis of a planar passive dynamic walking biped system using *Spatial Operator Algebra* (SOA). This method is very useful to compute kinematics and dynamics of all types of complex robotic systems, which also reveals the force and velocity distributions of links and joints. It is also known in the literature as a high performance algorithm. Therefore, it is important to utilize this method to have a true understanding of passive dynamical walking. To explain this method in steps, we have introduced first the SOA method for the kinematics of serial manipulators on a fixed platform. After that, the effect of moving platform has been discussed. In kinematic analysis, the equations also have been composed for the cooperating manipulators on a moving platform, which is a complex system.

This thesis is focused on a planar passive dynamic walker, which is modeled based on the aforementioned method. Since the system operates in 2D space, some required adjustments to make the SOA work on 2D space has been represented. Due to the nature of human legs, the biped is modeled with knees and a knee locking system using *pseudo joint* technique, which prevents the knees from folding forward. The most important contribution of this thesis is the calculation of constraint torques and forces on knees and the feet when both touch the ground, respectively.

Consequently, we have implemented SOA method on a planar passive dynamic walking biped robot, which could provide deep insight of this kind of systems. The latter results show the dynamical analysis of the system.

İKİ BACAKLI PASİF DİNAMİK YÜRÜYEN BİR ROBOTUN DİNAMİK ANALİZİ

ÖZET

İki bacakli robotlar üzerine yapılan çalışmalar robotik ve tıbbi alanda büyük gelişmeler yaratmaktalar. Uzun yıllardır devam eden bu calışmalar tıpta özellikle protez bacakların gelişimi, robotik alanda ise insansı robotlar üzerinde hatrı sayılır ölçüde yenilikler getirmiştir.

İki bacaklı robotlar birçok yönden araştırma konusu olmakla birlikte, bu tez kapsamında bizim ilgilendiklerimiz pasif dinamik yürüyen robotlardır. Pasif dinamik yürüme kavramı ile ifade edilmek istenen şey, iki bacaklı bir sistemin yerçekimi kuvveti etkisinde eğimli bir yüzeyde yürüme hareketini gerçekleştirebilmesidir. Sistem sahip olduğu ağırlık ve üzerinde bulunduğu yüzeyin eğimi sayesinde eğim yönünde düşme hareketi yapar ve ilk konfigürasyona bağlı olarak bir bacak diğerini takip edecek şekilde harekete geçer. Hareketin başlangıcında yerde olan bacağın, diğer bacaktan sonra tekrar yerle temas etmesiyle yürüme hareketinin ilk devri tamamlanmış olur. Sistem devrilene kadar bu harekete devam eder. Burada dikkate çarpan kısım, yürüme hareketinin sistemin tasarımı gereği doğal bir hareket olduğudur. İki bacaklı bir sistem uygun başlangıç koşullarında sadece yerçekimi kuvveti ile yürüyebilir.

Pasif dinamik yürüyen robotlar üzerinde yapılan araştırmaların getirdiği avantaj ise daha az enerji harcayan insansı robotların önünü açıyor olmasıdır. Kendiliğinden yürüme devrini tamamlayabilen bir sistem, daha az eyleyici ve kontrol mekanizmasıyla daha uzun mesafeleri yürüyebilecektir. Tüm bu tasarrufların yanı sıra, daha az enerji daha az güç üniteleri gerektireceğinden, insansı robotun taşıması gereken pil grubunun boyutları daha ufak olacaktır. Tüm bunlar daha karmaşık işleri daha az enerji ile yapabilen, yürüme kabiliyetine sahip insansı robotların gelişimine büyük katkı sağlamaktadır.

Yaklaşık olarak pasif dinamik yürüme kavramının ortaya atılmasıyla aynı zamanlara rastlayan Uzaysal Operatör Algoritması (UOA) ise, karmaşık robotik sistemlerin kolayca modellenmesine olanak sağlayan, temeli Newton-Euler hesaplamalarına dayandığı için herhangi bir link veya eklemin kuvvet veya hız bilgilerini göz önüne serebilen, özyinelemeli yapısı sayesinde kendinden önceki algoritmalara oranla daha hızlı hesaplama yapabilen bir yöntemdir. Uzaysal kavramı, açısal ve doğrusal vektörlerin bir arada tanımlanması nedeniyle kullanılmaktadır. Özyinelemeli yapısı sayesinde *n. dereceden,* $O(n)$, hesaplama yapan algoritmalar sınıfına girmektedir. Normalde robotik sistemlerin kinematik ve dinamik hesaplamaları genelleştirilmiş koordinat sayısının, *n*, küpü şeklinde artmakta iken, özyinelemeli algoritmalarda doğrusal olarak artmaktadır.

Teze ilk olarak bu iki konu hakkında daha önceden yapılmış önemli çalışmaları ve sonuçlarıyla yaptıkları katkıları sıralayarak başlanmıştır. Ardından sırasıyla UOA yöntemi ve bu yöntemin pasif dinamik robotlara uygulanışı anlatılmıştır.

UOA yöntemi tez içinde ikinci kısımda kinematik ve dinamik olmak üzere iki ana başlığa ayrılmıştır. Kinematik ifadeleri açıklarken öncelikle bir vektörün türevinin ne anlama geldiği; o vektörün, dönme veya öteleme ekseni ile vektörel çarpımıyla aynı şeyi ifade ettiği gösterildi. Ardından UOA için büyük önem teşkil eden hız yayılım matrisinin oluşturulması açıklandı. Ayrıca kuvvet dağılımında da kullandığımız bu matris, özyinemeli algoritmanın temelini oluşturmaktadır.

Robotik sistemler için kinematik denklemlerin çıkarımına, daha basit bir sistem olduğu için hareketsiz platform üzerindeki *n* serbestlik dereceli seri robotlarla başlandı. Bütün linkler için hız denklemleri yazıldıktan sonra ortak matris gösterimi ile ifade edildi. Ardından uç işlevci ile robotun geri kalanı arasında bağlantı sağlanarak ileri kinematik ve sonrasında ters kinematik bağlantılarına ulaşıldı. Sonrasında hareketli bir platform üzerindeki seri robotun kinematiği incelendi ve üç boyutlu uzayda hareketli platformun bütün sisteme altı serbestlik derecesi daha kattığı görüldü.

UOA yöntemi daha karmaşık sistemler için daha büyük bir fark yarattığından, ortak çalışan robotların kinematiği de bu tez kapsamında incelendi. Hareketsiz ve hareketli platform durumlarının ayrı ayrı dikkate alındığı bu kısımda, en dikkate çarpan yer kinematik sınırların belirlenmesidir. Ortak hareket eden manipülatör sisteminde, robotların tek bir yük taşıdığı örneğinden yola çıkarsak, bu yükün merkezinin yaptığı hareketi, izlenmesi gereken yörünge olarak kabul edebiliriz. Robotların uç işlevcileri ile katı cismin merkezi arasındaki mesafe sabit kalacağı gerçeği göz önüne alınarak oluşturulan kısıtlama ile bu merkez noktasından uç işlevcilere hız dağılımları gerçekleştirildi ve geri kalan işlemler tıpkı seri robotlarda olduğu gibi yapıldı. Sistem daha karmaşık ve çok sayıda robottan oluştuğu için, bildiğimiz ileri ve ters kinematik denklemlerini kullanabilmek ve işlemleri daha basit olarak gösterebilmek adına denklemler sonradan genelleştirilimiş matrisler şeklinde ifade edildi.

Ardından dinamik denklemlerin çıkarımı incelendi. Başlangıç olarak yine hareketsiz düzlemdeki *n* serbestlik dereceli seri robotlar üzerinde çalışıldı. Sırasıyla eklemlerdeki ivme ve kuvvetlerin hesaplanması gösterildikten sonra, yine tüm sistem için genel matrissel ifade çıkarıldı.

Üçüncü kısımda düzlemsel iki bacaklı robotik sistem örnek olarak ele alındı. Başta da bahsedildiği gibi pasif dinamik yürüme gerçekleştirebilen robot tasarımı ve bu robotun UOA kullanılarak incelenmesi amaçlanmıştır. Tezin içeriğinde UOA yöntemi üç boyutlu uzayı kapsayacak şekilde tanımlandığından ve iki boyutlu uzayda çalışan düzlemsel robotta bu yöntemin gereğinden fazla hesaplama yapacağı için, metodun örnek olarak seçilen modelin hareket ettiği iki boyutlu uzayda işlem yapmasını sağlayacak düzenlemeler gösterilmiştir. Pasif dinamik yürüyen robot ikisi ayak, ikisi diz, biri çatı olmak üzere beş dönel eklemden oluşmaktadır. Bu kısımda önemli olan bir diğer nokta ise diz kitleme mekanizmasıdır. Diz hareketinin doğal olarak taklit edilebilmesi için, dizin öne doğru bükülmesini engellemek amacıyla "*sözde eklem*" metodu kullanılmştır. Bu yönteme göre dizin üzerindeki bağlantı (uyluk) ile altındaki bağlantı (baldır) arasındaki açı değeri sıfır olduguğunda, bu açıyı sıfır derecede tutacak düzlemsel torklar hesaplanıp sistem ona göre çalıştırılmıştır.

Farklı çalışmalardan örneklerin de gösterildiği bu kısımda, tasarlanan robotun parametreleri ve yürüme hareketini sağlayacak sınırlamalara değinilmiştir. Şöyle ki, robot ayaklarından birini yerle temas ettirdiği anda bu ayak üzerindeki çizgisel hızlar sıfır tutulacak şekilde hesaplamalar yapılmıştır. Burada yapılan sınır kuvvetlerinin hesaplanması, bu tezin en can alıcı kısmını oluşturmaktadır. Genelde araştırmalar bir ayak yere bastığı an diğer ayağın otomatik olarak havaya kalktığını varsayarken, bu tezde ili ayağın da yerde oluğu durum, *zincir sistemi*, göz önüne alınmıştır. İlk adım atıldıktan sonra, arkada kalan ayağın, bizim tanımlamamıza göre taban noktası, yukarı yöndeki tepki kuvvetlerinin negatif değer aldığı ana kadar zaman boyunca *zincir sistemi* için gerekli hesaplamalar yapılmıştır. Bu sürenin sonunda arkada kalan ayağın adım atması gerekeceği için; uç ve taban noktaları başta olmak üzere sistemin tamamı aynalanmıştır. Buna tez içinde *"değişim kuralı"* adını verdik. Lineer bir T matrisi ile gerekli değerleri çarparak sistemin önceki taban noktasını yeni uç, eski uç noktasını yeni taban noktası olacak şekilde değiştirdik. Bu sayede robotun adım atan ayağı hep uç noktası olmuş oldu.

Dizlerde ve yere basan ayakta gerekli sınır torkları ve kuvvetleri, o noktalardaki ivmeleri sıfırlayacak şekilde hesaplanmıştır. Pozisyonları da sıfırlamak için, dizlerde kitleme mekanizmasının, yere basan ayakta ise yerin karakteristiğini temsil edecek PI kontrolörler tasarlanmıştır. Dördüncü bölümde buna yer verilmiştir.

Son bölümde sonuçlar ve önerilere yer verilmiştir. İlk sonuçlar robotun kinematik yeterliliği hakkındadır. Sonraki sonuçlar robotun dinamik analizini göstermektedir. Başlangıç koşullarından ziyade sınır tork ve kuvvetlerinin hesaplanması ve buna göre robotun analizinin yapılması tezin amacını oluşturduğu için, robotun adım atmasını sağlayacak herhangi bir başlangıç koşulu seçilmiştir. Simulasyonlar MATLAB-Simulink ortamında gerçeklenmiş olup, yukarı yöne z, sağ yöne y, sayfa düzleminde bize doğru olan yöne de x, denilmiştir. Adım atan ayağın z yönündeki pozisyonu, dizlerde ve yere basan ayakta oluşan kısıt ve tepki kuvvetleri ve tabanda oluşan kuvvetler sonuçlar bölümünde sunulmuştur. Bu bölümde ayrıca gelecek araştırma fikirlerine yer verilmiştir.

1. INTRODUCTION

Robotic systems consist of several links connected with joints to each other. The movement of these systems is studied using kinematics and dynamics. Kinematics is a way to calculate the motion of a system without focusing on the causes of the movement. Dynamics, on the other hand, investigates the main reason of this movement. In real-life applications, the manipulators operate in a 3D task space, which means three angular, and three linear vectors, i.e. the information of velocity, acceleration, force and torque, for each component. For this reason, the spatial vector representation brings a great simplicity to understand the insight of the robotic system during an operation. Spatial operator algebra (SOA) is one of them, which is based on Newton-Euler formulation and allows analyzing the dynamics of a complex system. Since biped walking systems are two legged robots, they can be treated as one of them.

In this thesis, we applied the SOA to complex systems, and then presented a methodology for dynamical analysis of a planar passive walking biped robot.

1.1 Purpose of Thesis

Dynamical analysis of biped walking is important for both robotic and medical research. The outcome of such studies in the medical area help certain group of patients to recover in a way to accomplish tasks like walking. Another important outcome is in the field of designing artificial limbs like prosthetic legs. However, the most challenging part of this design is efficiency. In this regard, research on passive walking dynamics holds an important place for designing efficient limbs.

On the other hand, Spatial Operator Algebra (SOA) presented a novel improvement both mathematically and computationally on multi-body kinematics and dynamics of robotic systems. Since, it is based on Newton-Euler formulation; it provides insight into the structure of rigid-body by analyzing the system using vector representation. The propagation of velocity and force vectors in a recursive manner helps to speed up the calculations by representing the kinematic and dynamic behavior of each rigid link.

Generally speaking, using the SOA for passive dynamic walking systems would be more useful to investigate the force distributions on each component of entire body, and also it could propose a new outlook in this area.

1.2 Literature Review

Before starting to explain the methods and models that we have used in this thesis, it is important to look at the history in literature. This is not only useful for comprehension of the purpose of this study, but also for finding its place between relative studies. Therefore, this section is dedicated for literature review about spatial operator algebra method and passive dynamic walking, respectively.

1.2.1 Spatial operator algebra

The spatial operator algebra (SOA) is a method based on calculation of spatial vectors of a multi-body system. The spatial (rotational and translational) vectors have been used in many researches, mostly as advanced works of order $n, O(n)$, algorithms. The usual way of calculation the rigid body equations of a system uses order n^3, which means the computation time increases as a cubic function of the number of system generalized coordinates, n. On the other hand, order n algorithms provide a linear relationship between number of numerical computations and the number of generalized coordinates [1].

The first studies in $O(n)$ formulation of multi-body dynamics were based on Newton-Euler formulation and Walker and Orin's study [2] was one of them. They also compared different methods for calculating the rigid body equations, and the results show that the recursive procedure provided a major improvement in decreasing the number of computations. Later on, Anderson and Critchley [3] improved their $O(n)$ method. More researches on $O(n)$ has been done, and Jain [5] showed that they are all related to each other.

The roots of spatial operators come from the Ball's theory of screws [6]. Featherstone used first time spatial algebra to present his method based on algorithms, who also published a review paper [7], which reveals the development of

Newton-Euler based algorithms. Rodriguez [8] used a new method for calculation of robot dynamics using Kalman filtering and Bryson-Frazier fixed time-interval smoothing technique. His spatially recursive calculation method for link dynamics of a rigid-body system proposed not only better understanding of robot dynamics, but also improvement in computation efficiency, which became the base of SOA. He implemented his spatial recursive method on open- and closed-chain systems [9, 10]. Jain [5] shaped the spatial operator algebra (SOA) in his study, where he also built the connection between other methods on rigid body motion. Rodriguez, Jain and Kreutz-Delgado improved and implemented SOA for several types of the multi-body system design and dynamics [11-14]. Another research of Jain and Rodriguez [15] was on sensitivity analysis for rigid-body systems based on spatial operators. More recently, Yesiloglu [1] used SOA to model the complex topology systems with high efficiency. The mathematical equations and formulations in this thesis are mostly based on this study. One of his contributions, new "*pseudo joint*" concept, is also used in this thesis.

Featherstone [16] and Jain [17] can be considered as base sources to understand the mathematical background of SOA.

