

# International Conference on Differential & Difference Equations and Applications 2015

## Abstract Book

Departamento de Ciências Exactas e Naturais  
Academia Militar  
Amadora, Portugal  
May 18 - 22, 2015



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TCor. Carlos Caravela, Academia Militar, Portugal

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## Organizing Institutions



## Our sponsors







Dear Colleagues.

Welcome at the *International Conference on Differential and Difference Equations & Applications 2015*.

The main aim of this conference is to promote, encourage, cooperate, and bring together researchers in the fields of differential and difference equations. All areas of differential & difference equations will be represented with special emphasis on applications. It will be mathematically enriching and socially exciting event.

List of registered participants consists of 169 persons from 45 countries.

The five-day scientific program runs from May 18 (Monday) till May 22, 2015 (Friday). It consists of invited lectures (plenary lectures and invited lectures in sections) and contributed talks in the following areas:

Ordinary differential equations,

Partial differential equations,

Numerical methods and applications, other topics.

The social program, including trips around Lisbon (Sintra, Queluz, Cabo da Roca and Estoril) as well a visit at the Benposta Palace, is organized too.

We hope that will be mathematically enriching and socially exciting event.

We hope that you will find scientific as well as social program interesting and fruitful.

Organizing Committee





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# 1

## Program

### 1.1 General information

#### **Conference site**

The conference is held at the Academia Militar, campus of Amadora, Av. Conde Castro Guimarães, Amadora.

#### **Office**

The office of the conference is located:

- in Hotel Vip Executive Diplomático, Sunday, May 17, 10:00-12:00 and 15:00-19:00,
- in Academia Militar (Amadora), it is open from Monday, May 18, daily from 11:45 a.m.(building A in the map).

If you have any questions, please feel free to ask any member of the conference staff.

#### **Where to eat**

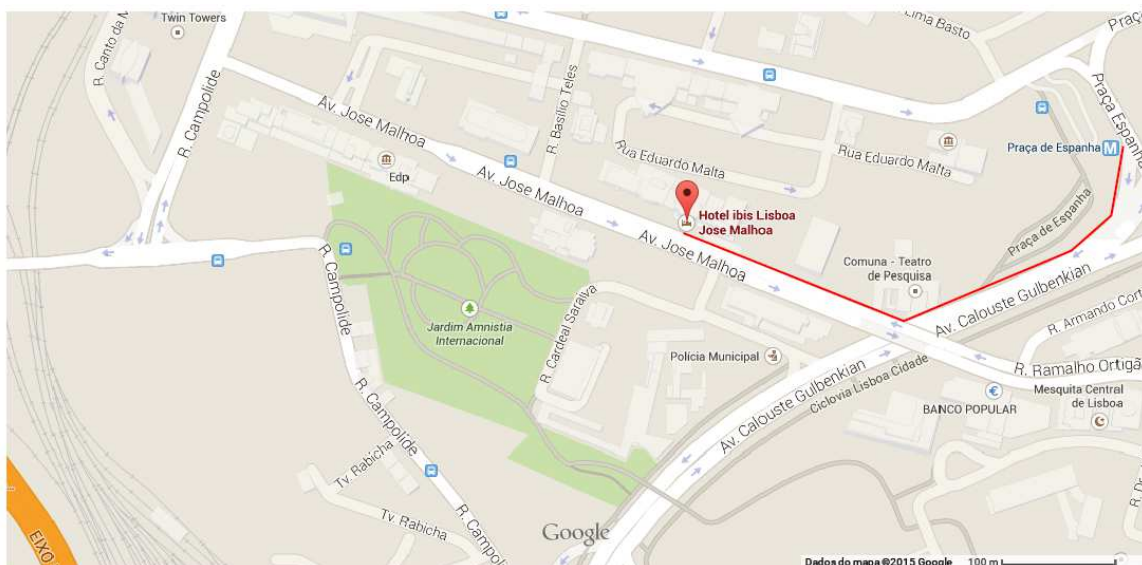
For the participants who stay in Hotel Ibis Lisboa José Malhoa, Hotel Vip Executive Diplomático, Hotel Miraparque and Hotel Açores Lisboa, breakfasts are served there.

Coffee, tea, mineral water, cakes and cookies will be served (free of charge) in the main lobby of the building Aula Magna (building A in the map) during the morning and afternoon coffee breaks.

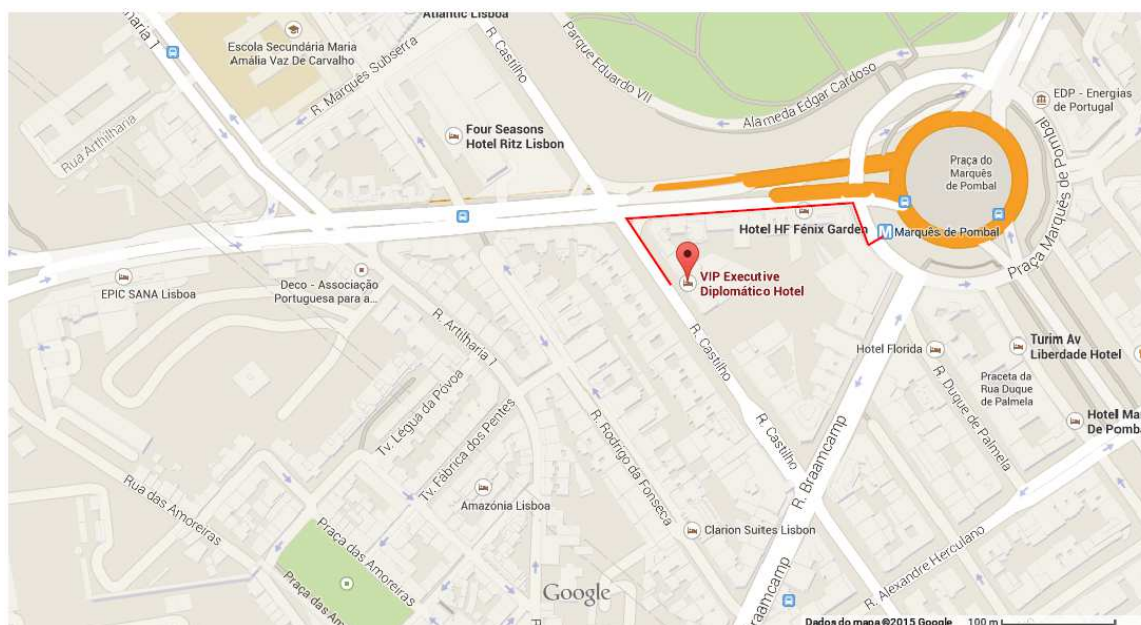
The participants can have lunch at the Academia Militar (free of charges).

