İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

SEMI ACTIVE SUSPENSION CONTROL WITH MAGNETORHEOLOGICAL DAMPERS

M.Sc. Thesis by
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Department : Mechanical Engineering

Programme : Automotive Engineering

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

MAGNETOREOLOJİK DAMPERLİ YARI AKTİF SÜSPANSİYON KONTROLÜ

YÜKSEK LİSANS TEZİ

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KASIM 2008

FOREWORD

First of all, I really want to thank my instructor *Prof. Dr. Ata Muğan* for helping me at my most desperate time, sharing his immense and wide engineering experience and knowledge, giving clues about making wise decisions in my life, providing the opportunity to be Research Assistant in Mechanics division and teaching me not only how to be a good engineer but also how to be a good person.

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November 2008 Yüksel Can Yabansu

Mechanical Engineer

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ABBREVIATIONS

ER : Electrorheological
LQR : Linear Quadratic Regulator
MR : Magnetorheological
RMS : Root Mean Square
ECS : Electronic Suspension Control System
FFT : Fast Fourier Transform

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SEMI ACTIVE SUSPENSION CONTROL WITH MAGNETORHEOLOGICAL DAMPERS

SUMMARY

In this study, control algorithms are applied to semi active suspension system with MR (magnetorheologic) dampers to improve both handling and comfort as much as it is possible. First of all, the suspension types are explained that have been today. A brief comparison is made between the suspension types that are explained and the semi active suspension system that is studied in this project is evaluated in details. The damper types that are generally put in semi active suspension are explained and the MR damper is decided to be used in this project. And the working principle and structure of MR dampers are presented briefly.

First, the mathematical model that the control algorithms will be applied to is decided and the governing equations are derived. Then, the control types are decided and the algorithms according to the mathematical model are prepared. In this project, switch control, Skyhook control, State feedback control and LQR (Linear quadratic Regulator) control systems are used. Conventional passive suspension system and semi active suspension system with MR damper flow diagrams are drawn in SIMULINK module of MATLAB software. The disturbance that comes from the road is created in various forms. Following, responses of different control types for different disturbance types are compared with the responses of conventional passive system gives. Also, the comparisons with passive suspension system are made with different values of each specific controller to see the behavior of that controller for different parameters. Finally, all controllers are compared on the same ground by using the responses of conventional passive suspension system.

MAGNETOREOLOJİK DAMPERLİ YARI AKTİF SÜSPANSİYON KONTROLÜ

ÖZET

Bu çalışmada motorlu taşıtlarda günümüzde aranan konfor ve yol tutuşunu mümkün olabildiğince aynı anda arttırmak için MR (Magnetoreolojik) damperli yarı aktif süspansiyonlar üzerinde kontrol uygulamaları yapılmıştır. Öncelikle şimdiye kadar kullanılan ve günümüzde kullanılmakta olan süspansiyon çeşitleri kısa olarak anlatılmıştır. İncelenen bu süspansiyon çeşitlerinin kısa olarak karşılaştırması yapılmış ve projede kullanılmış olan yarı aktif süspansiyon çeşidi ayrıntılı olarak incelenmiştir. Yarı aktif süspansiyonlarda kullanılan damper çeşitleri kısa olarak anlatılmış olup, en uygun olan MR (manyetoreolojik) damperin kullanılmasına karar verilmiştir. MR damperin kısaca yapısı ve çalışma prensibi anlatılmıştır.

Önce kontrol uygulamalarının yapılacağı matematik modele karar verilmiş ve denklemleri çıkarılmıştır. Daha sonra hangi kontrol çeşitlerinin kullanılacağına karar verilmiş ve çıkarılan matematik modele göre algoritmalar hazırlanmıştır. Çalışmada anahtar kontrolü, Skyhook kontrolü, durum geri beslemeli kontrol ve LQR (Linear Quadratic Regulator) kontrolü kullanılmıştır. Klasik pasif süspansiyon sistemi ve MR damperli yarı aktif süspansiyon için SIMULINK diagramları hazırlanmıştır. Daha sonra belirlenmiş olan yoldan gelen bozucu sinyaller hazırlanmıştır. Uygulanan tüm kontrol tiplerinin yoldan gelen bozucu sinyal tiplerine göre verdiği cevaplar pasif sistemin verdiği cevaplar ile karşılaştırılmıştır. Titreşim geçirgenlik oranları her kontrolcünün en iyi değerine gore incelenmiştir. Aynı zamanda pasif sistemle yapılan karşılaştırmalar bu kontrolcülerin farklı değerleri için de yapılmış ve o kontrolcünün farklı parametreler için nasıl cevap verdikleri gözlenmiştir. En son olarak tüm kontrolcüler pasif sisteme göre verdikleri cevaplara bakılarak karşılaştırılmıştır.

1. INTRODUCTION

Since the industrial machines were developed, vibration isolation has become a major problem for human beings. Especially in motor vehicles, comfort is one of the most important factors that the customers are interested in. Because of the demand for more comfort and handling factors, automotive firms started to make big investments for the research studies that are related with vibration isolation and suspension systems.

On the other hand, the harming effects of vibrations to human body were proved by researchers too. Prolonged exposures to vibrations contributed to the health disorders in human body. Even though there is not a certain work, research or theory on which vibration causes a certain injury, it is almost agreed by researchers that health disorders and failures are related to the magnitude and frequency of the vibration that is resulting from the road disturbances. These failures and disorders appear because of the vibrations transmitted through solid materials. And in the last century, suspension systems were developed to isolate the vibrations such that they prevent the vibrations to be transmitted to the vehicle body.

Nowadays, three types of suspension systems are used: passive suspension, active suspension, semi active suspension. These systems are described in the succeeding sections.

2. SUSPENSION SYSTEM TYPES

New suspension systems have been developed or existing systems are improved as the demand for comfort increased. In general, three suspension systems have been used to isolate the vibrations resulting from road. These are passive, active and semi active suspension systems.

2.1 Passive Suspension System

At the beginning, all vehicles had passive suspension systems. Even today, most of the vehicles still have passive suspension systems. These system have some springs and dampers to isolate vibrations. A passive suspension system is shown in Figure 2.1. Working principle is based on energy dissipation in the damper. The most important disadvantage of the passive suspension system is the trade-off between road holding capability and the comfort. When it comes to improve the road holding capability, the comfort decreases and vice versa. Decreasing the damping coefficient enhances the comfort but in this case wheel deflection increases which decreases the road holding capability of the vehicle.

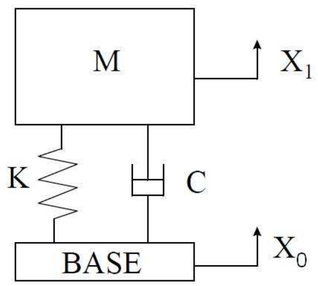

Figure 2.1: Passive suspension system model.

In practice, passive dampers do not have only one damper coefficient but two. The passive suspension damper mechanism consists of two orifices closed with springs at the end of each orifice. Because of this mechanism, the system uses different orifices when the piston travels up or down. So the system has two different damper coefficients depending on the sign of the relative velocity of the base with respect to the body. When the piston travels upwards, the spring on the upper chamber does pressure to the orifice onto which the spring is mounted, so the orifice gets closed leading the oil to travel to the lower chamber from the other orifice (Figure 2.2). But when an MR damper control system malfunctions, it works as a passive suspension system with one damper coefficient. To make the comparisons easier, in this study only one passive damper coefficient is taken into account.

Figure 2.2 shows the behavior of the passive damper containing two orifices and therefore two damper coefficients. A comparison between the passive dampers with one damping coefficient and two damping coefficients can be seen in Figure 2.3, Figure 2.4 and Figure 2.5 for body accelerations, displacements and suspension deflections.

According to the results, in sum, the difference between two orifice dampers and one orifice dampers did not yield to large differences. Two orifice damper has 1500 kg/s damper coefficient if the suspension deflection is positive and 800 kg/s damper coefficient if the suspension deflection is negative. According to these two orifice damper parameters, the difference in acceleration is 5 percent and in displacement the difference is 8 percent.

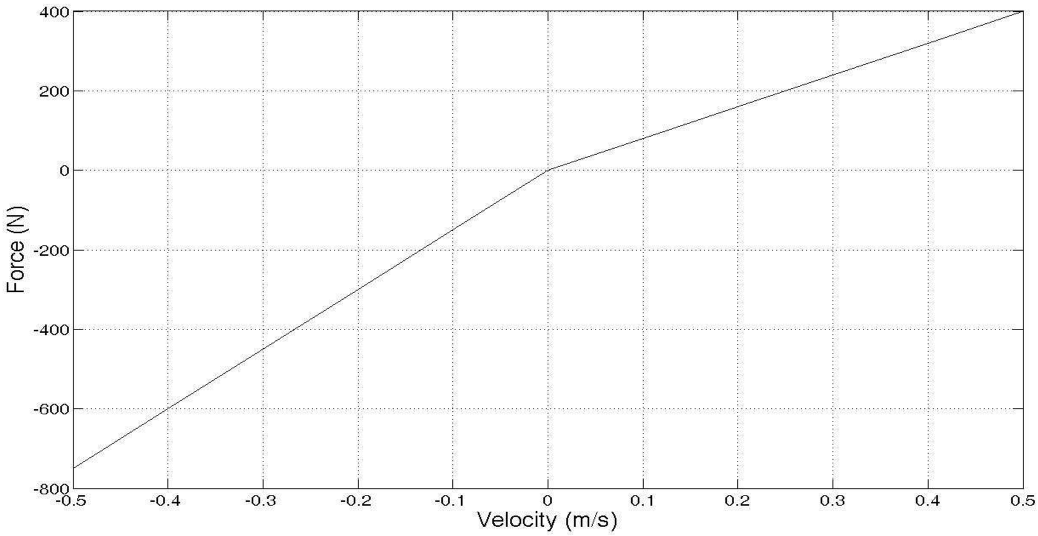

Figure 2.2: Damper coefficient distribution of passive damper with two damper coefficients.

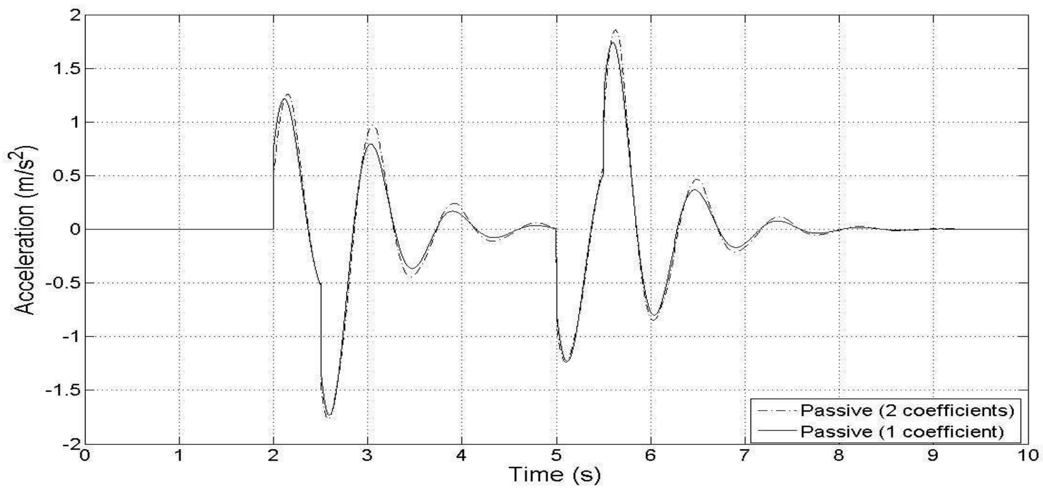

Figure 2.3: Body accelerations of passive dampers with one and two damping coefficients under the step excitation.

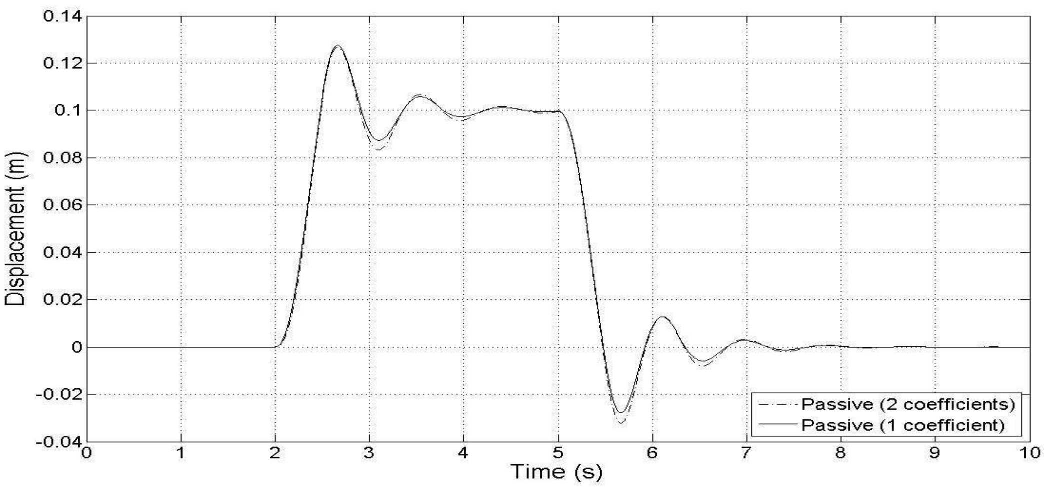

Figure 2.4: Body displacements of passive dampers with one and two damping coefficients under the step excitation.

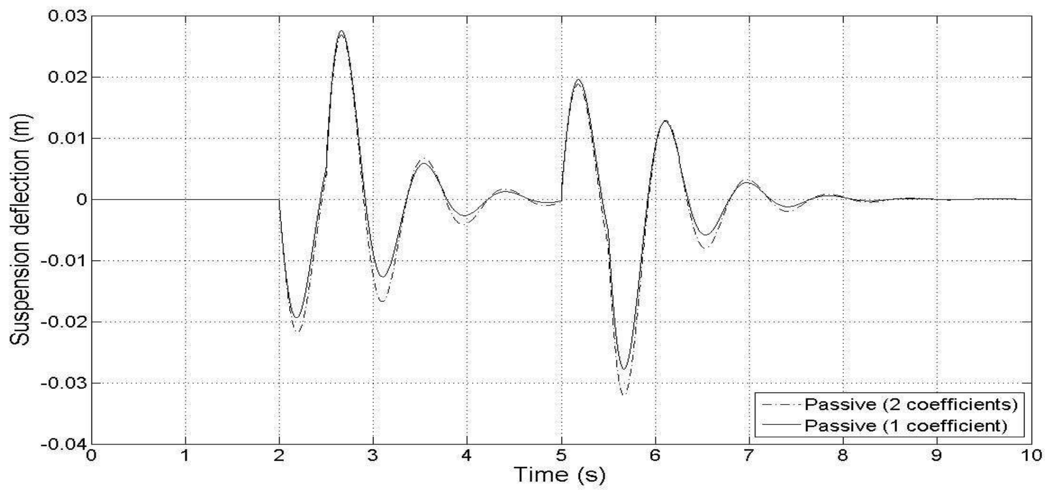

Figure 2.5: Suspension deflections of passive dampers with one and two damping coefficients under the step excitation.

2.2 Active Suspension System

Active suspension systems are the best suspension system type if we compare its performance with those of passive suspension systems and semi active suspension systems. The working principle of the active suspension system is based on a control system that is activated to directly control the force generated by the suspension system. The control system will usually intervene to displacements and parameters. Since there is a continuous force generation and intervene to vehicle parameters, both road holding capability and comfort factors can be achieved.

Although it has good performance when compared with passive and semi active suspension systems, this can be attained only with complex control systems. Active suspension can only be applied to a vehicle with parametric measurements. But this makes the control system and structure more complex than passive and semi active systems. Also it is needed to add external energy to generate the force so this makes the system complex and expensive. Active suspension is not used in practice in today's vehicles because of its disadvantages.

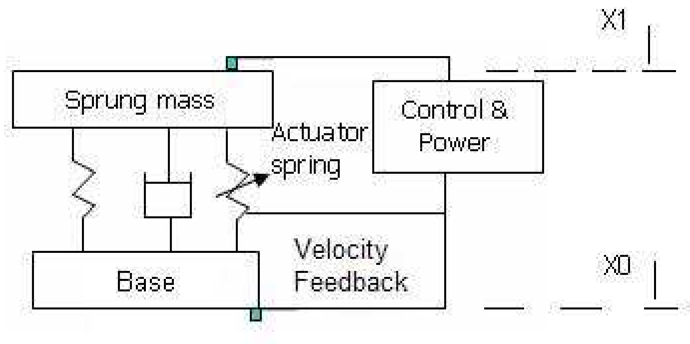

Figure 2.6: Active suspension system model.

In active systems, there is always a damping force generated by the control system and actuator. With active damping coefficient C_{active} and the vertical suspension velocity $\dot{x}_1$, the damping force of an active suspension system can be given by;

$$F_d = C_{active}\dot{x}_1 \quad \textbf{(2.1)}$$

2.3 Semi-active Suspension System

Semi active suspension systems can be considered as a transition from passive suspension systems to active suspension systems. The basic working principle is based on the active control of the damping. Unlike the active suspension systems, it does not require much energy. This is because the system only controls amount of energy dissipation. Since only energy dissipation is allowed, it consumes less energy and does not require complex control systems, some military and luxury vehicles contain semi active suspension systems at present.

These advantages also made the semi active suspension system very popular in other engineering disciplines too. Today many tall buildings are being constructed on semi active suspension systems with MR (magnetorheological) dampers to isolate the vibration that is created by the earthquakes, and there are many studies and projects to implement the semi active systems in the bridges to prevent the big displacements originated from the winds and weights of the objects on them.

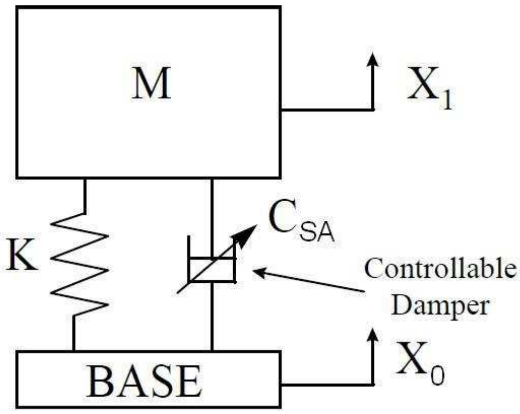

Figure 2.7: Semi active suspension system model.

If we express the semi active suspension working principle mathematically, where C_{SA} is the semi active damping coefficient;

$$F_d = \begin{cases} C_{SA}.\dot{x}_1 \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) > 0 \\ 0 \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) \leq 0 \end{cases} \quad \textbf{(2.2)}$$

According to this equation, the semi active damping force will be generated only if the product of relative velocity between the sprung mass and the base multiplied by the velocity of sprung mass is positive. This definition means the system will dissipate energy in damper. But in the contrary, the force becomes zero because only the active suspension system can generate forces and isolate the vibrations in negative condition. Semi active suspension systems do not provide good vibration isolation as active suspension systems do but when they are compared with respect to their costs and energy consumptions, semi active suspensions are much more efficient than active systems. Besides, semi active suspensions can work as a passive suspension when it is malfunctioned. If the control system or sensors get out of order, the semi active system still works as a passive system. So this makes semi active suspensions more reliable.

3. SEMI ACTIVE DAMPERS

In almost every vehicle of today, semi active systems have been used. Because of the structure and control requirements of semi active systems, the structure will surely need special dampers instead of a simple viscous damper used in passive suspension systems.

3.1 Friction Dampers

Working principle of these dampers depends on the simple friction rules. The energy is dissipated during the friction. By means of semi active control, the amount of friction can be adjusted by changing the nominal force that creates the friction force. These dampers are not used in suspension systems in today's vehicles. But they are related with the braking systems or some transmission systems.

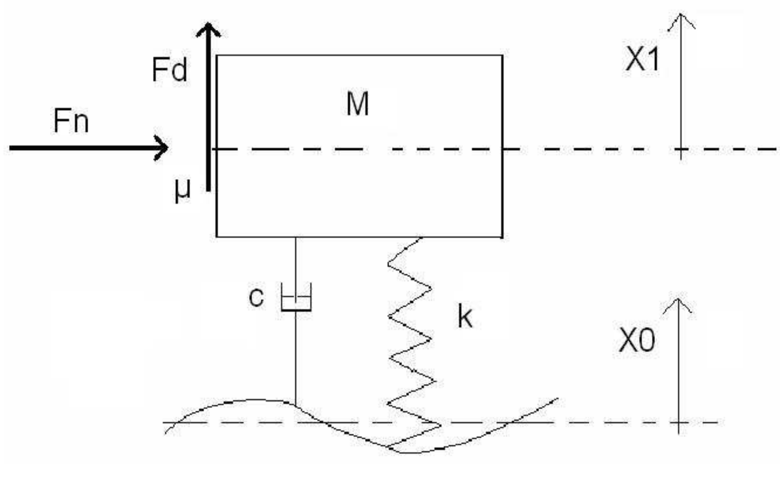

Figure 3.1: Working principle of a friction damper.

If a force F_n is applied to a mass by means of a pad with a desired amount, during the relative motion between the pad and the plate, a friction force will occur because of the friction between the pad and plate. So since a friction is present, a damping force F_d will exist. If we have the chance to change F_d, then it means we have a semi active damper. If we decrease the normal force, the friction force will decrease leading to a less damping force. On the contrary, if we increase the normal force, the friction force will increase leading to a higher damping force. But adjusting the normal force is not easy by means of control tools. So friction dampers are not preferred in semi active suspensions. But it has wide usage in industrial applications, washing machines and even in trains because of its low cost and easy maintenance.

Figure 3.2: A helical spring suspension with friction damper.

3.2 ER (Electrorheological) Dampers

ER dampers consist of ER fluids. The ER damper is a mixture of oil and particulates that are semiconducting. When an electric field is applied to the ER damper, the viscosity of the fluid increases. With the change of viscosity of the fluid in an ER damper, the variable damping can be obtained. This is provided by the electric field, because during the electric field, particulates get in an order and shaped as a line which causes that viscosity and subsequently the damper coefficient increase.

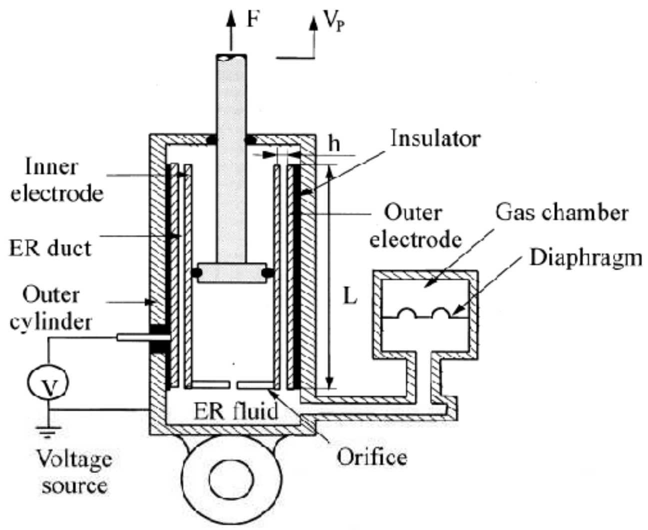

Figure 3.3: Structure of the ER damper.

At the first stages of the research studies on semi active damping, the researchers concentrated on ER dampers. ER dampers need so much voltage to influence the order of the particulates and ER dampers have longer response times when compared with the MR dampers. The advantages of wide operational temperatures of MR dampers also made people concentrate on MR dampers. While ER dampers have an operational temperature range between 10 °C and 40 °C, MR dampers showed an operational temperature between -40 °C and 150 °C. When the same amount of viscosity was present in both an ER damper and MR damper, the MR liquid showed a shear stress of 100 kPa while the ER liquid showed a shear stress of approximately 10 kPa.

3.3 MR (Magnetorheological) Dampers

Nowadays, in some military and luxury vehicles, semi active suspension systems having MR dampers are preferred because of their advantages when compared with other semi active damper types. The MR damper contains an MR fluid that consists of lubricated oil and particulates that is sensitive to magnetic field. When a magnetic field is applied to an MR damper, the particulates are arranged in the order of magnetic field lines leading the viscosity of the MR fluid to change. As the viscosity of the fluid increases, the damping coefficient increases.

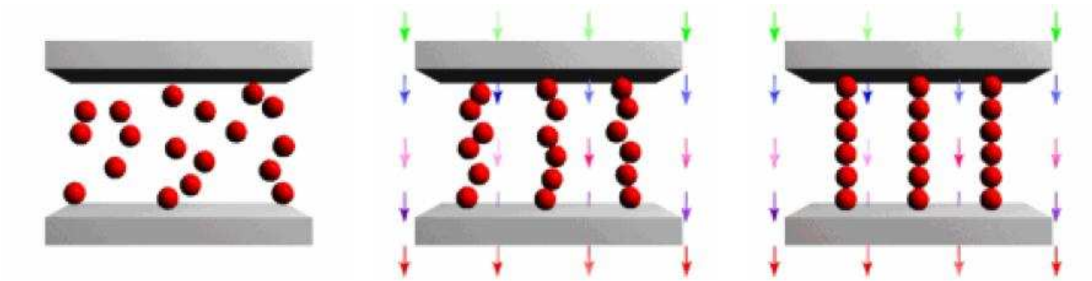

Figure 3.4: The arrangement steps of particulates in an MR damper.

3.3.1 Comparison of the MR dampers and ER dampers

MR dampers are widely used in many industrial applications today. Especially in automotive semi active suspensions, it became indispensable in the last years. The MR dampers have many advantages when they are compared with the ER dampers. The most significant advantages are for the less affection by impurities, smaller power supply, larger yield stress and wider range of operable temperature. MR dampers can operate in a wider range of temperature and does not need a high voltage power supply and its stability is not affected by impurities in the fluid. During the manufacturing of the dampers some impurities and dirt can be found. Also some surfactants, dispersants and friction modifiers are put into the MR damper to improve stability, seal and bearing life. In ER dampers the impurities and additives can affect the arrangement of the particles and the electric field that modifies the viscosity of the ER fluid. So ER dampers must be manufactured and used carefully while MR dampers can be used in harder environment situations and manufactured simpler. Power requirements are better for MR dampers as well. In ER dampers, electric field that modifies the viscosity of the ER fluid needs too much voltage. So increment in the viscosity costs too much energy in contrary to the MR dampers. MR dampers work with magnetic fields instead of an electric field. The magnetic field in an MR damper can easily be formed with very small electric currents. The particulates immediately can be arranged in the order of lines of magnetic fields. So this makes MR dampers to be energy saving. The main differences between the ER and MR dampers can be seen in Table 3.1.

Table 3.1: Comparison of MR dampers and ER dampers

Property	MR Fluids	ER Fluids
Max yield stress	50-100 kPa	2-5 kPa
Maximum field	-250 kA/m	4kV/mm
Apparent plastic viscosity	0.1-10 Pa-s	0.1-1.0 Pa-s
Operable temperature range	(-40) - 150 °C	+10 - 90 °C
Stability	Unaffected by most impurities	Cannot tolerate impurities
Density	3-4 g/cm^3	1-2 g/cm^3
Maximum energy density	0.1 Joules/cm^3	0.001 Joules/cm^3
Power supply	2-50 V, 1-2 A	2000-5000V, 1 - 10 mA

3.3.2 Working principle of an MR damper

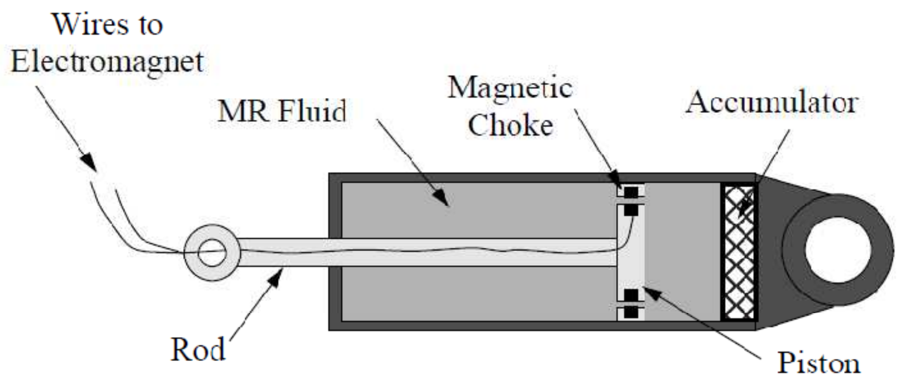

Figure 3.5: Scheme of MR damper.