1.2.2 Passive dynamic walking

Passive dynamic walking task for bipeds has been studied by the scientists for many decades. The basic idea was constructing a biped system, which can move, even walk only by using the gravitational force. Before starting the history of passive dynamic walking of two-legged systems, it would be wise to learn how the legged robots and the studies about them involved in time. For this purpose, Raibert's study constitutes a novel research in this field [18]. He showed a criminology starting from 1850s with Chebyshev's works towards 1980s. These milestones have great importance to understand the development of the legged-machines and to link up all history towards the beginning of two-legged systems with passive dynamic walking. Completely actuated biped systems are not very efficient, since they hold actuators on their each joint. Hence, for the human-like robots it is important to carry out the passive dynamic walking principles. This term first used by Ted McGeer, who is one of the pioneers on passive walking area. The goal was constructing limbs, which are capable of holding and moving the body without any external source of energy, so the need of actuators and their control would be minimized. The idea based on a two-legged system, which can sustain a walking cycle on a shallow slope by using only the gravitational force. McGeer's studies presented that the analysis of the physics of two-legged 2D biped machines is straightforward and the results of theoretical facts are successfully applicable [19, 20]. He also stated that the passive dynamic walking makes powered machines more efficient while adding power sources to a passive walker, in his study they were small motors for clearance the feet during recovery, causes a negligible effect on the system. His further research [21] showed that the passive walking rules on 2D biped machine work also for 3D legged machines with knees on a shallow slope. His work presented the advantages of biped with knees in stability, efficiency and control of walking routine compared to biped with straight legs, since knee-jointed form did not need to be actuated for retraction of the legs. This study of him, which was inspired by Mochon et. al [22], also showed how to numerically generate different passive cycles based on a first impulse on the feet using a *Poincaré map*, or as he called, a *stride function* . Ruina et al. [23] confirmed McGeer's studies and showed that a 3D biped with unstable configuration can also move stably. Goswami et al. [24] worked on a similar system to McGeer's biped [3], which they called as *compass gait walker*, with same kinematics, but using the overlook of robotics. They indicated that perception of biped systems with a robotic look could be very helpful to understand the human locomotion, so they modeled the system as acrobat and pendubot, based on the same kinematics of a double pendulum system, which was used as the main model in previous researches. This research also differs from McGeer's study by means of using non-linear equations instead of linearized mathematical model. It showed that such a mechanism with non-linear dynamics could follow a stable limit cycle with unequal step lengths. An important outcome of this study was that only the slope of the inclined plane determines all the types of gaits. Garcia et al. [25] used a simpler two-legged model compared to McGeer's and Goswami's studies by splitting the mass of the system between hip and feet, and assuming the former is much greater than the latter one. This work presented that the dynamics of such a system relies mostly on design parameters like mass distribution and length rather than its control strategy and showed that the simplest passive dynamic walker can achieve a stable cyclic motion. Schwab and Wisse [26] used the region (or basin) of attraction of the motion and the cases when failure occurs for the simplest walking model. Their study presented that a stable cycle of walking exists in a small boundary of slope angle. The results showed that the region of attraction of the simplest walking model, which is not directly related to the stability of the cycle motion, is very small; hence, in practical applications the model has to be initiated very carefully on a flat and rigid surface. An important outcome of this research revealed that the best robust design is always the one with largest region of attraction. Collins, Wisse and Ruina [27] have built the first two-legged machine with human-like motions. Their 3D machine was inspired of McGeer's 2D passive walker and had counter-swinging arms linked to the opposing legs, which improved the stability in lateral directions. Wisse [28] presented the gait synthesis and designing steps of bipeds in his work. His study was a very useful source to see the bigger picture for evolution of the studies on human walking from the ancient times to present day. It includes the results of the studies that have been explaining from beginning of this chapter and analyzes several types of passive dynamic systems like skateboards and bicycles. There are also several studies [29-31] that present how to apply passive dynamics of bipeds on several active humanoids by using less control and consuming less energy. Atkeson et al. [32] investigated the positive effect of swing leg retraction on gait stability, which resulted confirmatory when the speed of retraction is not rapid and in case of fast walking routine. Another research combines several efficient bipeds based on passive dynamic walkers from different universities [33]. An interesting research subject on passive dynamic walkers is the shape of the feet. In real applications, the shape of feet is usually semicircular or arc. Wisse and Anderson [34] proposed how this becomes an disadvantage for a standing position of a passive dynamic walker, and how to replace it by flat feet with a torsion spring. The results provided more human-like look to this kind of systems. A similar study made by Wang et al. [35] and another one but with segmented feet design using toe joints [36]. Another research using flat feet and compliant ankles by Wang et al. [37] presented the role of ground contact angle in passive dynamic walking. Wisse et al. [38] studied the stability of simplest passive dynamic walkers by means of falling forward or backward. The results showed that there is no solution for falling backward; on the other hand, the falling forward problem could be solved by adjusting the swing leg speed. Passive dynamic walkers are sensitive mechanisms against the changes in environment, so called disturbances. Hobbelen and Wisse [39] indicated that the measurement of disturbance rejection is more important than measurement of speed or energy efficiency. Therefore, this study introduced the idea of *gait sensitivity*, which is a measurement method of disturbance rejection based on disturbances and gait characteristics chosen by the designer. The proposed method showed improvement in calculation time and prediction of the disturbances for simple and more complex bipeds compared to previous researches, which mostly based on basin of attraction or *Floquet multipliers*. Another study of the same researchers [40] presented a way of avoiding from disturbances during the walking process of biped mechanism by using swing-leg retraction, which defines the backwards movement of swing leg prior to foot impact. The study has been implemented on three different types of passive dynamic walkers by also adding the idea of gait sensitivity, and results showed that the optimal case of rejection occurs when the retraction is on a mild-level speed. Wisse et al. [41] studied the case of passive walking biped machine with an upper body, which needed to be stabilized while the system was walking successfully. They presented bisecting hip mechanism as a solution, which behaves as an inverted pendulum and allows a passive swing leg movement in the same time. Another research on adding an upper body as a bisecting hip was done by Asano and Luo [42]. Additionally scientists used some control techniques on passive dynamic walking robots, like reinforcement learning [43] or dynamic programming [44]. The idea of passive dynamic running is also an interesting subject. The first studies inspired by McGeer again [45] and examined by different researchers in recent time [46, 47]. There are also another researches, which studies the effect of swinging arm on the biped walking. Collins, et al. [48] showed that arm swinging is a natural movement caused by walking, and has great impact on balancing the vertical torques. Actually the study revealed that: “*Vertical ground reaction moment was most affected by arm swinging and increased by 63 per cent without it*.”

1.3 Notation

The notation in this thesis is not different from S.Murat Yesiloglu’s dissertation [1]. All left-superscripts indicate the numbers of associated manipulator, the left-subscripts present the dimensions and the right-subscripts show the numbers of rigid link. Vectors in 3D represented with one arrow, in 6D they are shown with two arrows on the upside. The underlined vectors mean a serial of variables or vectors. The matrices are shown with bold capital letters or calligraphic fonts. The left-superscripts have been used only for the general representation. In application we have used only one manipulator system, hence there is no left-superscript in biped equations.

2. KINEMATICS AND DYNAMICS OF MANIPULATORS USING SOA

2.1 Purpose

In this chapter, we will first investigate the velocity propagation on a rigid link. After that, we will derive the kinematic equations of an *n* DoF serial manipulator both on a fixed and moving platform, using spatial vectors. The main purpose of this chapter is obtaining the kinematic equations for *p* number of cooperating manipulators on a moving platform. In many robotic applications, the cooperating manipulators are being used in both industry and scientific researches. These systems consist of several serial manipulators, which are operating simultaneously and their tip points follow the same trajectory. For this reason, we will define a *kinematic constraint*, which will hold the tip points of different manipulators for a common task. Finally, we will calculate the dynamic equations of serial manipulators. The calculation steps for both kinematics and dynamics are inspired by S. Murat Yesiloglu's dissertation including the notation, and they are available in [1].

2.2 Basic Information and Calculation of Propagation Matrix

A rigid multi-body system consists of rigid links. Hence, we will study first the velocity transformation across a rigid link. Let us consider the link in Figure 2.1:

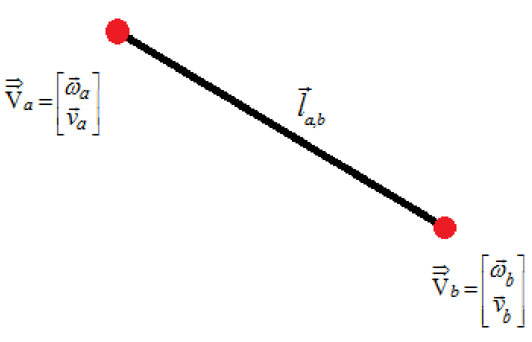


Figure 2.1 : Velocity propagation on a rigid link.

The angular velocity is same for all the points on a rigid link. Therefore, $\vec{\omega}_a = \vec{\omega}_b$ equality will be true. The linear velocity propagation depends on the time derivative of the vector from point a to b . It can be defined as,

$$\vec{v}_b = \vec{v}_a + \frac{\mathrm{d}}{\mathrm{dt}}\vec{l}_{a,b} \tag{2.1}$$

The second term on the right hand side of the equation (2.1) represents the time derivation of a vector. This concept is important, because we will use it in further calculations of both kinematics and dynamics.

2.2.1 Time derivation of a vector

Time derivation of a vector defines the change of its direction in time. Let say the vector $\vec{\mathrm{P}}(t)$ is rotating around the axis of rotation, $\vec{\omega}$, with a constant velocity $\dot{\theta}$ and radius r. The position of $\vec{\mathrm{P}}(t)$ after Δt time will change as in Figure 2.2:

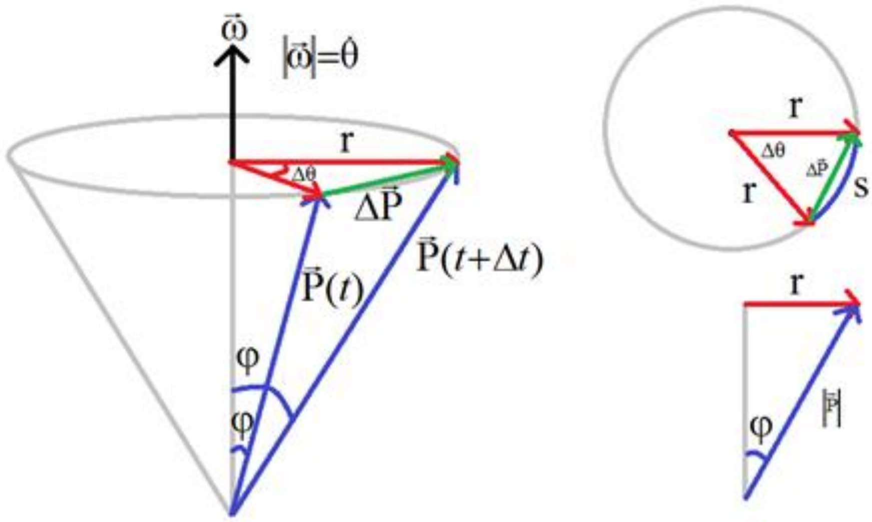


Figure 2.2 : Time derivation of a vector.

The following calculations will lead us to the meaning of derivation of a vector. The change in $\vec{\mathrm{P}}$ can be described as

$$\Delta\vec{\mathrm{P}} = \vec{\mathrm{P}}(t+\Delta t) - \vec{\mathrm{P}}(t)$$
$$\frac{d\vec{\mathrm{P}}(t)}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{\mathrm{P}}}{\Delta t}\right) \tag{2.2}$$

The arc s is to be calculated as,

$$s = r.\Delta\theta = \left\|\vec{P}\right\| \sin\varphi.\Delta\theta \quad \textbf{(2.3)}$$

If we follow the calculation steps as follow,

$$\left\| \frac{d\vec{P}(t)}{dt} \right\| = \left\| \lim_{\Delta t \to 0} \frac{\Delta\vec{P}}{\Delta t} \right\| = \lim_{\Delta t \to 0} \left\| \frac{\Delta\vec{P}}{\Delta t} \right\| = \lim_{\Delta t \to 0} \frac{\left\|\Delta\vec{P}\right\|}{\Delta t} \quad \textbf{(2.4)}$$

Then the following equation holds

$$\lim_{\Delta t \to 0} \frac{\left\|\vec{P}\right\| \Delta\theta \ \sin\varphi}{\Delta t} = \left\|\vec{P}\right\| \sin\varphi \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \textbf{(2.5)}$$

where

$$\begin{aligned} &\lim_{\Delta t \to 0} \left\|\Delta\vec{P}\right\| \to s \\ &\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \dot{\theta} = \left\|\vec{\omega}\right\| \end{aligned} \quad \textbf{(2.6)}$$

Therefore the magnitude of vector $\vec{P}(t)$ will be,

$$\left\| \frac{d\vec{P}(t)}{dt} \right\| = \left\|\vec{P}\right\| \left\|\vec{\omega}\right\| \sin\varphi \quad \textbf{(2.7)}$$

The right hand side of (2.7) represents the cross product between vectors $\vec{P}(t)$ and $\vec{\omega}$. Therefore, the time derivative of a vector is the cross product between its axis of rotation and itself, depending on the direction of rotation,

$$\frac{d\vec{P}(t)}{dt} = \vec{\omega} \times \vec{P}(t) \quad \textbf{(2.8)}$$

2.2.2 Velocity translation on a rigid link and propagation matrix

Now if we consider again the rigid link equation in (2.1), and a manipulator, which consists of multiple rigid links as in Figure 2.3.

The angular and linear velocities will be propagated from one joint to another as

$${}^{i}\vec{\omega}_{k} = {}^{i}\vec{\omega}_{k-1} + {}^{i}\vec{h}_{k}\,{}^{i}\dot{\theta}_{k} \quad \textbf{(2.9)}$$

$${}^{i}\vec{v}_{k} = {}^{i}\vec{v}_{k-1} + {}^{i}\vec{\omega}_{k-1} \times {}^{i}\vec{l}_{k-1,k} = \vec{v}_{k-1} - {}^{i}\vec{l}_{k-1,k} \times {}^{i}\vec{\omega}_{k-1} \quad \textbf{(2.10)}$$

where, i represents the number of manipulators, k is the number of joints, and ${}^{i}\vec{h}_{k}$ represents the axis of rotation vector on joint k.

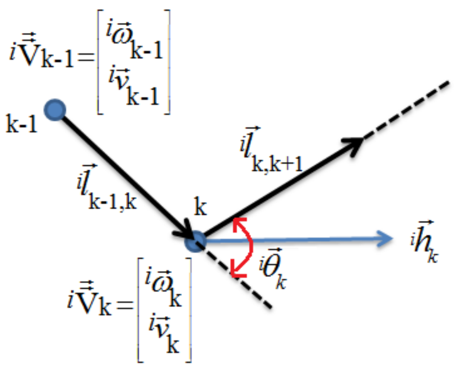


Figure 2.3 : Velocity propagation on a rigid link.

Using the skew-symmetric property for cross product representation, (2.10) becomes

$$ {}^{i}\vec{v}_{k} = \vec{v}_{k-1} - {}^{i}\hat{l}_{k-1,k}\, {}^{i}\vec{\omega}_{k-1} \qquad \textbf{(2.11)} $$

where

$$ {}^{i}\hat{l}_{k-1,k} = {}^{i}\vec{l}_{k-1,k} \times = \begin{bmatrix} 0 & -{}^{i}l_{(k-1,k)_z} & {}^{i}l_{(k-1,k)_y} \\ {}^{i}l_{(k-1,k)_z} & 0 & -{}^{i}l_{(k-1,k)_x} \\ -{}^{i}l_{(k-1,k)y} & {}^{i}l_{(k-1,k)_z} & 0 \end{bmatrix} $$

If we define ${}^{i}\vec{\vec{V}}_{k} \in \mathbb{R}^{6}$ as spatial velocity vector, which consists of ${}^{i}\vec{\omega}_{k} \in \mathbb{R}^{3}$ and ${}^{i}\vec{v}_{k} \in \mathbb{R}^{3}$, then the following expression will be true:

$$ {}^{i}\vec{\vec{V}}_{k} = \begin{bmatrix} {}^{i}\vec{\omega}_{k} \\ {}^{i}\vec{v}_{k} \end{bmatrix} = \begin{bmatrix} I & 0 \\ -{}^{i}\hat{l}_{k-1,k} & I \end{bmatrix} \begin{bmatrix} {}^{i}\vec{\omega}_{k-1} \\ {}^{i}\vec{v}_{k-1} \end{bmatrix} + \begin{bmatrix} {}^{i}\vec{h}_{k} \\ 0 \end{bmatrix} {}^{i}\dot{\theta}_{k} \qquad \textbf{(2.12)} $$

The equation (2.12) represents the spatial velocity propagation on a rigid link from the predecessor joint to the successor one. We can put (2.12) into a matrix from,

$$ {}^{i}\vec{\vec{V}}_{k} = {}^{i}\Phi_{k,k-1}\, {}^{i}\vec{\vec{V}}_{k-1} + {}^{i}\vec{\mathrm{H}}_{k}\, {}^{i}\dot{\theta}_{k} \qquad \textbf{(2.13)} $$

where ${}^{i}\Phi_{k,k-1}$ is the velocity propagation matrix from joint *k-1* to *k*. ${}^{i}\vec{\mathrm{H}}_{k}$ represents here the spatial rotation and translation matrix.