## 1.2 Maps

Maps including the hotels recommended by the conference and the nearest Metro Station



Hotel Ibis Lisboa José Malhoa and Metro Station (Praça de Espanha)

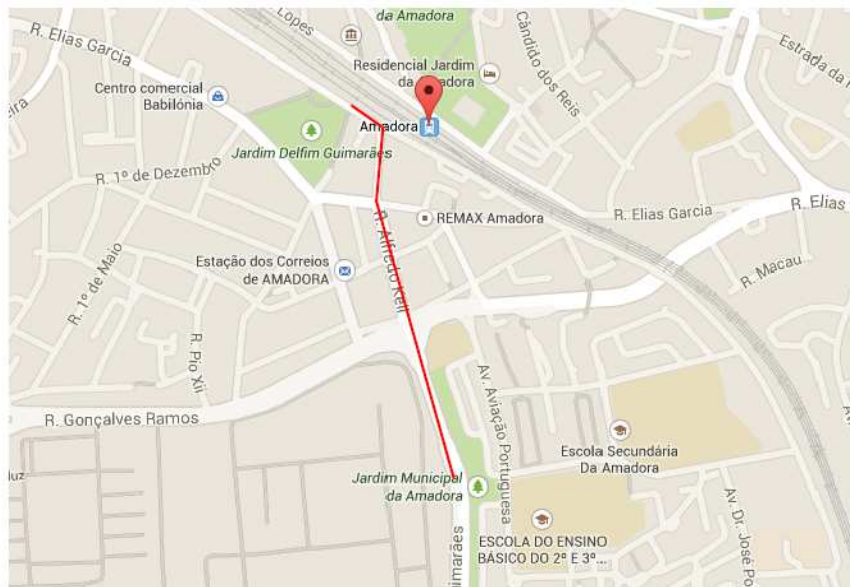


Hotel Vip Executive Diplomático and Metro Station (Marquês de Pombal)



Hotel Miraparque and the Metro Station (Parque)

Map including the conference site and the train station



## 1.3 Social Program

The ICDDEA 2015 will start at May 18, with the Open Ceremony

The Open Ceremony will have two moments: welcome message of the Commandant of Military Academy and a short lecture by Professor Ardeshir Guran about the importance of the difference and differential equations on mechanics, medicine and military applications.

During the conference week, four social events are planned: Welcome Party, a military and equestrian performance by the cadets of the Military Academy, an excursion and Farewell Party. The attendance in Welcome Party, the military and equestrian performance and Farewell Party is free of charge for all registered participants and accompanying persons. The price of the excursion is 50 EUR per person.

### Welcome Party

Monday, May 18, there are a Welcome Party in the headquarter the Military Academy whit a visit at the Palácio da Bemposta.

### Military and equestrian performance

Tuesday, May 19, there are military and equestrian performances by the cadets of the Military Academy in Amadora.

### Excursion

Departure from Amadora and stop in:

- The Queluz National Palace is a Portuguese 18th-century palace located at Queluz in the Lisbon District. One of the last great Rococo buildings to be designed in Europe, the palace was conceived as a summer retreat for Dom Pedro of Braganza, later to become husband and then king consort to his own niece, Queen Maria I.
- The Sintra National Palace also called Town Palace is located in the town of Sintra. It is the best preserved medieval Royal Palace in Portugal, having been

inhabited more or less continuously at least from the early 15th up to the late 19th century. It is an important tourist attraction and is part of the Cultural landscape of Sintra, designated a World Heritage Site by UNESCO.

- Cabo da Roca (Cape Roca) is a cape which forms the westernmost extent of mainland Portugal and continental Europe (and by definition the Eurasian land mass).
- Praia do Guincho (Guincho Beach) is a popular Atlantic beach. The beach has preferred surfing conditions and is popular for surfing, windsurfing, and kitesurfing.
- Cascais is a coastal town. It is a cosmopolitan suburb of the Portuguese capital and one of the richest municipalities in Portugal. The former fishing village gained fame as a resort for Portugal's royal family in the late 19th century and early 20th century. Nowadays, it is a popular vacation spot for both Portuguese and foreign tourists.
- The Gardens of the Casino Estoril located just 20 minutes from downtown Lisbon. Large garden that extends from the Estoril Casino to the Avenida Marginal. Space well maintained, with extensive lawns, many trees, benches, fountains and some Lagoes. The view of the sea accompanies perfectly, walk or any physical activity.

### **Farwell Party**

The farewell party will take place at Força Aérea Restaurante in Monsanto.

This place has a large outdoor area, which will give the visitors the opportunity to enjoy a time of relaxation and leisure.

## **1.4 Conference Proceedings**

The Conference Proceedings will be published in a special issue of Springer edited by the guest editors Sandra Pinelas, Zuzana Dosla, Peter Kloeden and Ondrej Dosly.

All submitted papers will go through a regular peer review process.

The length of the contribution for plenary and invited speakers should not exceed 16 pages. The length of the contribution for other participants should not exceed 8 pages. The papers have to be prepared using the Springer style (see Instructions for authors, <http://sites.google.com/site/sandrapinelas/icddea-2015/j-proceedings>) and submitted as a pdf file to the address *icddea.2015@gmail.com*.

The deadline for submission is June 30, 2015.



## 1.5 Scientific Program in Details

Monday - May 18

**Room** Grande Auditório

10:00 - 10:45 Opening Ceremony

10:45 - 11:15 *Coffee-Break*

PLENARY LECTURES

**Room** Grande Auditório

**Chairmen:** Zuzana Došlá

11:15 - 12:00 Pavel Drabek

INVERSE POSITIVITY AND NEGATIVITY PROPERTY  
OF THE FOURTH ORDER OPERATOR  
WITH VARIABLE STIFFNESS COEFFICIENT

CONTRIBUTED TALKS

**Room** Pequeno Auditório

**Chairmen:** Petr Zemánek

14:00 - 14:30 Fevzi Erdogan

ON THE NUMERICAL SOLUTION OF SINGULARLY PERTURBED  
REACTION DIFFUSION EQUATIONS

14:30 - 15:00 Mehmet Gıyas Sakar

A NEW METHOD FOR SOLVING TIME-FRACTIONAL  
PARTIAL DIFFERENTIAL EQUATIONS

15:00 - 15:30 Onur Saldır

A NOVEL APPROACH TO SOLUTION OF SINGULARLY  
PERTURBED CONVECTION-DIFFUSION PROBLEM

15:30:16:00 Dina Tavares

ISOPERIMETRIC PROBLEMS WITH DEPENDENCE ON FRACTIONAL DERIVATIVES

16:00 - 16:30 *Coffee-Break*

**Chairmen:** Gabriella Bognar

16:30 - 17:00 Laszlo Csizmadia

ON THE TRAJECTORIES OF THE LINEAR SWINGING MODEL

17:00 - 17:30 Janusz Zieliński

RINGS AND FIELDS OF CONSTANTS OF CYCLIC FACTORIZABLE DERIVATIONS

**Room 105****Chairmen:** Roman Šimon Hilscher

- 14:00 - 14:30 Mihály Pituk  
ASYMPTOTIC FORMULAS FOR THE SOLUTIONS  
OF A LINEAR DELAY DIFFERENTIAL EQUATION
- 14:30 - 15:00 Jan Čermák  
STABILITY REGIONS FOR LINEAR FRACTIONAL  
DIFFERENTIAL SYSTEMS WITH A CONSTANT DELAY
- 15:00 - 15:30 Luděk Nechvátal  
ON STABILITY REGION FOR CERTAIN FRACTIONAL DIFFERENCE SYSTEM
- 15:30:16:00 Mehmet Ünal  
STABILITY IN DELAY DYNAMIC EQUATIONS BY FIXED POINT THEORY:  
NONLINEAR CASE
- 16:00 - 16:30 *Coffee-Break*