The suspension dampers are designed for the energy dissipation created at the orifice in the damper. When the piston of the damper tries to move forwards and backwards, a resistance occurs as a function of the velocity of the piston. By applying a magnetic field, the viscosity of an MR fluid can be changed. So the damping coefficient changes with the modification of the viscosity. The accumulator at the end of the MR damper acts like a spring in the damper. If the fluid expands because of temperature increase, the accumulator will decrease in size. And during the fluid transfer from down to up or up to down, accumulator will act like fluid so this will prevent cavitations. Cavitations are not desired because it affects the damping coefficient and damping force.

Figure 3.6: An example of MR damper of LORD Corporation.

3.4 Dampers with Controllable Orifice

In this type of semi active dampers, the damper coefficient changes with the area of the orifice. If the area of the orifice increases, the fluid passes through the orifice easier leading the damper having a lower damper coefficient. If higher damper coefficients are desired, the orifice area is reduced. The reduced orifice area makes the damper fluid pass through the orifice less leading the damper coefficient to a higher value. These dampers are used in early semi active suspension systems in vehicles.

Figure 3.7: Dampers with controllable orifice

4. MATHEMATICAL MODEL AND ROAD DISTURBANCE SIGNALS

In this project, a quarter car model is used. The vibration isolation performances of the controllers are evaluated according to this mathematical model. And the performances of the controllers are analyzed by using two different road disturbance signals. In this part, how to create these disturbance signals and the mathematical suspension model on which this thesis is based will be explained.

4.1 Quarter Car Model

This model is the simplest model that is used to examine the vibration and bounces in vertical direction. It is only used to show the displacement, accelerations etc. in the vertical direction. We did not add the tire dynamics into the model because we assumed that the suspension system is mounted to experimental setup and the road disturbances are assumed to be exciting directly to the suspension system.

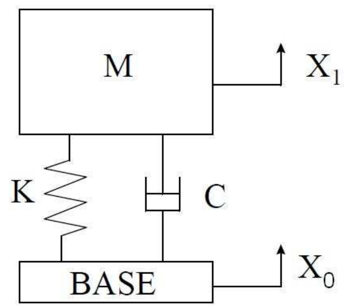

Figure 4.1: Quarter car model.

Let "c" denote the damper coefficient, "x_1" the sprung mass displacement, "x_0" the base displacement, "k" the spring coefficient, "m" the sprung mass. Then the mathematical model of the passive suspension system and the acceleration of the sprung mass can be written as follows:

$$m\ddot{x}_1 + c(\dot{x}_1 - \dot{x}_0) + k(x_1 - x_0) = 0 \quad \textbf{(4.1)}$$

$$\ddot{x}_1 = \frac{c}{m}(\dot{x}_1 - \dot{x}_0) - \frac{k}{m}(x_1 - x_0) \quad \textbf{(4.2)}$$

4.2 Road Disturbance Signals

In this study, two different road excitation signals are used. The signals are generated with MATLAB/SIMULINK toolbox blocks. Firstly, the first signal is the product of a set of sine waves and the second one is a simple step shaped obstacle.

4.2.1 Random road excitation of sine waves

This signal is used in this study to see the performance of the controllers as the system is advancing on a road with disturbances having different amplitudes. The signal is created in MATLAB/SIMULINK by multiplying 4 different sine waves. The SIMULINK diagram of the random sine road excitation and the shape of the disturbance can be seen in Figure 4.2 and 4.3 respectively.

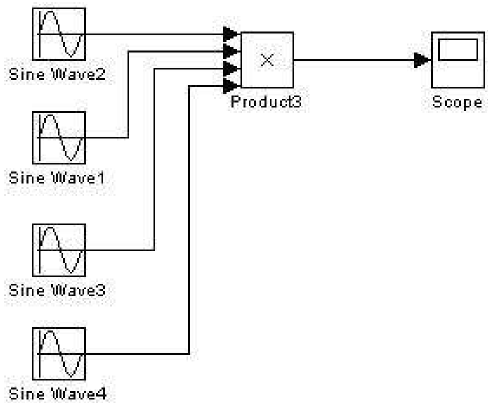

Figure 4.2: Block diagram of random sinus road excitation

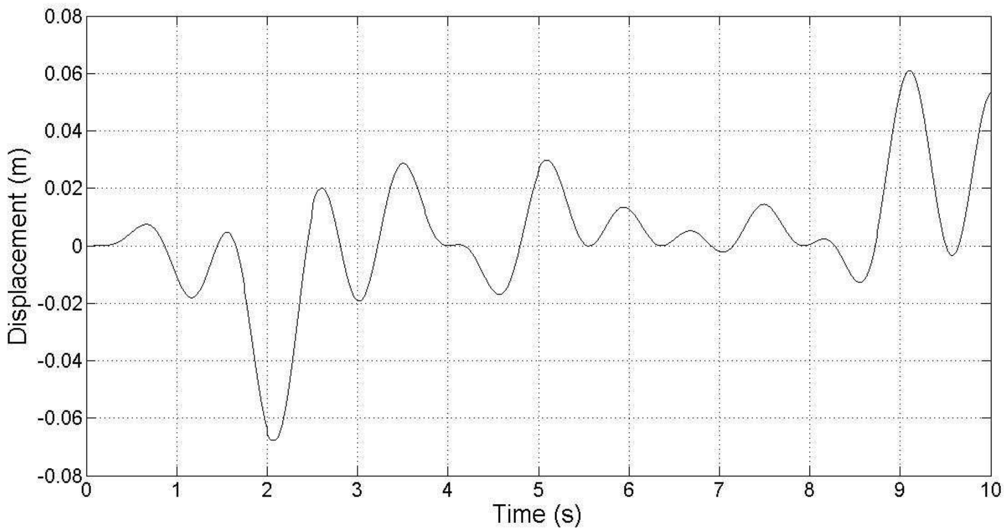

Figure 4.3: Random road excitation of sine waves

4.2.2 Step shaped obstacle

This signal is used in this thesis to observe the performance of the controllers when the suspension system encounters sharp obstacles such as a 10 cm obstacle. This is mostly used to see how quick the controlled system recovers itself to its prior position after being subjected to the obstacle. The signal is created in MATLAB/SIMULINK by drawing the signal in the signal builder. Below in Figure 4.4, the signal builder block can be seen, and in Figure 4.5, step shaped obstacle can be seen as a MATLAB plot.

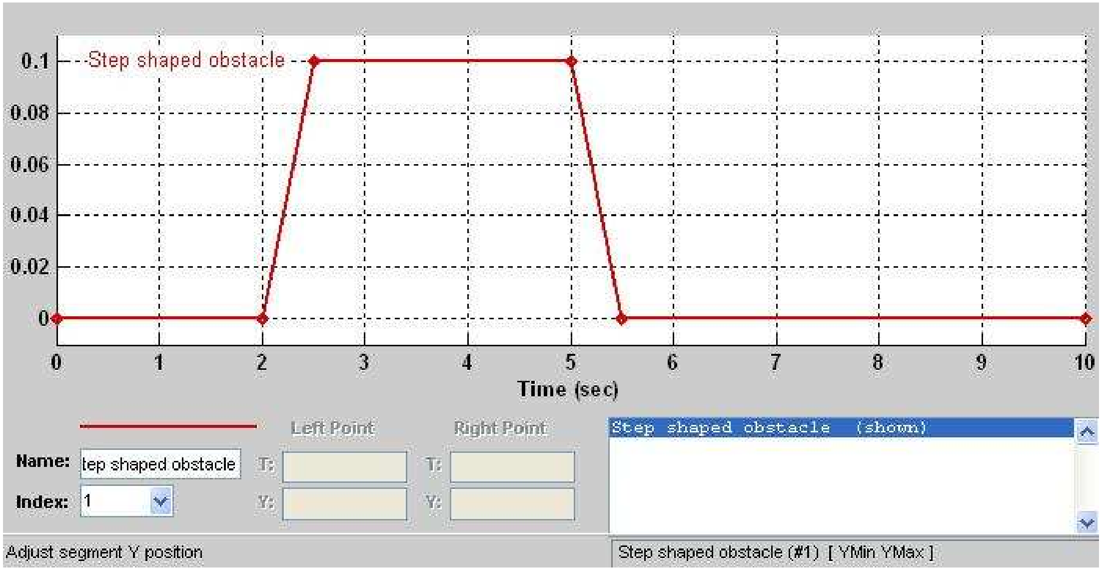

Figure 4.4: Signal builder block.

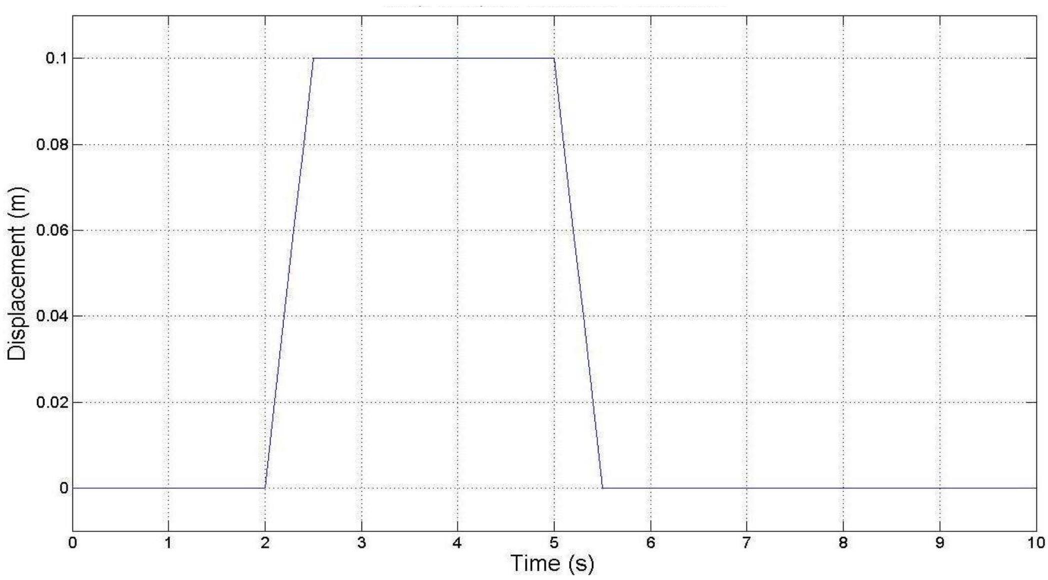

Figure 4.5: Step shaped obstacle excitation.

5. SEMI ACTIVE CONTROL ALGORITHMS

In this section, the aim is to find the best control algorithm for all types of road disturbances that are used in this thesis. Many controllers are implemented into a semi active suspension system and their performances are evaluated for two different road excitations. The results for different controller parameters are noted and shown. For the random sine road excitation, the RMS values are calculated to quantify the performance of different controllers.

All semi active control systems will be compared with the passive system. Hence, beforehand it will be good to explain the passive suspension system briefly.

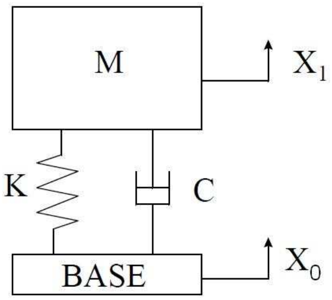

Figure 5.1: Passive suspension system.

The passive system has a constant damper coefficient and it works with only one damping coefficient for all of its life time. This limits the improvement of both the road holding and comfort. We can define the passive suspension system by assigning a constant damper coefficient to the damper value. Below given values are for a typical mid-size car.

$C_{passive} = 1290\,kg/s$

$k = 19960N/m$

$m = 365kg$

$$m\ddot{x}_1 + c_{passive}(\dot{x}_1 - \dot{x}_0) + k(x_1 - x_0) = 0 \qquad \textbf{(5.1)}$$

$$\ddot{x}_1 = -\frac{c_{passive}}{m}(\dot{x}_1 - \dot{x}_0) + \frac{k}{m}(x_1 - x_0) \quad \textbf{(5.2)}$$

In Figure 5.2, the block diagram of the passive suspension system that is used in comparison with the other semi active suspension systems can be seen. The block diagram is drawn in SIMULINK.

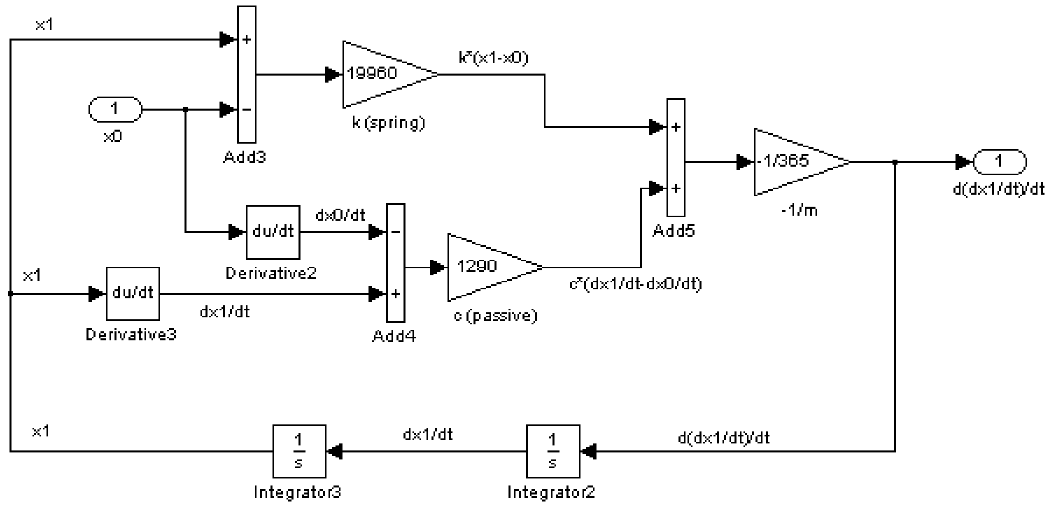

Figure 5.2: SIMULINK diagram of the passive suspension system.

5.1 Semi-active ON/OFF System

This control algorithm is easy enough to understand and can be applied to a quarter car model. It is based on switching the damper coefficient to one of the desired values if the conditions of that value are met. The system is not versatile in terms of performance because only two damping values can be selected with this control system so this limits the use of two specific damping value for all kinds of road disturbances.

The control algorithm lets us use only two damping values. It is a simple switch system. c_{soft} is selected as the same with passive suspension system and chard is selected according to the second order system with damping ratio of $\xi = 0.65$. With c_{soft} damping coefficient in soft mode and c_{hard} damping coefficient in hard mode, semi active ON/OFF system control algorithm can be described as:

$$c_{soft} = 1290 kg/s$$

$$c_{hard} = 3511 kg/s$$

$$c = \begin{cases} c_{hard} \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) > 0 \\ c_{soft} \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) \leq 0 \end{cases} \quad \textbf{(5.3)}$$

$$m\ddot{x}_1 + \begin{Bmatrix} c_{hard} \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) > 0 \\ c_{soft} \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) \leq 0 \end{Bmatrix}(\dot{x}_1 - \dot{x}_0) + k(x_1 - x_0) = 0 \quad \textbf{(5.4)}$$

The system changes its damping value according to the velocity relationship of the base and sprung mass. With $\dot{x}_1$ velocity of sprung mass, $\dot{x}_0$ velocity of the base and $(\dot{x}_1 - \dot{x}_0)$ the relative velocity between the sprung mass and base, the algorithm decides which damping to be used. If the product of the relative velocity and sprung mass velocity is positive, the system chooses the hard mode of damper. But if the product of the relative velocity and the sprung mass velocity is negative, the system chooses the soft mode.

The mathematical model is nearly the same as the passive suspension system mathematical model; the difference is that two different damping values are employed. The block diagram of a semi active ON/OFF system is drawn in SIMULINK.

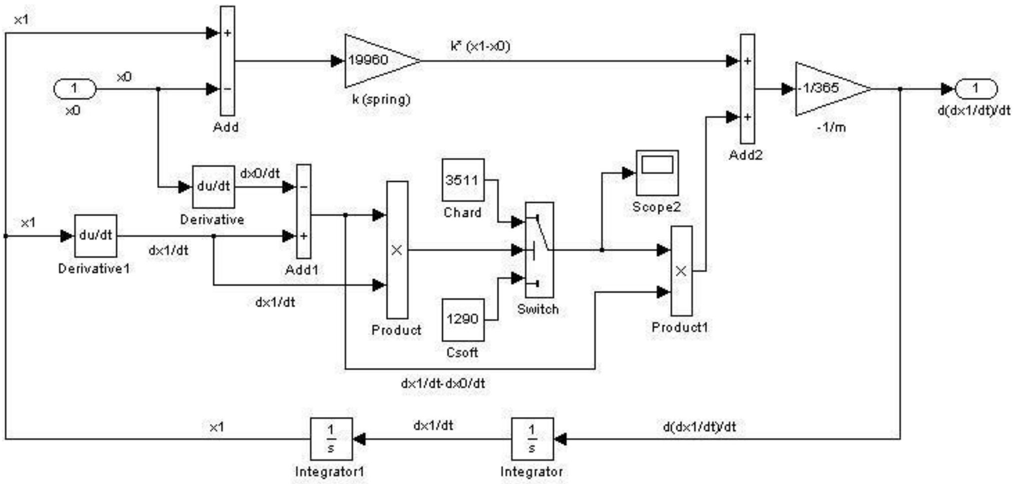

Figure 5.3: SIMULINK diagram of semi active ON/OFF system.

All graphs are drawn for the values of c_{soft} = 1290 kg/s and c_{hard} = 3511 kg/s. The performance of the control system will be evaluated in response to both the random sine road excitation and step shaped obstacle excitation. Other results are given in tables to compare briefly all the results for all values of the ON/OFF system. It can be seen easily that a trade off is present between acceleration, displacement and suspension deflection. The ON/OFF system yielded different results under the sine and step excitations.

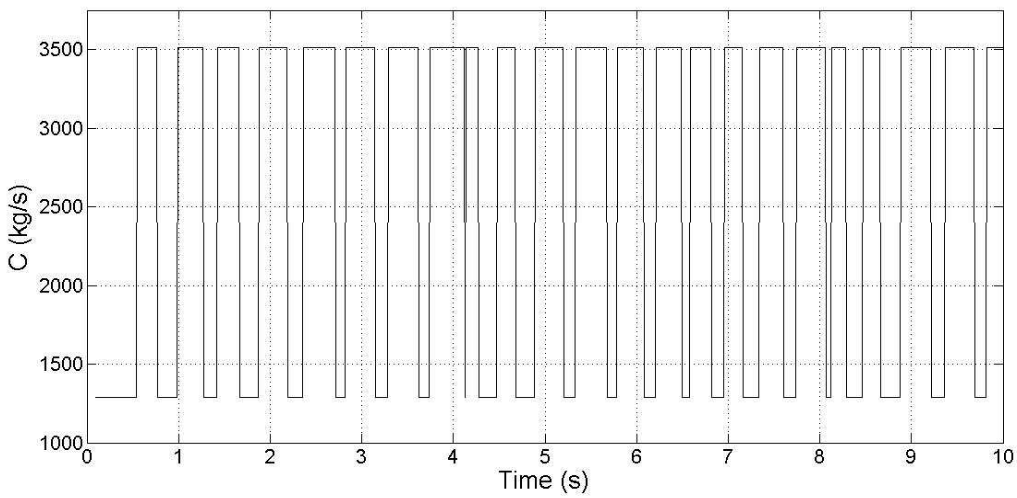

Figure 5.4: Damper coefficient of the ON/OFF system under the sine excitation

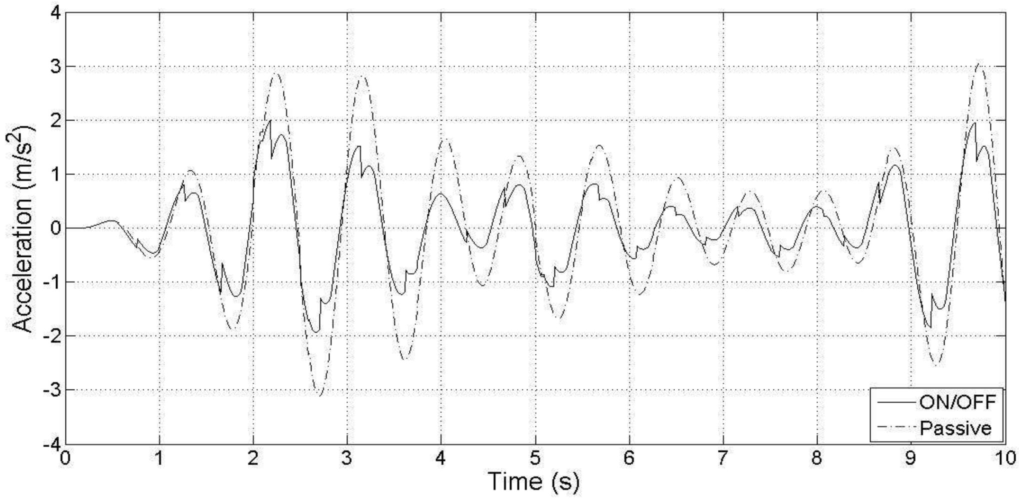

Figure 5.5: Body acceleration of the ON/OFF and passive systems under the sine excitation.

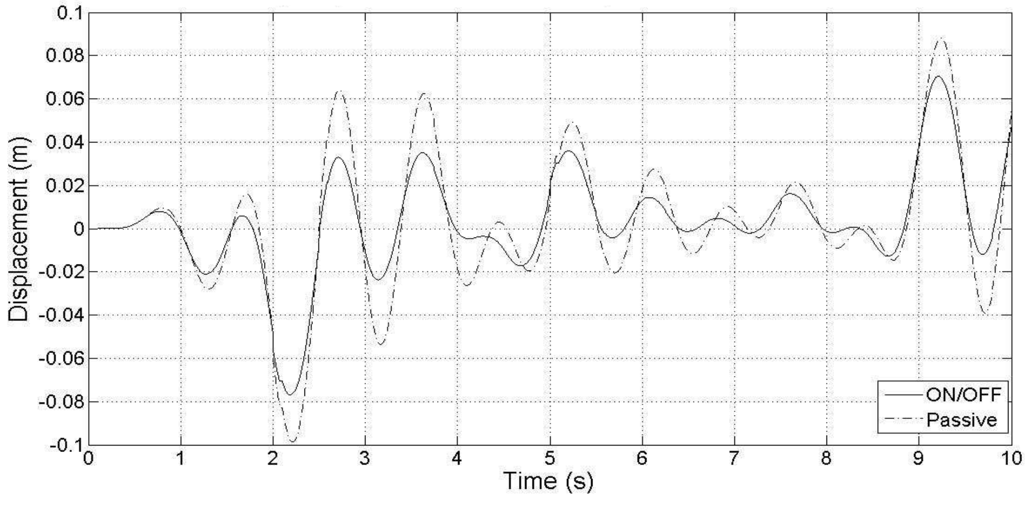

Figure 5.6: Body displacement of the ON/OFF and passive systems under the sine excitation.

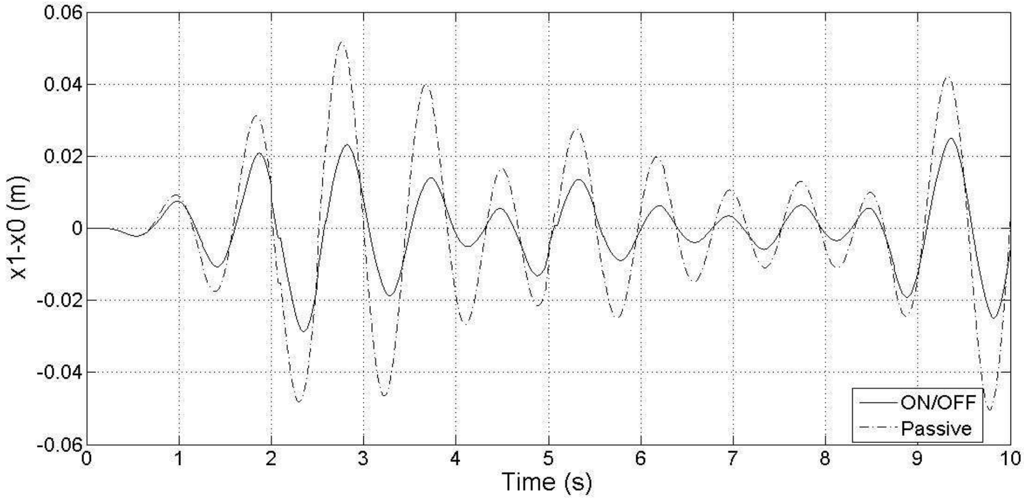

Figure 5.7: Suspension deflections of the ON/OFF and passive systems under the sine excitation.

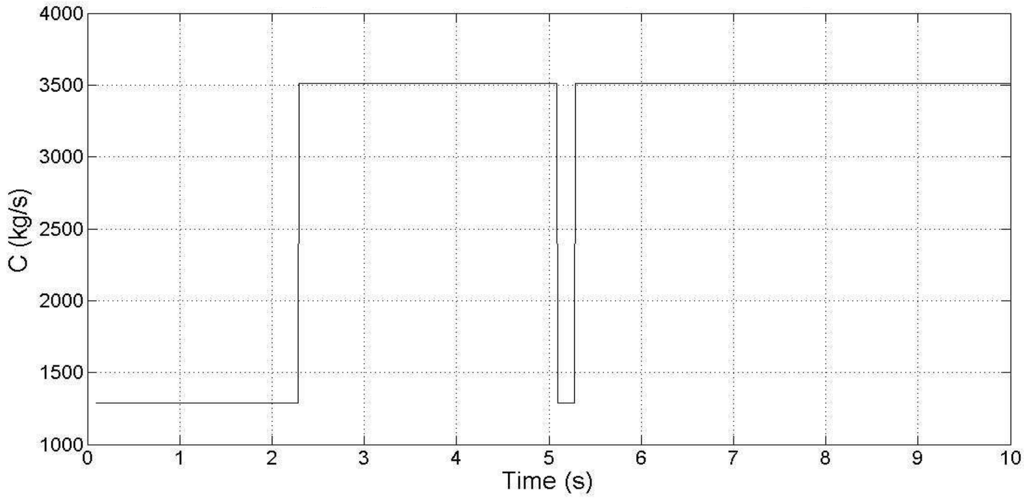

Figure 5.8: Damper coefficient of the ON/OFF system under the step excitation.

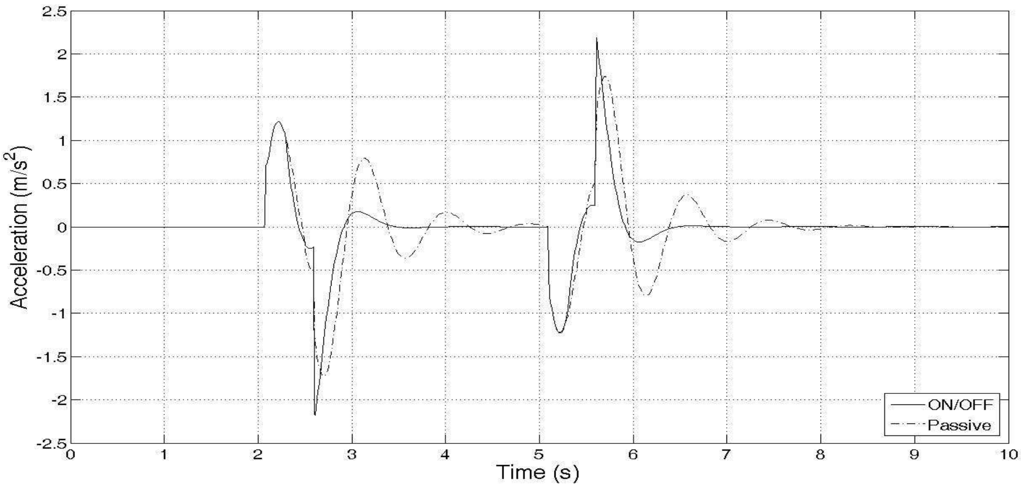

Figure 5.9: Body accelerations of the ON/OFF and passive systems under the step excitation.