2.3 Kinematics

In this section, we will first derive the kinematic equations for a serial *n* DoF manipulator using spatial vectors. We will see how to calculate the velocities of each joint from base through tip point in a recursive manner. After having the forward and inverse kinematic representation, we will apply the same algorithm for cooperating manipulators.

2.3.1 Kinematics of a serial manipulator on a fixed platform

If we consider that a manipulator consists of n rigid links, the velocity propagation for each joint can be calculated based on (2.13). Let us assume first that the manipulator stands on a fixed platform, which means the platform velocity is zero.

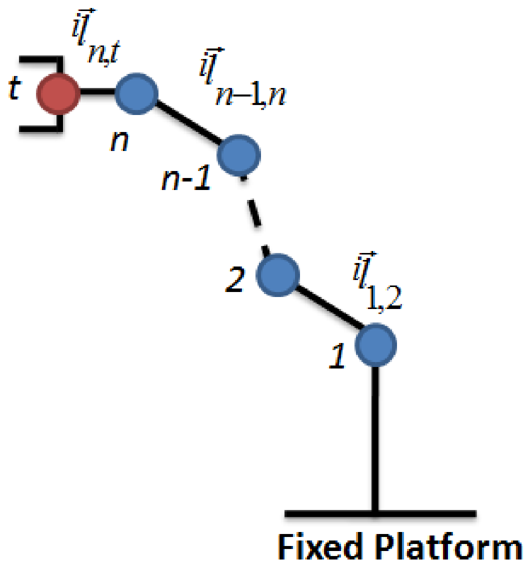


Figure 2.4 : *n* DoF serial manipulator on a fixed platform.

Then the spatial velocity equations for each joint will be as follows,

$$
\begin{aligned}
&{}^{i}\vec{\vec{V}}_0 = 0 \\
&{}^{i}\vec{\vec{V}}_1 = {}^{i}\vec{H}_1\, {}^{i}\dot{\theta}_1 \\
&{}^{i}\vec{\vec{V}}_2 = {}^{i}\Phi_{2,1}\, {}^{i}\vec{\vec{V}}_1 + {}^{i}\vec{H}_2\, {}^{i}\dot{\theta}_2 = {}^{i}\Phi_{2,1}\, {}^{i}\vec{H}_1\, {}^{i}\dot{\theta}_1 + {}^{i}\vec{H}_2\, {}^{i}\dot{\theta}_2 \\
&{}^{i}\vec{\vec{V}}_3 = \Phi_{3,2}\, {}^{i}\vec{\vec{V}}_2 + {}^{i}\vec{H}_3\, {}^{i}\dot{\theta}_3 = \Phi_{3,2}\left(\Phi_{2,1}\, {}^{i}\vec{H}_1\, {}^{i}\dot{\theta}_1 + {}^{i}\vec{H}_2\, {}^{i}\dot{\theta}_2\right) + {}^{i}\vec{H}_3\, {}^{i}\dot{\theta}_3
\end{aligned}
\qquad \textbf{(2.14)}
$$

To improve this calculation step for next joint numbers, we need to examine the multiplication of $\Phi_{k+1,k}$ and $\Phi_{k,k-1}$ matrices.

$$
\Phi_{k+1,k}\Phi_{k,k-1} = \begin{bmatrix} I & 0 \\ -{}^{i}\hat{l}_{k,k+1} & I \end{bmatrix} \begin{bmatrix} I & 0 \\ -{}^{i}\hat{l}_{k-1,k} & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ -{}^{i}\hat{l}_{k,k+1} - {}^{i}\hat{l}_{k-1,k} & I \end{bmatrix} \qquad \textbf{(2.15)}
$$

Since,

$$-{}^i\hat{l}_{k,k+1} - {}^i\hat{l}_{k-1,k} = \left(-{}^i l_{k,k+1}\times\right) - \left({}^i l_{k-1,k}\times\right) = -\left({}^i l_{k-1,k} + {}^i l_{k,k+1}\right)\times = {}^i l_{k-1,k+1}\times = -{}^i\hat{l}_{k-1,k+1}$$

By utilizing this in (2.15),

$$\Phi_{k+1,k}\Phi_{k,k-1} = \begin{bmatrix} I & 0 \\ -{}^i\hat{l}_{k-1,k+1} & I \end{bmatrix} = \Phi_{k+1,k-1} \quad \textbf{(2.16)}$$

Now, if we use this property of Φ matrix, (2.14) becomes,

$${}^i\vec{\vec{V}}_3 = {}^i\Phi_{3,2}\,{}^i\Phi_{2,1}\,{}^i\vec{H}_1\,{}^i\dot{\theta}_1 + {}^i\Phi_{3,2}\,{}^i\vec{H}_2\,{}^i\dot{\theta}_2 + {}^i\vec{H}_3\,{}^i\dot{\theta}_3 = {}^i\Phi_{3,1}\,{}^i\vec{H}_1\,{}^i\dot{\theta}_1 + {}^i\Phi_{3,2}\,{}^i\vec{H}_2\,{}^i\dot{\theta}_2 + {}^i\vec{H}_3\,{}^i\dot{\theta}_3 \quad \textbf{(2.17)}$$

If we keep doing this calculation for each joint, finally we get the velocity of the nth joint:

$${}^i\vec{\vec{V}}_n = {}^i\Phi_{n,1}\,{}^i\vec{H}_1\,{}^i\dot{\theta}_1 + {}^i\Phi_{n,n-2}\,{}^i\vec{H}_2\,{}^i\dot{\theta}_2 + \ldots + {}^i\vec{H}_n\,{}^i\dot{\theta}_n \quad \textbf{(2.18)}$$

If we put these equations in the matrix form, we get,

$$\underset{6nx1}{\begin{bmatrix} {}^i\vec{\vec{V}}_1 \\ {}^i\vec{\vec{V}}_2 \\ {}^i\vec{\vec{V}}_3 \\ \vdots \\ {}^i\vec{\vec{V}}_n \end{bmatrix}} = \underset{6nx6n}{\begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ {}^i\Phi_{2,1} & I & 0 & \cdots & 0 \\ {}^i\Phi_{3,1} & {}^i\Phi_{3,2} & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ {}^i\Phi_{n,1} & {}^i\Phi_{n,2} & {}^i\Phi_{n,3} & \cdots & I \end{bmatrix}} \underset{6nxn}{\begin{bmatrix} {}^i\vec{H}_1 & 0 & 0 & \cdots & 0 \\ 0 & {}^i\vec{H}_2 & 0 & \cdots & 0 \\ 0 & 0 & {}^i\vec{H}_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & {}^i\vec{H}_n \end{bmatrix}} \underset{nx1}{\begin{bmatrix} {}^i\dot{\theta}_1 \\ {}^i\dot{\theta}_2 \\ {}^i\dot{\theta}_3 \\ \vdots \\ {}^i\dot{\theta}_n \end{bmatrix}} \quad \textbf{(2.19)}$$

Therefore, the general matrix form of spatial velocity vectors for each joint is,

$${}^i\underline{V} = {}^i\Phi\,{}^i H\,{}^i\underline{\dot{\theta}} \quad \textbf{(2.20)}$$

where, ${}^i\underline{V}$ is a 6n-by-1, ${}^i\Phi$ is a 6n-by-6n, ${}^i H$ is a 6n-by-n, and ${}^i\underline{\dot{\theta}}$ is an n-by-1 matrix.

So far, we have found the spatial velocity vectors for each joint. To find the velocity vector of tip point of the manipulator, we need to define a propagation matrix, ${}^i\Phi_{t,n}$ from joint number *n* through tip point *t*:

$${}^i\vec{\vec{V}}_t = {}^i\Phi_{t,n}\,{}^i\vec{\vec{V}}_n \quad \textbf{(2.21)}$$

The velocity propagation from the first joint through the tip point will be then,

$$ {}^{i}\vec{\vec{V}}_t = \begin{bmatrix} 0 & 0 & 0 & \cdots & {}^{i}\Phi_{t,n} \end{bmatrix} \begin{bmatrix} {}^{i}\vec{\vec{V}}_1 \\ {}^{i}\vec{\vec{V}}_2 \\ {}^{i}\vec{\vec{V}}_3 \\ \vdots \\ {}^{i}\vec{\vec{V}}_n \end{bmatrix} \qquad \textbf{(2.22)} $$

The matrix form will be,

$$ {}^{i}\vec{\vec{V}}_t = {}^{i}\Phi_t \, {}^{i}\underline{V} \qquad \textbf{(2.23)} $$

where ${}^{i}\Phi_t = \begin{bmatrix} 0 & 0 & 0 & \cdots & {}^{i}\Phi_{t,n} \end{bmatrix}$ and it is a 6-by-6n matrix. Placing (2.20) into (2.23) will give us the relation between tip point velocities and joint velocities, which we call it *Jacobian*:

$$ {}^{i}\vec{\vec{V}}_t = {}^{i}\Phi_t \, {}^{i}\Phi \, {}^{i}\mathrm{H} \, {}^{i}\dot{\theta} = {}^{i}J \, {}^{i}\dot{\theta} $$

$$ {}^{i}J = {}^{i}\Phi_t \, {}^{i}\Phi \, {}^{i}\mathrm{H} \qquad \textbf{(2.24)} $$

As shown in (2.24), we will calculate the *Jacobian Matrix* using propagation matrices and spatial rotation and translation matrix. We wil use *Jacobian* both for forward dynamics,

$$ {}^{i}\vec{\vec{V}}_t = {}^{i}J \, {}^{i}\dot{\theta} \qquad \textbf{(2.25)} $$

and for inverse dynamics,

$$ {}^{i}\dot{\theta} = {}^{i}J^{\#} \, {}^{i}\vec{\vec{V}}_t \qquad \textbf{(2.26)} $$

where ${}^{i}\mathrm{J}^{\#}$ represents the pseudo inverse of *Jacobian*.

2.3.2 Kinematics of a serial manipulator on a moving platform

The moving platform, or base, brings six more DoF to the serial manipulator, which can be presented as an additional term to velocity of tip point. This term can be calculated using the propagation matrix from base through tip:

$$ {}^{i}\vec{\vec{V}}_t = {}^{i}J \, {}^{i}\dot{\theta} + {}^{i}\Phi_{t,b} \, {}^{i}\vec{\vec{V}}_b \qquad \textbf{(2.27)} $$

where, ${}^{i}\Phi_{t,b}$ is the propagation matrix, and ${}^{i}\vec{\vec{V}}_b$ is the spatial velocity vector of the platform.

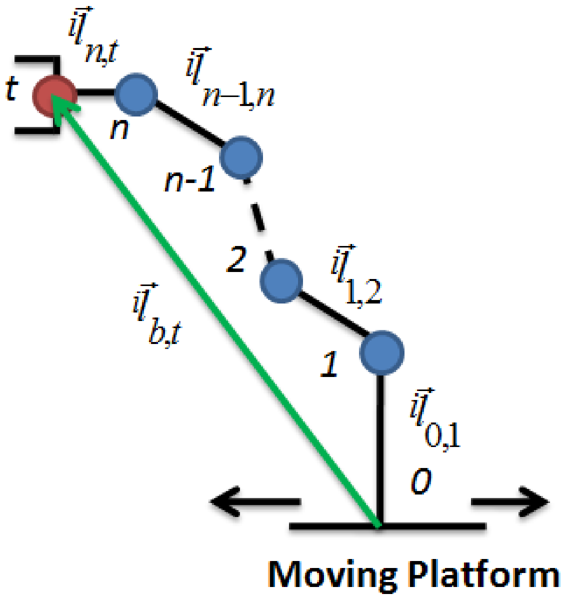


Figure 2.5 : *n* DoF serial manipulator on a moving platform.

There are two ways to calculate $^{i}\Phi_{t,b}$; it could be either finding the $^{i}\vec{l}_{t,b}$ by adding the lengths of each link together,

$$^{i}\vec{l}_{t,b} = \sum_{k=0}^{n} {}^{i}\vec{l}_{k,k+1}$$

$$^{i}\Phi_{t,b} = \begin{bmatrix} I & 0 \\ -{}^{i}\hat{l}_{b,t} & I \end{bmatrix} \quad \textbf{(2.28)}$$

or we can use the multiplication property of matrix, as shown in (2.16),

$$^{i}\Phi_{t,b} = {}^{i}\Phi_{t}\, {}^{i}\Phi\, {}^{i}\Phi_{b} \quad \textbf{(2.29)}$$

where,

$$^{i}\Phi_{b} = \begin{bmatrix} {}^{i}\Phi_{1,0} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

2.3.3 Kinematics of cooperating manipulators

The kinematic equations for each cooperating manipulator are the same with the equations of a serial manipulator. One crucial difference is the *kinematic constraint*, which will allow the manipulators to work together.

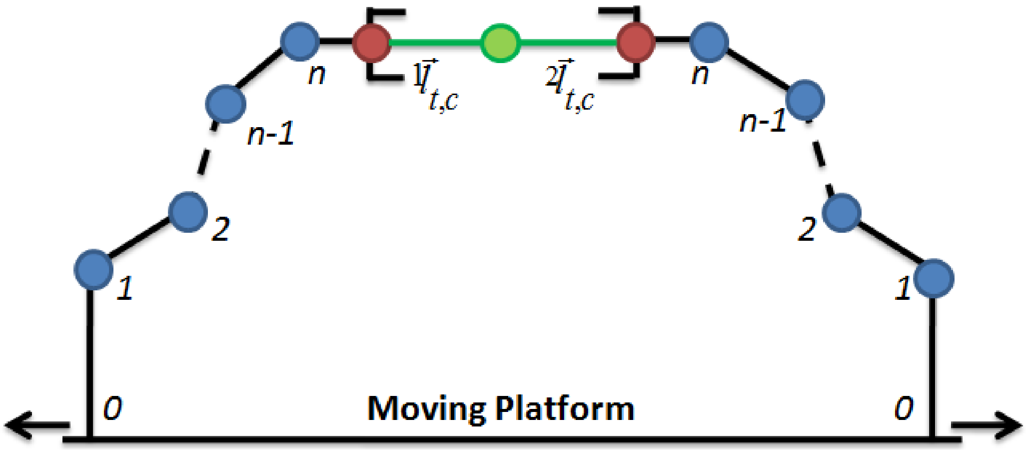


Figure 2.6 : Cooperating manipulators on a moving platform.

We define a kinematic constraint, which makes the several manipulators to make the same operation like holding an object or tracking a trajectory together. Let us say the manipulators are holding a rigid object and carrying it while tracking a certain trajectory. The velocity vector of the object is $\vec{\vec{V}}_c$. It is obviously that we can transform the velocity from the object to the tip points of the manipulators by defining a new Φ matrix:

$$ {}^i\vec{\vec{V}}_t = {}^i\Phi_{t,c}\vec{\vec{V}}_c \tag{2.30} $$

The ${}^i\Phi_{t,c}$ matrix will include the kinematic constraint. Since the manipulators are holding an object, the distance between the tip points and center of the rigid object will never change (Figure 2.6). For one manipulator, the ${}^i\Phi_{t,c}$ will be,

$$ {}^i\Phi_{t,c} = \begin{bmatrix} I & 0 \\ -{}^i\hat{l}_{c,t} & I \end{bmatrix} \tag{2.31} $$

If we generalize (2.30) for p number of manipulators, the kinematic constraint of the system will be,

$$ \begin{bmatrix} {}^1\vec{\vec{V}}_t \\ {}^2\vec{\vec{V}}_t \\ {}^3\vec{\vec{V}}_t \\ \vdots \\ {}^p\vec{\vec{V}}_t \end{bmatrix} = \begin{bmatrix} {}^1\Phi_{t,c} \\ {}^2\Phi_{t,c} \\ {}^3\Phi_{t,c} \\ \vdots \\ {}^p\Phi_{t,c} \end{bmatrix} \vec{\vec{V}}_c $$

$$\underline{V}_t = \Phi_{t,c} \vec{\vec{V}}_c \tag{2.32}$$

So far, we calculated the spatial tip velocity vector of each manipulator. Now to find the kinematic equations of each arm, we will follow the same steps for each one, as we did for a serial manipulator. If we consider the cooperating manipulator system operates on a moving platform as in Figure 2.6, then we can utilize (2.27) for each manipulator,

$$\begin{aligned}
{}^1\vec{\vec{V}}_t &= {}^1J\,{}^1\dot{\theta} + {}^1\Phi_{t,b}\vec{\vec{V}}_b \\
{}^2\vec{\vec{V}}_t &= {}^2J\,{}^2\dot{\theta} + {}^2\Phi_{t,b}\vec{\vec{V}}_b \\
{}^3\vec{\vec{V}}_t &= {}^3J\,{}^3\dot{\theta} + {}^3\Phi_{t,b}\vec{\vec{V}}_b \\
\vdots \quad & \quad \vdots \qquad\quad \vdots \\
{}^p\vec{\vec{V}}_t &= {}^pJ\,{}^p\dot{\theta} + {}^p\Phi_{t,b}\vec{\vec{V}}_b
\end{aligned} \tag{2.33}$$