**Chairmen:** Ivan Dražić

- 16:30 - 17:00 Irina Astashova  
ON ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO REGULAR  
AND SINGULAR NONLINEAR DIFFERENTIAL EQUATIONS
- 17:00 - 17:30 Evgeny Galakhov and Olga Salieva  
BLOW-UP SITUATION FOR SOME DIFFERENTIAL INEQUALITIES  
WITH SHIFTED ARGUMENT

**Room 211****Chairmen:** Małgorzata Migda

- 14:00 - 14:30 Afonso F. Tsandzana  
HOMOGENIZATION OF A COMPRESSIBLE CAVITATION MODEL
- 14:30 - 15:00 Faustino Maestre  
A CORRECTOR FOR THE WAVE EQUATION IN A BOUNDED DOMAIN
- 15:00 - 15:30 Gabriela Holubová  
BIFURCATIONS FROM POSITIVE STATIONARY SOLUTIONS  
IN SPECIAL SUSPENSION BRIDGE MODELS
- 15:30:16:00 Petr Stehlík  
VARIATIONAL METHODS AND DISCRETE REACTION-DIFFUSION EQUATION
- 16:00 - 16:30 *Coffee-Break*

**Chairmen:** Valery Gaiko

- 16:30 - 17:00 Lubomira Softova  
PARABOLIC OBSTACLE PROBLEM IN GENERALIZED MORREY SPACES
- 17:00 - 17:30 Jonáš Volek  
MAXIMUM PRINCIPLES FOR DISCRETE REACTION-DIFFUSION EQUATIONS

**Room 218****Chairmen:** Aurelian Cernea

- 14:00 - 14:30 Mirosława Zima  
ON POSITIVE SOLUTIONS OF SECOND-ORDER NONLOCAL  
SINGULAR BOUNDARY VALUE PROBLEM
- 14:30 - 15:00 Milan Tvrdý  
PERIODIC OSCILLATIONS RELATED TO THE LIEBAU PHENOMENA
- 15:00 - 15:30 Irena Rachůnková  
BOUNDARY VALUE PROBLEMS WITH IMPULSES  
AT STATE-DEPENDENT MOMENTS
- 15:30:16:00 F. Adrián Fernández Tojo  
PROBLEMS WITH NONLOCAL NEUMANN CONDITIONS:  
A TOPOLOGICAL APPROACH
- 16:00 - 16:30 *Coffee-Break*

**Chairmen:** Andrejs Reinfelds

- 16:30 - 17:00 Martina Langerová  
ON THE SECOND ORDER IMPULSIVE PERIODIC PROBLEM AT RESONANCE
- 17:00 - 17:30 Nicholas Fewster-Young  
A PRIORI BOUNDS & EXISTENCE RESULTS FOR SINGULAR BVPS WITH AN  
APPLICATION TO THOMAS-FERMI EQUATIONS WHEN THE ATOM IS NEUTRAL

**Room 306****Chairmen:** Volodymyr Sushch

- 14:00 - 14:30 Yarema Prykarpatsky  
FINITE-DIMENSIONAL REDUCTION OF DISCRETE DYNAMICAL SYSTEMS
- 14:30 - 15:00 Sergey Kryzhevich  
TOPOLOGICAL TURBULENCE AND OTHER PROPERTIES  
IN ONE-DIMENSIONAL MODEL OF PERCUSSION DRILLING
- 15:00 - 15:30 Kenneth Palmer  
CHAOS IN A MODEL FOR MASTING
- 15:30:16:00 Natascha Neumärker  
RATIONAL MAPS AND DIFFERENCE EQUATIONS OVER FINITE FIELDS
- 16:00 - 16:30 *Coffee-Break*

**Chairmen:** Magda Rebelo

- 16:30 - 17:00 David Shoikhet  
FIXED POINTS AND COMPLEX DYNAMICS
- 17:00 - 17:30 Jasvinder Singh Virdi  
FOURTH ORDER DYNAMICAL INVARIANTS IN ONE-DIMENSION  
FOR CLASSICAL AND QUANTUM DYNAMICAL SYSTEMS

**Room 315****Chairmen:** Jozef Dzurina

- 14:00 - 14:30 Barbara Szyszka  
AN INTERVAL VERSION OF FINITE DIFFERENCE METHOD  
FOR THE WAVE EQUATION
- 14:30 - 15:00 Denis Hautesserres  
ANALYTICAL INTEGRATION OF THE OSCULATING LAGRANGE PLANETARY  
EQUATIONS IN THE ELLIPTIC ORBITAL MOTION - J2, 3RD BODY,  
SRP, + ATMOSPHERIC DRAG - SOFTWARE NADIA
- 15:00 - 15:30 Gabil Amirali (Amiraliyev)  
ANALYSIS OF DIFFERENCE APPROXIMATIONS TO DELAY  
PSEUDO-PARABOLIC EQUATIONS
- 15:30:16:00 Ilhame Amirali  
NUMERICAL METHOD FOR A SINGULARLY PERTURBED DELAY  
INTEGRO-DIFFERENTIAL EQUATION
- 16:00 - 16:30 *Coffee-Break*
- Chairmen:** Jana Krejčová
- 16:30 - 17:00 Stepan Manko  
ON SPECTRUM OF MAGNETIC GRAPHS
- 17:00 - 17:30 Mark Elin  
LOCAL GEOMETRY OF TRAJECTORIES OF PARABOLIC TYPE SEMIGROUPS

## Tuesday - May 19

### PLENARY LECTURES

**Room** Grande Auditório;

**Chairmen:** Andrzej Szulkin

09:00 - 09:45 Ioannis Stravroulakis  
OSCILLATIONS OF DELAY AND DIFFERENCE EQUATIONS  
WITH SEVERAL VARIABLE COEFFICIENTS AND ARGUMENTS

09:45 - 10:30 Ardeshir Guran  
RECENT RESULTS ON STABILITY, VIBRATIONS AND  
CONTROL OF MECHANICAL STRUCTURES

10:00 - 10:30 *Coffee-Break*

### INVITED LECTURES

**Room** Grande Auditório;

**Chairmen:** Stefan Hilger

11:00 - 11:45 Istvan Gyori  
ON A POPULATION MODEL EQUATION  
WITH DELAYED BIRTH AND UNDELAYED INHIBITATION TERM

11:45 - 12:30 Leonid Berezansky  
GLOBAL EXPONENTIAL STABILITY  
FOR NONLINEAR DELAY DIFFERENTIAL SYSTEMS

**Room** Pequeno Auditório;

**Chairmen:** Ağacık Zafer

11:00 - 11:45 Luis Sanchez  
TRAVELLING WAVES AND CRITICAL SPEED  
IN THE PRESENCE OF NONLINEAR DIFFUSION

11:45 - 12:30 Alberto Cabada  
CONSTANT SIGN SOLUTIONS OF DIFFERENTIAL EQUATIONS  
WITH INVOLUTIONS

## CONTRIBUTED TALKS

**Room** Pequeno Auditório**Chairmen:** Mehmet Ünal

14:00 - 14:30 Vedat Ertürk

DYNAMICAL ANALYSIS OF THE HAAVELMO GROWTH CYCLE MODEL  
WITH FRACTIONAL DERIVATIVE

14:30 - 15:00 Hiroyuki Usami

ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS  
OF A KIND OF LANCHESTER-TYPE SYSTEM