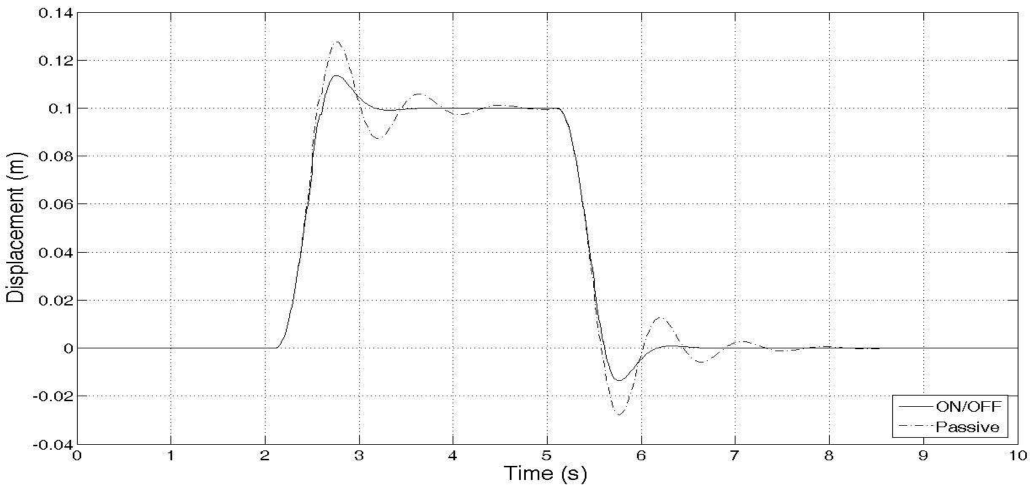

Figure 5.10: Body displacements of the ON/OFF and passive systems under the step excitation.

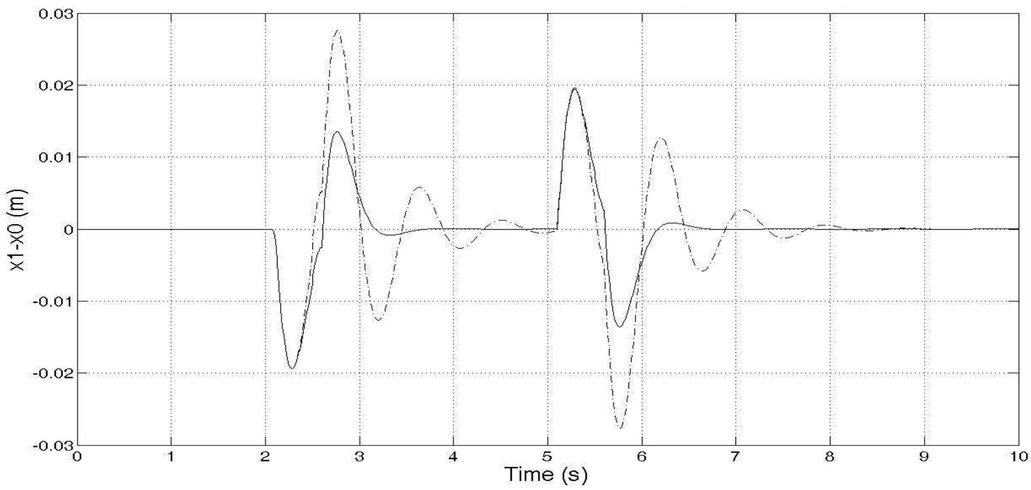

Figure 5.11: Suspension deflections of the ON/OFF and passive systems under the step excitation.

To compare the performance of the ON/OFF system for different C_{hard}/C_{soft} values, the RMS values of the acceleration, displacement and suspension deflection must be found; because being under random sinus road excitation, it is not easy to compare the results by looking at the ratios of the peak points. Six C_{hard}/C_{soft} ratios are selected and the RMS results are calculated for the accelerations, displacements and suspension deflections. Improvements are made in comparison to the passive system and the results are given in terms of percentage of improvements of the RMS values. This gives us the opportunity to evaluate the results affected by C_{hard}/C_{soft} ratios.

Table 5.1: RMS values of the body acceleration of the ON/OFF and passive systems for different C_{hard}/C_{soft} values under the sine excitation.

C_{hard}/C_{soft}	Passive (RMS)	ON/OFF (RMS)	Improvement (%)
2000/1290	1,27106	0,997222	21,54
2500/1290	1,268896	0,894209	29,53
3000/1290	1,267689	0,827039	34,76
3511/1290	1,267623	0,779925	38,47
4000/1290	1,268496	0,751081	40,79
4500/1290	1,26913	0,722538	43,07

Table 5.2: RMS values of the body displacement of the ON/OFF and passive systems for different C_{hard}/C_{soft} values under the sine excitation.

C_{hard}/C_{soft}	Passive (RMS)	ON/OFF (RMS)	Improvement (%)
2000/1290	0,032041	0,027497	14,18
2500/1290	0,032037	0,025728	19,69
3000/1290	0,031929	0,024391	23,61
3511/1290	0,032004	0,023543	26,44
4000/1290	0,031994	0,02282	28,67
4500/1290	0,032042	0,022275	30,48

Table 5.3: RMS values of the suspension deflection of the ON/OFF and passive systems for different C_{hard}/C_{soft} values under the sine excitation.

C_{hard}/C_{soft}	Passive (RMS)	ON/OFF (RMS)	Improvement (%)
2000/1290	0,020763	0,015064	27,45
2500/1290	0,020757	0,012859	38,05
3000/1290	0,020762	0,011389	45,14
3511/1290	0,02076	0,010338	50,20
4000/1290	0,02076	0,009631	53,61
4500/1290	0,020759	0,009072	56,30

Following, the performance of the ON/OFF system is evaluated under step shaped obstacle excitation on the second step. In this part, as in the sine excitation process, six different C_{hard}/C_{soft} values are used to find the improvement of the semi active ON/OFF system against the passive system. The improvements in terms of the percentages are found by using the points of highest values for the following outputs: acceleration, displacement and suspension deflection. In every C_{hard}/C_{soft} ratio, the semi active system reached the steady state solution earlier than the passive system. So the behavior of the ON/OFF system is evaluated for the points where the maximum acceleration, displacement and suspension deflection occur.

Table 5.4: Maximum acceleration values of the ON/OFF and passive suspension systems for different C_{hard}/C_{soft} values under the step excitation.

C_{hard}/C_{soft}	Passive (m/s^2)	ON/OFF (m/s^2)	Improvement (%)
2000/1290	1,7322	1,681	2,96
2500/1290	1,7322	1,7772	-2,60
3000/1290	1,7322	1,9736	-13,94
3511/1290	1,7322	2,1816	-25,94
4000/1290	1,7322	2,3943	-38,22
4500/1290	1,7322	2,6364	-52,20

Table 5.5: Maximum displacement values of the ON/OFF and passive suspension systems for different C_{hard}/C_{soft} values under the step excitation.

C_{hard}/C_{soft}	Passive (m)	ON/OFF (m)	Improvement (%)
2000/1290	0,1276	0,122	4,39
2500/1290	0,1276	0,1188	6,90
3000/1290	0,1276	0,116	9,09
3511/1290	0,1276	0,1135	11,05
4000/1290	0,1276	0,1115	12,62
4500/1290	0,1276	0,1096	14,11

Table 5.6: Maximum suspension deflection values of the ON/OFF and passive suspension systems for different C_{hard}/C_{soft} values under the step excitation.

C_{hard}/C_{soft}	Passive (m)	ON/OFF (m)	Improvement (%)
2000/1290	0,0276	0,022	20,29
2500/1290	0,0276	0,0188	31,88
3000/1290	0,0276	0,016	42,03
3511/1290	0,0276	0,0135	51,09
4000/1290	0,0276	0,0115	58,33
4500/1290	0,0276	0,00963	65,11

5.2 Skyhook Control Law

The skyhook control system is one of the most popular control systems that is used to control the semi-active suspension damper. It is widely used in semi-active damper control studies and it is observed that it eliminates the tradeoff between the resonance frequency control and high frequency control. In Skyhook control scheme, it is assumed that the damper between the base and the sprung mass is fixed to a fictional point in the sky. It must be known that this configuration is not possible in real life and is a fictional assumption. The skyhook model behaves as it generates a force to reduce the velocity of the sprung mass but conventional models aim to reduce the relative velocity between the sprung mass and the base.

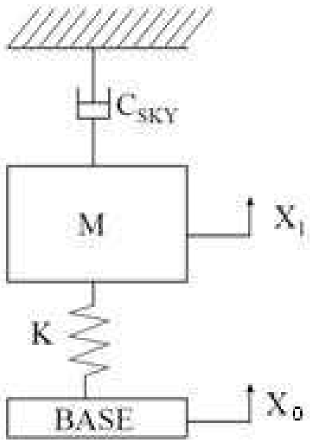

Figure 5.12: The model for the skyhook system.

We need to emulate the damper, shown as fixed to a fictional point in the sky, as it behaves in the conventional mass, spring and damper system. Hence, firstly we need to define the speed of the sprung mass relative to the base.

$$\dot{x}_{10} = \dot{x}_1 - \dot{x}_0 \qquad \textbf{(5.5)}$$

This relative velocity value is positive for two cases as follows: if the sprung mass and the base are separating from each other or the velocity of the sprung mass is bigger than the one of base when they are traveling in the same direction. If we consider the force that is provided by the skyhook damper, we can see that it is in the negative X_1 direction. So we can write the skyhook damper force as follows;

$$F_{SKY} = C_{SKY}\dot{x}_1 \qquad \textbf{(5.6)}$$

Now we need to translate this equation to an equation as this force is provided by a semi-active damper.

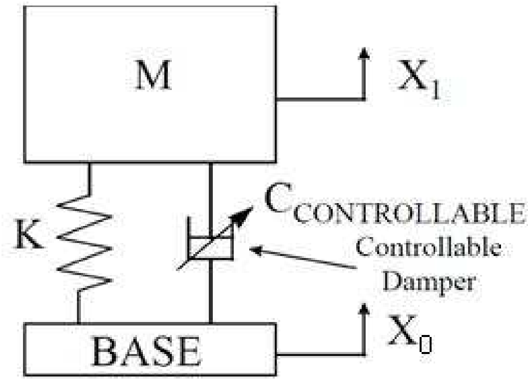

Figure 5.13: Model of a semi active system that is defined with skyhook parameters.

As it can be seen in equation 5.7, we defined the skyhook damper as a controllable semi-active damper. Hence, we can define the real semi-active damper coefficient in terms of the skyhook damper coefficient.

$$F_{CONTROLLABLE} = C_{CONTROLLABLE} . \dot{x}_{10} \quad \textbf{(5.7)}$$

$$C_{CONTROLLAB\,LE} = C_{SKY} \frac{\dot{x}_1}{\dot{x}_{10}} \quad \textbf{(5.8)}$$

To obtain an algorithm that defines the change of the semi-active damper coefficient according to the base speed and the relative speed of the sprung mass with respect to base, C_{SKY} (Skyhook damper coefficient) must be found. This constant can be considered as the damper constant of the passive suspension system. If the second order system is checked, the natural frequency and damper coefficient of the system can be found by the parameter values given below.

$k = 19960 N / m$

$m = 365 kg$

For damping ratio ξ = 0.65, the damping coefficient of a second order system can be found. The second order system defines a conventional passive suspension system. The natural frequency and damper coefficient will be defined with the parameters of this system.

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \textbf{(5.9)}$$

The undamped natural frequency of a second order system is:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{19960}{365}} = 7.395 rad/s \cong 1.177 Hz \quad \textbf{(5.10)}$$

$$\frac{C_{SKY}}{m} = 2.\xi.\omega_n \Rightarrow C_{SKY} = 2.\xi.\omega_n.m = 2x0.65x7.395 \cong 3511 kg/s \quad \textbf{(5.11)}$$

In skyhook control, the damping value is changed continuously by modifying the constant damping C_{SKY}. First it can be recognized as an ON/OFF system because of the switch between C_{SA} (semi active damper coefficient) and C_{min} (minimum damper coefficient of the skyhook system) values. Because C_{SKY} modified with the relative velocity between sprung mass and base and the velocity of sprung mass itself, the system will always have variable damping during all process. As it must be known that a skyhook model like shown in Figure 5.12 is not possible in practice, it provides us to have variable damping. By fixing the damper to a fictional point in the sky, the system has a condition that damping force changes with only sprung mass velocity not with the relative velocity between the sprung mass and base.

$$c = \begin{Bmatrix} c_{SA} \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) > 0 \\ c_{\min} \rightarrow \dot{x}_1(\dot{x}_1 - \dot{x}_0) \leq 0 \end{Bmatrix} \quad \textbf{(5.12)}$$

In condition given in equation 5.10, the system will have 2 different coefficients during the process. C_{min} is a single value that provides the damping; it does not change and works only when the product of the relative velocity and sprung mass velocity is negative. C_{SA} is the main damping coefficient; if the product of the relative velocity and sprung mass velocity is positive, the system will have this semi active damping coefficient. This coefficient is formed with velocities by applying the skyhook rule to a constant C_{SKY} value that we found by applying our parameters to the second order system.

$$c_{\min} = 800 kg/s$$

$$F_d = c_{SKY}.\dot{x}_1 \quad \textbf{(5.13)}$$

$$F_d = c_{SKY}.\dot{x}_1 = c_{SA}.(\dot{x}_1 - \dot{x}_0) \quad \textbf{(5.14)}$$

$$c_{SA} = c_{SKY} \frac{\dot{x}_1}{(\dot{x}_1 - \dot{x}_0)} \quad \textbf{(5.15)}$$

The semi active damper coefficient will be C_{SA}. But it will change with the variation of the relative velocity and sprung mass velocity. But the damper coefficient will have limits, because the viscosity of the damper fluid cannot reach to too high values. For these, the semi active damper coefficient can be defined as follows;

$$c_{SA} = \left\{ \begin{array}{l} c_{\max} \rightarrow c_{SKY} \dfrac{\dot{x}_1}{(\dot{x}_1 - \dot{x}_0)} > c_{\max} \\ c_{SKY} \dfrac{\dot{x}_1}{(\dot{x}_1 - \dot{x}_0)} \rightarrow c_{passive} < c_{SKY} \dfrac{\dot{x}_1}{(\dot{x}_1 - \dot{x}_0)} \leq c_{\max} \\ c_{passive} \rightarrow c_{SKY} \dfrac{\dot{x}_1}{(\dot{x}_1 - \dot{x}_0)} \leq c_{passive} \end{array} \right\} \quad \textbf{(5.16)}$$

The model of the skyhook system is drawn in SIMULINK. The skyhook model is defined in the subsystem of block diagram.

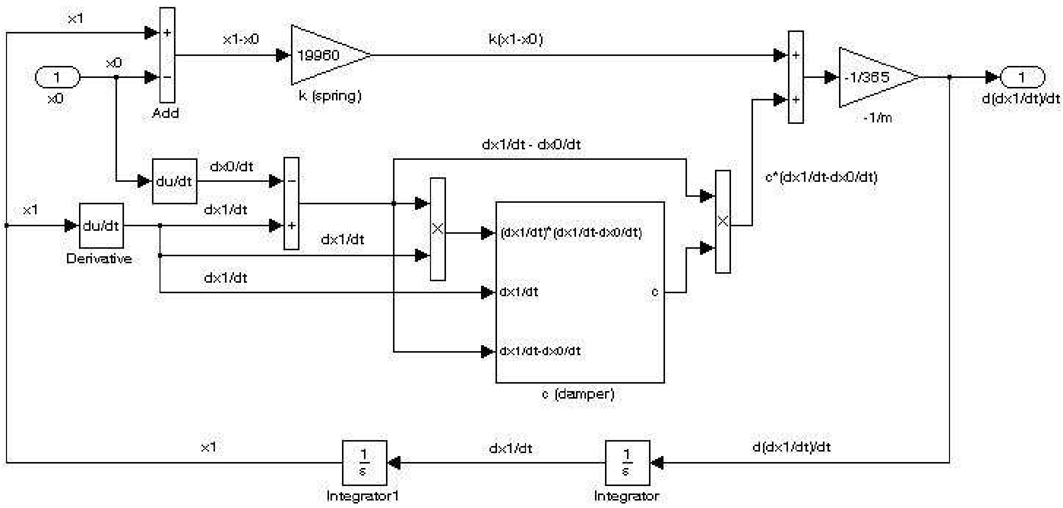

Figure 5.14: SIMULINK diagram of a semi active Skyhook system.

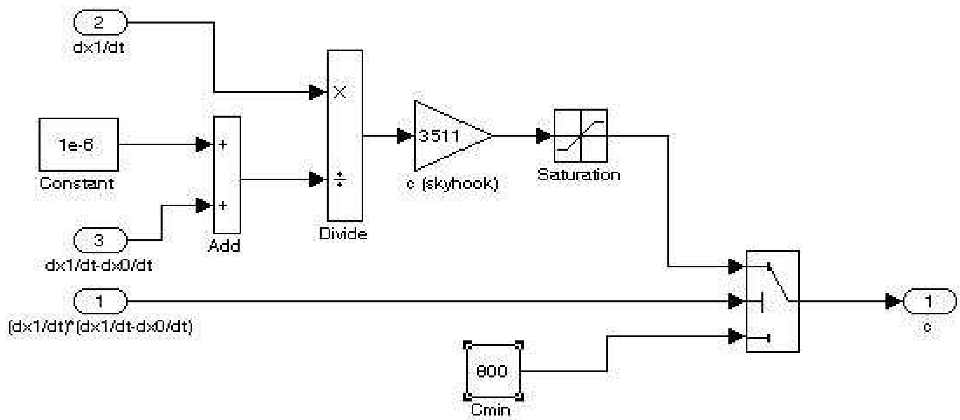

Figure 5.15: Semi active damper subsystem that shows the skyhook algorithm.

All graphs are drawn with value of underdamped case $\xi = 0.65$ damping ratio. The performance of the control system will be evaluated with both the random sine road excitation and step shaped obstacle excitation. The results will be shown in terms of the acceleration, body displacement and suspension deflection. The performance of a skyhook system is compared with the passive suspension system. The behavior of the skyhook control system is evaluated for different damping ratios. Thus, we will be able to determine the trade-off between the parameters such as body acceleration which defines the comfort, body displacements and suspension deflections determining the road holding capability.

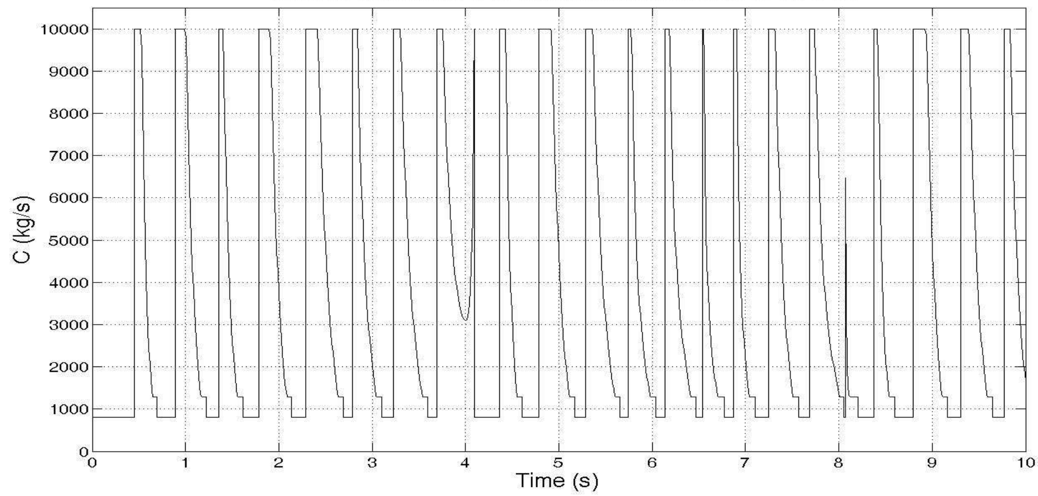

Figure 5.16: Damping coefficient of the skyhook system under the sine excitation.

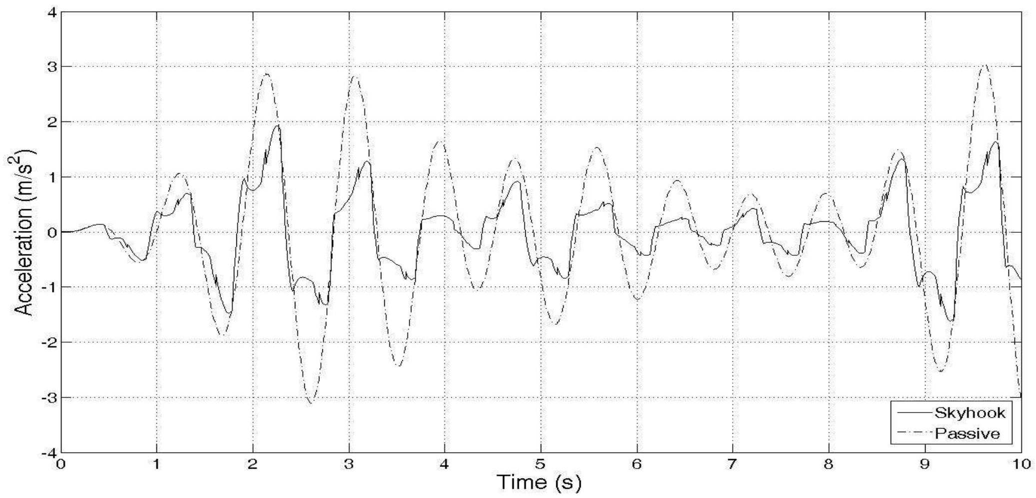

Figure 5.17: Body accelerations of the skyhook and passive systems under the sine excitation.

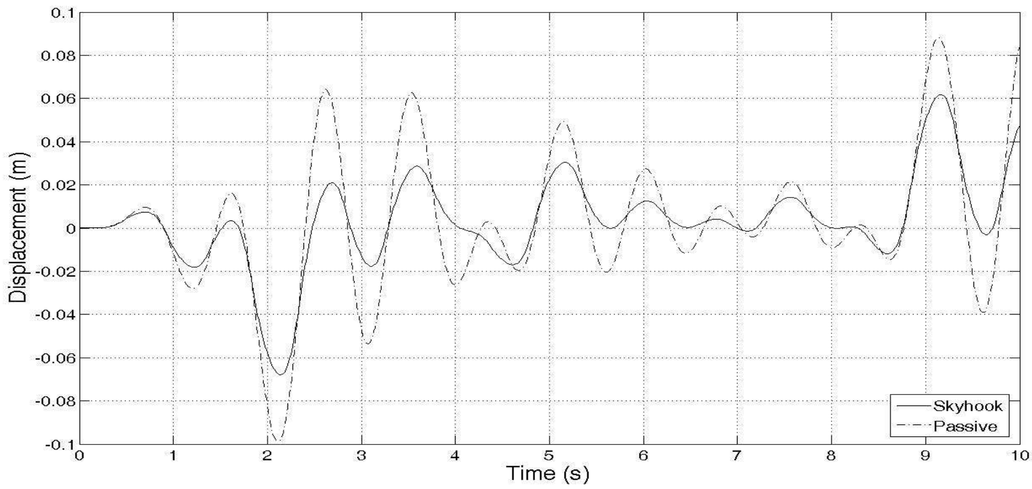

Figure 5.18: Body displacements of the skyhook and passive systems under the sine excitation.

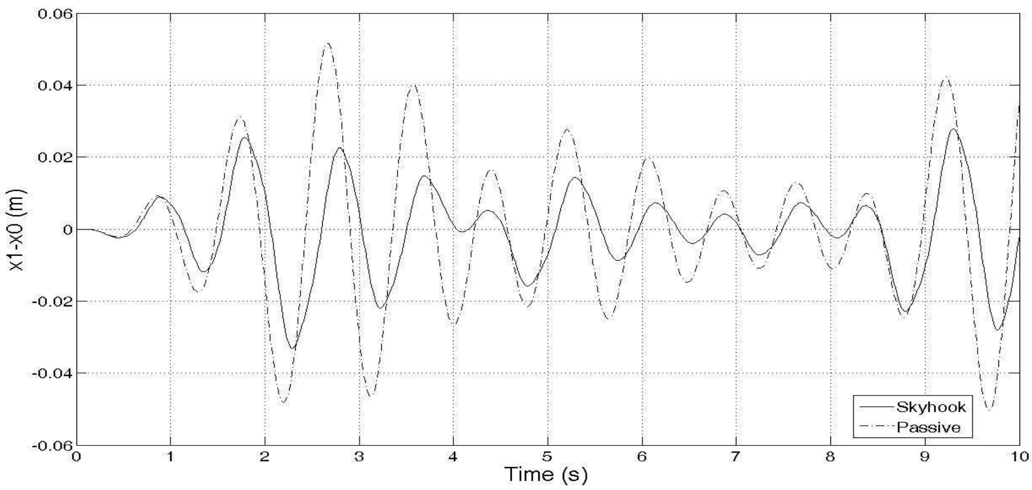

Figure 5.19: Suspension deflections of the skyhook and passive systems under the sine excitation.

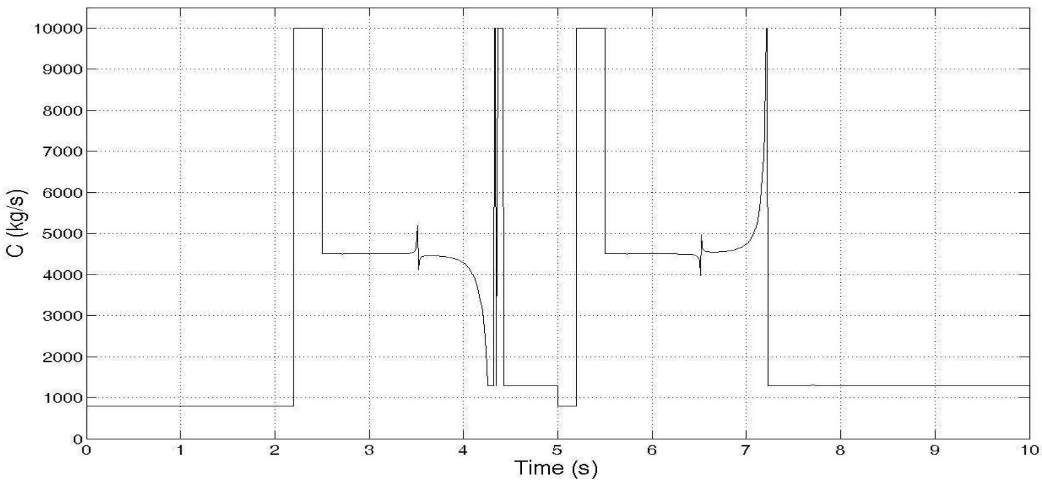

Figure 5.20: Damping coefficient of the skyhook system under the step excitation.