If we reconfigure (2.33) into a matrix form, we get

$$\begin{bmatrix} {}^1\vec{\vec{V}}_t \\ {}^2\vec{\vec{V}}_t \\ {}^3\vec{\vec{V}}_t \\ \vdots \\ {}^p\vec{\vec{V}}_t \end{bmatrix} = \begin{bmatrix} {}^1\mathrm{J} & 0 & 0 & \cdots & 0 \\ 0 & {}^2\mathrm{J} & 0 & \cdots & 0 \\ 0 & 0 & {}^3\mathrm{J} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & {}^p\mathrm{J} \end{bmatrix} \begin{bmatrix} {}^1\dot{\theta} \\ {}^2\dot{\theta} \\ {}^3\dot{\theta} \\ \vdots \\ {}^p\dot{\theta} \end{bmatrix} + \begin{bmatrix} {}^1\Phi_{t,b} \\ {}^2\Phi_{t,b} \\ {}^3\Phi_{t,b} \\ \vdots \\ {}^p\Phi_{t,b} \end{bmatrix} \vec{\vec{V}}_b \tag{2.34}$$

This representation reveals the relationship between joints and tip velocities, yet it is different from the classical notation of forward dynamics, as in (2.25). For this reason, first, we need to redefine the following matrices as,

$$\underline{V} = \begin{bmatrix} \vec{\vec{V}}_b \\ {}^1\mathrm{V} \\ {}^2\mathrm{V} \\ \vdots \\ {}^p\mathrm{V} \end{bmatrix} \qquad \dot{\theta} = \begin{bmatrix} \vec{\vec{V}}_b \\ {}^1\dot{\theta} \\ {}^2\dot{\theta} \\ \vdots \\ {}^p\dot{\theta} \end{bmatrix} \tag{2.35}$$

$$\Phi = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ {}^1\Phi\Phi_b & {}^1\Phi & 0 & \cdots & 0 \\ {}^2\Phi\Phi_b & 0 & {}^2\Phi & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ {}^p\Phi\Phi_b & 0 & 0 & \ldots & {}^p\Phi \end{bmatrix} \tag{2.36}$$

$$\mathrm{H} = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ 0 & {}^{1}\mathrm{H} & 0 & \cdots & 0 \\ 0 & 0 & {}^{2}\mathrm{H} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & {}^{p}\mathrm{H} \end{bmatrix} \quad \textbf{(2.37)}$$

$$\Phi = \begin{bmatrix} 0 & {}^{1}\Phi_{t} & 0 & \cdots & 0 \\ 0 & 0 & {}^{2}\Phi_{t} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & {}^{p}\Phi_{t} \end{bmatrix} \quad \textbf{(2.38)}$$

Therefore, the new Jacobian will be,

$$J = \begin{bmatrix} {}^{1}\Phi_{t,b} & {}^{1}J & 0 & \dots & 0 \\ {}^{2}\Phi_{t,b} & 0 & {}^{2}J & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ {}^{p}\Phi_{t,b} & 0 & 0 & \dots & {}^{p}J \end{bmatrix} \quad \textbf{(2.39)}$$

Now, using the new representation we have the classic forward kinematics equation,

$$\vec{\vec{\mathrm{V}}}_{t} = J\dot{\theta} \quad \textbf{(2.40)}$$

2.4 Dynamics

In this section, we will first start with basic information in dynamics of a particle and a rigid link. Later on, we derive the dynamic equations of a serial *n* DoF manipulator using spatial vectors. In the beginning, we calculate the joint accelerations, and later the joint forces. After that, we find the equation of motion in both forward and inverse dynamics form.

2.4.1 Preliminaries in dynamics of a rigid link

The dynamics of a particle infer its angular and linear momentums, which occur due to the torques and forces on the particle, respectively. Let us consider a particle on a rigid body in frame *B*, where the reference base frame is *A* (Figure 2.7).

The mass of the particle will be,

$$m = \frac{1}{2}\int_{\Omega} \rho(r)d\Omega \quad \textbf{(2.41)}$$

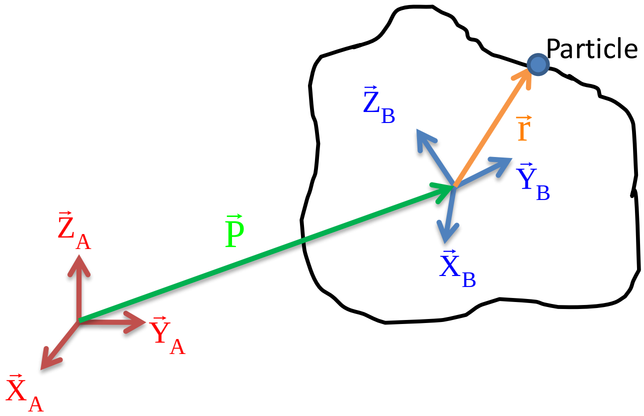


Figure 2.7 : Dynamics of a particle.

where, ρ is the mass density and Ω is the volume. The kinetic energy will be,

$$T = \frac{1}{2}\int_{\Omega} \rho(r)\|\vec{v}\|^2 d\Omega \tag{2.42}$$

where $\vec{v}$ represents the linear velocity and,

$$\vec{v} = \dot{\vec{\mathrm{P}}} + \dot{\mathrm{R}}\mathrm{r} \tag{2.43}$$

where R is the rotation matrix. If we place (2.43) into (2.42), the kinetic energy equation becomes,

$$T = \frac{1}{2}m\left\|\dot{\vec{\mathrm{P}}}\right\|^2 + \frac{1}{2}\omega^{\mathrm{T}} I\omega \tag{2.44}$$

where I is the inertia and,

$$I = mr^2 = \int_{\Omega} \rho(r) r^2 d\Omega \tag{2.45}$$

Now, we know that the time derivative of angular and linear momentum will give the torque and force, respectively:

$$\tau = \frac{d}{dt}(I\omega)$$
$$f = \frac{d}{dt}(mv) \tag{2.46}$$

For the rigid body transformation, we will use the propagation matrix, which we found for kinematics. In fact, Jain's table [17] would be very useful before we start to calculate link accelerations and forces:

Table 2.1 : Rigid body transformations [17].

	Point ***k***	Point ***k+1***
Spatial Velocities	$\vec{\vec{V}}_k$	$\vec{\vec{V}}_{k+1} = \Phi_{k+1,k}\vec{\vec{V}}_k$
Spatial Forces	$\vec{\vec{F}}_k = \Phi^{T}{}_{k+1,k}\vec{\vec{F}}_{k+1}$	$\vec{\vec{F}}_{k+1}$
Spatial Inertias	$M_k = \Phi^{T}{}_{k+1,k} M_{k+1} \Phi_{k+1,k}$	M_{k+1}
Kinetic Energy	$T_k = \frac{1}{2}\vec{\vec{V}}_k^{T} M_k \vec{\vec{V}}_k$	$T_{k+1} = \frac{1}{2}\vec{\vec{V}}_{k+1}^{T} M_{k+1} \vec{\vec{V}}_{k+1}$

2.4.2 Calculation of link accelerations and forces of a serial manipulator

In dynamic model of the system, first we need to take the time derivatives of the angular (2.9) and linear velocities (2.10). For the angular acceleration,

$$
\begin{aligned}
{}^i\dot{\vec{\omega}}_k &= {}^i\dot{\vec{\omega}}_{k-1} + {}^i\vec{h}_k\,{}^i\ddot{\theta}_k + {}^i\dot{\vec{h}}_k\,{}^i\dot{\theta}_k \\
{}^i\dot{\vec{\omega}}_k &= {}^i\dot{\vec{\omega}}_{k-1} + {}^i\vec{h}_k\,{}^i\ddot{\theta}_k + {}^i\vec{\omega}_{k-1} \times {}^i\vec{\omega}_k
\end{aligned} \qquad \textbf{(2.47)}
$$

where we utilized (2.8) for the time derivation of vector ${}^i\vec{h}_k$ and the relation in (2.9)

$$
\begin{aligned}
{}^i\dot{\vec{h}}_k &= {}^i\vec{\omega}_k \times {}^i\vec{h}_k \\
{}^i\vec{h}_k\,{}^i\dot{\theta}_k &= {}^i\vec{\omega}_k - {}^i\vec{\omega}_{k-1}
\end{aligned}
$$

For the linear acceleration, we take the time derivative of (2.10)

$$
{}^i\dot{\vec{v}}_k = {}^i\dot{\vec{v}}_{k-1} + {}^i\dot{\vec{\omega}}_{k-1} \times {}^i\vec{l}_{k-1,k} + {}^i\vec{\omega}_{k-1} \times {}^i\dot{\vec{l}}_{k-1,k}
$$

By using (2.8) for the time derivative of vector ${}^i\vec{l}_{k-1,k}$

$$
{}^i\dot{\vec{v}}_k = {}^i\dot{\vec{v}}_{k-1} - {}^i\vec{l}_{k-1,k} \times {}^i\dot{\vec{\omega}}_{k-1} + {}^i\vec{\omega}_{k-1} \times ({}^i\vec{\omega}_{k-1} \times {}^i\vec{l}_{k-1,k}) \qquad \textbf{(2.48)}
$$

Now, if we put (2.47) and (2.48) into the matrix form to get the spatial acceleration vector,

$$
{}^i\dot{\vec{\vec{V}}}_k = \begin{bmatrix} {}^i\dot{\vec{\omega}}_k \\ {}^i\dot{\vec{v}}_k \end{bmatrix} = \begin{bmatrix} I & 0 \\ -{}^i\hat{l}_{k-1,k} & I \end{bmatrix} \begin{bmatrix} {}^i\dot{\vec{\omega}}_{k-1} \\ {}^i\dot{\vec{v}}_{k-1} \end{bmatrix} + \begin{bmatrix} {}^i\vec{h}_k \\ 0 \end{bmatrix} {}^i\ddot{\theta}_k + \begin{bmatrix} {}^i\vec{\omega}_{k-1} \times {}^i\vec{\omega}_k \\ {}^i\vec{\omega}_{k-1} \times ({}^i\vec{\omega}_k \times {}^i\vec{l}_{k-1,k}) \end{bmatrix} \qquad \textbf{(2.49)}
$$

The more general representation will be like,

$$
{}^i\dot{\vec{\vec{V}}}_k = {}^i\Phi_{k,k-1}\,{}^i\dot{\vec{\vec{V}}}_{k-1} + {}^i\vec{\vec{H}}_k\,{}^i\ddot{\theta}_k + {}^i\vec{\vec{a}}_k \qquad \textbf{(2.50)}
$$

where, ${}^{i}\vec{\vec{a}}_k$ represents the spatial bias accelerations.

Now let us implement (2.50) for each link of the manipulator. First, we assume that the arm (manipulator) stands on a fixed platform. Hence, ${}^{i}\dot{\vec{\vec{\mathrm{V}}}}_0 = 0$. The acceleration vectors of the other links are to be calculated as follows,

$$
\begin{aligned}
&{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_0 = 0\\
&{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_1 = {}^{i}\vec{\vec{\mathrm{H}}}_1\,{}^{i}\ddot{\theta}_1 + {}^{i}\vec{\vec{a}}_1\\
&{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_2 = {}^{i}\Phi_{2,1}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_1 + {}^{i}\vec{\vec{\mathrm{H}}}_2\,{}^{i}\ddot{\theta}_2 + {}^{i}\vec{\vec{a}}_2 = {}^{i}\Phi_{2,1}\left({}^{i}\vec{\vec{\mathrm{H}}}_1\,{}^{i}\ddot{\theta}_1 + {}^{i}\vec{\vec{a}}_1\right) + {}^{i}\vec{\vec{\mathrm{H}}}_2\,{}^{i}\ddot{\theta}_2 + {}^{i}\vec{\vec{a}}_2\\
&{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_3 = {}^{i}\Phi_{3,2}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_2 + {}^{i}\vec{\vec{\mathrm{H}}}_3\,{}^{i}\ddot{\theta}_3 + {}^{i}\vec{\vec{a}}_3 = {}^{i}\Phi_{3,2}\left[{}^{i}\Phi_{2,1}\left({}^{i}\vec{\vec{\mathrm{H}}}_1\,{}^{i}\ddot{\theta}_1 + {}^{i}\vec{\vec{a}}_1\right) + {}^{i}\vec{\vec{\mathrm{H}}}_2\,{}^{i}\ddot{\theta}_2 + {}^{i}\vec{\vec{a}}_2\right] + {}^{i}\vec{\vec{\mathrm{H}}}_3\,{}^{i}\ddot{\theta}_3 + {}^{i}\vec{\vec{a}}_3\\
&\qquad = {}^{i}\Phi_{3,1}\left({}^{i}\vec{\vec{\mathrm{H}}}_1\,{}^{i}\ddot{\theta}_1 + {}^{i}\vec{\vec{a}}_1\right) + {}^{i}\Phi_{3,2}\left({}^{i}\vec{\vec{\mathrm{H}}}_2\,{}^{i}\ddot{\theta}_2 + {}^{i}\vec{\vec{a}}_2\right) + {}^{i}\vec{\vec{\mathrm{H}}}_3\,{}^{i}\ddot{\theta}_3 + {}^{i}\vec{\vec{a}}_3\\
&\qquad \vdots\\
&{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_n = {}^{i}\Phi_{n,1}\left({}^{i}\vec{\vec{\mathrm{H}}}_1\,{}^{i}\ddot{\theta}_1 + {}^{i}\vec{\vec{a}}_1\right) + {}^{i}\Phi_{n,2}\left({}^{i}\vec{\vec{\mathrm{H}}}_2\,{}^{i}\ddot{\theta}_2 + {}^{i}\vec{\vec{a}}_2\right) + \ldots + {}^{i}\vec{\vec{\mathrm{H}}}_n\,{}^{i}\ddot{\theta}_n + {}^{i}\vec{\vec{a}}_n
\end{aligned}
\tag{2.51}
$$

Now if we stack up the entire link accelerations,

$$
\begin{bmatrix} {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_1 \\ {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_2 \\ {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_3 \\ \vdots \\ {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_n \end{bmatrix}
=
\begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ {}^{i}\Phi_{2,1} & I & 0 & \cdots & 0 \\ {}^{i}\Phi_{3,1} & {}^{i}\Phi_{3,2} & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ {}^{i}\Phi_{n,1} & {}^{i}\Phi_{n,2} & {}^{i}\Phi_{n,3} & \ldots & I \end{bmatrix}
\left\{
\begin{bmatrix} {}^{i}\vec{\vec{\mathrm{H}}}_1 & 0 & 0 & \cdots & 0 \\ 0 & {}^{i}\vec{\vec{\mathrm{H}}}_2 & 0 & \cdots & 0 \\ 0 & 0 & {}^{i}\vec{\vec{\mathrm{H}}}_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & {}^{i}\vec{\vec{\mathrm{H}}}_n \end{bmatrix}
\begin{bmatrix} {}^{i}\ddot{\theta}_1 \\ {}^{i}\ddot{\theta}_2 \\ {}^{i}\ddot{\theta}_3 \\ \vdots \\ {}^{i}\ddot{\theta}_n \end{bmatrix}
+
\begin{bmatrix} {}^{i}\vec{\vec{a}}_1 \\ {}^{i}\vec{\vec{a}}_2 \\ {}^{i}\vec{\vec{a}}_3 \\ \vdots \\ {}^{i}\vec{\vec{a}}_n \end{bmatrix}
\right\}
\tag{2.52}
$$

Moreover, the general matrix form of acceleration vectors for each links will be

$$
{}^{i}\dot{\underline{\mathrm{V}}} = {}^{i}\Phi\left({}^{i}\mathrm{H}\,{}^{i}\ddot{\underline{\theta}} + {}^{i}\underline{a}\right) \tag{2.53}
$$

For the forces, on the other hand, the calculations will be done from tip towards base, instead of from base towards tip as in the calculations of velocity propagation, because of the boundary conditions of a two point boundary value problem [1]. For the torque propagation,

$$
{}^{i}\vec{\tau}_k = {}^{i}\vec{\tau}_{k+1} + {}^{i}\vec{l}_{k,k+1} \times \vec{f}_{k+1} + {}^{i}\vec{l}_{k,c} \times {}^{i}\dot{\vec{v}}_k\,{}^{i}m_k + \frac{d}{dt}\left({}^{i}\mathrm{I}_k\,{}^{i}\vec{\omega}_k\right) \tag{2.54}
$$

where, ${}^{i}\vec{l}_{k,c}$ represents the distance vector from the origin of the link frame k to the center of mass of the rigid link, and ${}^{i}\mathrm{I}_k$ is the link inertia. The term due to rotation can be expressed by using the idea of time derivation of a vector:

$$\frac{d}{dt}\left({}^{i}\mathrm{I}_k\,{}^{i}\vec{\omega}_k\right) = {}^{i}\dot{\mathrm{I}}_k\,{}^{i}\vec{\omega}_k + {}^{i}\mathrm{I}_k\,{}^{i}\dot{\vec{\omega}}_k = {}^{i}\vec{\omega}_k \times {}^{i}\mathrm{I}_k\,{}^{i}\vec{\omega}_k + {}^{i}\mathrm{I}_k\,{}^{i}\dot{\vec{\omega}}_k$$

The first and second terms of the right hand side of (2.54) come from the successor joint *k+1*, the third and last terms come from due to translation and rotation, respectively.