15:00 - 15:30 Tomoyuki Tanigawa

ASYMPTOTIC ANALYSIS OF STRONGLY MONOTONE SOLUTIONS  
OF NONLINEAR DIFFERENTIAL EQUATIONS

15:30:16:00 Zdeněk Opluštil

ON OSCILLATION AND NONOSCILLATION TO CERTAIN  
TWO-DIMENSIONAL SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS16:00 - 16:30 *Coffee-Break***Chairmen:** Lorena Saavedra

16:30 - 17:00 Jozef Dzurina

OSCILLATION OF FOURTH-ORDER TRINOMIAL  
DELAY DIFFERENTIAL EQUATIONS

17:00 - 17:30 Bonita Lawrence

CONVERGENCE OF SOLUTIONS OF DYNAMIC EQUATIONS ON TIME SCALES

**Room** 105**Chairmen:** Irena Rachůnková

14:00 - 14:30 Ruslan Surmanidze

SMALL VALENCE INVARIANT TENSOR FIELDS ON SOME HOMOGENEOUS  
RIEMANN SPACES AND SHUR'S POLYNOMIALS

14:30 - 15:00 Malkhaz Bakuradze

EXPLICIT COMPUTATION OF SOME RATIONAL  
AND INTEGRAL COMPLEX GENERA

15:00 - 15:30 Werner Varnhorn

ON EXTENSIONS OF SERRIN'S CONDITION  
FOR THE NAVIER-STOKES EQUATIONS

15:30:16:00 Magda Rebelo

COMPARATIVE STUDY OF NUMERICAL METHODS  
FOR THE TIME-FRACTIONAL DIFFUSION EQUATION16:00 - 16:30 *Coffee-Break***Chairmen:** Zeynep Kayar

16:30 - 17:00 Anwar Hussein

SPLITTING METHODS FOR OPTION PRICING IN A GENERALISED  
BLACK-SCHOLES MODEL

17:00 - 17:30 Sanjukta Das

EXISTENCE OF SOLUTION AND APPROXIMATE CONTROLLABILITY OF A  
SECOND-ORDER NEUTRAL STOCHASTIC DIFFERENTIAL EQUATION  
WITH STATE DEPENDENT DELAY DEVIATED ARGUMENT

**Room 215****Chairmen:** Gabriela Holubová

- 14:00 - 14:30 Petr Zemánek  
SYMPLECTIC SYSTEMS WITH ANALYTIC DEPENDENCE ON  
SPECTRAL PARAMETER RIEMANN SPACES AND SHUR'S POLYNOMIALS
- 14:30 - 15:00 Roman Šimon Hilscher  
THEORY OF RECESSIVE SOLUTIONS AT INFINITY  
FOR NONOSCILLATORY SYMPLECTIC DIFFERENCE SYSTEMS
- 15:00 - 15:30 Julia Elyseeva  
NONOSCILLATION CRITERIA FOR DISCRETE TRIGONOMETRIC SYSTEMS
- 15:30:16:00 Candace M. Kent  
PIECEWISE-DEFINED DIFFERENCE EQUATIONS: OPEN PROBLEM
- 16:00 - 16:30 *Coffee-Break*

**Chairmen:** Petr Stehlík

- 16:30 - 17:00 Peter Šepitka  
PRINCIPAL AND ANTIPRINCIPAL SOLUTIONS  
OF LINEAR HAMILTONIAN SYSTEMS
- 17:00 - 17:30 Roman Kozlov  
FIRST INTEGRALS OF ODES VIA  $\lambda$ -SYMMETRIES

**Room 218****Chairmen:** Hüseyin Koçak

- 14:00 - 14:30 José G. Espín Buendía  
ATTRACTING FIXED POINTS FOR THE BUCHNER-ZEBROWSKI EQUATION:  
THE ROLE OF SCHWARZIAN DERIVATIVE
- 14:30 - 15:00 Ioannis Dassios  
SINGULAR LINEAR SYSTEMS OF FRACTIONAL NABLA DIFFERENCE EQUATIONS
- 15:00 - 15:30 David Shoikhet  
FIXED POINTS AND COMPLEX DYNAMICS
- 15:30:16:00 Volodymyr Sushch  
A DISCRETE DIRAC-KÄHLER EQUATION ON A DOUBLE COMPLEX
- 16:00 - 16:30 *Coffee-Break*

**Chairmen:** Martina Langerová

- 16:30 - 17:00 Tatiana A. Shaposhnikova  
HOMOGENIZATION OF A VARIATIONAL INEQUALITY FOR  
THE P-LAPLACE OPERATOR WITH NONLINEAR RESTRICTION  
ON THE PERFORATED PART OF THE BOUNDARY IN CRITICAL CASE
- 17:00 - 17:30 Nadezhda Rautian  
SPECTRAL ANALYSIS OF INTEGRO-DIFFERENTIAL EQUATIONS  
ARISING IN VISCOELASTICITY THEORY

**Room 306****Chairmen:** David C. Ni

14:00 - 14:30 Valery Cherepennikov  
SMOOTH SOLUTIONS OF LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

14:30 - 15:00 Rodica Luca Tudorache  
EXISTENCE OF POSITIVE SOLUTIONS FOR A SYSTEM  
OF FRACTIONAL BOUNDARY VALUE PROBLEMS

15:00 - 15:30 Aldona Dutkiewicz  
ON THE CONVERGENCE OF SUCCESSIVE APPROXIMATIONS  
FOR A FRACTIONAL DIFFERENTIAL EQUATION IN BANACH SPACES

15:30:16:00 Gabriella Bognar  
EXISTENCE RESULTS FOR A NONLINEAR BOUNDARY LAYER PROBLEM

16:00 - 16:30 *Coffee-Break*

**Chairmen:** Laszlo Csizmadia

16:30 - 17:00 Jana Burkotová  
SINGULAR LINEAR BVPs WITH UNSMOOTH DATA

17:00 - 17:30 Josef Rebenda  
INITIAL VALUE PROBLEM FOR PARTIAL DIFFERENTIAL EQUATIONS:  
CONVERGENCE ANALYSIS OF AN ITERATIVE SCHEME

**Room 315****Chairmen:** Jan Milewshi

14:00 - 14:30 Ana Paiva  
ON A ONE EQUATION TURBULENT MODEL OF RANS TYPE  
WITH STRONG NONLINEAR FEEDBACKS

14:30 - 15:00 Andrey Zvyagin  
SOLVABILITY OF TERMOVISCOELASTICITY PROBLEM FOR VOIGT MODEL

15:00 - 15:30 Victor Zvyagin  
ON A WEAK SOLVABILITY OF A SYSTEM OF THERMOVISCOELASTICITY  
OF THE OLDROID TYPE

15:30:16:00 Vladimir Vasilyev  
ON SOME BOUNDARY VALUE PROBLEMS FOR DIFFERENCE EQUATIONS

16:00 - 16:30 *Coffee-Break*

**Chairmen:** Irina Astashova

16:30 - 17:00 Pridon Dvalishvili  
CONTINUOUS DEPENDENCE OF THE MINIMUM OF THE BOLZA TYPE  
FUNCTIONAL ON THE PARAMETERS IN THE OPTIMAL CONTROL  
PROBLEM WITH DISTRIBUTED DELAY