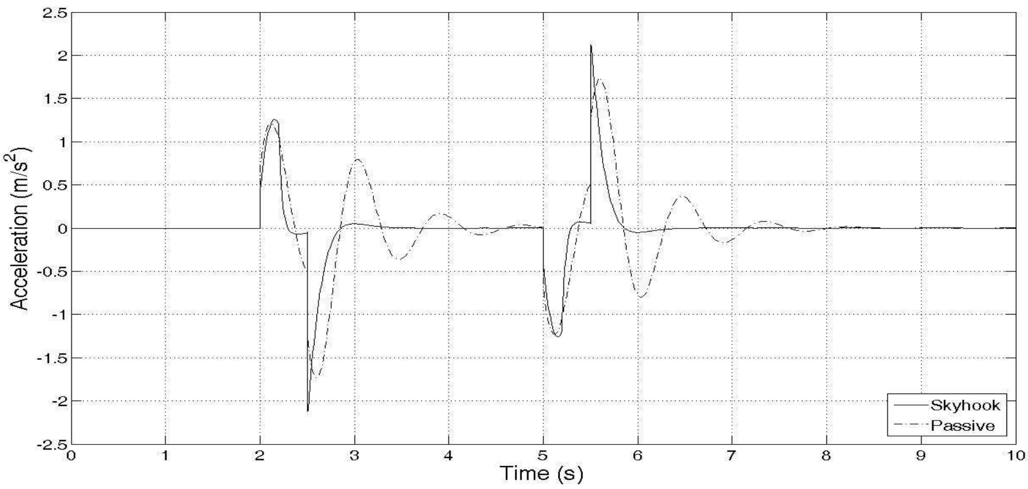

Figure 5.21: Body accelerations of the skyhook and passive systems under the step excitations.

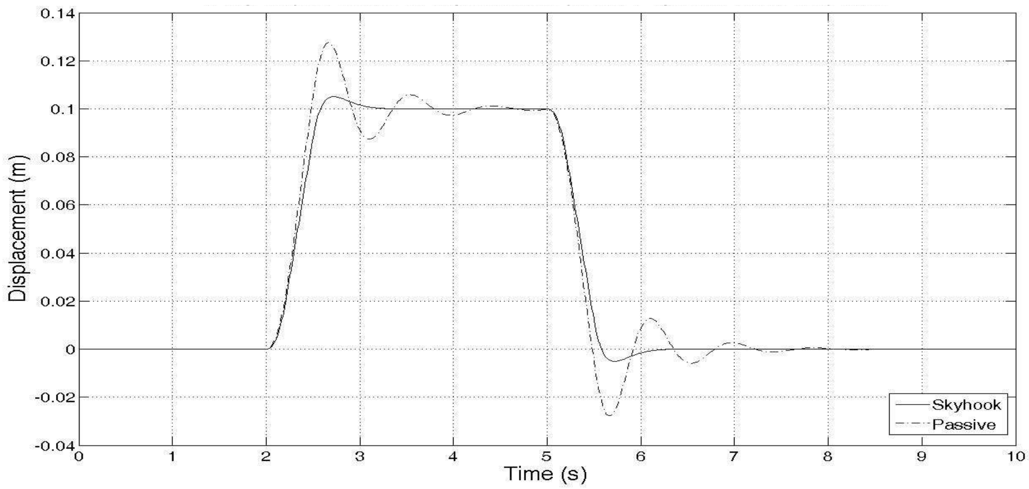

Figure 5.22: Body displacements of the skyhook and passive systems under the step excitation.

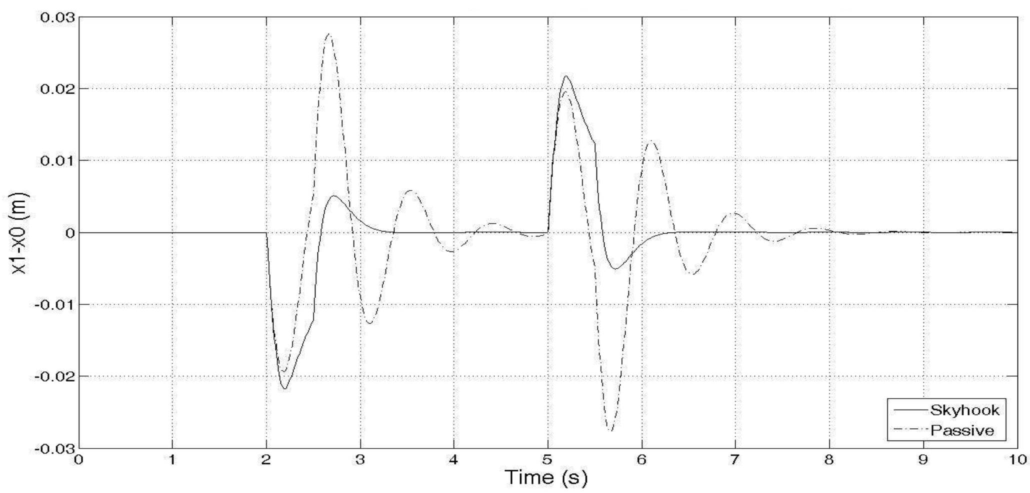

Figure 5.23: Suspension deflections of the skyhook and passive systems under the step excitation.

To compare the performance of the skyhook system for every C_{SKY} value, the RMS values of the acceleration, displacement and suspension deflection must be found. Finding the RMS values will make the evaluation of performances easier for the sine excitation. Six C_{SKY} ratios are selected and the RMS results are found for the accelerations, displacements and suspension deflections. Improvements are made according to the passive system and the results are given in percentages. This gives us the opportunity to evaluate the results affected by the parameter C_{SKY}.

Table 5.7: RMS values of the body acceleration of the skyhook and passive systems for different C_{SKY} values under the sine excitation.

C_{SKY}	Passive (RMS)	Skyhook (RMS)	Improvement (%)
2000	1,292708	0,826337	36,08
2500	1,288151	0,736905	42,79
3000	1,28538	0,689453	46,36
3511	1,296764	0,664607	48,75
4000	1,286271	0,641049	50,16
4500	1,29013	0,633889	50,87

Table 5.8: RMS values of the body displacement of the skyhook and passive systems for different C_{SKY} values under the sine excitation.

C_{SKY}	Passive (RMS)	Skyhook (RMS)	Improvement (%)
2000	0,032677	0,024467	25,12
2500	0,032553	0,022701	30,26
3000	0,032512	0,021669	33,35
3511	0,033161	0,02151	35,13
4000	0,032517	0,020562	36,77
4500	0,032687	0,020367	37,69

Table 5.9: RMS values of the suspension deflection of the skyhook and passive systems for different C_{SKY} values under the sine excitation.

C_{SKY}	Passive (RMS)	Skyhook (RMS)	Improvement (%)
2000	0,021123	0,015142	28,32
2500	0,02106	0,013605	35,40
3000	0,021054	0,012639	39,97
3511	0,021323	0,011942	43,99
4000	0,020989	0,011363	45,86
4500	0,021071	0,011037	47,62

The performance of the skyhook system is evaluated under the step excitation to see its behavior for sudden impacts. Different C_{SKY} values are used to find the improvement of the skyhook system against passive system. The improvements in terms of percentages calculated for the points with highest values for the following variables; acceleration, displacement and suspension deflection. For C_{SKY} ratio, semi active system reached to steady state earlier than the passive system. So the behavior of the skyhook system is evaluated at the points where the maximum acceleration, displacement and suspension deflection occur.

Table 5.10: Maximum body acceleration values of the skyhook and passive suspension systems for different C_{SKY} values under the step excitation.

C_{SKY}	Passive (m/s^2)	Skyhook (m/s^2)	Improvement (%)
2000	1,731979	1,130697	34,72
2500	1,731979	1,151236	33,53
3000	1,731979	1,277632	26,23
3511	1,731979	1,554667	10,24
4000	1,731979	1,830684	-5,70
4500	1,731979	2,120208	-22,42

Table 5.11: Maximum body displacement values of the skyhook and passive suspension systems for different C_{SKY} values under the step excitation.

C_{SKY}	Passive (m)	Skyhook (m)	Improvement (%)
2000	0,127583	0,114779	10,04
2500	0,127583	0,112145	12,10
3000	0,127583	0,109938	13,83
3511	0,127583	0,10803	15,33
4000	0,127583	0,106465	16,55
4500	0,127583	0,105082	17,64

Table 5.12: Maximum suspension deflection values of the skyhook and passive suspension systems for different C_{SKY} values under the step excitation.

C_{SKY}	Passive (m)	Skyhook (m)	Improvement (%)
2000	0,027583	0,014779	46,42
2500	0,027583	0,012145	55,97
3000	0,027583	0,009938	63,97
3511	0,027583	0,00803	70,89
4000	0,027583	0,006465	76,56
4500	0,027583	0,005082	81,58

The maximum C damper coefficient value affects the acceleration, displacement and suspension deflection values. So the performance of the skyhook system is evaluated for different maximum saturation levels as well. The results for both the sine and step excitations are given. For values more than 15000 kg/s saturation levels, the results begin to oscillate too much that do not give acceptable values. All table results are obtained with the C damper value of 3511 kg/s. 3511 kg/s value is found for $\xi = 0.65$ the damping ratio.

Table 5.13: RMS values of the body acceleration of the skyhook and passive system for different saturation levels under the sine excitation.

Saturation	Passive (RMS)	Skyhook (RMS)	Improvement (%)
15000	1,29362	0,635473	50,88
10000	1,29362	0,656836	49,22
5000	1,29362	0,738841	42,89

Table 5.14: RMS values of the body displacement of the skyhook and passive system for different saturation levels under the sine excitation.

Saturation	Passive (RMS)	Skyhook (RMS)	Improvement (%)
15000	0,03273	0,020586	37,10
10000	0,03273	0,02095	35,99
5000	0,03273	0,022669	30,74

Table 5.15: RMS values of the suspension deflection of the skyhook and passive system for different saturation levels under the sine excitation.

Saturation	Passive (RMS)	Skyhook (RMS)	Improvement (%)
15000	0,021111	0,012063	42,86
10000	0,021111	0,011866	43,79
5000	0,021111	0,011876	43,74

Table 5.16: Maximum body acceleration values of the skyhook and passive system for different saturation levels under the step excitation.

Saturation	Passive (m/s^2)	Skyhook (m/s^2)	Improvement (%)
15000	1,731979	1,364275	21,23
10000	1,731979	1,554667	10,24
5000	1,731979	1,965426	-13,48

Table 5.17: Maximum body displacement values of the skyhook and passive system for different saturation levels under the step excitation.

Saturation	Passive (m)	Skyhook (m)	Improvement (%)
15000	0,127583	0,106801	16,29
10000	0,127583	0,10803	15,33
5000	0,127583	0,111293	12,77

Table 5.18: Maximum suspension deflection values of the skyhook and passive system for different saturation levels under the step excitation.

Saturation	Passive (m)	Skyhook (m)	Improvement (%)
15000	0,027583	0,006801	75,34
10000	0,027583	0,00803	70,89
5000	0,027583	0,011293	59,06

5.3 State Feedback Control with Pole Placement

State feedback control is a method employed in feedback control systems to place the closed loop poles of a plant in pre-determined locations in the s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system.

It is shown that if the system considered is completely state controllable, then poles of the closed loop system may be placed at any desired locations by means of the state feedback through an appropriate state feedback gain matrix. (K. Ogata, 2001)

To find the state feedback gain matrix, closed loop poles must be selected. These poles can be selected according to the analyses made with the frequency response and transient response. The closed loop poles are selected according to these tools to obtain desired speed, bandwidth, damping ratio, etc. With this pole assignment method, the system must be proved that it meets the necessary and sufficient conditions. According to these conditions, closed loop poles can be placed in any desired location which makes the system completely state controllable (K. Ogata, 2002).

First we will present the state space equations of a simple system. With x state vector, y output signal, u control signal, A nxn constant matrix, B, nx1 constant matrix, C 1xn constant matrix, D constant, the state space representation can be written as follows;

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad \textbf{(5.17)}$$

The control signal must be chosen as if the input signal is determined by an instantaneous state. If K is defined as state feedback gain matrix, the state feedback control signal will be as follows;

$$u = -Kx \qquad \textbf{(5.18)}$$

With the state feedback system, the output is returned to the zero reference input because of the state feedback. But in general, the output is not always zero. Hence that output is returned to a zero reference signal to reach the output closer to zero in the next step. The system can be defined as;

$$\dot{x}(t) = (A - BK).x(t) \quad \textbf{(5.19)}$$

$$\dot{x}(t) = e^{(A-BK)} x(0) \quad \textbf{(5.20)}$$

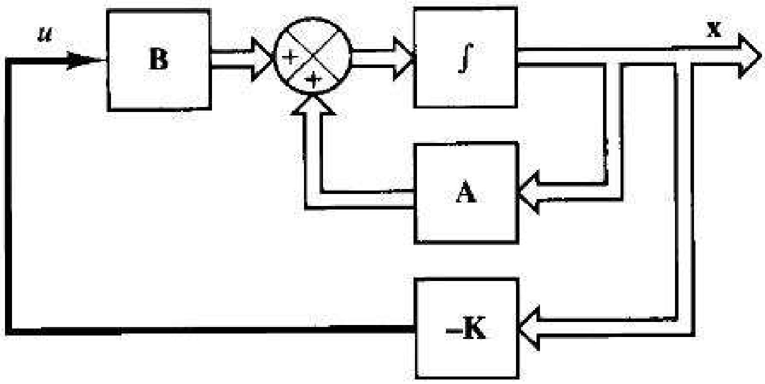

Figure 5.24: Closed looped control system with u = -Kx

x(0) defines the initial state. Stability and transient response characteristics of the system are determined by the eigenvalues of (A-BK) matrix. If the state feedback gain matrix is chosen good, the (A-BK) matrix will be a stable matrix and x(t) will reach zero in infinite time. If the poles of this matrix are at the left half plane, the system will find its stability. But in order to choose the poles arbitrarily, the system must satisfy the completely state controllable condition.

To place the eigenvalues of (A-BK) arbitrarily, the system must be completely state controllable. To approve the complete controllability, the controllability matrix M must have a rank of n.

$$RankM = Rank\left[B \vdots AB \vdots ...... \vdots A^{n-1}B\right] = n \quad \textbf{(5.21)}$$

The model of the system that is used in semi active control was defined in equation 4.1. This equation can be rewritten in state space form by applying the semi active control.

$$m\ddot{x}_1 = k(x_0 - x_1) + c(\dot{x}_0 - \dot{x}_1) \quad \textbf{(5.22)}$$

$$\ddot{x}_1 = \frac{k}{m}\dot{x}_0 + \frac{c}{m}x_0 - \frac{c}{m}\dot{x}_1 - \frac{k}{m}x_1 \quad \textbf{(5.23)}$$

$$\dot{x}_1 = y \quad \textbf{(5.24)}$$

$$\dot{y} = \frac{k}{m}\dot{x}_0 + \frac{c}{m}x_0 - \frac{c}{m}\dot{x}_1 - \frac{k}{m}x_1 \quad \textbf{(5.25)}$$

Then, state feedback closed loop control can be defined as below;

$$\begin{Bmatrix} \dot{x}_1 \\ y \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ y \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} u \tag{5.26}$$

u is the input signal and [k1 k2] is the state feedback matrix. According to these values, the damper coefficient can be defined with equation 5.31.

$$u = -Kx \tag{5.27}$$

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ y \end{Bmatrix} \tag{5.28}$$

$$u = -\frac{c}{m} y \tag{5.29}$$

$$-\frac{c}{m} y = -k_1 x_1 - k_2 y \tag{5.30}$$

$$c = k_1 m . \frac{x_1}{y} + k_2 m \tag{5.31}$$

Also the state controllability of the system is checked. The controllability is made with MATLAB. The "ctrb" command gives the result of controllability matrix M. The rank of M is 2 which satisfies the complete state controllability.

In state feedback control, since the complete state controllability is satisfied, the poles can be chosen arbitrarily. For this, damping ratio is taken into account. For a second order system, the undamped natural frequency can be calculated as;

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{5.32}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{19960}{365}} = 7.395 rad/s \cong 1.177 Hz \tag{5.33}$$

The place for the poles can be found with the rules of second order system that are defined below;

$$s_1 = \omega_n (\xi + \sqrt{\xi^2 - 1}) \tag{5.34}$$

$$s_2 = \omega_n (\xi - \sqrt{\xi^2 - 1}) \tag{5.35}$$

The state feedback matrix K is determined by using the Ackermann's formula. To this end, determination, MATLAB program is used. The "acker" command gives the state feedback matrix according to the defined closed loop poles.

The block diagram of the state feedback control is drawn in SIMULINK. The state feedback control applied to a viscous damper is inside the subsystem of the block diagram.

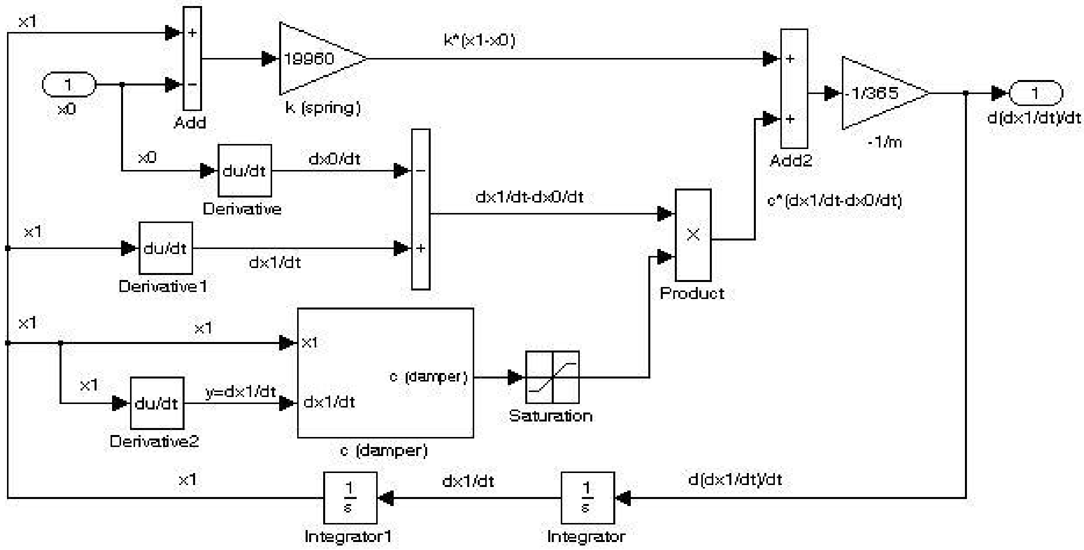

Figure 5.25: SIMULINK diagram of the semi active state feedback control system.

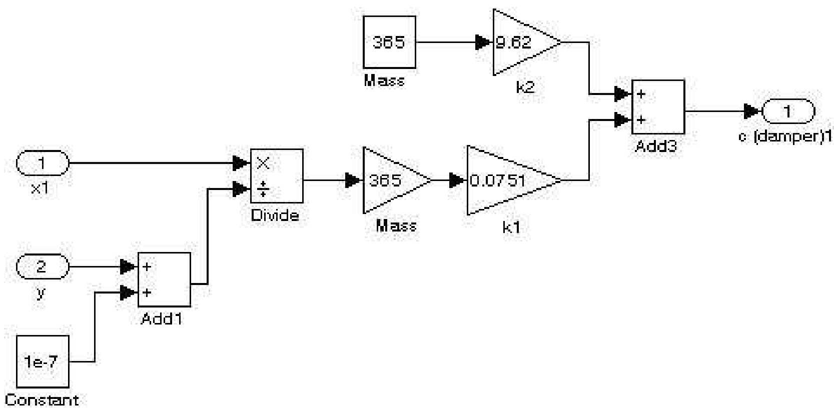

Figure 5.26: Semi active damper subsystem that shows the state feedback control.

All plots are drawn according to the poles defined with the undamped natural frequency that is found for $\xi = 0.65$ damping ratio. The performance of the control system will be evaluated with both the random sine road excitation and step shaped obstacle excitation. The performance of the state feedback control is compared with the passive suspension system. The behavior of the state feedback pole assignment is evaluated for different damping ratios.

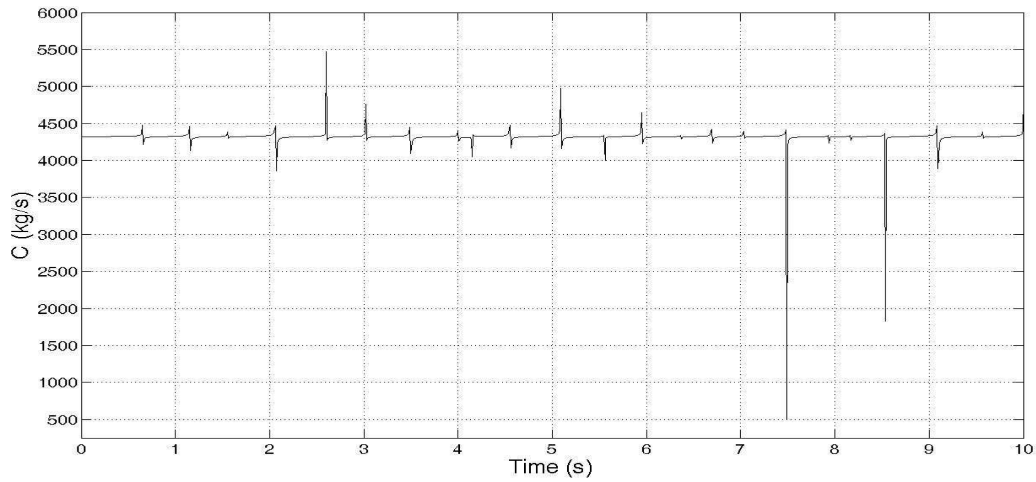

Figure 5.27: Damping coefficient for the pole assignment under the sine excitation.

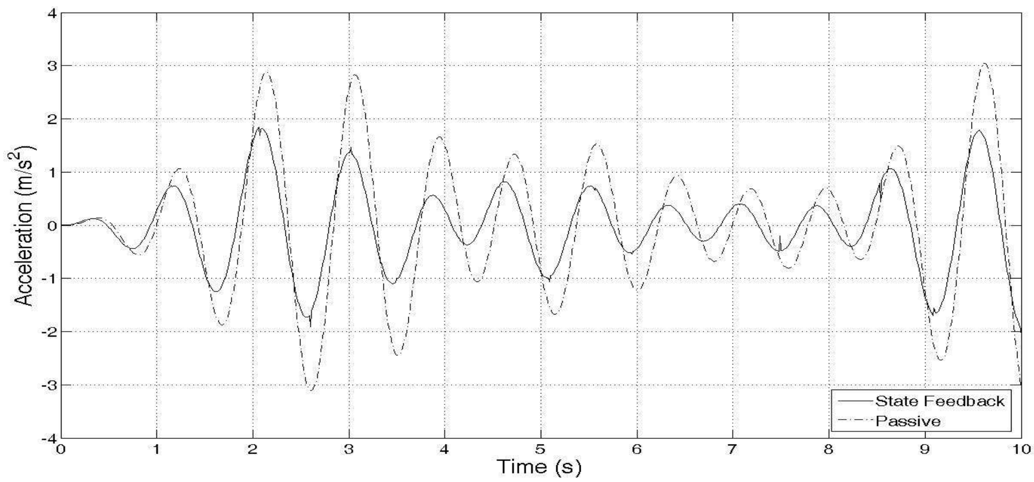

Figure 5.28: Body accelerations for the pole assignment and passive systems under the sine excitation.

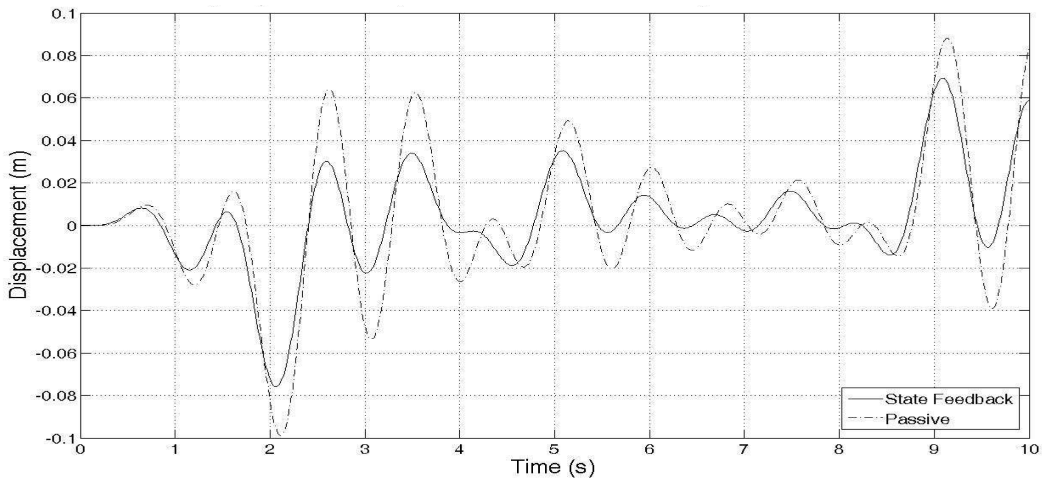

Figure 5.29: Body displacements for the pole assignment and passive systems under the sine excitation.

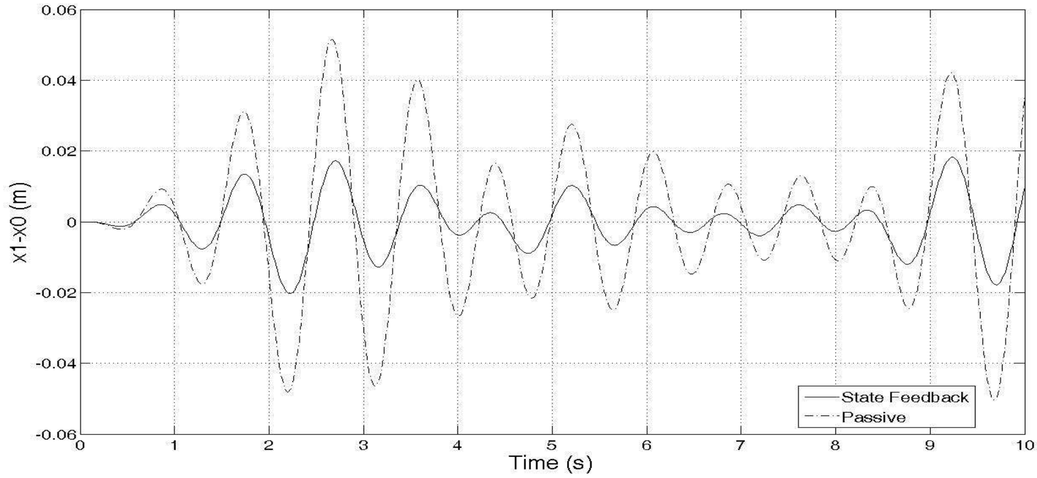

Figure 5.30: Suspension deflections for pole assignment and passive systems under the sine excitation.

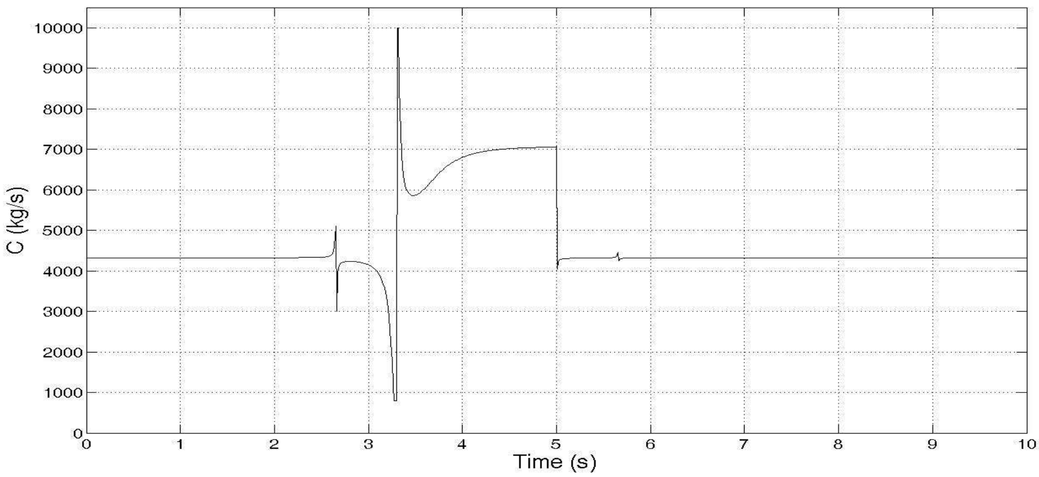

Figure 5.31: Damping coefficient for the pole assignment under the step excitation.