For the force propagation,

$${}^{i}\vec{f}_k = \vec{f}_{k+1} + {}^{i}m_k \frac{d}{dt}\left({}^{i}\vec{v}_k + {}^{i}\vec{\omega}_k \times {}^{i}\vec{l}_{k,c}\right) \tag{2.55}$$

where,

$$\frac{d}{dt}\left({}^{i}\vec{v}_k + {}^{i}\vec{\omega}_k \times {}^{i}\vec{l}_{k,c}\right) = {}^{i}\dot{\vec{v}}_k + {}^{i}\vec{\omega}_k \times {}^{i}\vec{l}_{k,c} + {}^{i}\vec{\omega}_k \times \left({}^{i}\vec{\omega}_k \times {}^{i}\vec{l}_{k,c}\right)$$

Now, if we put (2.54) and (2.55) in the same matrix, we get the spatial forces,

$${}^{i}\vec{\vec{\mathrm{F}}}_k = \begin{bmatrix} {}^{i}\vec{\tau}_k \\ {}^{i}\vec{f}_k \end{bmatrix}$$

$${}^{i}\vec{\vec{\mathrm{F}}}_k = \begin{bmatrix} I & {}^{i}\hat{l}_{k,k+1} \\ 0 & I \end{bmatrix}\begin{bmatrix} {}^{i}\vec{\tau}_{k+1} \\ {}^{i}\vec{f}_{k+1} \end{bmatrix} + \begin{bmatrix} {}^{i}\mathrm{I}_k & {}^{i}\vec{l}_{k,c} \times {}^{i}m_k \\ -{}^{i}\vec{l}_{k,c} \times {}^{i}m_k & {}_{3x3}I\,{}^{i}m_k \end{bmatrix}\begin{bmatrix} {}^{i}\dot{\vec{\omega}}_k \\ {}^{i}\dot{\vec{v}}_k \end{bmatrix} + \begin{bmatrix} {}^{i}\vec{\omega}_k \times {}^{i}\mathrm{I}_k\,{}^{i}\vec{\omega}_k \\ {}^{i}m_k\,{}^{i}\vec{\omega}_k \times ({}^{i}\vec{\omega}_k \times {}^{i}\vec{l}_{k,c}) \end{bmatrix} \tag{2.56}$$

which can also be shown as

$${}^{i}\vec{\vec{\mathrm{F}}}_k = {}^{i}\Phi^{\mathrm{T}}{}_{k+1,k}\,{}^{i}\vec{\vec{\mathrm{F}}}_{k+1} + {}^{i}\mathrm{M}_k\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_k + {}^{i}\vec{\vec{b}}_k \tag{2.57}$$

where, ${}^{i}\vec{\vec{\mathrm{F}}}_k$, ${}^{i}\mathrm{M}_k$ and ${}^{i}\vec{\vec{b}}_k$ represents link spatial forces, link mass matrix and the remainder terms, respectively.

Now if we implement (2.57) for each link of the manipulator starting from the n^{th} link, we get the spatial forces for each link,

$$
\begin{aligned}
&{}^i\vec{\vec{F}}_n = {}^i\Phi^{T}{}_{t,n}\,{}^i\vec{\vec{F}}_t + {}^iM_n\,{}^i\dot{\vec{\vec{V}}}_n + {}^i\vec{\vec{b}}_n \\
&{}^i\vec{\vec{F}}_{n-1} = {}^i\Phi^{T}{}_{n,n-1}\,{}^i\vec{\vec{F}}_n + {}^iM_{n-1}\,{}^i\dot{\vec{\vec{V}}}_{n-1} + {}^i\vec{\vec{b}}_{n-1} = {}^i\Phi^{T}{}_{n,n-1}\left({}^i\Phi^{T}{}_{t,n}\,{}^i\vec{\vec{F}}_t + {}^iM_n\,{}^i\dot{\vec{\vec{V}}}_n + {}^i\vec{\vec{b}}_n\right) + {}^iM_{n-1}\,{}^i\dot{\vec{\vec{V}}}_{n-1} + {}^i\vec{\vec{b}}_{n-1} \\
&\qquad = {}^i\Phi^{T}{}_{t,n-1}\,{}^i\vec{\vec{F}}_t + {}^i\Phi^{T}{}_{n,n-1}\left({}^iM_n\,{}^i\dot{\vec{\vec{V}}}_n + {}^i\vec{\vec{b}}_n\right) + {}^iM_{n-1}\,{}^i\dot{\vec{\vec{V}}}_{n-1} + {}^i\vec{\vec{b}}_{n-1} \\
&{}^i\vec{\vec{F}}_{n-2} = {}^i\Phi^{T}{}_{n-1,n-2}\,{}^i\vec{\vec{F}}_{n-1} + {}^iM_{n-2}\,{}^i\dot{\vec{\vec{V}}}_{n-2} + {}^i\vec{\vec{b}}_{n-2} \\
&\qquad = {}^i\Phi^{T}{}_{t,n-2}\,{}^i\vec{\vec{F}}_t + {}^i\Phi^{T}{}_{n,n-2}\left({}^iM_n\,{}^i\dot{\vec{\vec{V}}}_n + {}^i\vec{\vec{b}}_n\right) + {}^i\Phi^{T}{}_{n-1,n-2}\left({}^iM_{n-1}\,{}^i\dot{\vec{\vec{V}}}_{n-1} + {}^i\vec{\vec{b}}_{n-1}\right) + {}^iM_{n-2}\,{}^i\dot{\vec{\vec{V}}}_{n-2} + {}^i\vec{\vec{b}}_{n-2} \\
&\qquad \vdots \\
&{}^i\vec{\vec{F}}_1 = {}^i\Phi^{T}{}_{t,1}\,{}^i\vec{\vec{F}}_t + {}^i\Phi^{T}{}_{n,1}\left({}^iM_n\,{}^i\dot{\vec{\vec{V}}}_n + {}^i\vec{\vec{b}}_n\right) + {}^i\Phi^{T}{}_{n-1,1}\left({}^iM_{n-1}\,{}^i\dot{\vec{\vec{V}}}_{n-1} + {}^i\vec{\vec{b}}_{n-1}\right) + \ldots + {}^iM_1\,{}^i\dot{\vec{\vec{V}}}_1 + {}^i\vec{\vec{b}}_1
\end{aligned}
$$

If we stack up the entire link spatial forces,

$$
\begin{bmatrix} {}^i\vec{\vec{F}}_1 \\ {}^i\vec{\vec{F}}_2 \\ {}^i\vec{\vec{F}}_3 \\ \vdots \\ {}^i\vec{\vec{F}}_n \end{bmatrix} = \begin{bmatrix} I & {}^i\Phi_{2,1} & {}^i\Phi_{3,1} & \cdots & {}^i\Phi_{n,1} \\ 0 & I & {}^i\Phi_{3,2} & \cdots & {}^i\Phi_{n,2} \\ 0 & 0 & I & \cdots & {}^i\Phi_{n,3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & I \end{bmatrix} \left\{ \begin{bmatrix} {}^iM_1 & 0 & 0 & \cdots & 0 \\ 0 & {}^iM_2 & 0 & \cdots & 0 \\ 0 & 0 & {}^iM_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & {}^iM_n \end{bmatrix} \begin{bmatrix} {}^i\dot{\vec{\vec{V}}}_1 \\ {}^i\dot{\vec{\vec{V}}}_2 \\ {}^i\dot{\vec{\vec{V}}}_3 \\ \vdots \\ {}^i\dot{\vec{\vec{V}}}_n \end{bmatrix} + \begin{bmatrix} {}^i\vec{\vec{b}}_1 \\ {}^i\vec{\vec{b}}_2 \\ {}^i\vec{\vec{b}}_3 \\ \vdots \\ {}^i\vec{\vec{b}}_n \end{bmatrix} + {}^i\Phi^{T}{}_t\vec{\vec{F}}_t \right\} \tag{2.58}
$$

The more generalized form will be,

$$
{}^i\underline{F} = {}^i\Phi^{T}\left({}^iM\,{}^i\dot{\underline{V}} + {}^i\underline{b} + {}^i\Phi_t{}^{T}\,{}^i\vec{\vec{F}}_t\right) \tag{2.59}
$$

Now we have the spatial force equation of one rigid arm. To find the applied torques, let us first place (2.53) in (2.60)

$$
{}^i\underline{F} = {}^i\Phi^{T}\left({}^iM\,{}^i\Phi\,{}^iH\,{}^i\ddot{\underline{\theta}} + {}^iM\,{}^i\Phi\,{}^i\underline{a} + {}^i\underline{b} + {}^i\Phi_t{}^{T}\,{}^i\vec{\vec{F}}_t\right) \tag{2.60}
$$

Since the applied torques are the projections of the link spatial forces along the axis of rotation,

$$
{}^i\underline{\tau} = {}^iH^{T}\,{}^i\underline{F} \tag{2.61}
$$

If we place (2.61) in (2.62), we find the applied torque equation

$$
{}^i\underline{\tau} = {}^iH^{T}\,{}^i\Phi^{T}\,{}^iM\,{}^i\Phi\,{}^iH\,{}^i\ddot{\underline{\theta}} + {}^iH^{T}\,{}^i\Phi^{T}\left({}^iM\,{}^i\Phi\,{}^i\underline{a} + {}^i\underline{b}\right) + {}^iH^{T}\,{}^i\Phi^{T}\,{}^i\Phi_t{}^{T}\,{}^i\vec{\vec{F}}_t
$$

$$
{}^i\underline{\tau} = {}^i\mathcal{M}\,{}^i\ddot{\underline{\theta}} + {}^iC + {}^iJ^{T}\,{}^i\vec{\vec{F}}_t \tag{2.62}
$$

where, ${}^{i}\mathcal{M}$, ${}^{i}C$ and ${}^{i}J^{\mathrm{T}}$ represents generalized mass matrix, bias terms and transpose of *Jacobian* matrix, respectively. The equation of motion regarding the forward dynamic is,

$$ {}^{i}\ddot{\underline{\theta}} = {}^{i}\mathcal{M}^{-1}\left({}^{i}\overline{\tau} - {}^{i}J^{\mathrm{T}}\,{}^{i}\vec{\vec{\mathrm{F}}}_{t}\right) \tag{2.63} $$

where

$$ {}^{i}\overline{\tau} = {}^{i}\tau - {}^{i}C $$

2.4.3 Dynamics of a serial manipulator on moving platform

In this case, we will consider a manipulator on a moving massless platform. We will configure our equations by adding the platform acceleration in (2.51),

$$
\begin{aligned}
{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{0} &= {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} \\
{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{1} &= {}^{i}\Phi_{1,0}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} + {}^{i}\vec{\vec{\mathrm{H}}}_{1}\,{}^{i}\ddot{\theta}_{1} + {}^{i}\vec{\vec{a}}_{1} \\
{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{2} &= {}^{i}\Phi_{2,1}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{1} + {}^{i}\vec{\vec{\mathrm{H}}}_{2}\,{}^{i}\ddot{\theta}_{2} + {}^{i}\vec{\vec{a}}_{2} = {}^{i}\Phi_{2,1}\left({}^{i}\Phi_{1,0}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} + {}^{i}\vec{\vec{\mathrm{H}}}_{1}\,{}^{i}\ddot{\theta}_{1} + {}^{i}\vec{\vec{a}}_{1}\right) + {}^{i}\vec{\vec{\mathrm{H}}}_{2}\,{}^{i}\ddot{\theta}_{2} + {}^{i}\vec{\vec{a}}_{2} \\
&= {}^{i}\Phi_{2,0}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} + {}^{i}\Phi_{2,1}\left({}^{i}\vec{\vec{\mathrm{H}}}_{1}\,{}^{i}\ddot{\theta}_{1} + {}^{i}\vec{\vec{a}}_{1}\right) + {}^{i}\vec{\vec{\mathrm{H}}}_{2}\,{}^{i}\ddot{\theta}_{2} + {}^{i}\vec{\vec{a}}_{2} \\
&\vdots \\
{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{n} &= {}^{i}\Phi_{n,0}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} + {}^{i}\Phi_{n,1}\left({}^{i}\vec{\vec{\mathrm{H}}}_{1}\,{}^{i}\ddot{\theta}_{1} + {}^{i}\vec{\vec{a}}_{1}\right) + {}^{i}\Phi_{n,2}\left({}^{i}\vec{\vec{\mathrm{H}}}_{2}\,{}^{i}\ddot{\theta}_{2} + {}^{i}\vec{\vec{a}}_{2}\right) + \ldots + {}^{i}\vec{\vec{\mathrm{H}}}_{n}\,{}^{i}\ddot{\theta}_{n} + {}^{i}\vec{\vec{a}}_{n}
\end{aligned} \tag{2.64}
$$

where base point acceleration, the term ${}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b}$, includes both platform and bias accelerations,

$$ {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} = {}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{p} + \vec{\vec{a}}_{b} $$

where, ${}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{p}$ is the platform acceleration and $\vec{\vec{a}}_{b}$ is the acceleration term that includes gravity. If we reconfigure the equations according to this idea, the equation of motion will be,

$$ {}^{i}\ddot{\theta} = {}^{i}\mathcal{M}\left({}^{i}\overline{\tau} - {}^{i}\mathcal{M}_{b}\,{}^{i}\dot{\vec{\vec{\mathrm{V}}}}_{b} - {}^{i}J^{\mathrm{T}}\,{}^{i}\vec{\vec{\mathrm{F}}}_{t}\right) \tag{2.65} $$

where,

$$ {}^{i}\mathcal{M} = {}^{i}\mathrm{H}^{\mathrm{T}}\,{}^{i}\Phi^{\mathrm{T}}\,{}^{i}\mathrm{M}\,{}^{i}\Phi\,{}^{i}\mathrm{H} $$

$$ {}^{i}C = {}^{i}\mathrm{H}^{\mathrm{T}}\, {}^{i}\Phi^{\mathrm{T}} \left({}^{i}\mathrm{M}\, {}^{i}\Phi\, {}^{i}\boldsymbol{a} + {}^{i}\boldsymbol{b} \right) $$

$$ {}^{i}\mathcal{M}_{b} = {}^{i}\mathrm{H}^{\mathrm{T}}\, {}^{i}\Phi^{\mathrm{T}}\, {}^{i}\mathrm{M}\, {}^{i}\Phi\, {}^{i}\Phi_{b} $$

For the dynamics of more complex systems, [1] can be referred.

3. BIPEDS AND PASSIVE DYNAMIC WALKING

3.1 Purpose

The human locomotion has been an interesting topic to study for several decades. People, and of course scientists have curiosity on human-like robotic systems, which have human characteristics. Biped walking is one of these features, which is also a physical system to be investigated. The natural movement of human walking is an outcome of the gravity effect on the body; means on a shallow slope plane, a biped system can successfully make its walking cycle only using the gravitational force. In literature this kind of systems are called as “passive dynamic walkers” [19]. Obviously, achieving a walking routine without any external source or control mechanism is very useful for both robotic applications and development of the artificial prosthetic limbs. Since it will reduce the cost of operation, the biped or human-like-walking robots will have smaller battery packages and easier controllers, while people will use less energy to achieve the daily walking tasks, those need prosthetic legs for these purposes.

In this chapter, we will use the spatial operator algebra on a 2D planar passive walker to analyze the force distribution on each joint and contact point, feet of the legs, with surface. The second section explains the design parameters and the model of the walker. Section three will show how to implement the SOA on the chosen model. Section four presents a novel approach, proposed by [1], and called “pseudo joint” method, which is used to constrain the movement of knees for knee-locking. In section five we have explained the closed-chain dynamics, which appears when the two feet of robot touches the ground. Once the force vector of the base point on $\uparrow$ direction has positive value, the kinematics of the robot has been reconfigured by switching rule using a transformation matrix, T, and walking period will continue in this order. The following chart in Figure 3.1 reveals the programming logic.