17:00 - 17:30 Samaneh Soradi Zeid  
APPLICABLE METHOD FOR SOLVING THE SYSTEM  
OF FRACTIONAL DIFFERENTIAL EQUATIONS



## Wednesday - May 20

### PLENARY LECTURES

**Room** Grande Auditório

**Chairmen:** Pavel Drabek

09:00 - 09:45 Andrzej Szulkin  
GROUND STATES FOR PROBLEMS OF BREZIS-NIRENBERG TYPE  
WITH CRITICAL AND SUPERCRITICAL EXPONENT

### INVITED LECTURES

**Room** Grande Auditório

**Chairmen:** Tibor Krisztin

09:45 - 10:30 Martin Bohner  
INVERSE PROBLEMS FOR  
STURM-LIOUVILLE DIFFERENCE EQUATIONS

10:30 - 11:00 *Coffee-Break*

11:00 - 11:45 Ondřej Došlý  
OSCILLATION THEORY OF EVEN-ORDER LINEAR  
AND HALF-LINEAR DIFFERENTIAL EQUATIONS

11:45 - 12:30 Stefan Hilger  
OPERATOR CALCULUS FOR  $(q,h)$ -DIFFERENCE EQUATIONS

**Room** Pequeno Auditório

**Chairmen:** Adélia Sequeira

09:45 - 10:30 Hugo Beirão da Veiga  
CONCERNING THE EXISTENCE OF CLASSICAL SOLUTIONS  
TO THE STOKES SYSTEM.  
ON THE MINIMAL ASSUMPTIONS PROBLEM

10:30 - 11:00 *Coffee-Break*

11:00 - 11:45 Stevo Stević  
ON SOME PERTURBATIONS OF PRODUCT-TYPE DIFFERENCE  
EQUATIONS AND SYSTEMS OF DIFFERENCE EQUATIONS

11:45 - 12:30 Tomás Caraballo  
LOCAL AND GLOBAL STABILITY OF 2D NAVIER-STOKES EQUATIONS  
WITH DELAYS

.

## Thursday - May 21

### PLENARY LECTURES

**Room** Grande Auditório

**Chairmen:** Martin Bohner

09:00 - 09:45 Michel Chipot  
NON HOMOGENEOUS BOUNDARY VALUE PROBLEMS  
FOR THE STATIONARY NAVIER-STOKES EQUATIONS  
IN 2-D SYMMETRIC SEMI-INFINITE OUTLETS

### INVITED LECTURES

**Room** Grande Auditório

**Chairmen:** Haydar Akca

09:45 - 10:30 Stefan Siegmund  
DYNAMICAL SYSTEMS ON GRAPHS

10:30 - 11:00 *Coffee-Break*

11:00 - 11:45 Peter Kloeden  
CONSTRUCTION OF FOWARD ATTRACTORS  
IN NONAUTONOMOUS DYNAMICAL SYSTEMS

11:45 - 12:30 Allaberen Ashyralyev  
ON SOURCE IDENTIFICATION PROBLEM FOR A TELEGRAPH  
PARTIAL DIFFERENTIAL AND DIFFERENCE EQUATIONS

**Room** Pequeno Auditório

**Chairmen:** Ioannis Stravroulakis

9:45 - 10:30 Nating M. Atakishiyev  
ON A DISCRETE NUMBER OPERATOR AND ITS  
EIGENVECTORS ASSOCIATED WITH THE 5D  
DISCRETE FOURIER TRANSFORM

10:30 - 11:00 *Coffee-Break*

11:00 - 11:45 Ağacık Zafer  
ASYMPTOTIC INTEGRATION OF NONLINEAR DIFFERENTIAL EQUATIONS

11:45 - 12:30 Tibor Krisztin  
PERIODIC SOLUTIONS OF A DIFFERENTIAL EQUATION  
WITH A QUEUEING DELAY

## CONTRIBUTED TALKS

**Room** Pequeno Auditório**Chairmen:** Faustino Maestre

14:00 - 14:30 David C. Ni  
CHARACTERIZATION OF ROOTS OF EXTENDED BLASCHKE PRODUCTS  
VIA ITERATION

14:30 - 15:00 Jana Krejčová  
MINIMAL AND MAXIMAL SOLUTIONS OF FOURTH-ORDER  
NONLINEAR DIFFERENCE EQUATIONS

15:00 - 15:30 Małgorzata Migda  
PROPERTIES OF SOLUTIONS OF HIGHER-ORDER NEUTRAL  
DIFFERENCE EQUATIONS

15:30 - 16:00 A. K. Tripathy  
NEW OSCILLATION CRITERIA FOR NONLINEAR FOURTH ORDER  
DELAY DIFFERENCE EQUATIONS

16:00 - 16:30 *Coffee-Break*

**Chairmen:** Ana Paiva

16:30 - 17:00 Jaqueline G. Mesquita  
PERIODIC AVERAGING THEOREM FOR QUANTUM CALCULUS

17:00 - 17:30 Aurelian Cernea  
ON THE EXISTENCE OF SOLUTIONS FOR SOME NONLINEAR  
 $q$ -DIFFERENCE INCLUSIONS

**Room** 105**Chairmen:** Milan Tvrdý

14:00 - 14:30 Rafael Marques  
EXISTENCE OF SOLUTION RESULT FOR MEASURE DIFFERENTIAL EQUATIONS  
WITH KURZWEIL-HENSTOCK INTEGRABLE RIGHT-HAND SIDES

14:30 - 15:00 Giselle A. Monteiro  
REMARKS ON CONTINUOUS DEPENDENCE OF SOLUTION  
OF ABSTRACT GENERALIZED DIFFERENTIAL EQUATIONS

15:00 - 15:30 Antonín Slavík  
WELL-POSEDNESS RESULTS FOR ABSTRACT GENERALIZED DIFFERENTIAL  
EQUATIONS AND MEASURE FUNCTIONAL DIFFERENTIAL EQUATIONS

15:30 - 16:00 Delfim F. M. Torres  
A GENERAL DELTA-NABLA CALCULUS OF VARIATIONS ON TIME SCALES  
WITH APPLICATION TO ECONOMICS

16:00 - 16:30 *Coffee-Break*

**Chairmen:** Alka Chadha

16:30 - 17:00 Liana Karalashvili  
INTERPOLATION METHOD OF SHALVA MIKELADZE  
FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

17:00 - 17:30 Elvin Azizbayov  
REPRESENTATION OF THE SOLUTION OF A BIHARMONIC EQUATION  
WITH A DELAY

**Room 211****Chairmen:** Mirosława Zima

- 14:00 - 14:30 Sema Servi  
SOLVING HEAT EQUATIONS FOR REDUCED DIFFERENTIAL  
TRANSFORM METHOD WITH FIXED GRID SIZE
- 14:30 - 15:00 Octavio Vera  
ON THE STABILIZATION FOR A SCHRÖDINGER EQUATION  
WITH DOUBLE POWER NONLINEARITY
- 15:00 - 15:30 Andrei Faminskii  
INTERNAL REGULARITY OF SOLUTIONS TO CERTAIN QUASILINEAR  
DISPERSIVE EVOLUTION EQUATIONS
- 15:30 - 16:00 Nelida Črnjarić-Žic  
FINITE DIFFERENCE FORMULATION FOR THE MODEL  
OF A COMPRESSIBLE VISCOUS AND HEAT-CONDUCTING FLUID  
WITH SPHERICAL SYMMETRY