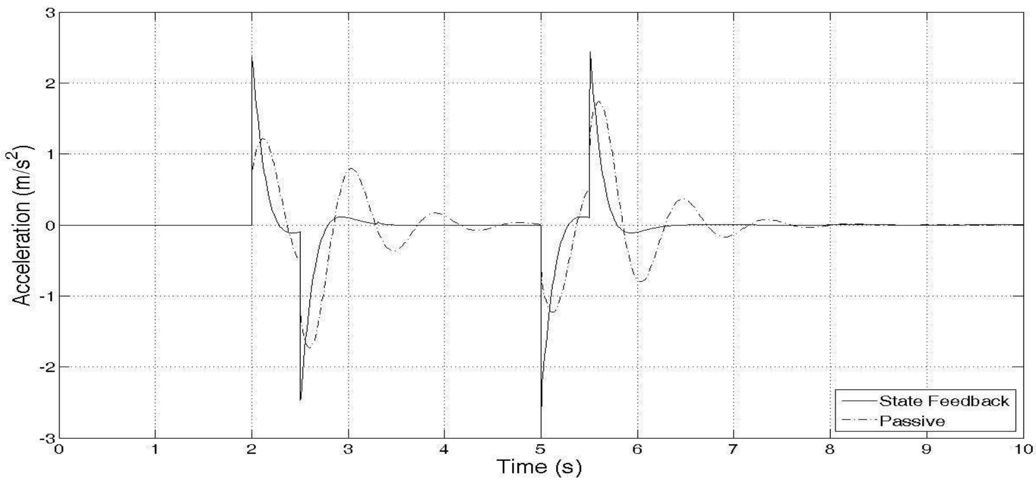

Figure 5.32: Body accelerations for the pole assignment and passive systems under the step excitation.

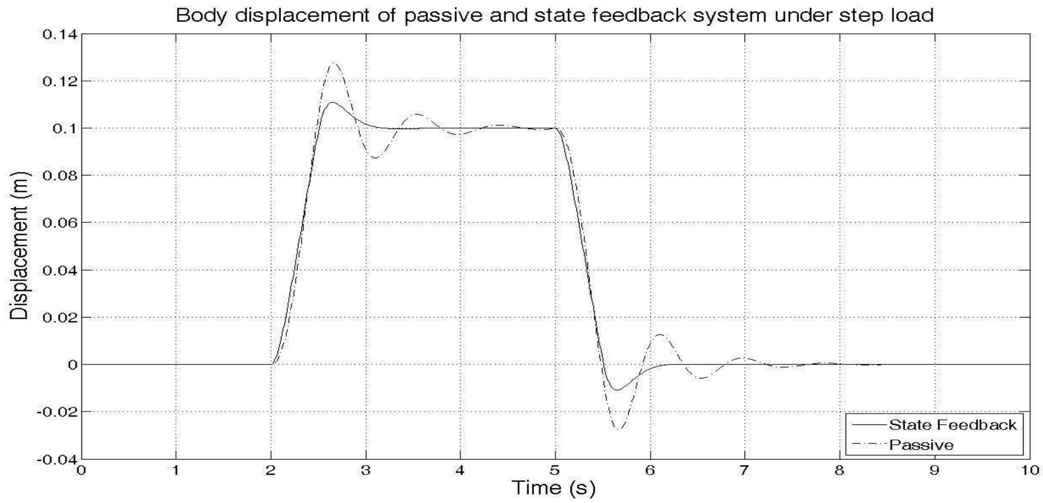

Figure 5.33: Body displacements for the pole assignment and passive systems under the step excitations.

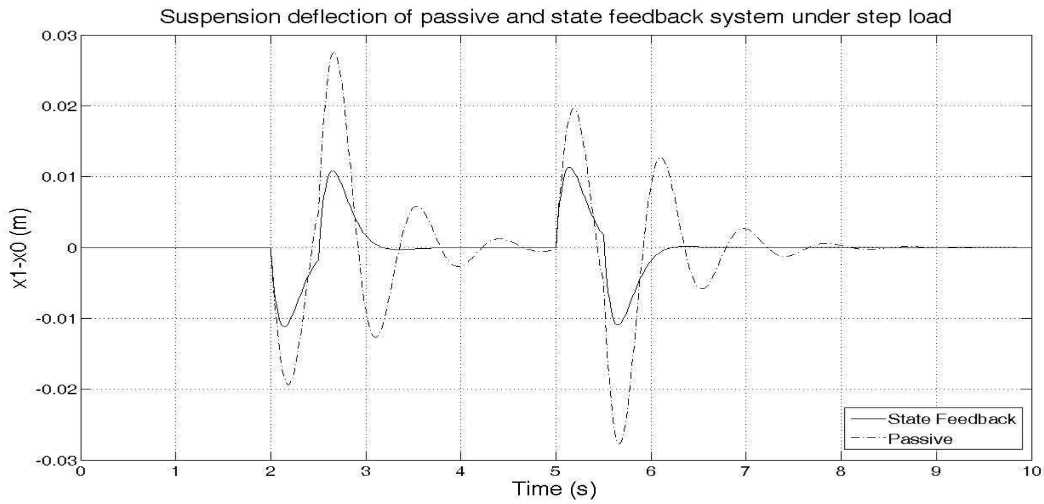

Figure 5.34: Suspension deflections for the pole assignment and passive systems under the step excitation.

The performance of the state feedback pole assignment for different ξ damping ratios is evaluated with the help of RMS values. RMS values are found for sine excitations to make the comparison between the state feedback pole assignment and passive suspension systems. Eleven different ξ damping ratio values are taken into account to see the behavior of the system under the sine and step excitations. For the damping ratio, $\xi = 2$, the system response does not change because of the saturation. The damping coefficient values pass beyond the saturation level 10000 kg/s for all process time.

Table 5.19: RMS values of the body acceleration for the pole assignment and passive systems for different ξ values under the sine excitation.

ξ (damping ratio)	Poles	k1 / k2	Passive (RMS)	State Feedback (RMS)	Improvement (%)
0,5	-3,7 ± 6,4086 i	0,0751 / 7,4	1,286266	0,878061	31,74
0,6	-4,44 ± 5,92 i	0,0751 / 8,88	1,286556	0,822401	36,08
0,7	-5,18 ± 5,2847 i	0,0751 / 10,36	1,286418	0,781624	39,24
0,8	-5,92 ± 4,44 i	0,0751 / 11,84	1,283949	0,749622	41,62
0,9	-6,66 ± 3,2256 i	0,0751 / 13,32	1,286148	0,73256	43,04
1	-7,4 / -7,4	0,0751 / 14,8	1,285491	0,716583	44,26
1,2	-13,7886 / -3,914	0,0751 / 17,76	1,288839	0,695293	46,05
1,4	-17,6105 / -3,1095	0,0751 / 20,72	1,282668	0,678129	47,13
1,6	-21,0826 / -2,5974	0,0751 / 23,68	1,288567	0,673993	47,69
1,8	-24,3953 / -2,2447	0,0751 / 26,64	1,283027	0,662813	48,34
2	-276172 / -1,9828	0,0751 / 29,6	1,283038	0,661405	48,45

Table 5.20: RMS values of the body displacement for the pole assignment and passive systems for different ξ values under the sine excitation.

ξ (damping ratio)	Poles	k1 / k2	Passive (RMS)	State Feedback (RMS)	Improvement (%)
0,5	-3,7 ± 6,4086 i	0,0751 / 7,4	0,032616	0,025905	20,58
0,6	-4,44 ± 5,92 i	0,0751 / 8,88	0,032717	0,025018	23,53
0,7	-5,18 ± 5,2847 i	0,0751 / 10,36	0,032694	0,024307	25,65
0,8	-5,92 ± 4,44 i	0,0751 / 11,84	0,032494	0,023606	27,35
0,9	-6,66 ± 3,2256 i	0,0751 / 13,32	0,032712	0,023466	28,26
1	-7,4 / -7,4	0,0751 / 14,8	0,032645	0,023111	29,21
1,2	-13,7886 / -3,914	0,0751 / 17,76	0,032586	0,022592	30,67
1,4	-17,6105 / -3,1095	0,0751 / 20,72	0,032464	0,022286	31,35
1,6	-21,0826 / -2,5974	0,0751 / 23,68	0,032655	0,022224	31,94
1,8	-24,3953 / -2,2447	0,0751 / 26,64	0,032441	0,021939	32,37
2	-276172 / -1,9828	0,0751 / 29,6	0,032444	0,021914	32,46

Table 5.21: RMS values of the suspension deflection for the pole assignment and passive systems for different ξ values under the sine excitation.

ξ (damping ratio)	Poles	k1 / k2	Passive (RMS)	State Feedback (RMS)	Improvement (%)
0,5	-3,7 ± 6,4086 i	0,0751 / 7,4	0,020811	0,011336	45,53
0,6	-4,44 ± 5,92 i	0,0751 / 8,88	0,020801	0,009657	53,57
0,7	-5,18 ± 5,2847 i	0,0751 / 10,36	0,020808	0,008431	59,48
0,8	-5,92 ± 4,44 i	0,0751 / 11,84	0,020794	0,007452	64,16
0,9	-6,66 ± 3,2256 i	0,0751 / 13,32	0,020806	0,006705	67,77
1	-7,4 / -7,4	0,0751 / 14,8	0,020814	0,006091	70,74
1,2	-13,7886 / -3,914	0,0751 / 17,76	0,020841	0,005119	75,44
1,4	-17,6105 / -3,1095	0,0751 / 20,72	0,020801	0,004425	78,73
1,6	-21,0826 / -2,5974	0,0751 / 23,68	0,020842	0,003901	81,28
1,8	-24,3953 / -2,2447	0,0751 / 26,64	0,020804	0,003478	83,28
2	-276172 / -1,9828	0,0751 / 29,6	0,020804	0,003384	83,73

Performance of the state feedback pole assignment is evaluated under the step excitation as well to see its behavior for impact displacements. Different ξ damping ratio values are selected to see how much the maximum acceleration, displacement and suspension deflections values are affected. For each ξ damping ratio, semi active system reached to steady state earlier than the passive system. So behavior of the state feedback pole assignment is evaluated for the maximum acceleration, displacement and suspension deflection points.

Table 5.22: Maximum body acceleration values for the pole assignment and passive systems for different ξ values under the step excitation.

ξ (damping ratio)	Poles	k1 / k2	Passive (m/s^2)	State Feedback (m/s2)	Improvement (%)
0,4	2,96 ± 6,7822 i	0.0751 / 5.92	1,732039	1,630997	5,83
0,5	-3,7 ± 6,4086 i	0,0751 / 7,4	1,732045	1,775494	-2,51
0,6	-4,44 ± 5,92 i	0,0751 / 8,88	1,732048	1,983048	-14,49
0,7	-5,18 ± 5,2847 i	0,0751 / 10,36	1,732021	2,204697	-27,29
0,8	-5,92 ± 4,44 i	0,0751 / 11,84	1,732005	2,468838	-42,54
0,9	-6,66 ± 3,2256 i	0,0751 / 13,32	1,731999	2,7437	-58,41
1	-7,4 / -7,4	0,0751 / 14,8	1,731997	3,024219	-74,61
1,2	-13,7886 / -3,914	0,0751 / 17,76	1,731998	3,596472	-107,65
1,4	-17,6105 / -3,1095	0,0751 / 20,72	1,732041	4,171961	-140,87
1,6	-21,0826 / -2,5974	0,0751 / 23,68	1,732038	4,759327	-174,78
1,8	-24,3953 / -2,2447	0,0751 / 26,64	1,73203	5,342535	-208,46
2	-276172 / -1,9828	0,0751 / 29,6	1,732027	5,49193	-217,08

Table 5.23: Maximum body displacement values for the pole assignment and passive systems for the different ξ values under the step excitation.

ξ (damping ratio)	Poles	k1 / k2	Passive (m)	State Feedback (m)	Improvement (%)
0,4	2,96 ± 6,7822 i	0.0751 / 5.92	0,127584	0,119906	6,02
0,5	-3,7 ± 6,4086 i	0,0751 / 7,4	0,127584	0,116753	8,49
0,6	-4,44 ± 5,92 i	0,0751 / 8,88	0,127584	0,114311	10,40
0,7	-5,18 ± 5,2847 i	0,0751 / 10,36	0,127583	0,112379	11,92
0,8	-5,92 ± 4,44 i	0,0751 / 11,84	0,127583	0,110813	13,14
0,9	-6,66 ± 3,2256 i	0,0751 / 13,32	0,127583	0,109516	14,16
1	-7,4 / -7,4	0,0751 / 14,8	0,127583	0,108439	15,01
1,2	-13,7886 / -3,914	0,0751 / 17,76	0,127583	0,106748	16,33
1,4	-17,6105 / -3,1095	0,0751 / 20,72	0,127583	0,105506	17,30
1,6	-21,0826 / -2,5974	0,0751 / 23,68	0,127583	0,10456	18,05
1,8	-24,3953 / -2,2447	0,0751 / 26,64	0,127583	0,103828	18,62
2	-276172 / -1,9828	0,0751 / 29,6	0,127583	0,103689	18,73

Table 5.24: Maximum suspension deflection values for the pole assignment and passive systems for different ξ values under the step excitation.

ξ (damping ratio)	Poles	k1 / k2	Passive (m)	State Feedback (m)	Improvement (%)
0,4	2,96 ± 6,7822 i	0.0751 / 5.92	0,027584	0,019906	27,83
0,5	-3,7 ± 6,4086 i	0,0751 / 7,4	0,027584	0,016753	39,27
0,6	-4,44 ± 5,92 i	0,0751 / 8,88	0,027584	0,014311	48,12
0,7	-5,18 ± 5,2847 i	0,0751 / 10,36	0,027583	0,012379	55,12
0,8	-5,92 ± 4,44 i	0,0751 / 11,84	0,027583	0,010813	60,80
0,9	-6,66 ± 3,2256 i	0,0751 / 13,32	0,027583	0,009516	65,50
1	-7,4 / -7,4	0,0751 / 14,8	0,027583	0,008439	69,41
1,2	-13,7886 / -3,914	0,0751 / 17,76	0,027583	0,006748	75,54
1,4	-17,6105 / -3,1095	0,0751 / 20,72	0,027584	0,005506	80,04
1,6	-21,0826 / -2,5974	0,0751 / 23,68	0,027583	0,00456	83,47
1,8	-24,3953 / -2,2447	0,0751 / 26,64	0,027583	0,003828	86,12
2	-276172 / -1,9828	0,0751 / 29,6	0,027583	0,003689	86,63

Because all MR dampers have an upper limit for the maximum damper coefficient C, saturation is set on the systems to simulate the effect of these limits. Hence, the performance of state feedback pole assignment is also evaluated under different saturation levels as well. But in state feedback control k_1 gain defines the change in damper coefficient, while k_2 defines the base C damping coefficient. So most of the time damper coefficient becomes in the vicinity of the base value defined by k_2. Hence, changing saturation level to a different value does not change performance in terms of accelerations, displacements and suspension deflections while the C damper value is not oscillating much.

Saturation levels affect the limit that state feedback pole assignment can use the maximum damping ratio. Below, the plots are given that show the limits which the maximum damping ratio can be used. Beyond those points, increasing the damping ratio does not change the acceleration, displacement and suspension deflection performances.

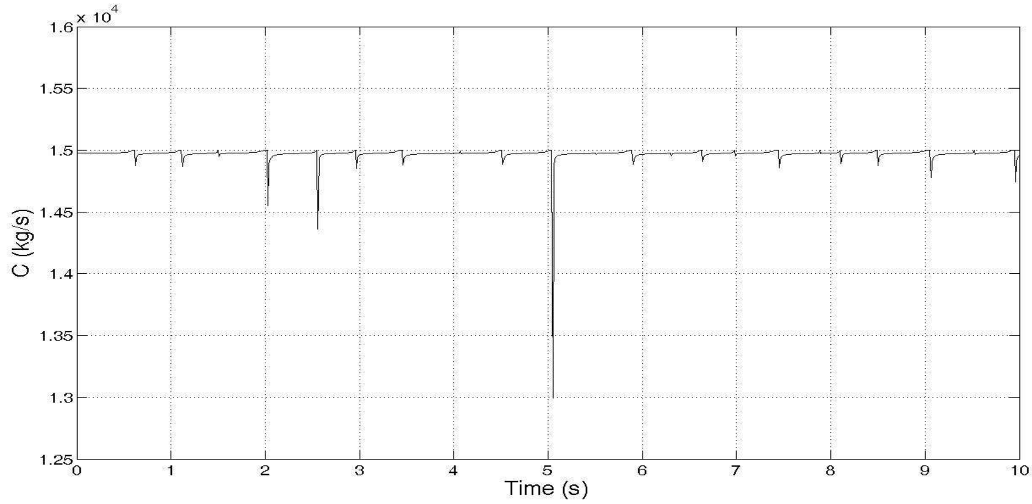

Figure 5.35: Maximum damping ratio that can be selected with 15000 kg/s saturation limit.

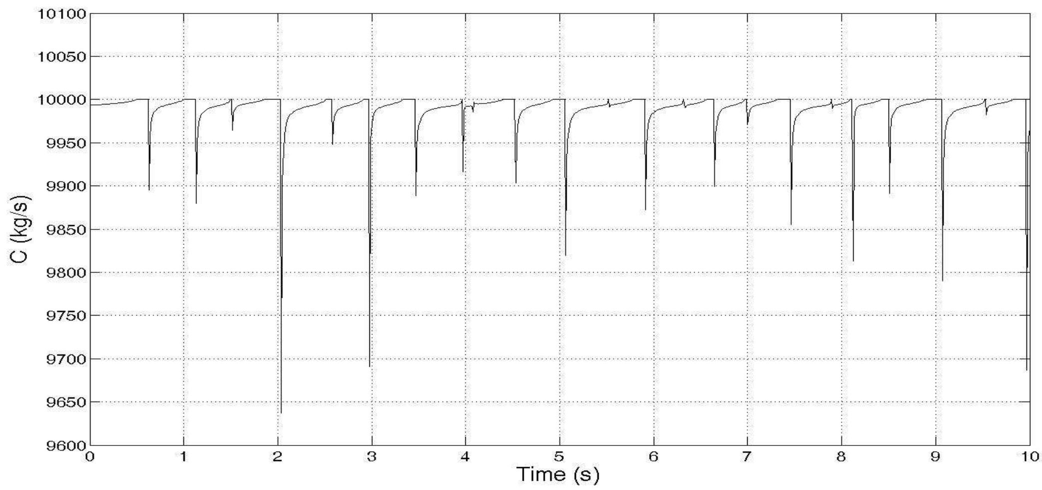

Figure 5.36: Maximum damping ratio that can be selected with 10000 kg/s saturation limit.

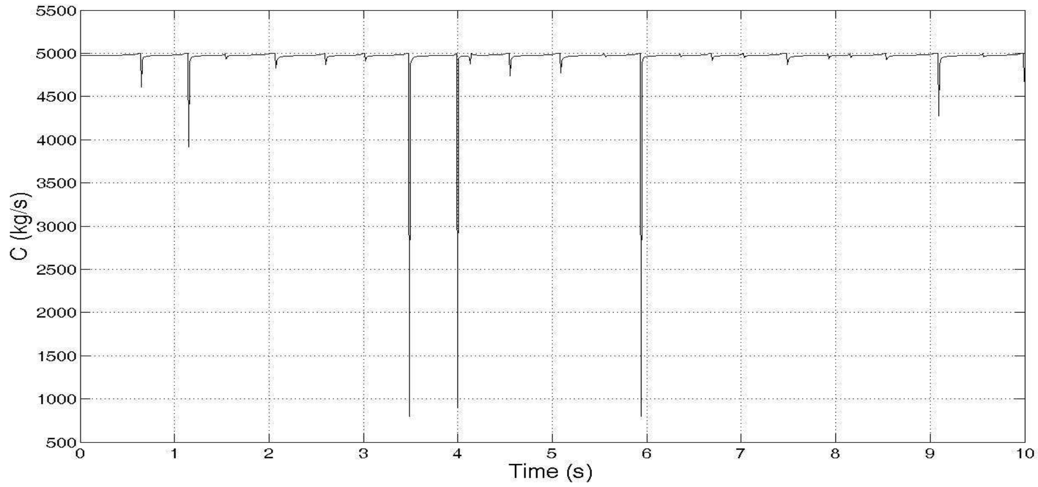

Figure 5.37: Maximum damping ratio that can be selected with 5000 kg/s saturation limit.

Natural frequency of the system affects the performance of the pole assignment system as well. Hence, for different natural frequency values, performance of the pole assignment system is evaluated under the sine and step excitations.

Table 5.25: RMS values of the body acceleration for the pole assignment and passive systems for different ω_n values under the sine excitation.

ω_n	k1 / k2	Passive (RMS)	State Feedback (RMS)	Improvement (%)
2	-50,6849 / 2,8	1,289003	1,223366	5,09
3	-45,6849 / 4,2	1,286054	1,162903	9,58
4	-38,6849 / 5,6	1,282346	1,096769	14,47
6	-18,6849 / 8,4	1,285227	0,939985	26,86
8	9,3151 / 11,52	1,28857	0,77093	40,17
10	45,3151 / 14	1,289591	0,754255	41,51
12	89,3151 / 16,8	1,289602	0,761478	40,95
15	170,3151 / 21	1,289296	0,771143	40,19

Table 5.26: RMS values of the body displacement for the pole assignment and passive systems for different ω_n values under the sine excitation.

ω_n	k1 / k2	Passive (RMS)	State Feedback (RMS)	Improvement (%)
2	-50,6849 / 2,8	0,03273	0,033874	-3,50
3	-45,6849 / 4,2	0,032682	0,032949	-0,82
4	-38,6849 / 5,6	0,032437	0,031653	2,42
6	-18,6849 / 8,4	0,032508	0,02825	13,10
8	9,3151 / 11,52	0,032725	0,023674	27,66
10	45,3151 / 14	0,032757	0,023356	28,70
12	89,3151 / 16,8	0,032701	0,023199	29,06
15	170,3151 / 21	0,032749	0,023129	29,37

Table 5.27: RMS values of the suspension deflection for the pole assignment and passive systems for different ω_n values under the sine excitation.

ω_n	k1 / k2	Passive (RMS)	State Feedback (RMS)	Improvement (%)
2	-50,6849 / 2,8	0,020807	0,021375	-2,73
3	-45,6849 / 4,2	0,020804	0,020223	2,79
4	-38,6849 / 5,6	0,020785	0,018819	9,46
6	-18,6849 / 8,4	0,020829	0,01419	31,87
8	9,3151 / 11,52	0,020798	0,008348	59,86
10	45,3151 / 14	0,020819	0,008512	59,11
12	89,3151 / 16,8	0,020823	0,008637	58,52
15	170,3151 / 21	0,020812	0,008789	57,77

Table 5.28: Maximum body acceleration values for the pole assignment and passive systems for different ω_n values under the step excitation.

ω_n	k1 / k2	Passive (m/s^2)	State Feedback (RMS)	Improvement (%)
2	-50,6849 / 2,8	1,732149	1,971576	-13,82
3	-45,6849 / 4,2	1,732154	1,917887	-10,72
4	-38,6849 / 5,6	1,732031	1,822897	-5,25
6	-18,6849 / 8,4	1,732046	1,614208	6,80
8	9,3151 / 11,52	1,732006	2,816604	-62,62
10	45,3151 / 14	1,731998	2,808583	-62,16
12	89,3151 / 16,8	1,731998	2,808583	-62,16
15	170,3151 / 21	1,731998	2,808583	-62,16

Table 5.29: Maximum body displacement values for the pole assignment and passive systems for the different ω_n values under the step excitation.

ω_n	k1 / k2	Passive (m)	State Feedback (RMS)	Improvement (%)
2	-50,6849 / 2,8	0,127585	0,134098	-5,10
3	-45,6849 / 4,2	0,127585	0,133169	-4,38
4	-38,6849 / 5,6	0,127583	0,131527	-3,09
6	-18,6849 / 8,4	0,127584	0,127917	-0,26
8	9,3151 / 11,52	0,127583	0,109185	14,42
10	45,3151 / 14	0,127582	0,10927	14,35
12	89,3151 / 16,8	0,127583	0,10927	14,35
15	170,3151 / 21	0,127583	0,10927	14,35

Table 5.30: Maximum suspension deflection values for the pole assignment and passive systems for different ω_n values under the step excitation.

ω_n	k1 / k2	Passive (m)	State Feedback (RMS)	Improvement (%)
2	-50,6849 / 2,8	0,027585	0,034098	-23,61
3	-45,6849 / 4,2	0,027585	0,033169	-20,24
4	-38,6849 / 5,6	0,027583	0,031527	-14,30
6	-18,6849 / 8,4	0,027583	0,027917	-1,21
8	9,3151 / 11,52	0,027584	0,009185	66,70
10	45,3151 / 14	0,027583	0,00927	66,39
12	89,3151 / 16,8	0,027583	0,00927	66,39
15	170,3151 / 21	0,027583	0,00927	66,39

5.4 LQR (Linear Quadratic Regulator) System

In optimal regulator problem, the aim of the system is to find a K gain matrix that minimizes a performance index. The system equations and the gain matrix can be defined as follows:

$$\dot{x} = Ax + Bu \quad \textbf{(5.36)}$$

$$u = -Kx \quad \textbf{(5.37)}$$

Then, if these are the system and the gain matrix K, optimal control can be provided by minimizing the performance index defined as;

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \quad \textbf{(5.38)}$$

where Q and R are nxn real symmetric matrices. While Q defines the expenditure of energy of states, R defines the expenditure of energy of control signals. These matrices show the error and expenditure of energy of the related parameters (Ogata, 2001).

If the elements of the gain matrix K can be found for the minimum performance index, then u=-Kx becomes optimal. This is called optimal control law.

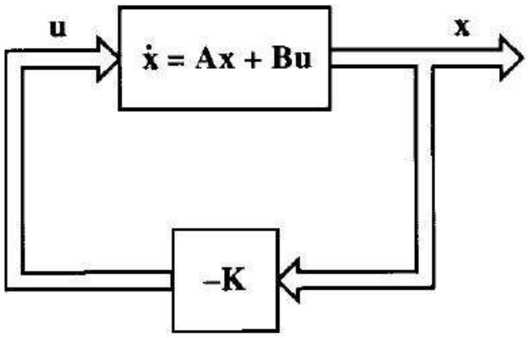

Figure 5.38: Optimal regulator system.

In optimal control, if the A-BK matrix is stable, then this method always gives zero asymptotic error. To find the optimal K gain matrix, the reduced-matrix Riccati equation (5.39) must be solved for the P matrix. If P matrix exists, the system or A-BK matrix is stable (Ogata, 2001). Please check Modern Control Engineering, Ogata (2001) for details.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad \textbf{(5.39)}$$

We can define the P matrix for our system as defined with the following equations below;

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}$$

$$B = [0 \quad 1]$$

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt = x^T P x \qquad \textbf{(5.40)}$$

If a positive-definite P matrix can be found from the reduced-matrix Riccati equation, then the system or A-BK matrix is stable.

LQR diagram is the same as state feedback pole assignment SIMULINK diagram. The difference between them is in the calculation method of the K gain matrix.