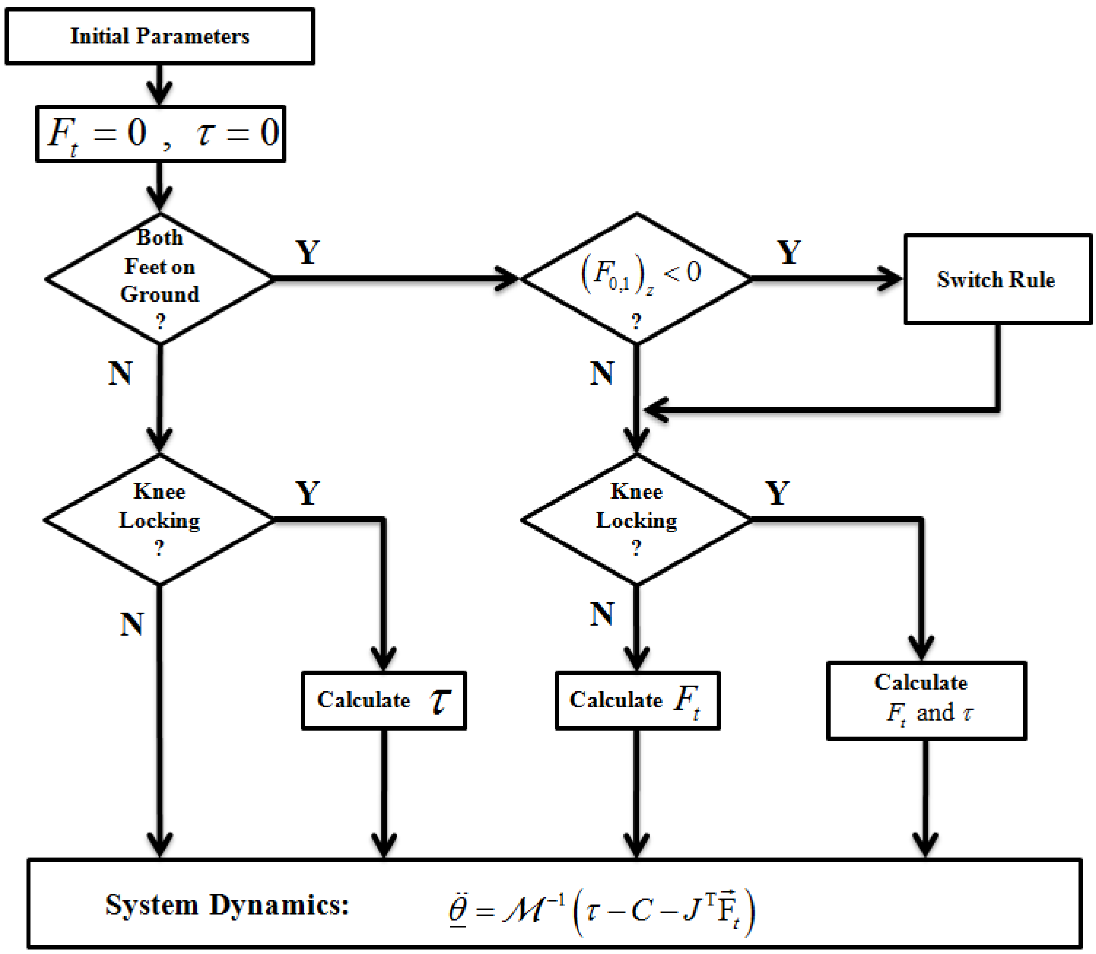


Figure 3.1 : Programming chart of walking session.

3.2 Passive Dynamic Walking and Model Design

In this section, we will explain how we chose the system model and the parameters. First of all, it is a planar passive dynamic walker, which makes only longitudinal motion, but no lateral. So the system will operate in 2D space, which means the task space will consist of 3 DoF element. Figure 3.2 shows the examples from different studies on planar kneed passive walkers. Our system, Figure 3.4, consists of one 5 DoF manipulator. The first joint, also the base point, is the first foot; and the 5th joint is second foot. The second and 4th joints are knees, which have a special locking system. The third joint is the hip.

The main purpose of using kneed-legged biped instead of simplest mechanism with straight legs, is first its appearance is more human like, and second; because of the *'foot-scuffing'* problem, as in Figure 3.3.

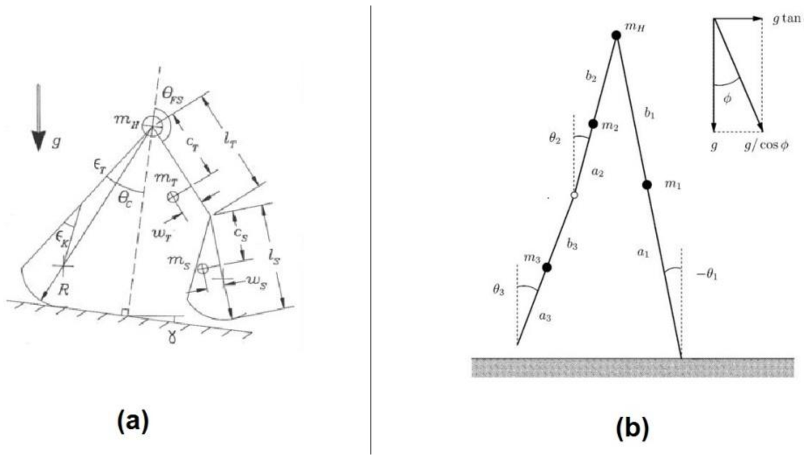


Figure 3.2 : Planar passive dynamic walkers with knees.
(a) McGeer [21], (b) Asano et al. [49].

In straight legged model, the swing leg remains below the floor level at around midstance. Knees will surely prevent from this case [28].

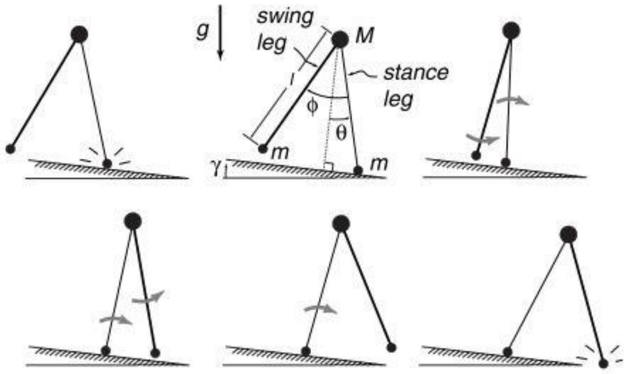


Figure 3.3 : Foot-scuffing problem [28].

Our legged system is shown in Figure 3.4, where the joints are rotational around x-axis, and displacement occurs on y and z-axes.

The first joint is the one of the foots and also the base point. The second and sixth joints represent the elasticy and the flexibility of the surface. The third and fifth joints are knees of biped system. The fourth joint is the hip. As it is seen from Figure 3.4, the leg in contact with surface is called as *stance leg*, and the free leg is called as *swinging leg*.

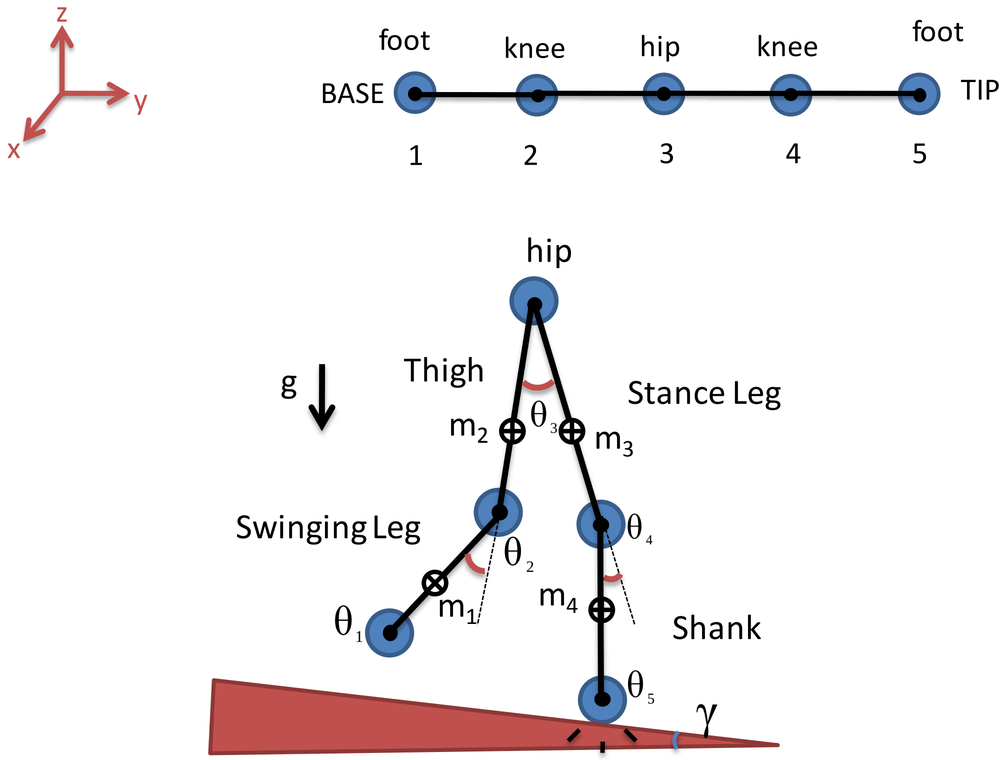


Figure 3.4 : Passive dynamic walking biped robot.

The paremeters used for simulations are chosen from [49], and listed in Table 3.1:

Table 3.1 : Parameters of biped model.

PARAMETER	DEFINITION	VALUE	UNIT
m_1	Swinging Leg –Shank Mass	2.00	kg
m_2	Swinging Leg –Thigh Mass	3.00	kg
m_3	Stance Leg –Shank Mass	2.00	kg
m_4	Stance Leg –Thigh Mass	3.00	kg
l_1	Swinging Leg –Shank Length	2.00	m
l_2	Swinging Leg –Thigh Length	2.00	m
l_3	Stance Leg –Shank Length	2.00	m
l_4	Stance Leg –Thigh Length	2.00	m
g	Gravity Acceleration	9.81	m/s^2

The center of mass and the geometric center of each leg are considered as coincide. γ represents the slope angle.

The one important point on biped design is *specific cost of transportation*. This dimensionless term means the amount of required energy to complete the walking task by carrrying the weight of the system over a distance. As in [29], it can be represented in equational form as

$$c_t = (energy\,used)/(weight \times range\ traveled)$$

where c_t represents the cost of transportation. One of the most efficient passive dynamic walking design was made by Collins, et al.[29] with $c_t \approx 0.2$. "*Cornell Ranger*", on the other hand, is a developed passive walking based robot and has the world record of 65.2 km non-stop walking [50],[51].

In this thesis we are interested in observing the force distributions of passive dynamic walking sequence using SOA. As a future work, the system can be developed for more efficient designs, using the knowledge of force and torque propagation on each elements by the help of SOA.

3.3 Implementation of SOA Method for Planar Bipeds

We considered our planar passive dynamic walking system as a form of serial manipulator on a moving platform. In second chapter we gave detailed calculations for this kind of systems. However, SOA method was built on the idea that the task space of tip points has 6 DoF. For a 2D planar system, this space has to be reduced into 3 DoF.

In task space we would write the forward kinematics like,

$$\begin{bmatrix} \vec{\omega}_t \\ \vec{v}_t \end{bmatrix} = \begin{bmatrix} J_{\omega_{X_1}} & J_{\omega_{X_2}} & J_{\omega_{X_3}} & J_{\omega_{X_4}} & J_{\omega_{X_5}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ J_{v_{Y_1}} & J_{v_{Y_2}} & J_{v_{Y_3}} & J_{v_{Y_4}} & J_{v_{Y_5}} \\ J_{v_{Z_1}} & J_{v_{Z_2}} & J_{v_{Z_3}} & J_{v_{Z_4}} & J_{v_{Z_5}} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix} \qquad \textbf{(3.1)}$$

It is obvious that there is no need to calculate a part of Jacobian, which is covered by a red rectangle in (3.1), and which represents the rotations around *y* and *z* axes, and translation on *x* axis. Hence, the task space of tip point of a leg can be reduced into 3 DoF space:

$$\begin{bmatrix} \vec{\omega}_t \\ \vec{v}_t \end{bmatrix} = \begin{bmatrix} \omega_{t_X} \\ v_{t_Y} \\ v_{t_Z} \end{bmatrix} \qquad \textbf{(3.2)}$$

And this holds for each joint,

$$\vec{V}_k = \begin{bmatrix} \omega_{k_X} \\ v_{k_Y} \\ v_{k_Z} \end{bmatrix} \qquad \textbf{(3.3)}$$

Another important point is that we need to reconfigure Φ and H matrices for new configuration. In our model, all rotations are in $\vec{x}$, and translations on $\vec{y}$ and $\vec{z}$. Then H matrix of each rotational joint will be,

$$\vec{H}_k = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \textbf{(3.4)}$$

and for translational joints,

$$\vec{H}_k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \; or \; \vec{H}_k = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \textbf{(3.5)}$$

according to the relevant axis. Then for the 5 DoF planar biped system, H matrix will be,

$$H^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad \textbf{(3.6)}$$

The calculation of Φ matrix was based on cross product of vectors (2.11) – (2.13), which was defined in 6 DoF task space. We need to find how to calculate Φ for a planar manipulator system. Let us start by obtaining Φ for a rigid link between joint *a* and joint *b* in 3D space. The length vector from joint *b* to *a* is,

$$\vec{l}_{b,a} = \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} \qquad \textbf{(3.7)}$$

Utilizing (2.11) – (2.13) , we can find the velocity propagation from joint *b* to *a*,

$$\begin{bmatrix} \omega_{a_X} \\ \omega_{a_Y} \\ \omega_{a_Z} \\ v_{a_X} \\ v_{a_Y} \\ v_{a_Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & l_z & -l_y & 1 & 0 & 0 \\ -l_z & 0 & l_x & 0 & 1 & 0 \\ l_y & -l_x & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_{b_X} \\ \omega_{b_Y} \\ \omega_{b_Z} \\ v_{b_X} \\ v_{b_Y} \\ v_{b_Z} \end{bmatrix} \tag{3.8}$$

Since we ignored the rotation around y and z axes, and the translation on x axis, the equation (3.8) can be formed as follows,

$$\begin{bmatrix} \omega_{a_X} \\ \omega_{a_Y} \\ \omega_{a_Z} \\ v_{a_X} \\ v_{a_Y} \\ v_{a_Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & l_z & -l_y & 1 & 0 & 0 \\ -l_z & 0 & l_x & 0 & 1 & 0 \\ l_y & -l_x & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_{b_X} \\ \omega_{b_Y} \\ \omega_{b_Z} \\ v_{b_X} \\ v_{b_Y} \\ v_{b_Z} \end{bmatrix} \rightarrow \begin{bmatrix} \omega_{a_X} \\ v_{a_Y} \\ v_{a_Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -l_z & 1 & 0 \\ l_y & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_{b_X} \\ v_{b_Y} \\ v_{b_Z} \end{bmatrix} \tag{3.9}$$

Therefore,

$$\Phi_{a,b} = \begin{bmatrix} 1 & 0 & 0 \\ -l_z & 1 & 0 \\ l_y & 0 & 1 \end{bmatrix} \tag{3.10}$$

Since cross product is defined only for 3D space, we will define a cross product operation in 2D space as,

$$\vec{l}_{b,a} \times = \hat{l}_{b,a} = \begin{bmatrix} -l_z \\ l_y \end{bmatrix} \tag{3.11}$$

Additionally, we need to reconfigure the base point acceleration, bias accelerations, inertias, mass matrix for k^{th} joint, like

$$\vec{a}_b = \begin{bmatrix} 0 \\ g \sin\gamma \\ g \cos\gamma \end{bmatrix} \tag{3.12}$$

$$\vec{a}_k = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \dot{\theta}_{k-1}^2 \vec{l}_{k-1,k} \tag{3.13}$$

$$\vec{b}_k = m_k \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \dot{\theta}_k^2 \vec{l}_{k,c} \quad \textbf{(3.14)}$$

$$I_k = m_k \left\| l_{k,c} \right\|^2 \quad \textbf{(3.15)}$$

$$\mathrm{M}_k = m_k \begin{bmatrix} \left\| l_{k,c} \right\|^2 & \hat{l}_{k,c}^{\mathrm{T}} \\ \hat{l}_{k,c} & {}_{2x2}\mathrm{I} \end{bmatrix} \quad \textbf{(3.16)}$$

where,

$$\left\| \vec{l}_{k,c} \right\| = \frac{1}{2} \left\| \vec{l}_{k-1,k} \right\| \quad \textbf{(3.17)}$$

3.4 Knee Locking Mechanism Using Pseudo Joint Technique

In kneed-bipeds, the knee joint is designed with a constraint, which prevents from an abnormal movement, i.e. bending forward, as in Figure 3.5 (a), and it needs to be locked, as in Figure 3.5 (b).