16:00 - 16:30 *Coffee-Break***Chairmen:** Yücel Çenesiz

- 16:30 - 17:00 Ivan Dražić  
3-D FLOW OF A COMPRESSIBLE VISCOUS MICROPOLAR FLUID  
WITH SPHERICAL SYMMETRY: STABILIZATION AND REGULARITY  
OF THE SOLUTION
- 17:00 - 17:30 Mariusz Bodzioch  
BEHAVIOR OF WEAK SOLUTIONS TO THE OBLIQUE DERIVATIVE PROBLEM  
FOR ELLIPTIC WEAK QUASI-LINEAR EQUATIONS IN A NEIGHBORHOOD OF  
A CONICAL BOUNDARY POINT

**Room 218****Chairmen:** Hiroyuki Usami

- 14:00 - 14:30 Hüseyin Koçak  
JUPITER'S BELTS, OUR OZONE HOLES, AND DEGENERATE TORI
- 14:30 - 15:00 Md. H. Ali Biswas  
NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS  
IN MODELING AND CONTROL OF INFECTIOUS DISEASE
- 15:00 - 15:30 Vivek Asthana  
ANALYSIS OF FREE CONVECTION DISSIPATIVE HEAT EFFECT  
ON MHD FLOW PAST AN INFINITE POROUS VERTICAL PLATE
- 15:30 - 16:00 Vafokul E. Ergashev  
BIFURCATION STADY OF 3-RD PRADATOR - PREY MODEL  
WITH SATURATION AND COMPETITION AFFECTS  
IN THE PRADATORS POPULATION
- 16:00 - 16:30 *Coffee-Break*
- Chairmen:** Călin-Constantin Șerban
- 16:30 - 17:00 Takeshi Taniguchi  
EXISTENCE AND ASYMPTOTIC BEHAVIOR OF WEAK SOLUTIONS TO  
NONLINEAR WAVE EQUATIONS WITH NONLINEAR BOUNDARY CONDITION
- 17:00 - 17:30 Helal Mahmoud  
ON EXISTENCE RESULTS FOR NONLINEAR FRACTIONAL DIFFERENTIAL  
EQUATIONS WITH FOUR-POINT BOUNDARY VALUE PROBLEM

**Room 306****Chairmen:** Ruslan Surmanidze

14:00 - 14:30 Duangkamol Poltem  
MODIFICATION OF ADOMAIN DECOMPOSITION METHOD FOR N-ORDER  
NONLINEAR DIFFERENTIAL EQUATIONS

14:30 - 15:00 Faten Toumi  
MONOTONE ITERATIVE TECHNIQUE FOR SYSTEMS OF NONLINEAR  
CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS

15:00 - 15:30 Alexander Makin  
SPECTRAL ANALYSIS FOR THE STURM-LIOUVILLE OPERATOR  
WITH DEGENERATE BOUNDARY CONDITIONS

15:30 - 16:00 Victor Vlasov  
SHARP ESTIMATES FOR SOLUTIONS OF SYSTEMS WITH AFTEREFFECT

16:00 - 16:30 *Coffee-Break*

**Chairmen:** Lahcene Guedda

16:30 - 17:00 Zeynep Kayar  
EXISTENCE AND UNIQUENESS OF SOLUTIONS OF INHOMOGENOUS  
BOUNDARY VALUE PROBLEM (BVP) FOR LINEAR IMPULSIVE  
FRACTIONAL DIFFERENTIAL EQUATIONS

17:00 - 17:30 Ali M. Atewi  
SOLVING FIRST-ORDER PERIODIC BVPs  
BASED ON THE REPRODUCING KERNEL METHOD

**Room 315****Chairmen:** José G. Espín Buendía

14:00 - 14:30 Abdullah M. S. Ajlouni  
RADIOACTIVE MATERIALS DISPERSION IN THE ATMOSPHERE

14:30 - 15:00 Atul Kumar  
ANALYSIS OF TWO-DIMENSIONAL TRANSPORT OF NUTRITION  
THROUGH HETEROGENEOUS POROUS MEDIUM  
WITH SPATIALLY RETARDATION FACTOR

15:00 - 15:30 Nugzar Kereselidze  
ABOUT ON A FEATURE OF THE OPTIMAL CONTROL PROBLEM  
IN MATHEMATICAL AND COMPUTER MODELS  
OF THE INFORMATION WARFARE

15:30 - 16:00 Manlika Rajchakit  
OPTIMAL GUARANTEED COST CONTROL FOR STOCHASTIC NEURAL NETWORKS

16:00 - 16:30 *Coffee-Break*

**Chairmen:** F. Adrián Fernández Tojo

16:30 - 17:00 Grienggrai Rajchakit  
OPTIMAL GUARANTEED COST CONTROL FOR UNCERTAIN NEUTRAL NETWORKS

17:00 - 17:30 Sebaheddin Şevgin  
ULAM STABILITY OF SOME VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

## Friday - May 22

### PLENARY LECTURES

**Room** Grande Auditório

**Chairmen:** Istvan Gyori

09:00 - 09:45 Juan J. Nieto  
FRACTIONAL DIFFERENTIAL EQUATIONS

### INVITED LECTURES

**Room** Grande Auditório

**Chairmen:** Ondřej Došlý

09:45 - 10:30 Haydar Akça  
GLOBAL EXPONENTIAL STABILITY OF NEUTRAL TYPE  
CELLULAR NEURAL NETWORKS WITH IMPULSES,  
TIME-VARYING AND DISTRIBUTED DELAYS