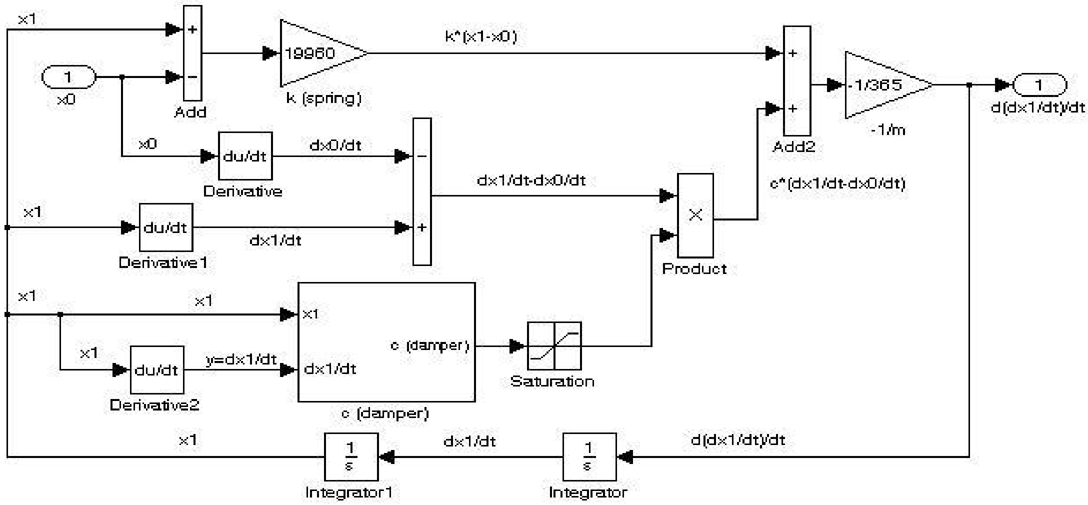

Figure 5.39: SIMULINK diagram of the semi active LQR control system.

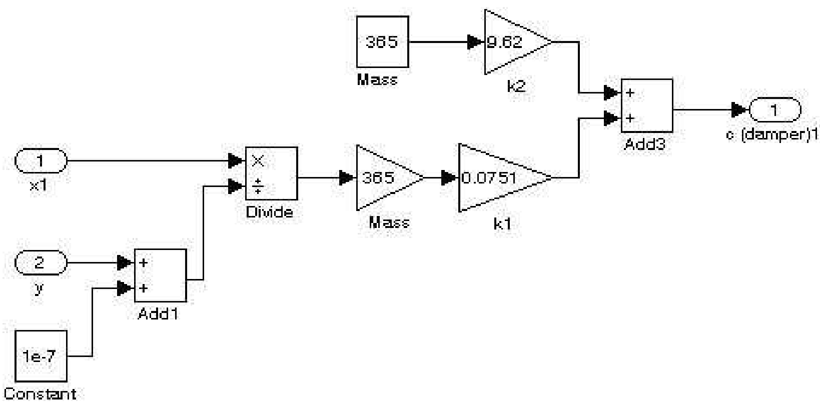

Figure 5.40: Semi active damper subsystem that shows the LQR control

The LQR system is evaluated under the step and sine excitations as we did in previous three control systems. For the sine excitation analysis, the gain matrix, the Q matrix and the R matrix are used as presented below:

$$K = \begin{bmatrix} 0.0914 & 10.0091 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1000 & 0 \\ 0 & 10000 \end{bmatrix}$$

$$R = \begin{bmatrix} 100 \end{bmatrix}$$

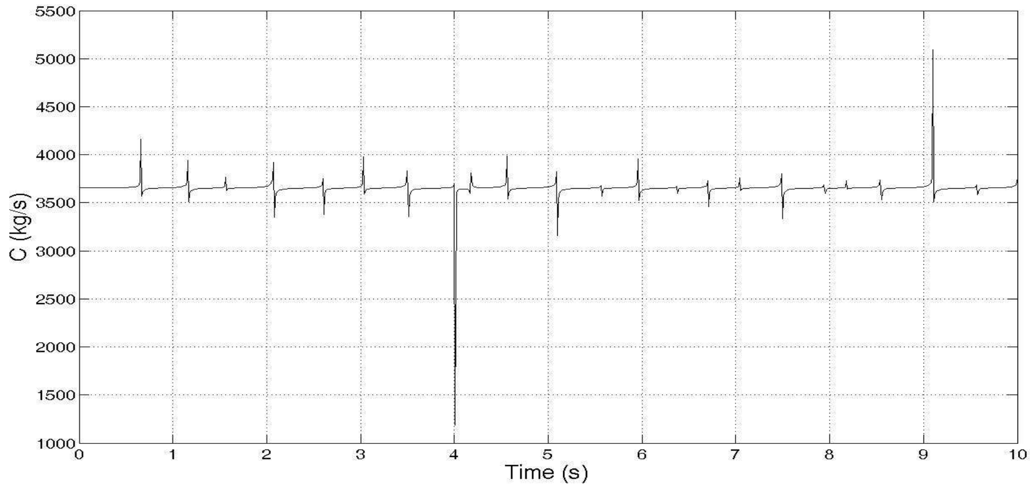

Figure 5.41: Damping coefficient of the LQR under the sine excitation.

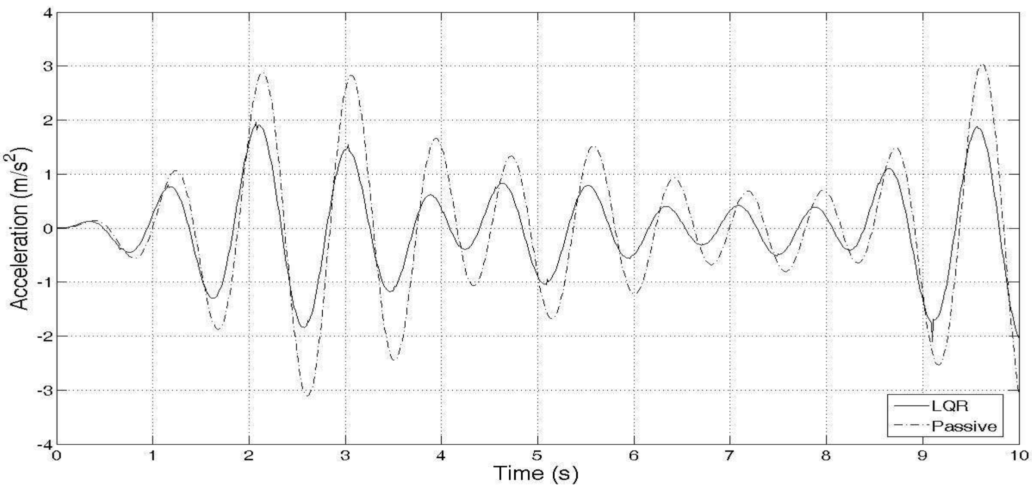

Figure 5.42: Body accelerations of the LQR and passive systems under the sine excitation.

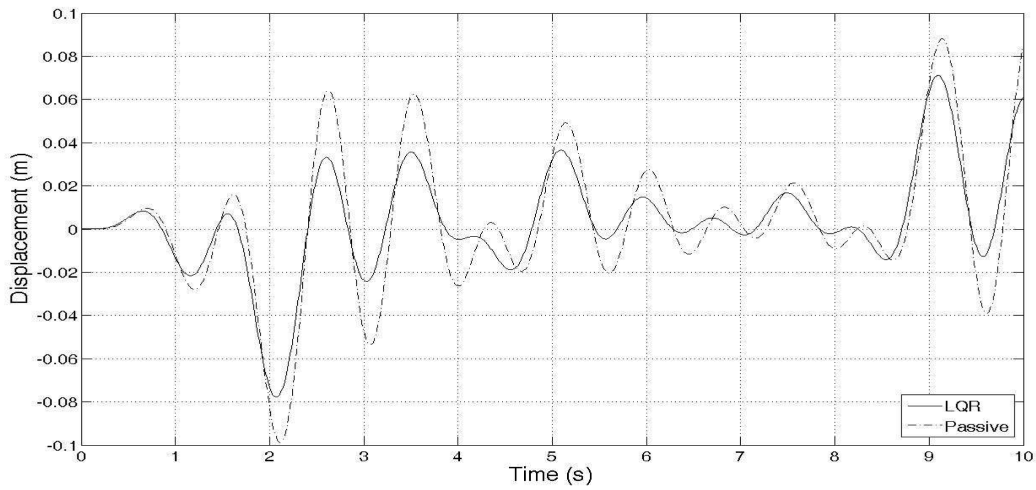

Figure 5.43: Body displacements of the LQR and passive systems under the sine excitation.

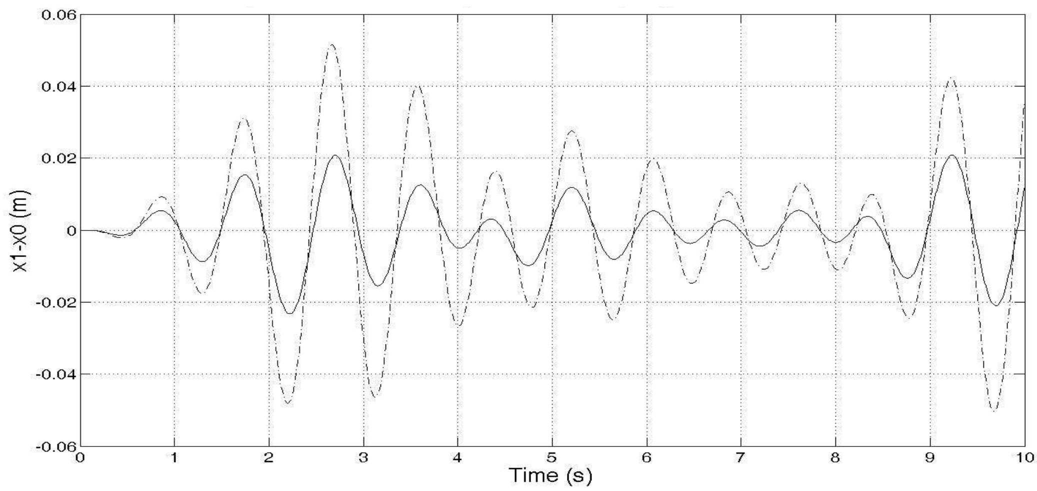

Figure 5.44: Suspension deflections of the LQR and passive systems under the sine excitation.

For the step excitation, the K gain matrix, the Q matrix and the R matrix are used that is given below:

$$K = [0.0914 \quad 5.4939]$$

$$Q = \begin{bmatrix} 1000 & 0 \\ 0 & 3000 \end{bmatrix}$$

$$R = [100]$$

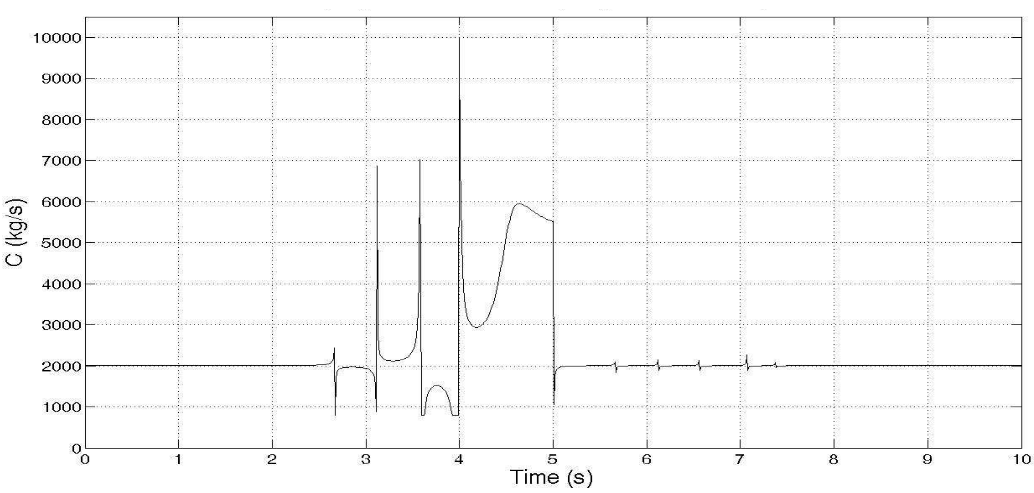

Figure 5.45: Damping coefficient of the LQR under the step excitation.

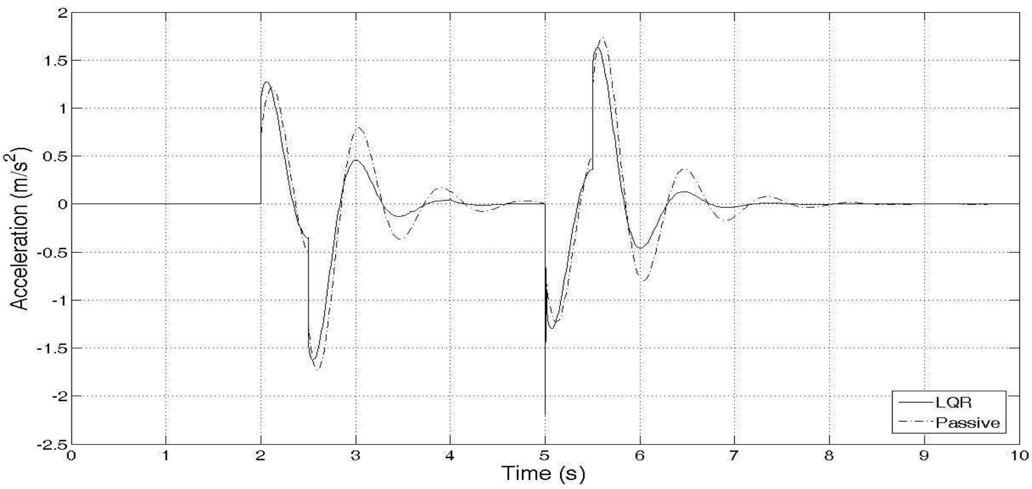

Figure 5.46: Body accelerations of the LQR and passive systems under the step excitation.

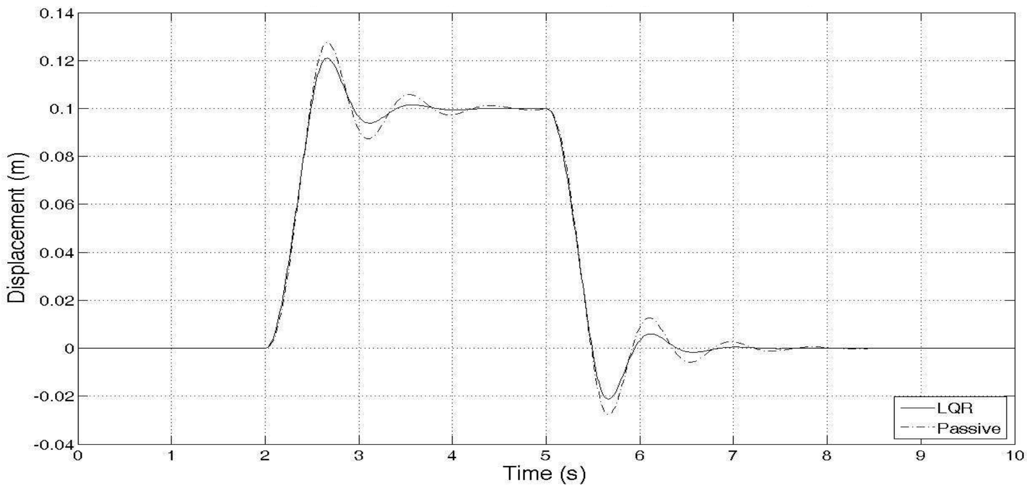

Figure 5.47: Body displacements of the LQR and passive systems under the step excitation.

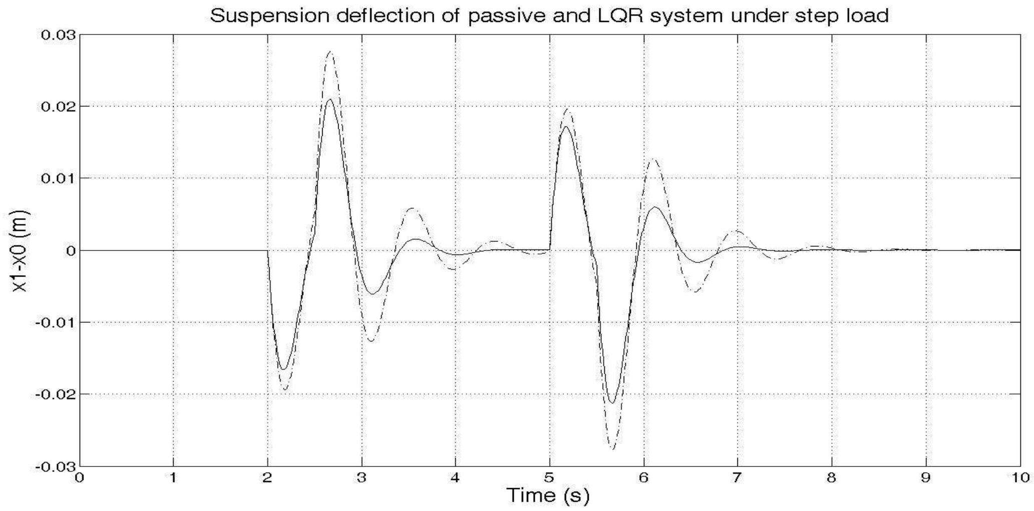

Figure 5.48: Suspension deflection of LQR and passive system under step load.

The performance of the LQR system is evaluated by changing three parameters. The changes in acceleration, body displacement and suspension deflection values are given in tables with the change of Q(1,1), Q(2,2) and R(1,1). When Q(1,1) changes, R(1,1) is held constant as 1 and Q(2,2) is held constant as 10. Every change of the parameter is calculated for both the sine and step excitations.

Table 5.31: RMS values of body acceleration for the LQR with different Q(1,1) values and passive system under the sine excitation.

Q(1,1)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
10	0,0914 / 3,191	1,285058	1,361809	-5,97
20	0,1826 / 3,2195	1,289451	1,355697	-5,14
30	0,2736 / 3,2476	1,285108	1,349524	-5,01
50	0,4553 / 3,3031	1,290739	1,348493	-4,47
75	0,6815 / 3,3709	1,286591	1,313671	-2,10
100	0,9068 / 3,4371	1,288253	1,299446	-0,87
150	1,3547 / 3,5650	1,292058	1,274176	1,38
200	1,7991 / 3,6876	1,285741	1,241177	3,47
300	2,6774 / 3,9185	1,287116	1,171791	8,96
400	3,5426 / 4,1334	1,287958	1,150186	10,70
500	4,3950 / 4,3348	1,284667	1,111286	13,50
750	6,4742 / 4,7905	1,285025	1,058026	17,66
1000	8,4850 / 5,1933	1,289304	1,008603	21,77
1500	12,3258 / 5,8866	1,293658	0,958243	25,93
2000	15,9581 / 6,4743	1,288504	0,919837	28,61

Table 5.32: RMS values of body displacement for the LQR with different Q(1,1) values and passive system under the sine excitation.

Q(1,1)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
10	0,0914 / 3,191	0,032545	0,033771	-3,77
20	0,1826 / 3,2195	0,032814	0,03387	-3,22
30	0,2736 / 3,2476	0,032657	0,03361	-2,92
50	0,4553 / 3,3031	0,032653	0,033262	-1,87
75	0,6815 / 3,3709	0,032693	0,032897	-0,62
100	0,9068 / 3,4371	0,032594	0,032442	0,47
150	1,3547 / 3,5650	0,032753	0,031868	2,70
200	1,7991 / 3,6876	0,032637	0,031187	4,44
300	2,6774 / 3,9185	0,032637	0,03014	7,65
400	3,5426 / 4,1334	0,032626	0,029276	10,27
500	4,3950 / 4,3348	0,032646	0,028736	11,98
750	6,4742 / 4,7905	0,032628	0,027341	16,20
1000	8,4850 / 5,1933	0,032785	0,026512	19,13
1500	12,3258 / 5,8866	0,032798	0,025184	23,21
2000	15,9581 / 6,4743	0,032676	0,02428	25,69

Table 5.33: RMS values of suspension deflection for the LQR with different Q(1,1) values and passive system under the sine excitation.

Q(1,1)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
10	0,0914 / 3,191	0,020816	0,022394	-7,58
20	0,1826 / 3,2195	0,020832	0,022186	-6,50
30	0,2736 / 3,2476	0,020796	0,02204	-5,98
50	0,4553 / 3,3031	0,020863	0,021634	-3,70
75	0,6815 / 3,3709	0,020821	0,021099	-1,34
100	0,9068 / 3,4371	0,020832	0,020619	1,02
150	1,3547 / 3,5650	0,02086	0,019699	5,57
200	1,7991 / 3,6876	0,020828	0,01895	9,02
300	2,6774 / 3,9185	0,020796	0,017536	15,68
400	3,5426 / 4,1334	0,020806	0,016461	20,88
500	4,3950 / 4,3348	0,020822	0,015789	24,17
750	6,4742 / 4,7905	0,020779	0,013945	32,89
1000	8,4850 / 5,1933	0,020833	0,012749	38,80
1500	12,3258 / 5,8866	0,02088	0,011354	45,62
2000	15,9581 / 6,4743	0,02079	0,010439	49,79

Table 5.34: Maximum body acceleration values for the LQR with different Q(1,1) values and passive system under the step excitation.

Q(1,1)	k1 / k2	Passive (m/s^2)	LQR (m/s^2)	Improvement (%)
10	0,0914 / 3,191	1,732162	1,782765	-2,92
20	0,1826 / 3,2195	1,732162	1,782182	-2,89
30	0,2736 / 3,2476	1,732162	1,781938	-2,87
50	0,4553 / 3,3031	1,732162	1,781549	-2,85
75	0,6815 / 3,3709	1,732162	1,781284	-2,84
100	0,9068 / 3,4371	1,732162	1,781227	-2,83
150	1,3547 / 3,5650	1,732162	1,7864	-3,13
200	1,7991 / 3,6876	1,732162	1,792553	-3,49
300	2,6774 / 3,9185	1,732162	1,815488	-4,81
400	3,5426 / 4,1334	1,732162	1,847995	-6,69
500	4,3950 / 4,3348	1,732162	1,888423	-9,02
750	6,4742 / 4,7905	1,732162	2,016787	-16,43
1000	8,4850 / 5,1933	1,732162	2,175607	-25,60
1500	12,3258 / 5,8866	1,732162	2,558439	-47,70
2000	15,9581 / 6,4743	1,732162	2,978703	-71,96

Table 5.35: Maximum body displacement values for the LQR with different Q(1,1) values and passive system under the step excitation.

Q(1,1)	k1 / k2	Passive (m)	LQR (m)	Improvement (%)
10	0,0914 / 3,191	0,127586	0,128909	-1,04
20	0,1826 / 3,2195	0,127586	0,12863	-0,82
30	0,2736 / 3,2476	0,127586	0,128363	-0,61
50	0,4553 / 3,3031	0,127586	0,127836	-0,20
75	0,6815 / 3,3709	0,127586	0,127198	0,30
100	0,9068 / 3,4371	0,127586	0,126579	0,79
150	1,3547 / 3,5650	0,127586	0,125397	1,72
200	1,7991 / 3,6876	0,127586	0,124297	2,58
300	2,6774 / 3,9185	0,127586	0,122322	4,13
400	3,5426 / 4,1334	0,127586	0,120564	5,50
500	4,3950 / 4,3348	0,127586	0,119014	6,72
750	6,4742 / 4,7905	0,127586	0,115803	9,24
1000	8,4850 / 5,1933	0,127586	0,113316	11,18
1500	12,3258 / 5,8866	0,127586	0,109804	13,94
2000	15,9581 / 6,4743	0,127586	0,107268	15,92

Table 5.36: Maximum suspension deflection values for the LQR with different Q(1,1) values and passive system under the step excitation.

Q(1,1)	k1 / k2	Passive (m)	LQR (m)	Improvement (%)
10	0,0914 / 3,191	0,027586	0,028909	-4,80
20	0,1826 / 3,2195	0,027586	0,02863	-3,78
30	0,2736 / 3,2476	0,027586	0,028363	-2,82
50	0,4553 / 3,3031	0,027586	0,027836	-0,91
75	0,6815 / 3,3709	0,027586	0,027198	1,41
100	0,9068 / 3,4371	0,027586	0,026579	3,65
150	1,3547 / 3,5650	0,027586	0,025397	7,94
200	1,7991 / 3,6876	0,027586	0,024297	11,92
300	2,6774 / 3,9185	0,027586	0,022322	19,08
400	3,5426 / 4,1334	0,027586	0,020564	25,45
500	4,3950 / 4,3348	0,027586	0,019014	31,07
750	6,4742 / 4,7905	0,027586	0,015803	42,71
1000	8,4850 / 5,1933	0,027586	0,013316	51,73
1500	12,3258 / 5,8866	0,027586	0,009804	64,46
2000	15,9581 / 6,4743	0,027586	0,007268	73,65

When Q(2,2) is changed, R(1,1) is held constant as 100 and Q(1,1) is held constant as 1000. The results can be seen for both cases that while Q(2,2) is less than Q(1,1) and Q(2,2) is larger than Q(1,1). The outputs are computed for different parameter values under both the sine and step excitations.

Table 5.37: RMS values of body acceleration for the LQR with different Q(2,2) values and passive system under the sine excitation.

Q(2,2)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
100	0,0914 / 1,0875	1,287728	1,734194	-34,67
500	0,0914 / 2,2766	1,285235	1,687909	-31,33
1000	0,0914 / 3,1910	1,285058	1,361809	-5,97
2000	0,0914 / 4,4925	1,286597	1,118943	13,03
3000	0,0914 / 5,4939	1,288143	1,010345	21,57
5000	0,0914 / 7,0840	1,291525	0,898718	30,41
10000	0,0914 / 10,0091	1,285739	0,787471	38,75
15000	0,0914 / 12,2549	1,288454	0,748767	41,89
25000	0,0914 / 15,8172	1,289146	0,708832	45,02

Table 5.38: RMS values of body displacement for the LQR with different Q(2,2) values and passive system under the sine excitation.

Q(2,2)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
100	0,0914 / 1,0875	0,032682	0,039856	-21,95
500	0,0914 / 2,2766	0,032556	0,038978	-19,73
1000	0,0914 / 3,1910	0,032545	0,033771	-3,77
2000	0,0914 / 4,4925	0,032674	0,029933	8,39
3000	0,0914 / 5,4939	0,032752	0,028161	14,02
5000	0,0914 / 7,0840	0,032815	0,026334	19,75
10000	0,0914 / 10,0091	0,032529	0,024251	25,45
15000	0,0914 / 12,2549	0,032725	0,023701	27,58
25000	0,0914 / 15,8172	0,032651	0,022889	29,90

Table 5.39: RMS values of suspension deflection for the LQR with different Q(2,2) values and passive system under the sine excitation.

Q(2,2)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
100	0,0914 / 1,0875	0,020838	0,0299	-43,49
500	0,0914 / 2,2766	0,020839	0,028926	-38,81
1000	0,0914 / 3,1910	0,020816	0,022394	-7,58
2000	0,0914 / 4,4925	0,020826	0,017165	17,58
3000	0,0914 / 5,4939	0,020822	0,014579	29,98
5000	0,0914 / 7,0840	0,020846	0,011763	43,57
10000	0,0914 / 10,0091	0,020824	0,008692	58,26
15000	0,0914 / 12,2549	0,020836	0,007247	65,22
25000	0,0914 / 15,8172	0,020852	0,005721	72,56

Table 5.40: Maximum body acceleration values for the LQR with different Q(2,2) values and passive system under the step excitation..

Q(2,2)	k1 / k2	Passive (m/s^2)	LQR (m/s^2)	Improvement (%)
100	0,0914 / 1,0875	1,732162	1,976731	-14,12
500	0,0914 / 2,2766	1,732162	1,958257	-13,05
1000	0,0914 / 3,1910	1,732162	1,782765	-2,92
2000	0,0914 / 4,4925	1,732162	1,647048	4,91
3000	0,0914 / 5,4939	1,732162	1,621845	6,37
5000	0,0914 / 7,0840	1,732162	1,734851	-0,16
10000	0,0914 / 10,0091	1,732162	2,152511	-24,27
15000	0,0914 / 12,2549	1,732162	2,544433	-46,89
25000	0,0914 / 15,8172	1,732162	3,21677	-85,71

Table 5.41: Maximum body displacement values for the LQR with different Q(2,2) values and passive system under the step excitation.