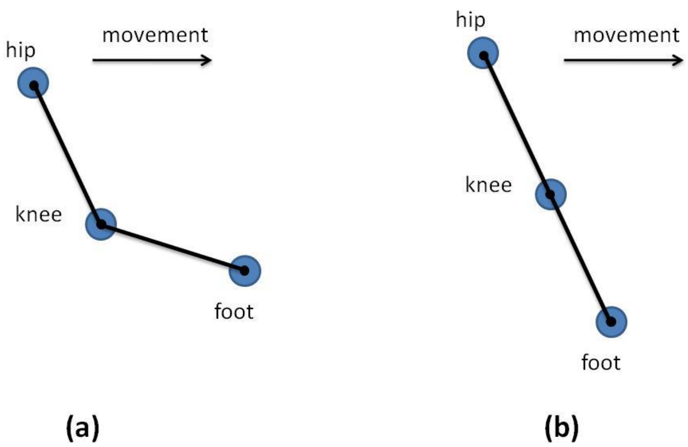


Figure 3.5 : Knee locking.

It is obvious that we have to constrain the knee joinst to make this possible. For this reason, we will use a novel approach, which was presented in [1], and called as "*pseudo joint*" method. Briefly, we will choose the knee joints as pseudo joinst and calculate the necessary torques to keep their angles at zero at all times before the retraction .

First, we need to obtain a linear operator, let us say S, to divide the joint space into two subspaces; real joints and pseudo joints. S will be obtained by rearranging the rows of $n \times n$ identity matrix, ${}_nI$, where n is the total DoF including pseudo joints, which are the kneed-joints in our case. S is an orthogonal matrix.

In our model *n=5* and pseudo joints are the 2[rd] and 4[th] ones. Let us first obtain the S matrix,

$$
{}_5I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \mathrm{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.18}
$$

If we multiply S with $\ddot{\theta}$,

$$
\underline{\ddot{\theta}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \\ \ddot{\theta}_5 \end{bmatrix} \rightarrow \mathrm{S}\underline{\ddot{\theta}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_3 \\ \ddot{\theta}_5 \\ \ddot{\theta}_2 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \overline{\ddot{\theta}} \\ \underline{\ddot{\theta}}_p \end{bmatrix} \tag{3.19}
$$

Bu using the orthogonal property of S matrix, the rearranged form of the inverse dynamics becomes:

$$
\ddot{\theta} = \mathrm{D}\overline{\tau} + \mathrm{E}
$$

$$
\mathrm{S}\ddot{\theta}_{augmented} = \left(\mathrm{SDS}^{\mathrm{T}}\right)\mathrm{S}\,\overline{\tau}_{augmented} + \mathrm{SE} \tag{3.20}
$$

$$
\begin{bmatrix} \ddot{\theta} \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} d_1 & d_2 \\ d_3 & d_4 \end{bmatrix} \begin{bmatrix} \overline{\tau} \\ \overline{\tau}_p \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{3.21}
$$

where,

$$
\underline{\ddot{\theta}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_3 \\ \ddot{\theta}_5 \end{bmatrix} \quad \underline{\ddot{\theta}}_p = \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_4 \end{bmatrix}
$$

Here, D and E matrices are obtained for augmented system. Our purpose is to find the torques at other joints, which are not pseudo joints, where $\ddot{\theta}_p = 0$. Therefore, the equation of motion for no-pseudo joints is achieved as:

$$\ddot{\theta}_r = \mathrm{D}_r \overline{\tau} + \mathrm{E}_r \quad \textbf{(3.22)}$$

where,

$$\begin{aligned} \mathrm{D}_r &= d_1 - d_2 d_4^{-1} d_3 \\ \mathrm{E}_r &= e_1 - d_2 d_4^{-1} e_2 \end{aligned} \quad \textbf{(3.23)}$$

provided that d_4 is full rank.

With this method we only ensure that the pseudo joint accelerations will remain zero. In order to keep the joint velocity and the position at desired values, we need to find the applied torques by using PI controller. This will be mentioned in the following chapter.

3.5 Chain Dynamics and Switch Rule

In this section we will show how to calculate the dynamics of a closed-chain system, which occours when the both feet of the biped robot touch the ground. Since the base point is always touching the ground during the first step, we need to check the tip point coordination. If $L_{tip_z} = 0$, it means that the second foot is on the ground too. In this moment, we need to calculate the dynamics of this closed system, where the linear velocities and accelerations of the tip point should be zero, in order to prevent its movement in YZ plane. The Figure 3.7 shows the transition from a serial manipulator system to a closed-chain system. It is obvious that the rear leg will need to step forward to complete the movement. Therefore, we switched the base and tip points by using a switching rule, when $F_{base_z} \leq 0$.

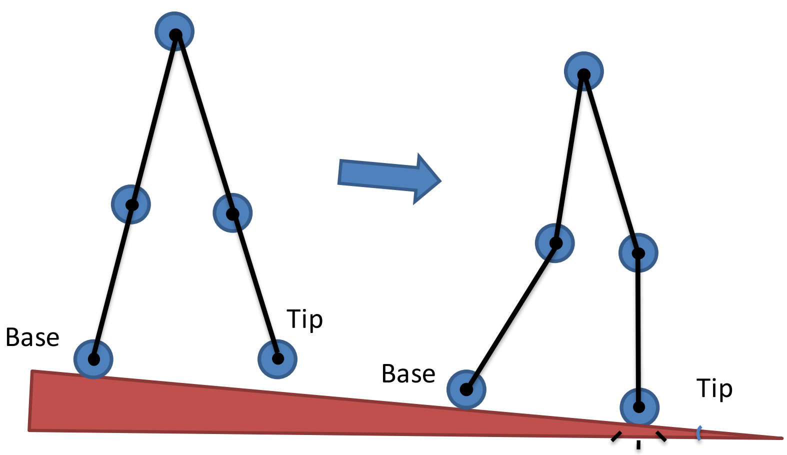


Figure 3.6 : Closed-chain system.

In closed-chain condition, the linear velocities and accelerations of the tip point have to be zero. This constraint can be showed in (3.26)

$$\begin{aligned} \vec{\mathrm{V}}_t &= J\underline{\dot{\theta}} = 0 \\ \dot{\vec{\mathrm{V}}}_t &= (J\underline{\ddot{\theta}} + \dot{J}\underline{\dot{\theta}}) = 0 \end{aligned} \tag{3.24}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Also the external forces on the tip point need to be constrained in order to keep it on the ground,

$$\vec{\mathrm{F}}_{t_{ACC}} = Af_c \tag{3.25}$$

By utilizing (3.24) and (3.25) in (2.64), we get the dynamic equation of the closed-chain sytem:

$$\begin{aligned} J\mathcal{M}^{-1}\left(\overline{\tau} - J^{\mathrm{T}}\,{}^{i}\vec{\mathrm{F}}_t\right) + \dot{J}\underline{\dot{\theta}} = 0 \\ \left(J\mathcal{M}^{-1}J^{\mathrm{T}}\right)A^{\mathrm{T}} f_c = J\mathcal{M}^{-1}\overline{\tau} + \dot{J}\underline{\dot{\theta}} \end{aligned}$$

$$f_c = A\left(J\mathcal{M}^{-1}J^{\mathrm{T}}\right)^{-1}\left(J\mathcal{M}^{-1}\left(\tau - C\right) + \dot{J}\underline{\dot{\theta}}\right) \tag{3.26}$$

Where $\tau = 0$, and from [1];

$$\dot{J}\underline{\dot{\theta}} = \Phi_t \Phi \underline{a} + \dot{\Phi}_t \Phi \mathrm{H} \underline{\dot{\theta}} \tag{3.27}$$

The external forces on the tip point will be:

$$\vec{\mathrm{F}}_{t_{ACC}} = \begin{bmatrix} 0 \\ f_c \end{bmatrix} \tag{3.28}$$

However, this will also ensure to keep the tip point accelerations at zero, but not its velocity and position. As for the knee locking, we can use a PI controller to find the external forces on tip point. This is mentioned in the following chapter.

The results will be valid from the first touch of the swinging feet to the ground and till $F_{base_z} \leq 0$. Once this happens, the base and tip points have to be replaced. Since the biped system is symmetrical in both kinematical and dynamical sense, we only need to define a transformation matrix to change the coordinate axes, the order of links, joints, angles, etc. Let us define this translation matrix, *T*, as following:

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(3.29)}$$

First, the switch rule will be ensured using T for joint angles, velocities and accelerations as in (3.30) :

$$\begin{aligned} \theta &= T\theta \\ \dot{\theta} &= T\dot{\theta} \\ \ddot{\theta} &= T\ddot{\theta} \end{aligned} \quad \textbf{(3.30)}$$

After that, we need to switch the coordinates of each link. It is obvious from the design of biped system, the link distance between 5th joint and tip point is zero, since 5th joint is also the tip point (Figure 3.7).

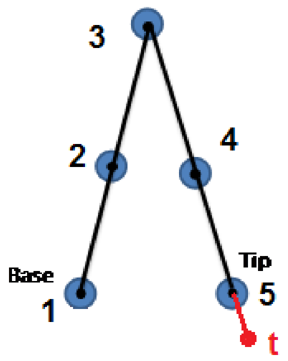


Figure 3.7 : The length of tip link is zero.

In SOA calculations, we use Φ_t, which includes $\vec{l}_{n,t} = 0$, to calculate J and other matrices. After taking a step, the base and tip points need to be changed as shwon in Figure 3.8 .

By considering this fact, we defined another translation matrix, T_c only for the coordinate axis y and z.

$$T_c = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \textbf{(3.31)}$$

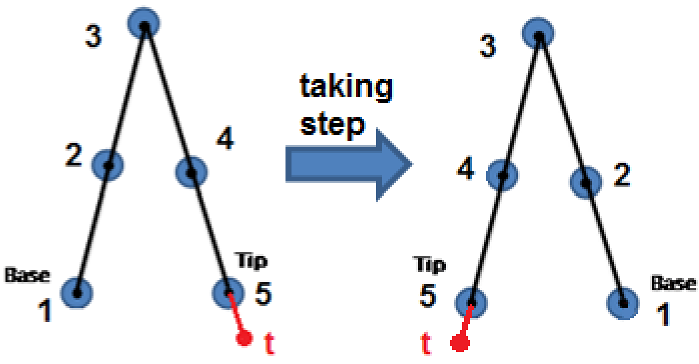


Figure 3.8 : Joint numbers and links after taking a step.

And now we can update the coordinates after taking a step:

$$\begin{aligned} y &= yT_c \\ z &= zT_c \end{aligned} \tag{3.32}$$

As a result of this method, the place of tip and base points will always change after chain dynamics completed. This also means that, if the front foot is the tip point at the initial conditions, then after first step the tip point will be the rear foot, and base point will be replaced in this manner.

4. EXTERNAL FORCES DUE TO BOUNDARY CONDITIONS

In this chapter, we focused on the calculation of external forces and torques due to the boundary conditions using PI controllers. In section 3.4, we have calculated the necessary torques on the knees to lock them using pseudo joint technique. This is shown as in τ_p Figure (4.1). However, this will result only the torques, which keeps the knee joint accelerations at zero, but not its velocity or angle. For this reason, we designed a PI controller, which will simulate the effect of the latches on the knees. The P and I values of the controller depends on the characteristics of this latch or knee mechanism. Obviously, we have chosen high values for P compared to I to decrease the rising time, while keeping the overshoot in a admissible range and keeping the steady state error at zero.

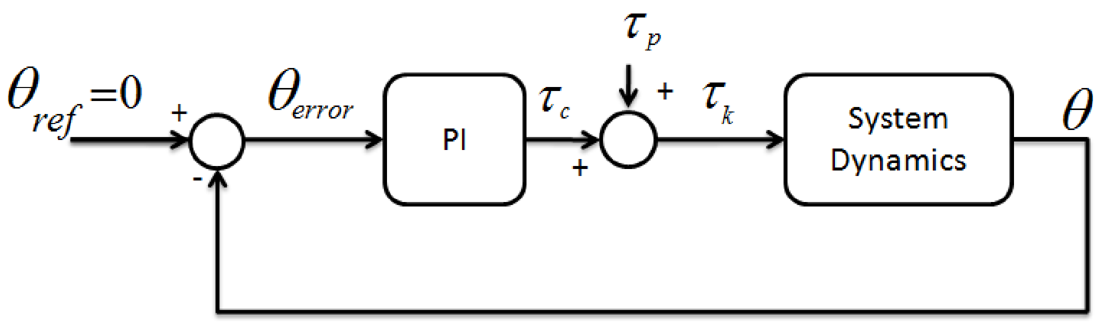


Figure 4.1 : Position control for knee joints.

Similarly, to control the position in *y* and *z* directions of foot touching the ground, we have used PI controllers. In section 3.5, we have calculated the constraint forces to set zero the tip point accelerations. This is shown as $F_{t_{ACC}}$ in Figure 4.2. However, this is not enough to keep the tip point at zero in *z* direction and at its last position in *y* direction. The PI controller in Figure 4.2 also represents the effect of ground like flexibility. Since we need to control the tip point in both directions, two different controllers have been created. But two of them are similar, hence we showed one general representation of these controllers in Figure 4.2, where d indicates the current position in mentioned directions. The P and I values of the controllers depend on the characteristics of the surface, and they have been chosen in the same logic as explained above for knees.

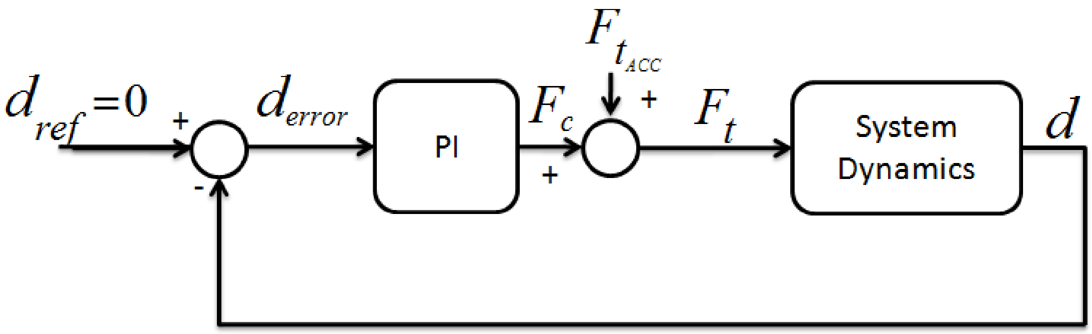


Figure 4.2 : Position control for tip point (foot touching the ground).

5. RESULTS, CONCLUSIONS AND RECOMMENDATIONS

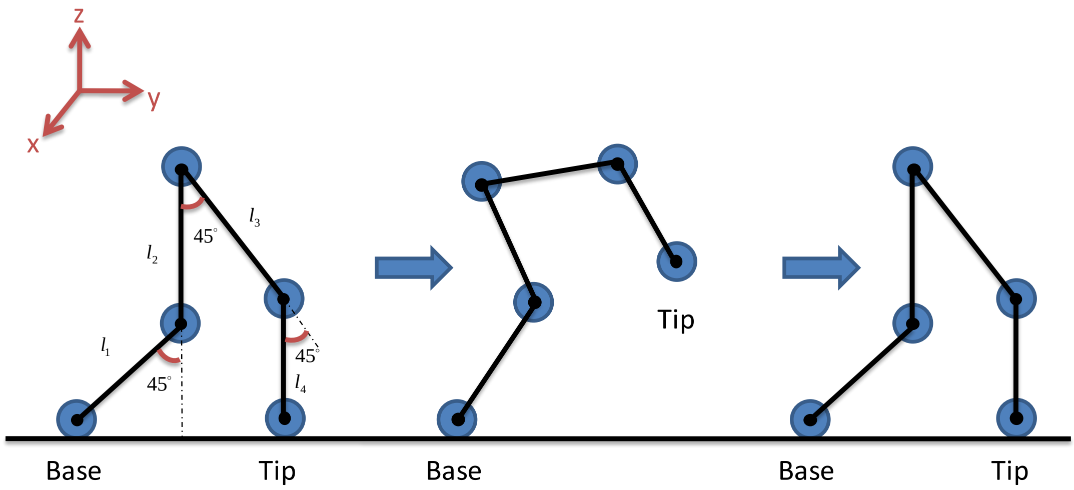
In this chapter, we will show the results of our simulations. The purpose of this thesis was implementing a novel approach, SOA, on a promising area of robotics, passive dynamic walking. The first results have been achieved about the kinematics of the system, and it revealed that the system is kinematically sufficient. After that, we have tested the passive dynamic walking session. The gravitational force is applied as only external force to the system on a shallow slope ground. Since our purpose is to analyze the passive dynamic walking for biped robots; we did not pay attention on the initial conditions, but focused on constraint forces for touching the ground, chain system, taking step and also knee locking. The results showed that the SOA algorithm provides insight look of the system, which allowed us to find constraint forces, more importantly during both feet are on the ground.

Finally, we have concluded our study and presented our recommendations including the possible future study ideas.