10:30 - 11:00 *Coffee-Break*

11:00 - 11:45 Zuzana Došlá  
COEXISTENCE PROBLEM FOR POSITIVE SOLUTIONS  
OF SECOND ORDER DIFFERENTIAL EQUATIONS

11:45 - 12:30 Felix Sadyrbaev  
OSCILLATORY SOLUTIONS OF BOUNDARY VALUE PROBLEMS

**Room** Pequeno Auditório

**Chairmen:** Allaberen Ashyralyev

09:45 - 10:30 Peter Bates  
SPECTRAL CONVERGENCE AND BIFURCATION  
IN NONLOCAL DIFFUSION-REACTION EQUATIONS

10:30 - 11:00 *Coffee-Break*

11:00 - 11:45 Sergey Korotov  
BISECTION-TYPE ALGORITHMS FOR MESH GENERATION AND ADAPTIVITY

11:45 - 12:30 Adélia Sequeira  
MODELING AND SIMULATIONS OF THE BLOOD COAGULATION PROCESS

## CONTRIBUTED TALKS

**Room** Pequeno Auditório**Chairmen:** Denis Hautesserres

14:00 - 14:30 Jan Milewshi

14:30 - 15:00 Andrejs Reinfelds  
STABILITY OF DYNAMIC SYSTEMS ON TIME SCALE15:00 - 15:30 Lorena Saavedra  
THE EIGENVALUE CHARACTERIZATION FOR DISCONJUGACY  
OF  $n$  ORDER LINEAR DIFFERENTIAL EQUATIONS15:30 - 16:00 Salima Bensebaa  
MULTIPLE POSITIVE SOLUTIONS FOR A  
FRACTIONAL BOUNDARY VALUE PROBLEM16:00 - 16:30 Lahcene Guedda  
ON THE STRUCTURE OF THE SOLUTION SET OF ABSTRACT INCLUSIONS  
WITH INFINITE DELAY IN A BANACH SPACE16:30 - 17:00 *Coffee-Break***Room** 105**Chairmen:** Vladimir Vasilyev14:00 - 14:30 Hilmi Ergoren  
IMPULSIVE DELAY FRACTIONAL DIFFERENTIAL EQUATIONS  
WITH VARIABLE MOMENTS14:30 - 15:00 Hassan A. Zedan  
SYMMETRY PROPERTIES FOR NONLINEAR SYSTEM  
OF FRACTIONAL DIFFERENTIAL EQUATIONS15:00 - 15:30 Yücel Çenesiz  
FRACTIONAL COMPLEX TRANSFORM FOR CONFORMABLE  
FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS15:30 - 16:00 Ayşe Betül Koç  
FIBONACCI TYPE COLLOCATION APPROACH FOR SOLVING SYSTEMS  
OF HIGH-ORDER LINEAR DIFFERENTIAL EQUATIONS16:30 - 17:00 *Coffee-Break*



**Room 211****Chairmen:** Jaqueline G. Mesquita

- 14:00 - 14:30 Mustafa Kudu  
ASYMPTOTIC ESTIMATES FOR SINGULARLY PERTURBED  
BOUNDARY VALUE PROBLEMS DEPENDING ON A PARAMETER
- 14:30 - 15:00 Ivan Staliarchuk  
MIXED PROBLEM FOR KLEIN-GORDON-FOCK-EQUATION  
WITH CURVE DERIVATIVES ON BOUNDS
- 15:00 - 15:30 Ganga Ram Gautam  
EXISTENCE OF MILD SOLUTIONS FOR IMPULSIVE FRACTIONAL  
FUNCTIONAL DIFFERENTIAL EQUATIONS OF ORDER  $\alpha \in (1, 2)$
- 15:30 - 16:00 Abdelhamid Ainouz  
HOMOGENIZATION OF A CONVECTION-DIFFUSION PROBLEM  
WITH REACTION IN BI-POROUS MEDIA
- 16:00 - 16:30 Moulay Driss Aouragh  
UNIFORM STABILIZATION OF A HYBRID SYSTEM OF ELASTICITY
- 16:30 - 17:00 *Coffee-Break*

**Room 218****Chairmen:** Natascha Neumärker

- 14:00 - 14:30 Ayed H. Al e'damat  
ANALYTICAL SOLUTIONS OF TWO-POINT BOUNDARY VALUE PROBLEMS:  
VOLTERRA INTEGRO-DIFFERENTIAL EQUATION
- 14:30 - 15:00 Mohammed H. Al-Smadi  
A RELIABLE ALGORITHM OF THE RKHS METHOD  
FOR SOLVING LANE-EMDEN TYPE EQUATIONS
- 15:00 - 15:30 Alka Chadha  
EXISTENCE OF THE MILD SOLUTION FOR NONLOCAL FRACTIONAL  
DIFFERENTIAL EQUATION OF SOBOLEV TYPE  
WITH ITERATED DEVIATING ARGUMENTS
- 15:30 - 16:00 Călin-Constantin Șerban  
DISCONTINUOUS PERTURBATIONS OF SINGULAR  $\phi$ -LAPLACIAN OPERATOR
- 16:00 - 16:30 Mehdi Benabdallah  
CONTROL OF SOME DEGENERATE  
DIFFERENTIAL SYSTEMS IN HILBERT SPACES
- 16:30 - 17:00 *Coffee-Break*

**Room 306****Chairmen:** Vafokul E. Ergashev

14:00 - 14:30 Valery Gaiko

LIMIT CYCLE BIFURCATIONS OF POLYNOMIAL  
DIFFERENTIAL AND DIFFERENCE EQUATIONS

14:30 - 15:00 Zagharide Z. El Abidine

COMBINED EFFECTS IN A SEMILINEAR FRACTIONAL  
TWO-POINT BOUNDARY VALUE PROBLEMS

15:00 - 15:30 Mouhcine Tilioua

ON THE LARGE TIME BEHAVIOUR OF A FERROELECTRIC SYSTEM

15:30 - 16:00 Fernane Khaireddine

A NUMERICAL METHOD FOR SOLVING LINEAR FREDHOLM  
BY THE RATIONALIZED HAAR (RH) FUNCTIONS16:30 - 17:00 *Coffee-Break*

## 1.6 Scientific Program - Tables

Monday, May 18

|               | Grande Aud.   |
|---------------|---------------|
| 10:00 - 10:45 | Open Cerimony |
| 10:45 - 11:15 | Coffee- Break |
| Chairman      | Z. Dosla      |
| 11:15 - 12:00 | P. Drabek     |
| 12:00 - 12:30 |               |
| 12:30 - 14:00 | Lunch         |

|               | Pequeno Aud.  | 105            | 211           |
|---------------|---------------|----------------|---------------|
| Chairman      | P. Zemanek    | R. S. Hilscher | M. Migda      |
| 14:00 - 14:30 | F. Erdogan    | M. Pituk       | A. Tsandzana  |
| 14:30 - 15:00 | M. Sakar      | J. Cermak      | F. Maestre    |
| 15:00 - 15:30 | O. Saldir     | L. Nechvatal   | G. Holubova   |
| 15:30 - 16:00 | D. Tavares    | M. Unal        | P. Stehlik    |
| 16:00 - 16:30 | Coffee- Break | Coffee- Break  | Coffee- Break |
| Chairman      | G. Bognar     | I. Drazic      | V. Gaiko      |
| 16:30 - 17:00 | L. Csizmadia  | I. Astashova   | L. Softova    |
| 17:00 - 17:30 | J. Zielinsky  | E. Galakhov    | J. Volek      |

|               | 218              | 306              | 315             |
|---------------|------------------|------------------|-----------------|
| Chairman      | A. Cernea        | V. Sushch        | J. Dzurina      |
| 14:00 - 14:30 | M. Zima          | Y. Prykarpatskyy | B. Szyszka      |
| 14:30 - 15:00 | M. Tvrdy         | S. Kryzhevich    | D. Hautesserres |
| 15:00 - 15:30 | I. Rachunkova    | K. Palmer        | G. Amirali      |
| 15:30 - 16:00 | A. F. Tojo       | N. Neumarker     | I. Amirali      |
| 16:00 - 16:30 | Coffee- Break    | Coffee- Break    | Coffee- Break   |
| Chairman      | A. Reinfelds     | M. Rebelo        | J. Krejcova     |
| 16:30 - 17:00 | M. Langerova     | D. Shoikhet      | S. Manko        |
| 17:00 - 17:30 | N. Fewster-Young | J. S. Viridi     | M. Elin         |