Q(2,2)	k1 / k2	Passive (m)	LQR (m)	Improvement (%)
100	0,0914 / 1,0875	0,127586	0,134188	-5,17
500	0,0914 / 2,2766	0,127586	0,133524	-4,65
1000	0,0914 / 3,1910	0,127586	0,128909	-1,04
2000	0,0914 / 4,4925	0,127586	0,123925	2,87
3000	0,0914 / 5,4939	0,127586	0,120969	5,19
5000	0,0914 / 7,0840	0,127586	0,11734	8,03
10000	0,0914 / 10,0091	0,127586	0,112785	11,60
15000	0,0914 / 12,2549	0,127586	0,110413	13,46
25000	0,0914 / 15,8172	0,127586	0,107789	15,52

Table 5.42: Maximum suspension deflection values for the LQR with different Q(2,2) values and passive system under the step excitation.

Q(2,2)	k1 / k2	Passive (m)	LQR (m)	Improvement (%)
100	0,0914 / 1,0875	0,027586	0,034188	-23,93
500	0,0914 / 2,2766	0,027586	0,033524	-21,53
1000	0,0914 / 3,1910	0,027586	0,028909	-4,80
2000	0,0914 / 4,4925	0,027586	0,023925	13,27
3000	0,0914 / 5,4939	0,027586	0,020969	23,99
5000	0,0914 / 7,0840	0,027586	0,01734	37,14
10000	0,0914 / 10,0091	0,027586	0,012785	53,65
15000	0,0914 / 12,2549	0,027586	0,010413	62,25
25000	0,0914 / 15,8172	0,027586	0,007789	71,76

Finally, when R(1,1) is changed, Q(2,2) is held constant as 1000 and Q(1,1) is held constant as 1000. The results can be seen both for R(1,1) is less than Q(1,1) and Q(2,2), and vice versa.

Table 5.43: RMS values of body acceleration for the LQR with different R(1,1) values and passive system under the sine excitation.

R(1,1)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
1	8,4850 / 31,89	1,288837	0,680893	47,17
5	1,7991 / 14,2688	1,294682	0,730023	43,61
10	0,9068 / 10,0903	1,294118	0,795116	38,56
20	0,4553 / 7,1352	1,288441	0,889193	30,99
30	0,3039 / 5,8259	1,289015	0,974379	24,41
50	0,1826 / 4,5128	1,288353	1,11766	13,25
100	0,0914 / 3,1910	1,285058	1,361809	-5,97
150	0,0609 / 2,6055	1,286542	1,551032	-20,56
250	0,0366 / 2,0182	1,287002	1,734082	-34,74

Table 5.44: RMS values of body displacement for the LQR with different R(1,1) values and passive system under the sine excitation.

R(1,1)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
1	8,4850 / 31,89	0,032644	0,022452	31,22
5	1,7991 / 14,2688	0,032899	0,023327	29,10
10	0,9068 / 10,0903	0,032939	0,024483	25,67
20	0,4553 / 7,1352	0,032758	0,026181	20,08
30	0,3039 / 5,8259	0,032728	0,027561	15,79
50	0,1826 / 4,5128	0,032805	0,029992	8,57
100	0,0914 / 3,1910	0,032545	0,033771	-3,77
150	0,0609 / 2,6055	0,032679	0,036922	-12,98
250	0,0366 / 2,0182	0,032705	0,039891	-21,97

Table 5.45: RMS values of suspension deflection for the LQR with different R(1,1) values and passive system under the sine excitation.

R(1,1)	k1 / k2	Passive (RMS)	LQR (RMS)	Improvement (%)
1	8,4850 / 31,89	0,020793	0,00402	80,67
5	1,7991 / 14,2688	0,02085	0,006324	69,67
10	0,9068 / 10,0903	0,020859	0,008591	58,81
20	0,4553 / 7,1352	0,020827	0,011579	44,40
30	0,3039 / 5,8259	0,020828	0,013771	33,88
50	0,1826 / 4,5128	0,020824	0,017074	18,01
100	0,0914 / 3,1910	0,020816	0,022394	-7,58
150	0,0609 / 2,6055	0,020824	0,026237	-25,99
250	0,0366 / 2,0182	0,020828	0,029897	-43,54

Table 5.46: Maximum body acceleration values for the LQR with different R(1,1) values and passive system under the step excitation.

R(1,1)	k1 / k2	Passive (m/s^2)	LQR (m/s^2)	Improvement (%)
1	8,4850 / 31,89	1,732027	5,49193	-217,08
5	1,7991 / 14,2688	1,731997	3,076064	-77,60
10	0,9068 / 10,0903	1,732024	2,23269	-28,91
20	0,4553 / 7,1352	1,732044	1,770645	-2,23
30	0,3039 / 5,8259	1,732039	1,642303	5,18
50	0,1826 / 4,5128	1,73203	1,64991	4,74
100	0,0914 / 3,1910	1,732162	1,782765	-2,92
150	0,0609 / 2,6055	1,732162	1,886393	-8,90
250	0,0366 / 2,0182	1,732162	1,976731	-14,12

Table 5.47: Maximum body displacement values for the LQR with different R(1,1) values and passive system under the step excitation.

R(1,1)	k1 / k2	Passive (m)	LQR (m)	Improvement (%)
1	8,4850 / 31,89	0,127583	0,103689	18,73
5	1,7991 / 14,2688	0,127583	0,107815	15,49
10	0,9068 / 10,0903	0,127583	0,11202	12,20
20	0,4553 / 7,1352	0,127584	0,116845	8,42
30	0,3039 / 5,8259	0,127584	0,11986	6,05
50	0,1826 / 4,5128	0,127583	0,12373	3,02
100	0,0914 / 3,1910	0,127586	0,128909	-1,04
150	0,0609 / 2,6055	0,127586	0,131792	-3,30
250	0,0366 / 2,0182	0,127586	0,134188	-5,17

Table 5.48: Maximum suspension deflection values for the LQR with different R(1,1) values and passive system under the step excitation.

R(1,1)	k1 / k2	Passive (m)	LQR (m)	Improvement (%)
1	8,4850 / 31,89	0,027583	0,003689	86,63
5	1,7991 / 14,2688	0,027583	0,007815	71,67
10	0,9068 / 10,0903	0,027583	0,01202	56,42
20	0,4553 / 7,1352	0,027584	0,016845	38,93
30	0,3039 / 5,8259	0,027584	0,01986	28,00
50	0,1826 / 4,5128	0,027583	0,02373	13,97
100	0,0914 / 3,1910	0,027586	0,028909	-4,80
150	0,0609 / 2,6055	0,027586	0,031792	-15,25
250	0,0366 / 2,0182	0,027586	0,034188	-23,93

5.5 Delay Effects of MR Dampers

In practice, the damper coefficient of an MR damper cannot be changed suddenly, because the MR dampers have a delay time in response to a magnetic field to change the viscosity of its MR fluid. In the conventional MR dampers today, the delay time in response to a signal changes between 20 ms and 40 ms.

In order to see the delay effect of MR dampers, the responses of Skyhook control law with and without delay are compared with each other. The skyhook control law with a saturation level of 10000 kg/s and a skyhook damper coefficient of 2000 kg/s are chosen to see the delay effect for 30 ms delay. Since the system decides the damper coefficient later than it should, the system has a damper coefficient softer than those of the ones without delay.

The system performance is summarized in Figures 5.49 to 5.56. In sum, delay effect improves the performance of control laws, in particular for suddenly changing excitations.

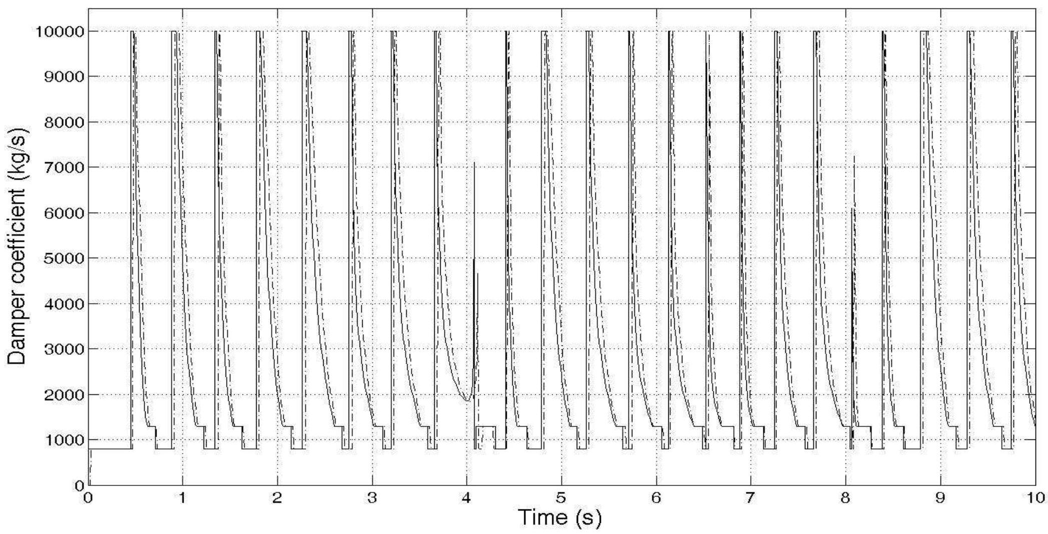

Figure 5.49: Damper coefficient of the skyhook control laws with and without delay under the sine excitation.

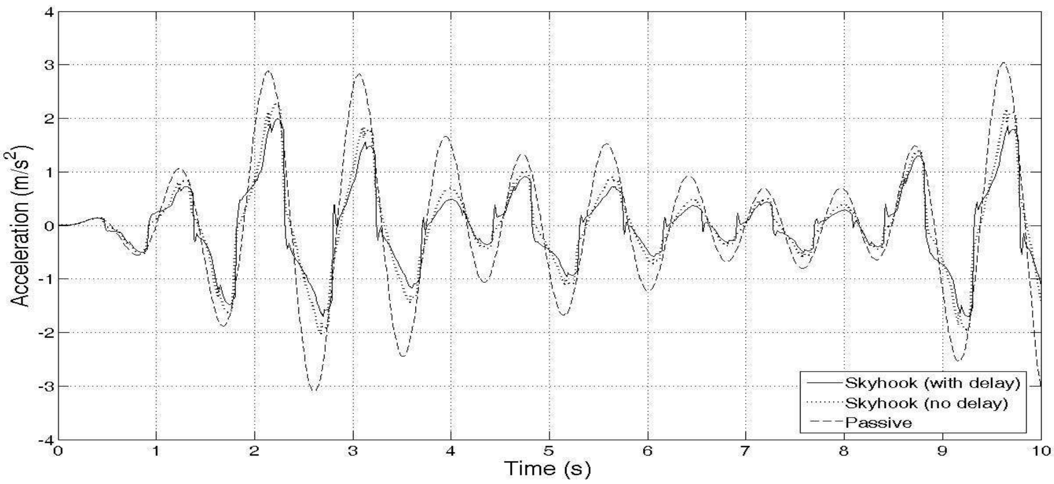

Figure 5.50: Body accelerations of the passive and skyhook systems with and without delay under the sine excitation.

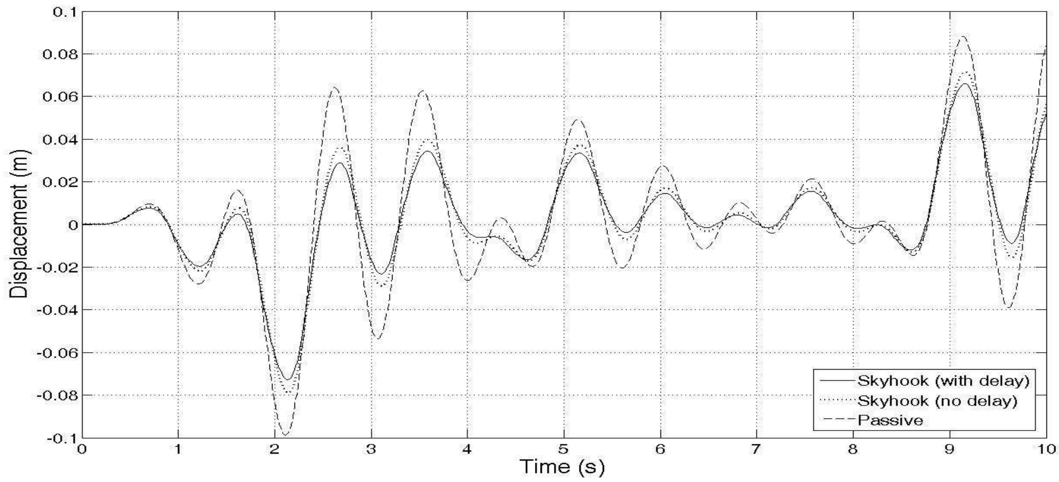

Figure 5.51: Body displacements of the passive and skyhook system with and without delay under the sine excitation.

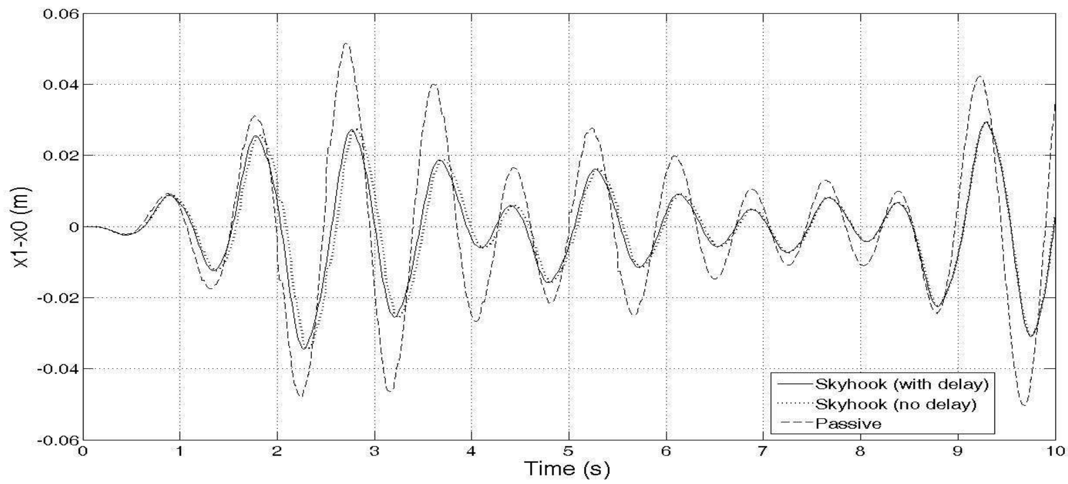

Figure 5.52: Suspension deflections of the passive and skyhook system with and without delay under the sine excitation.

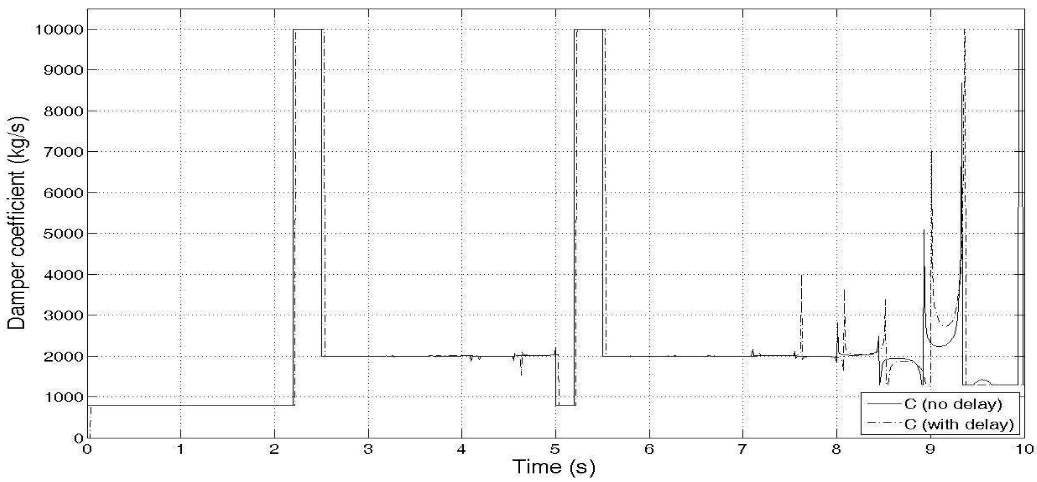

Figure 5.53: Damper coefficients of the passive and skyhook system with and without delay under the step excitation.

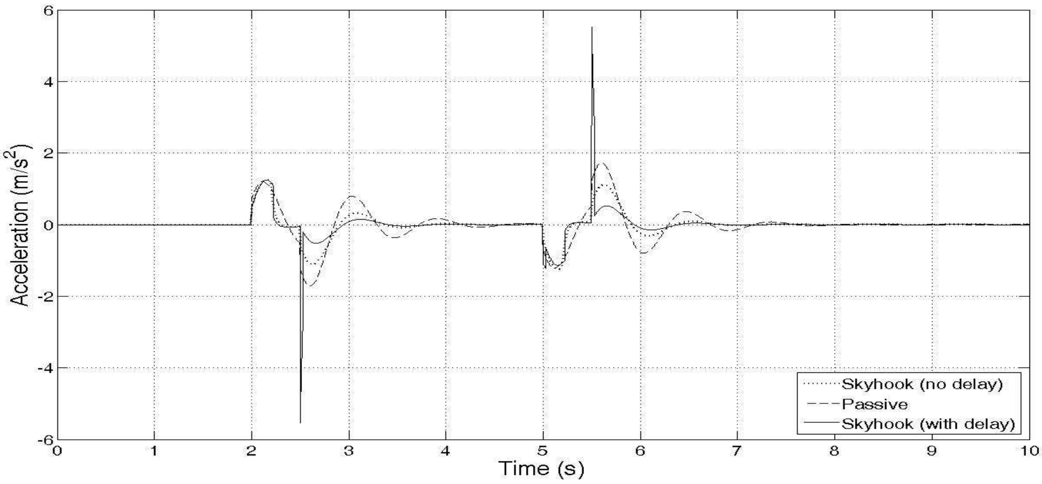

Figure 5.54: Body accelerations of the passive and skyhook systems with and without delay under the sine excitation.

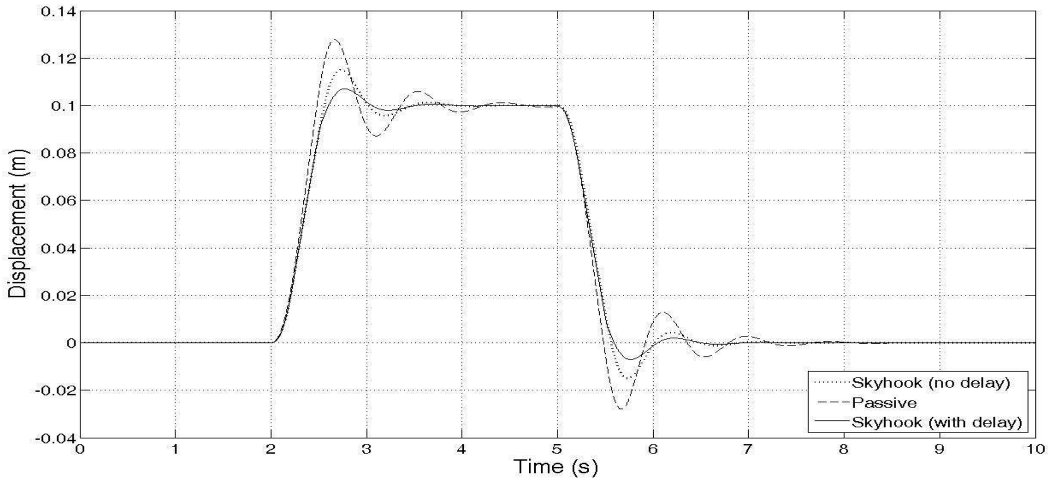

Figure 5.55: Body displacements of the passive and skyhook systems with and without delay under the sine excitation.

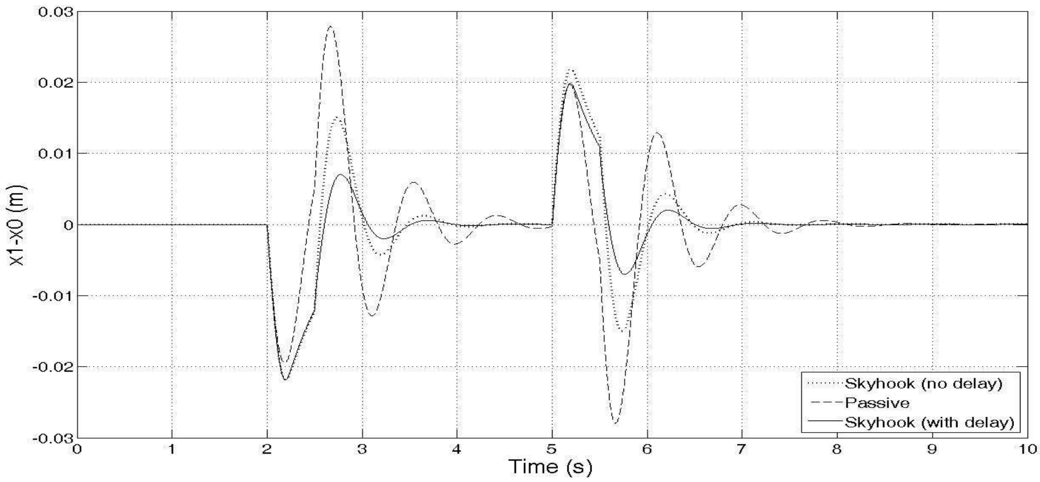

Figure 5.56: Suspension deflections of the passive and skyhook system with and without delay under the sine excitation.

5.6 Transmissibility of the Control Systems

All control systems are compared based on their transmissibility performances. The parameters for different control laws are selected to have the best performance of the selected control law. For the ON/OFF control law, 2500 kg/s hard mode damping is used. For the skyhook control law, skyhook damper coefficient of 3000 kg/s is selected. The state feedback pole assignment system uses the K gain matrix that is found with $\xi = 0.4$ damping ratios and the LQR system uses the K gain matrix found with Q(1,1)=1000, Q(2,2)=4000 and R(1,1)=100 matrix element values.

A simple sine signal is used to plot the graph in Figure 5.57. The excitation frequency is increased from 0 to 30 rad/s and the amplitude of 0.01 m is used for the sine signal. The equation of the signal that is used in transmissibility analysis can be written as;

$$x_0 = 0.01\sin(wt) \qquad \textbf{(5.41)}$$

According to Figure 5.57, the Skyhook control law gave the best transmissibility performance both below the resonance frequency of its own and high frequencies.

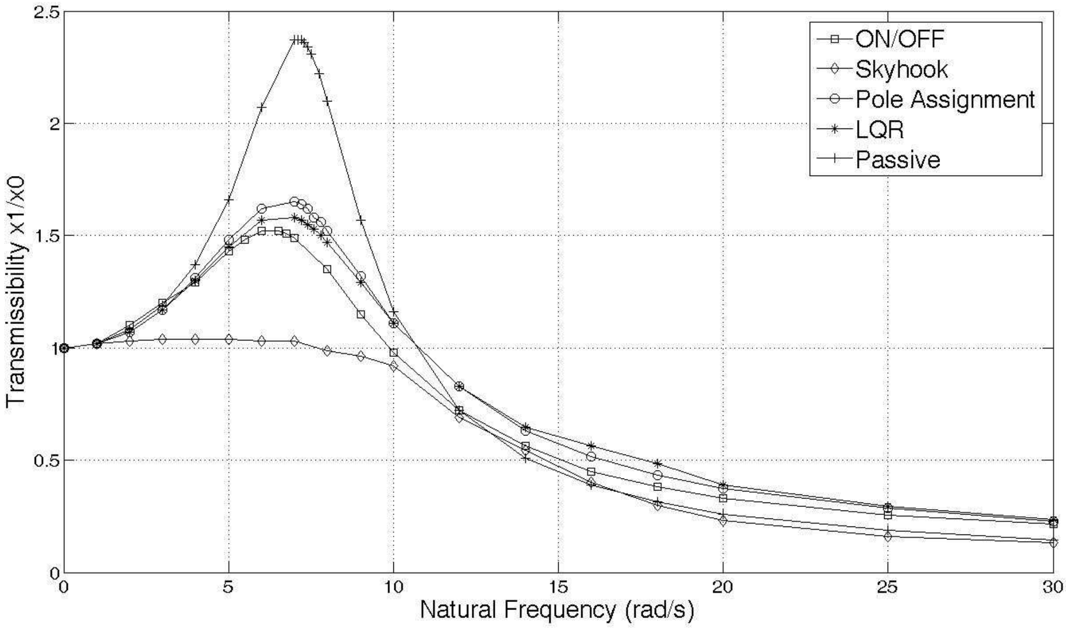

Figure 5.57: Transmissibility of the control systems.

5.7 Model of an MR Damper With Current Values

MR dampers can be modeled by several mathematical and statistical models. One of them is to model the MR damper with Lookup tables in SIMULINK toolbox of MATLAB. The damper characteristics can be seen in Figures 5.58 and 5.59. The algorithm used in this studyreads the data of the MR damper of the LORD corporation. The damper coefficients are given in Table 5.50 for corresponding current and velocity values. The damper coefficient values in Table 5.50 are found by dividing the force values in Table 5.49 by the corresponding velocity values. A SIMULINK diagram of the Lookup Table model of the MR damper model is given in Figure 5.60. Also an analysis is made with Skyhook control law with C_{SKY}=3000 kg/s value by using Lookup Table model to see the distribution of current under the step excitation.

Table 5.49: Force versus velocity values of the MR damper of LORD Corporation used in this study.

Current (A)	F (kN)						
	0,016 m/s	0,039 m/s	0,079 m/s	0,157 m/s	0,314 m/s	0,471 m/s	0,628 m/s
0	0,13	0,15	0,16	0,17	0,27	0,42	0,62
0,5	1,05	1,11	1,17	1,33	1,55	1,7	1,86
1	2,14	2,27	2,44	2,62	2,95	3,15	3,34
1,5	2,86	3,08	3,44	3,66	3,96	4,18	4,4
2	3,36	3,55	4,03	4,22	4,55	4,78	4,99
2,5	3,65	3,72	4,42	4,53	4,87	5,07	5,29

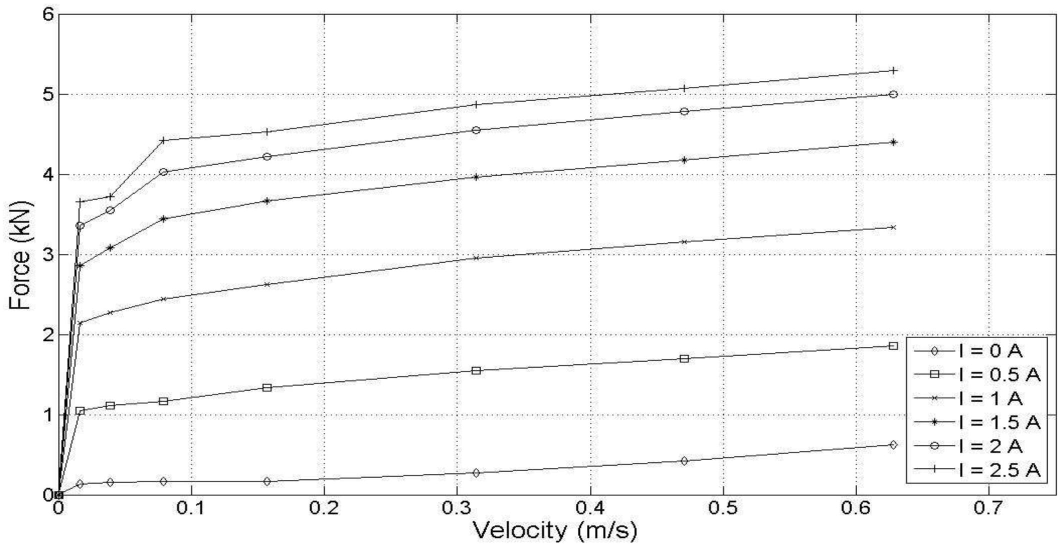

Figure 5.58: Force versus velocity values of an MR damper of LORD corporation.