5.1 Kinematics of a Biped System

In this section, we will present the results of forward and inverse kinematics for 5 DoF system. For this purpose, we commanded our biped to lift one of its legs, hold it for a while, and drop it to the starting position. This results are not directly related to the aim of this thesis, however before passive dynamic walking, we need to be sure that our codes generates true configuration matrices like H, Φ and J. Figure 5.1 shows the task, and Figure 5.2 presents the results.

Figure 5.2 shows that our robotic design is kinematically sufficient.



POSITION OF TIP

X	**0**
Y	$l_1 \sin(pi/4) + l_3 \sin(pi/4)$
Z	**0**

POSITION OF TIP

X	**0**
Y	$l_1 \sin(pi/4) + l_3 \sin(pi/4)$
Z	l

POSITION OF TIP

X	**0**
Y	$l_1 \sin(pi/4) + l_3 \sin(pi/4)$
Z	**0**

$$l = l_1 = l_2 = l_3 = l_4 = 0.5$$

Figure 5.1 : Example task for biped system.

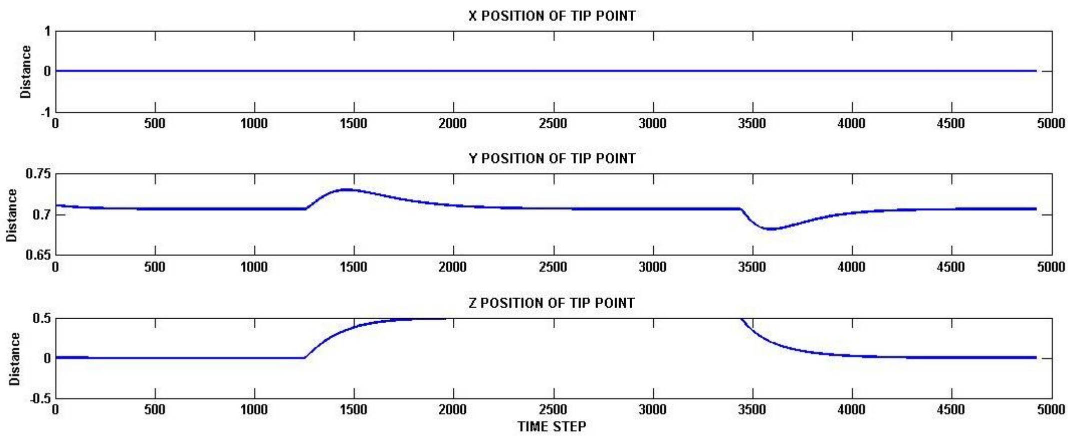
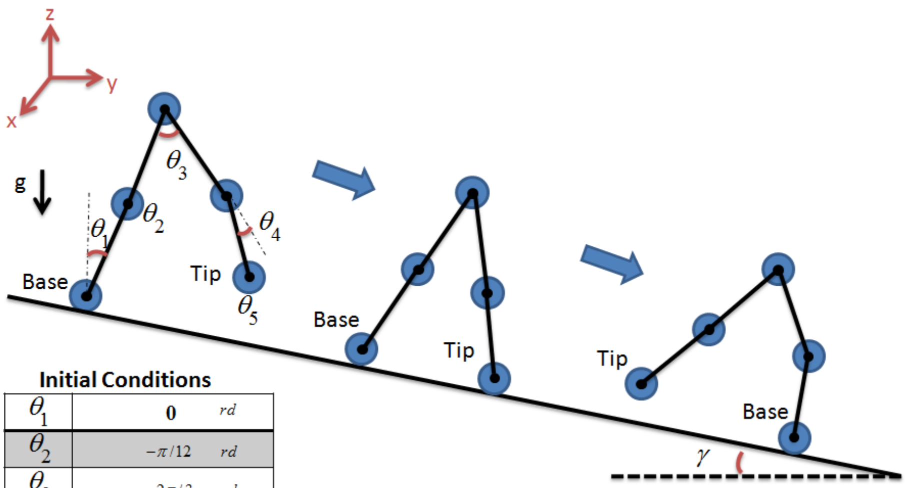


Figure 5.2 : Tip point results of leg lifting and dropping.

5.2 Dynamics of a Biped System

In this part, we will present the dynamical results of the designed biped system during its passive dynamic walking session. The robot starts with its initial conditions, as given in Figure 5.3 and acts only under the gravitational force on a shallow slope surface with calculations explained in Section 3.3. Once the front foot, in this case the tip point, touches the ground, the chain system dynamics are calculated. The calculations for first step continue, until the sign of reaction force on z direction of the base point become negative. This means that the switch rule needs to be applied to the system to change the moving foot, as shown in Section 3.5. and 4.2. During whole process, the knee locking needs to be maintained, as explained in Section 3.4 and 4.1.



Initial Conditions

θ_1	**0**	rd
θ_2	$-\pi/12$	rd
θ_3	$-2\pi/3$	rd
θ_4	$-\pi/8$	rd
θ_5	**0**	rd
γ	**-0.2**	rd
g	**9.81**	m/s^2

All initial velocities are zero.

Figure 5.3 : Taking a step of biped robot under the gravitational force.

The results of this process have been shown in the following figures. Figure 5.4 shows the level of tip point in z direction. Once the tip point touches the ground, the calculations for chain system dynamics take place for a short time, and then the first step is completed. After that switch rule takes place and the base and tip points are replaced by each other.

In this case, shown as in Figure 5.3, the tip point is the foot, which touches the ground first. For this reason, there will be an impact force on it, when it touches the ground. Figure 5.5 shows this result.

The reaction forces on base point (rare foot) are also important, because the sign of the force on z direction helps to decide when the switch rule needs to be applied. This decision is made only if the tip point is already touched the ground. The forces on base point have been shown in Figure 5.6.

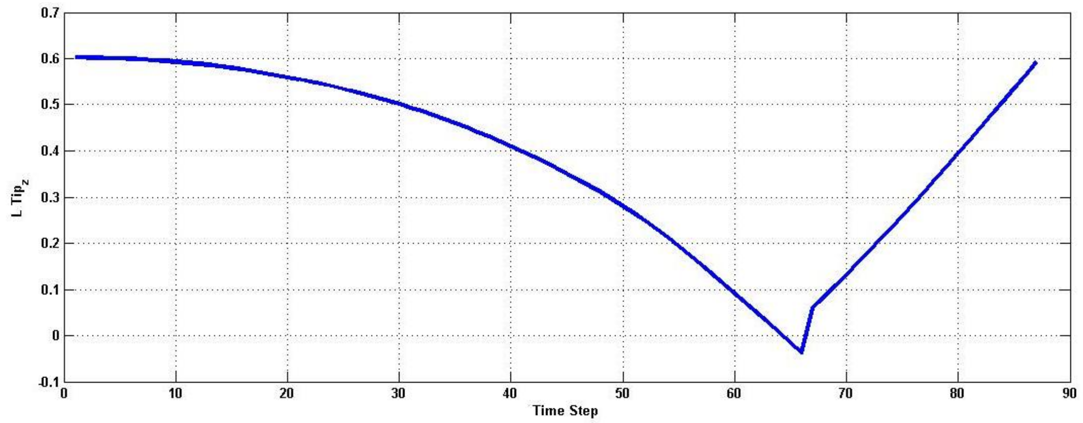


Figure 5.4 : The level of the tip point of the robot.

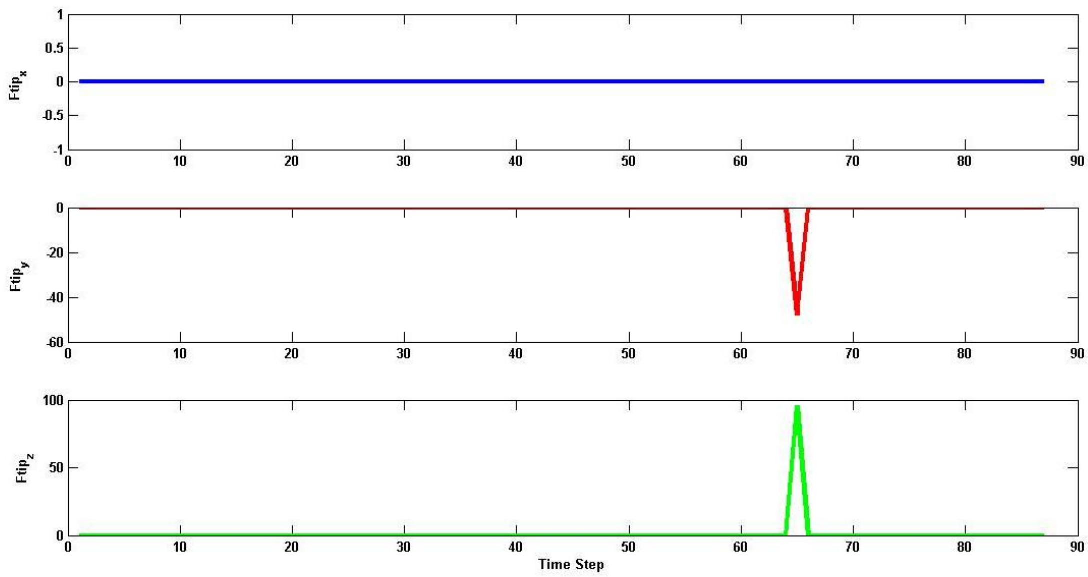


Figure 5.5 : Reaction forces on tip point (front foot).

Besides all of these results, another important result of this case is about knee locking (Figure 5.7). As shown in Section 3.4 and 4.1, it is necessary to apply torques on knees, i.e. second and fourth joints, when knee locking is needed. This will simulate the effect of the latch, which would prevent the knees bending into forward.

All simulations have been implemented in Matlab, using .m files, Simulink and VRML (3D Simulation) toolbox. Figure 5.8 shows an overview from VRML modeling, which is very close to Figure 5.3.

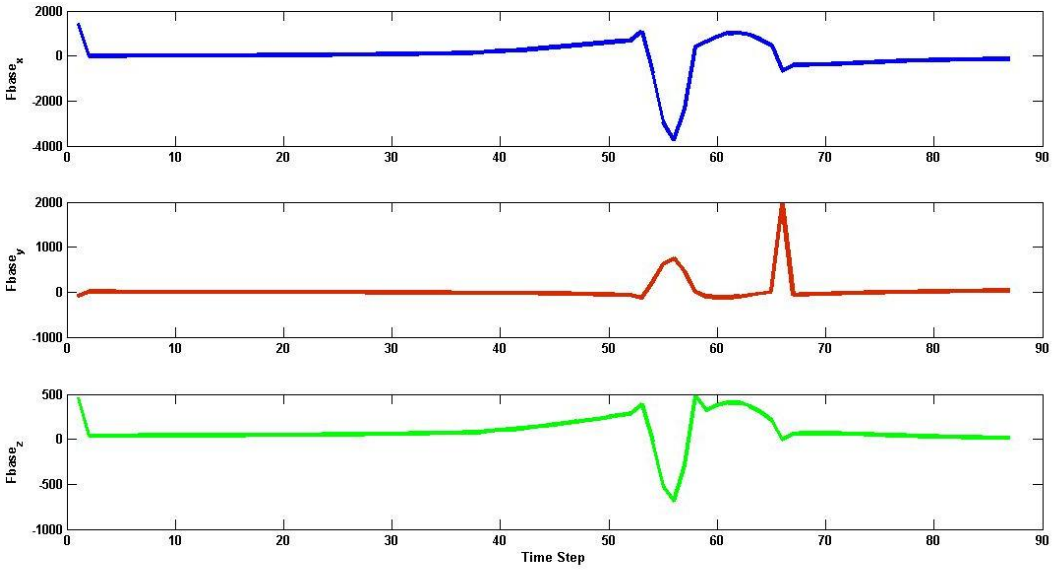


Figure 5.6 : Forces on base point (rare foot).

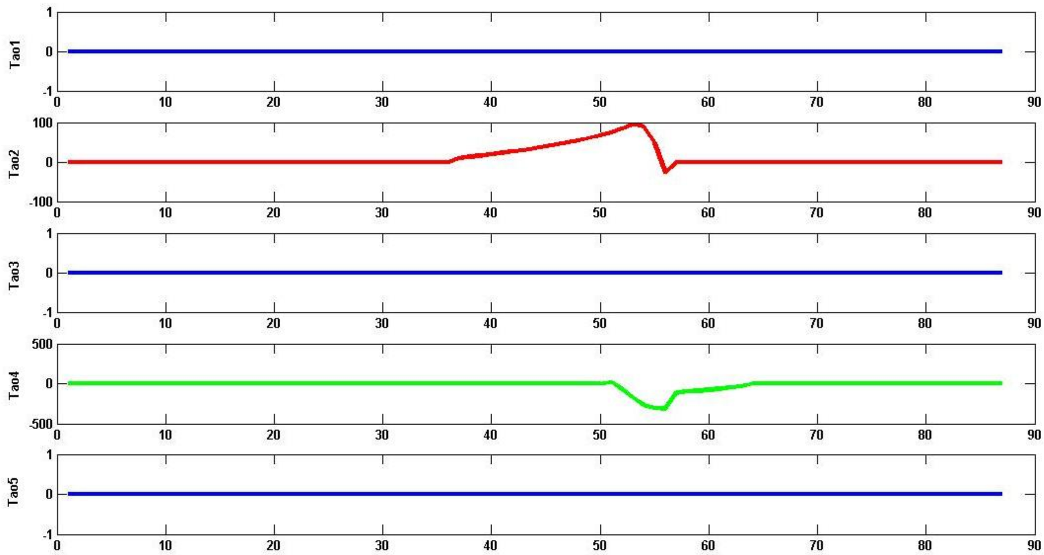


Figure 5.7 : Applied torques on the joints.

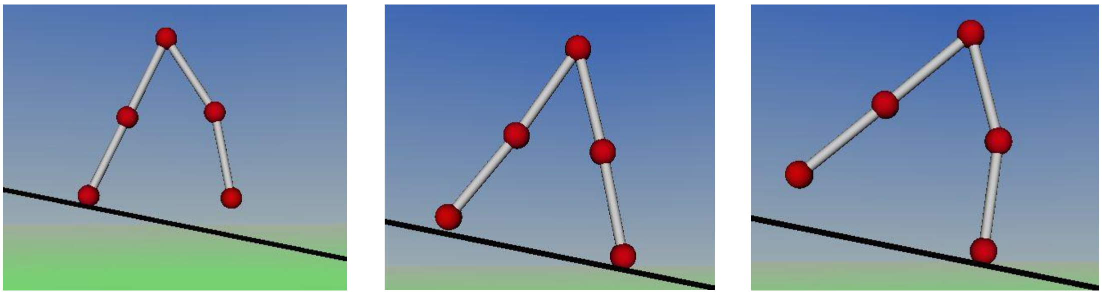


Figure 5.8 : Taking a step.

5.3 Conclusions and Recommendations

In this section we conclude our work and give some recommendations for future studies. In this thesis, we studied the dynamics of a planar passive walking biped robot using *Spatial Operator Algebra* (SOA) method. The advantage of this technique is that it provides insight into the structure of rigid-body by analyzing the system using vectoral representation. Because of its recursive manner, the calculation steps are fast for any complex system. That is why, we introduced this method for both kinematics and dynamics of any kind of system, but focusing on the serial manipulators, since we modeled our biped robot based on it. We designed a 5 DoF biped robot in 2D space, including knees. Since knee locking is a natural mechanism for a human leg, we used "*pseudo joint*" technique to make this virtually possible. The rearrangement of SOA method for 2D space is important, because it prevents the unnecessary calculations, and more relevant to the purpose of this thesis, since we want to implement this method for a planar passive dynamic walking biped. The first Matlab-Simulink result is on the kinematics of a 5 DoF biped robot, and it revealed that SOA method successfully works for bipeds. The latter results are about the dynamical analysis of the aforementioned system. Since the purpose of this thesis is to analyze the dynamics of a passive walking biped robot, instead of finding its proper initial conditions, we have focused on the calculation of constraint forces and torques on foot touching the ground and knees, respectively.

The pseudo joint technique to calculate the constraint torques to maintain the knee locking and chain system dynamics to find the constraint forces to keep the touching ground foot on the surface level are both efficient, when only the accelerations in aforementioned joints meant to be kept at zero. However, our purpose is to find the constraint torques and forces to keep their position at zero. For this reason, we have used PI controllers, which also simulated the characteristics of the latches for knees and surface for the feet. The **P** value represented the stiffness of the material (latch or ground), and **I** value eliminates the steady state errors.

For future studies, the 3D analysis of such systems with upper body and swinging arms can be done, since the SOA method has more advantage for more complex systems. This has been studied before by different scientists, however with SOA method we are able to analyze the forces on feet, joints and links of the biped system. This is important for designing this kind of systems. Besides, the realization of the system is also important, since this thesis focuses only the theoretical background of such systems and its implementation in Matlab-Simulink environment.

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