Table 5.50: Damper coefficient map of an MR damper of LORD Corporation for the corresponding current and velocity values.

Current (A)	C (kg/s)						
	0,016 m/s	0,039 m/s	0,079 m/s	0,157 m/s	0,314 m/s	0,471 m/s	0,628 m/s
0	8125	3846	2025	1083	859	891	987
0,5	65625	28461	14810	8471	4936	3609	2962
1	133750	58205	30886	16688	9394	6687	5318
1,5	178750	78974	43544	23312	12611	8874	7006
2	210000	91026	51013	26878	14490	10149	7946
2,5	228125	95385	55949	28854	15509	10764	8424

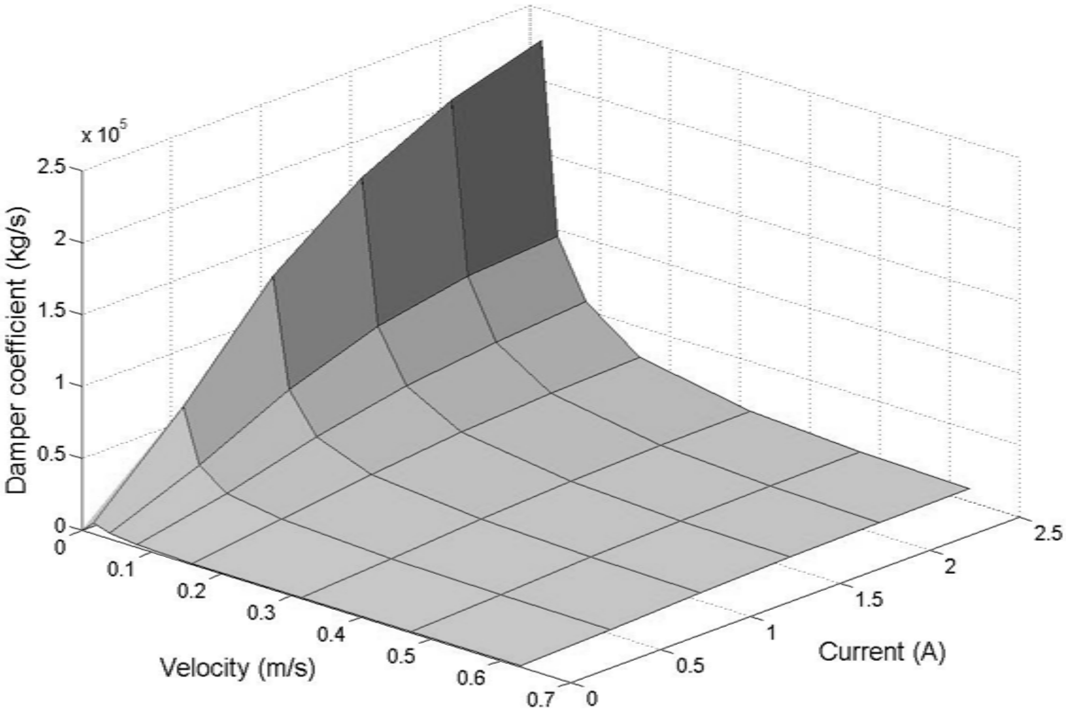

Figure 5.59: Damper coefficient map of an MR damper of LORD Corporation for the corresponding current and velocity values.

The Lookup Table model is applied to the Skyhook control law to see the current values and distribution. According to the diagram, the system first decides the velocity range that the suspension moves. Following, a new damper coefficient column is created for that specific suspension velocity. The model takes the damper coefficient value calculated by Skyhook algorithm and finds its place in the newly created damper coefficient column. Then, it finds the value of the current corresponding to value of the damper coefficient. In Figure 5.61, the current distribution that is applied to the damper by the Skyhook control law under the step excitation can be seen.

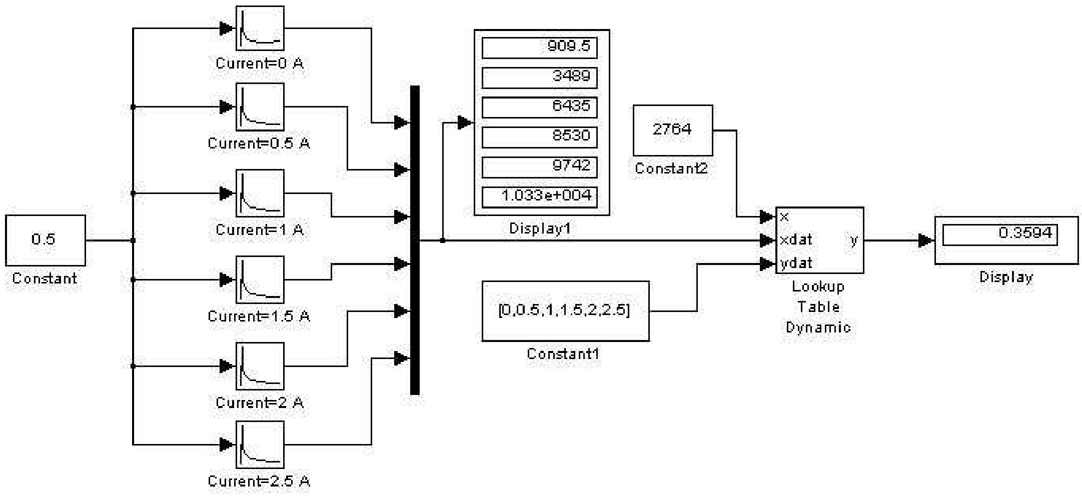

Figure 5.60: The SIMULINK diagram of Lookup Table model of an MR damper.

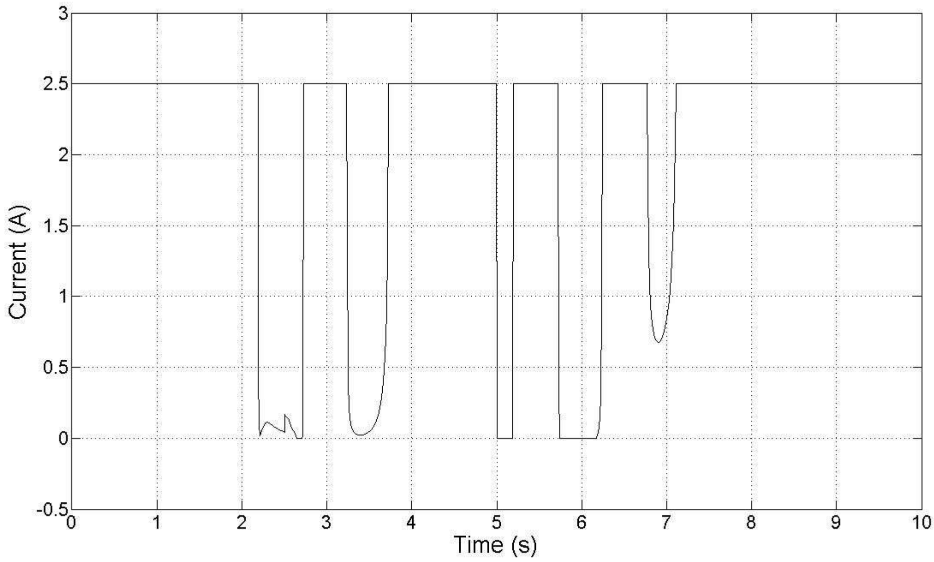

Figure 5.61: The distribution of current applied to the damper by the Skyhook control law under the step excitation.

5.8 Adaptive Control Models for Semi-active Suspension Systems

Adaptive control algorithms can be applied to the semi active suspension systems with MR dampers. For different types of road excitations, control systems can assign different damper coefficients to the damper to make an optimization of vibration isolation. Many patents and research studies are present today for the adaptive control models of semi active suspension models. Since the purpose of this study is to compare the performances of the different control algorithms, adaptive algorithms are not evaluated.

According to patent DE19804005, displacement can be determined by an accelerometer through an algorithm. With a second algorithm, road surface model can be estimated and the resonance region which the road signal corresponds can be found. For each road signal type, different control system parameter can be applied and the signal is guided through high or low-pass filters depending on the resonance region and the corresponding parts of the road excitation.

In patent DE4315917, the suspension deflection and the acceleration values are measured. The velocity of the damper and sprung mass can be found by differentiating the stroke of the damper and integrating the acceleration signal. The values found by differentiation and integration are summed to determine the road excitation. After this point a specific control signal can be produced to overcome the vibration of the road input.

US6202011 patent describes an ECS (Electronic Suspension Control System). This system contains wheel speed sensor to measure the speed of rear and front wheels, a throttle position sensor to measure the open status of the valve, a stop lamb for the brake, a rough road detecting algorithm which the datas that are gathered from the sensors mentioned before. The rough road detecting algorithm applies FFT (Fast Fourier Transform) to the inputted data. According to this data and the wheel speed, system decides control signal.

The system in US6164665 patent contains a system with a variable damper coefficient enhances both the road holding capability and comfort. Position sensors measures the level of the body relative to its axles. Electronic regulator manipulates the air amount inside the air spring to set the body height to the desired position. The signal calculated in the electronic regulator is a value of the acceleration of the body and helps to find the bad road parameter. After the determination of this signal, regulator calculates a shock absorption with the help of bad road parameter.

In this study, for different control algorithms can be applied. For the ON/OFF and Skyhook control laws, the systems can be switched between damper coefficients corresponding to smooth road and curb impact excitations. For the ON/OFF control law, the hard mode damping can be selected as 2000 kg/s and below to improve the curb impact performances and then, it can be increased to 5000 kg/s to get a better vibration isolation for high frequency low amplitude road excitation. For the skyhook control law, this adaptive control algorithm can be applied as well. According to the results in Section 5.2, for curb impact excitations, the skyhook control law can have a C_{SKY} value of 2000 kg/s or below and for continuous and smooth road excitations, it can be increased to 4000 kg/s for better performance under curb impact excitations.

For the state feedback pole assignment and LQR control laws, the adaptive control system can be used as well. Especially the LQR control law is the most suitable one for the adaptive control algorithm amongst the other control laws used in this study. Because the elements of the diagonal Q and R matrices can be changed to a specific value for the corresponding road excitation. In pole assignment, the K gain matrix can be changed to values for the damping ratio of 0.4 under curb impact excitations. For the sine excitations, the K gain matrix can be modified to the value for the damping ratio 1.6 and higher.

6. COMPARISON OF PERFORMANCES OF CONTROL SYSTEMS

All control systems that were analyzed in this thesis are evaluated for both the sine and step excitations. Each controller gives different results from each other. So the performances of the controllers are evaluated for different road conditions and usage.

6.1 ON/OFF System

The semi active ON/OFF system is a simple switch system that allows to use only two different damping coefficients. In other semi active systems that are used in this thesis, have infinite number of damper coefficients is possible.

The damping coefficient can take only two values because it is a switch system. If the condition of hard damper is met, then the damper has the harder damping mode. As it is seen in the plot of the damping coefficient, the damper travels between 3511 kg/s and 1290 kg/s. But the damper does not take values between 3511 kg/s and 1290 kg/s. In theory, damper coefficient rises from 1290 kg/s to 3511 kg/s and falls from 3511 kg/s to 1290 kg/s at the same time step. But in practice, it never falls or rises at the same time step. There is always a very small delay between the coefficient changes.

The ON/OFF system gives better results when it is compared with the passive system. In acceleration values that define the comfort, the semi active ON/OFF system has better RMS values as the hard damping value increases. At the damping coefficient value of 10000 kg/s, the acceleration can be improved at about 50 percent. But after this point, comfort increases too slowly. And beyond very large damping coefficient, the system starts to give worse comfort results. Setting the damping coefficient to 10000 kg/s gives a good result for the sine excitation; but at high frequency road disturbances, it does not give good results. This is because the system cannot manipulate itself to take a damping value between 10000 kg/s and 1290 kg/s. The system must take average values to damp the mild disturbance. When the system encounters with mild disturbance, the system has its damping coefficient set to 10000 kg/s so this makes it to show large responses at high frequencies.

For body displacements and suspension deflection that defines the road holding capability, as the damping coefficient increases, the displacement and suspension deflection decreases as well. As the damper hardens, the body cannot become too independent from the base. Because of this high dependency, the suspension deflection decreases leading to a high road holding capability.

But this capability increases the sensitivity to high impact disturbances. If the hard mode damping is increased too much, the system cannot damp the sudden impacts. As it can be seen in the Table 5.4, very large hard damping coefficient, makes the comfort response of the system worse. Hence, there is a tradeoff between this sudden impact response and road holding capability. For all values of hard damping mode that are evaluated in the tables that show the response of the ON/OFF system for the step shaped obstacle, the system reaches steady state earlier than passive system. Even when the system gives the same transient response characteristics, the semi active ON/OFF system reaches the steady state earlier. According to all of these results, the semi active ON/OFF system is better than the passive system for smooth road disturbances when compared with their accelerations and suspension deflections. But in transient response of high end impact disturbance, as the damping increases, the comfort drops dramatically for the ON/OFF system.

For the ON/OFF system, a damping coefficient in the vicinity of 2500 kg/s is suitable for a versatile motor vehicle. The responses of both the sine and step excitations are at the acceptable levels for hard mode damping coefficient of 2500 kg/s.

6.2 Skyhook System

The skyhook system is based on the modification of a constant damping coefficient as a function of the sprung mass velocity and the relative velocity between the sprung mass and base. It is nearly the same as the ON/OFF control system, because it works with a switch system as well. If the conditions are met, the hard mode damping is activated. If the product of the sprung mass velocity and the relative velocity is negative, then the system has minimum damping ratio.

The main difference between the ON/OFF system and skyhook system is the hard mode damping. The hard mode damping of the ON/OFF system is constant which is not constant in the skyhook system. In the skyhook system, a constant skyhook damping coefficient is selected for the model that the damper is fixed at a fictional point in the sky. When the skyhook system is transformed into a classical quarter car model, the skyhook damping coefficient is modified as a function of sprung mass velocity and relative velocity. At first look the system can be thought as a passive system but the skyhook damper is not used as a constant damper coefficient. The main damper coefficient of the system is determined by the algorithm that modifies the base skyhook damper with the ratio of sprung mass velocity and relative velocity. The skyhook system in this study is limited with 10000 kg/s damping coefficient to simulate the maximum damping limit which is assumed that the MR damper can reach.

The skyhook damper is selected according to a simple second order spring mass damper system. The performance of the system with respect to the acceleration, displacement and suspension deflection responses is evaluated for different damping ratios. In part 5.2, it can be seen that the damping ratio changes only the constant skyhook damper coefficient, hence the tables are formed with different skyhook damper coefficients.

In Table 5.7, it can be seen that after the damper coefficient value of 4500 - 5000 kg/s the acceleration defining the comfort cannot be improved anymore. The improvement increases too slowly after this point, but for a little improvement, there is no need to increase the damping too much since it will lead to large oscillations in acceleration performance and the system will not give good results under high impact disturbances.

The skyhook control system gives the most versatile performance in the vicinity of damper coefficient of 3000 kg/s. Good performance is obtained under both the step and sine excitations.

The skyhook system gives better results than the ON/OFF system for the body displacements and accelerations. If we compare the base hard mode damping values of Skyhook and ON/OFF systems, it can be easily seen that the skyhook system gives better improved results for the acceleration and body displacements but the suspension deflection performance is not as good as the acceleration and body displacement, because the hard mode damping has a constant value in the ON/OFF system; it cannot take intermediate values. As there are no intermediate damping values, the ON/OFF system applies harder damping coefficient than it is needed. Hence, the ON/OFF system does not give good results for the acceleration performance. But having higher damping value than the system needs in a specific time step provides ON/OFF system more road holding capability than the skyhook system has. Nonetheless, the difference is not too much than that of skyhook system in suspension deflection performance. Also a comparison between the skyhook and ON/OFF systems can be seen in tables below;

Table 6.1: Comparison of acceleration performances of the skyhook and ON/OFF systems under the sine excitation.

C_{hard} & $C_{skyhook}$	Skyhook (RMS)	ON/OFF (RMS)
2000	36,08	21,54
2500	42,79	29,53
3000	46,36	34,76
3511	48,75	38,47
4000	50,16	40,79
4500	50,87	43,07

Table 6.2: Comparison of body displacement performances of the skyhook and ON/OFF systems under the sine excitation.

C_{hard} & $C_{skyhook}$	Skyhook (RMS)	ON/OFF (RMS)
2000	25,12	14,18
2500	30,26	19,69
3000	33,35	23,61
3511	35,13	26,44
4000	36,77	28,67
4500	37,69	30,48

Table 6.3: Comparison of suspension deflection performances of the skyhook and ON/OFF systems under the sine excitation.

C_{hard} & $C_{skyhook}$	Skyhook (RMS)	ON/OFF (RMS)
2000	28,32	27,45
2500	35,40	38,05
3000	39,97	45,14
3511	43,99	50,20
4000	45,86	53,61
4500	47,62	56,30

The main performance difference can also be seen in the high end impact responses. The skyhook system has far better results than those of the ON/OFF system. For larger values, the maximum acceleration performances are deteriorating for both methods. But this deterioration in the maximum acceleration reduction is lower for the skyhook system. For lower hard mode damping values, the skyhook system also yields better maximum acceleration reduction. But for the ON/OFF system, system cannot modify the damper coefficient according to the level of impact because of the constant hard mode damping; and the corresponding performance is not satisfactory.

6.3 State Feedback Pole Assignment

With respect to the nature of the control system, the state feedback pole assignment and LQR control systems are different from the skyhook and ON/OFF system. In the skyhook and ON/OFF systems, if the conditions are not met, the model acts a passive suspension system meaning no electric current is sent into MR damper to change the viscosity of damper fluid.

In the control systems designed by pole assignment and LQR method, the semi active control system is continuously modifying the damping coefficient. And there is not a constant base damper coefficient that is exposed to change with an algorithm. The control system decides the damping coefficient according to equation (5.29). The difference between the LQR and pole assignment is not the block diagram but the values k_1 and k_2 of the K gain matrix.

In both the pole assignment and LQR methods, state feedback is used. The difference is that the K matrix is found by using different methods which are explained earlier.

For the pole assignment method, the results are taken with the damping ratios ranging from 0.5 to 2. With lower damping ratio values, the step responses are better than those of the higher values. Only with damping ratio 0.4 and lower values, good results can be obtained but this increases the settling time. The passive system reaches stability earlier than the pole assignment control system if the damping ratio is too low.

Comfort and road holding capability of the pole assignment method under the sine excitation increase as the damping ratio increases. If the damping ratio increases, the acceleration, body displacement and suspension deflection increase as well. But after damping ratio of 2, system cannot be improved anymore because the saturation prevents the damping coefficient to have values beyond 10000 kg/s. This is set because it is needed to simulate the limit of an actual MR damper.

By using second order system pole definitions, the first gain of matrix K does not change. But the second gain changes with the damping ratio. According to this, the base value of the damping coefficient is determined by the second gain. The second gain is larger than the first gain for all damping ratios. So the damping value will be around a value that is defined by the product of the mass and the second gain. After the value of 2, because the system will reach the saturation limit, pole assignment control system will begin to work as a passive system and the damper coefficient is set to 10000 kg/s. Saturation levels affect this property of the pole assignment system. If a better MR damper is used in this system which means having a higher saturation level, the system damping can have higher values. But this leads to peak points in the accelerations which is not desired. If the saturation level is reduced to 5000 kg/s, the system will not be able to use a damping value higher than 0.9; because the system will always have the same damper coefficient set at 0.9.

The pole assignment system has better results between the damping ratios of 0.4 and 0.5; leading to a performance which acceleration are reduced both under the step and sine excitations.

Natural frequency affects the performance of the pole assignment system as well. As the natural frequency increases, the values of gain matrix K increases leading to higher damping coefficient. But larger damping coefficient causes the system to give worse results under high end impact excitations. But in the vicinity of 12 rad/s, system performance cannot be improved anymore because of the saturation limit.

6.4 LQR System

The LQR system is nearly the same as pole assignment system. The only difference is the method to find the gain matrix K. We can adjust the gain matrix K as it is desired by changing the costs Q(1,1), Q(2,2) and R(1,1).

Q and R matrices are selected diagonal to make the calculations and trials easier. If these matrices are not selected as diagonal, Q(1,2) and Q(2,1) will change "y" shown in state space representation. But in diagonal form we can make this adjustment by changing only Q(2,2) value. So diagonal Q and R matrices grant us easiness in trials and analyses.

For the effect of Q(1,1) on the system, it is noticed that with higher Q(1,1) value, the system has larger acceleration, body displacement and suspension deflection values. But increasing the Q(1,1) affects the reduction in accelerations under high end impact loads.

For the effects of Q(2,2) on the system, it can be seen that increasing Q(2,2) improves the responses in terms of accelerations, displacements and suspension deflections. There is also some improvement for the high end impact acceleration reduction. The Q(2,2) value causes the same performance deterioration if it is increased too much.

For the effects of R(1,1) on the system, increasing R(1,1) value reduces the improvement of accelerations, body displacements and suspension deflections because the energy expenditure on the signals will increase.

For the LQR control system, the best results are obtained within the trials made in the thesis with Q(2,2) = 4000, Q(1,1) = 1000 and R(1,1) = 100.

For LQR control system, the values that are selected are not important. The important part of energy expenditure matrix selection is the relative magnitude of these costs. If the ratio between the R(1,1) and Q(1,1) is 100, then there will be no difference between the selections of 100 - 1 and 1 - 0.01 consecutively. The same rule is valid for the relationship between R(1,1) and Q(2,2) as well.

The LQR system is very flexible when it is compared with the pole assignment method in which the poles are selected according to the values of quarter car model parameters. For some cases, the pole assignment method gives better results with its higher damping ratios than the LQR control system gives. But this affects the high end impact disturbance; With the LQR system, the gain matrices K that are found with the pole assignment can be found too. Because there is no limitation to define the K matrix, by making several trials for the LQR control system, desired values can be obtained.

7. CONCLUSION AND DISCUSSION

In this thesis, mainly 4 different control systems were examined and applied to a semi active suspension system. For analyses, two different road disturbances were modeled and used for the evaluations of performances of the control systems. A quarter car model was selected to apply the control system on it. Results of all control systems were obtained by SIMULINK toolbox of MATLAB program. And all data was listed in tables and graphs.

According to the results, the Skyhook control law gave better and more flexible results than those of the ON/OFF system. Having the ability of damper coefficient assignment according to the disturbance provided the Skyhook control law better performance.

The pole assignment gave nearly the same results as the Skyhook control. But the pole assignment method has limitations because the gain k_1 is too low. Larger values of k_2 determines the base damping coefficient and k_1 determines the damper coefficient change. Because of low k_1, most of the time the pole assignment method acts as a passive system that has a damping coefficient determined by the product of the mass and k_2.

The LQR system has the most flexibility because it can be set as a pole assignment system by considering a cost function. By changing the parameters Q(1,1), Q(2,2) and R(1,1) on a trial-and-error base, the desired parameters can be found. This is very useful when we need a control system for a specific usage. With further studies the LQR system can be improved with certain conditions. By assigning different gain matrices for different road disturbances, it will give better results. By calculating the RMS of acceleration values for a time period, then the gain matrix can be adjusted, i.e., adaptive system. For example if the vehicle travels on a smooth section, then the gain matrix K can be modified to a larger value leading to a larger damper coefficient. If the vehicle travels on a road with large disturbances, then the gain matrix K can be modified to a lower value to damp the high impacts which increases the reduction in acceleration to improve comfort.

Also the control system can be tested with other controllers such as fuzzy logic and H_∞ controllers to find better results when compared with the ones analyzed in this thesis in future studies.

REFERENCES

Bouazara, M. and Richard, M.J, 2001. An optimization method designed to improve 3-D vehicle comfort and road holding capability through the use of active and semi-active suspensions. *European Journal Of Mechanics*, **20**, 509-520.

Bouazara, M., Richard, M.J. and Rakheja, S., 2006. Safety and comfort analysis of a 3-D vehicle model with optimal nonlinear active seat suspension. *Journal of Terramechanics*, **43**, 97-118.

Çakır, Ç., 2006. Optimization and Design of Passive and Semi-active Suspension Systems of Truck Cabin. *MSc Thesis*, ITU, Istanbul.

Carlson, D., Catanzarite, D.M. and Clair, K.A.St., Commercial Magneto-rheological Fluid Devices. *Lord Corporation*.

Carlson, J.D, 2001. What Makes A Good MR Fluid?. *8th International Conference on Electrorheological (ER) Fluids and Magneto-rheological (MR) Suspensions*, Nice, July 9-13.

Choi, S.B. and Kim, W.K., 2000. Vibration Control Of A Semi-active Suspension Featuring Electrorheological Fluid Dampers, *Journal of Sound and Vibration*, **234**, 537-546.

Choi, S.B., Lee, H.K. and Chang, E.G., 2001. Field test results of a semi-active ER suspension system associated with skyhook controller. *Mechatronics*, **11**, 345-353

Du, H., Sze, K.Y. and Lam, J., 2005. Semi- active H_∞ control of vehicle suspension with magneto-rheological dampers. *Journal of Sound and Vibration*, **283,** 981-996.

Fischer, D. and Isermann, R., 2004. Mechatronic semi-active and active vehicle suspensions. *Control Engineering Practice*, **12**, 1353-1367.

Guglielmino, E. and Edge, K.A., 2004. A controller friction damper for vehicle applications. *Control Engineering Practice*, **12**, 431-443.

Guglielmino, E., Sireteanu, T, Stammers, C.W, Ghita, G. and Giuclea, M., 2008, Semi active Suspension Control Improved Vehicle Ride and Road Friendliness. Springer, UK.

Jeon, Y.W., 2001. Electronic controlled suspension system using wheel speed, *United States Patent,* No: 6202011 dated 13.03.2001.

Jolly, M.R., Bender, J.W. and Carlson, J.D., Properties and Applications of Commercial Magnetorheological Fluids, *Lord Corporation.*

Kimura, T., Tobata, H., Fukuyama, K. and Akatsu, Y.,1993. Estimation circuit for road-induced vibrations acting on vehicle suspension, *Germany Patent,* No: 4315917 dated 18.11.1993.

Kwack, B.H. and SIM, J.S.,1998. Active control of suspension of car, *Germany Patent*, No: 19804005 dated 08.10.1998.

Lentz, U. and Meier, J., 2000. Vehicle suspension system with continuously adaptive shock absorption, *United States Patent*, No: 6164665 dated 26.12.2000.

McManus, S.J., Clair, K.A.St., Boileau, P.È, Boutin, J. and Rakheja, S, 2002. Evaluation of Vibration and Shock Attenuation Performance Of A Suspension Seat With a Semi Active Magnetorheological Fluid Damper. *Journal of Sound and Vibration*, **253**, 313-327.

Motta, D.S., Zampieri, D.E. and Pereira, A.K.A., 2000. Optimization of a Vehicle Suspension Using a Semi-Active Damper. *SAE Technical Paper Series*, **01-3304.**

Ogata, K., 2002, Modern Control Engineering, 4th Edition, Prentice Hall, USA.

Sassi, S, Cherif, K, Mezghani, L, Thomas, M. and Kotrane, A., 2005. An innovative magnetorheological damper for automotive suspension: from design to experimental characterization. *Smart Materials And Structures*, **14**, 811-822.

Spencer, B.F., Dyke, S.J., Sain, M.K. and Carlson, J.D., 1996. Phenomenological Model of a Magnetorheological Damper. *ASCE Journal of Engineering Mechanics*.

Url-1 <*http://cee.uiuc.edu/sstl/gyang2/Ch2.pdf*>, accessed at 14.09.2008

Url-2 <*http://scholar.lib.vt.edu/theses/available/etd-11997-164941/unrestricted/ Chapter2.pdf*>, accessed at 10.09.2008

Url-3 <*www.brlsi.org/proceed05/postgrad0505.html*>, accessed at 15.09.2008

Yao, G.Z., Yap, F.F., Chen, G., Li, W.H. and Yeo, S.H., 2002. MR damper and its application for semi-active control of vehicle suspension system. *Mechatronics*, **12**, 963-973.

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