Tuesday, May 19

|                      | Grande Aud.            | Pequeno Aud.      |
|----------------------|------------------------|-------------------|
| <b>Chairman</b>      | <i>A. Sulkin</i>       |                   |
| <b>9:00 - 9:45</b>   | <b>I. Stavroulakis</b> |                   |
| <b>9:45 - 10:30</b>  | <b>A. Guran</b>        |                   |
| <b>10:30 - 11:00</b> | <b>Coffee- Break</b>   |                   |
| <b>Chairman</b>      | <i>S. Hilger</i>       | <i>A. Zafer</i>   |
| <b>11:00 - 11:45</b> | <b>I. Gyori</b>        | <b>L. Sanchez</b> |
| <b>11:45 - 12:30</b> | <b>L. Berezansky</b>   | <b>A. Cabada</b>  |
| <b>12:30 - 14:00</b> | <b>Lunch</b>           | <b>Lunch</b>      |

|                      | Pequeno Aud.         | 105                  | 211                   |
|----------------------|----------------------|----------------------|-----------------------|
| <b>Chairman</b>      | <i>M. Unal</i>       | <i>I. Rachunkova</i> | <i>G. Holubova</i>    |
| <b>14:00 - 14:30</b> | <b>V. S. Erturk</b>  | <b>R. Surmanidze</b> | <b>P. Zemanek</b>     |
| <b>14:30 - 15:00</b> | <b>H. Usami</b>      | <b>M. Bakuradze</b>  | <b>R. S. Hilscher</b> |
| <b>15:00 - 15:30</b> | <b>T. Tanigawa</b>   | <b>W. Varnhorn</b>   | <b>J. Elyseeva</b>    |
| <b>15:30 - 16:00</b> | <b>Z. Oplustil</b>   | <b>M. Rebelo</b>     | <b>C. M. Kent</b>     |
| <b>16:00 - 16:30</b> | <b>Coffee- Break</b> | <b>Coffee- Break</b> | <b>Coffee- Break</b>  |
| <b>Chairman</b>      | <i>L. Saavedra</i>   | <i>Z. Kayar</i>      | <i>P. Stehlik</i>     |
| <b>16:30 - 17:00</b> | <b>J. Dzurina</b>    | <b>A. Hussein</b>    | <b>P. Sepitka</b>     |
| <b>17:00 - 17:30</b> | <b>B. Lawrence</b>   | <b>S. Das</b>        | <b>R. Kozlov</b>      |

|                      | 218                     | 306                     | 315                   |
|----------------------|-------------------------|-------------------------|-----------------------|
| <b>Chairman</b>      | <i>H. Koçak</i>         | <i>D. Ni</i>            | <i>J. Milewski</i>    |
| <b>14:00 - 14:30</b> | <b>J. E. Buendia</b>    | <b>V. Cherepennikov</b> | <b>A. Paiva</b>       |
| <b>14:30 - 15:00</b> | <b>I. Dassios</b>       | <b>R. L. Tudorache</b>  | <b>A. Zvyagin</b>     |
| <b>15:00 - 15:30</b> | <b>D. Shoikhet</b>      | <b>A. Dutkiewicz</b>    | <b>V. Zvyagin</b>     |
| <b>15:30 - 16:00</b> | <b>V. Sushch</b>        | <b>G. Bogнар</b>        | <b>V. Vasilyev</b>    |
| <b>16:00 - 16:30</b> | <b>Coffee- Break</b>    | <b>Coffee- Break</b>    | <b>Coffee- Break</b>  |
| <b>Chairman</b>      | <i>M. Langerova</i>     | <i>L. Csizmadia</i>     | <i>I. Astashova</i>   |
| <b>16:30 - 17:00</b> | <b>T. Shaposhnikova</b> | <b>J. Burkotova</b>     | <b>P. Dvalishvili</b> |
| <b>17:00 - 17:30</b> | <b>N. Rautian</b>       | <b>J. Rebenda</b>       | <b>S. Soradi Zeid</b> |

Wednesday, May 20

|                        | <b>Grande Aud.</b>   | <b>Pequeno Aud.</b>       |
|------------------------|----------------------|---------------------------|
| <b><i>Chairman</i></b> | <i>P. Drabek</i>     |                           |
| <b>9:00 - 9:45</b>     | <b>A. Sulkin</b>     |                           |
| <b><i>Chairman</i></b> | <i>T. Krisztin</i>   | <i>A. Sequeira</i>        |
| <b>9:45 - 10:30</b>    | <b>M. Bohner</b>     | <b>H. Beirão da Veiga</b> |
| <b>10:30 - 11:00</b>   | <b>Coffee- Break</b> | <b>Coffee- Break</b>      |
| <b>11:00 - 11:45</b>   | <b>O. Dosly</b>      | <b>S. Stevic</b>          |
| <b>11:45 - 12:30</b>   | <b>S. Hilger</b>     | <b>T. Caraballo</b>       |
| <b>12:30 - 14:00</b>   | <b>Lunch</b>         | <b>Lunch</b>              |

Thursday, May 21

|                      | <b>Grande Aud.</b>   | <b>Pequeno Aud.</b>    |
|----------------------|----------------------|------------------------|
| <b>Chairman</b>      | <i>M. Bohner</i>     |                        |
| <b>9:00 - 9:45</b>   | <b>M. Chipot</b>     |                        |
| <b>Chairman</b>      | <i>H. Akca</i>       | <i>I. Stavroulakis</i> |
| <b>9:45 - 10:30</b>  | <b>S. Siegmund</b>   | <b>N. Atakishiyev</b>  |
| <b>10:30 - 11:00</b> | <b>Coffee- Break</b> | <b>Coffee- Break</b>   |
| <b>11:00 - 11:45</b> | <b>P. Kloeden</b>    | <b>A. Zafer</b>        |
| <b>11:45 - 12:30</b> | <b>A. Ashyralyev</b> | <b>T. Krisztin</b>     |
| <b>12:30 - 14:00</b> | <b>Lunch</b>         | <b>Lunch</b>           |

|                      | <b>Pequeno Aud.</b>   | <b>105</b>             | <b>211</b>             |
|----------------------|-----------------------|------------------------|------------------------|
| <b>Chairman</b>      | <i>F. Maestre</i>     | <i>M. Tvrdy</i>        | <i>M. Zima</i>         |
| <b>14:00 - 14:30</b> | <b>D. Ni</b>          | <b>R. Marques</b>      | <b>S. Servi</b>        |
| <b>14:30 - 15:00</b> | <b>J. Krejcova</b>    | <b>G. Monteiro</b>     | <b>O. Vera</b>         |
| <b>15:00 - 15:30</b> | <b>M. Migda</b>       | <b>A. Slavik</b>       | <b>A. Faminskii</b>    |
| <b>15:30 - 16:00</b> | <b>A. K. Tripathy</b> | <b>D. Torres</b>       | <b>N. Crnjaric-Zic</b> |
| <b>16:00 - 16:30</b> | <b>Coffee- Break</b>  | <b>Coffee- Break</b>   | <b>Coffee- Break</b>   |
| <b>Chairman</b>      | <i>A. Paiva</i>       | <i>A. Chadha</i>       | <i>Y. Cenesiz</i>      |
| <b>16:30 - 17:00</b> | <b>J. Mesquita</b>    | <b>L. Karalashvili</b> | <b>I. Drazic</b>       |
| <b>17:00 - 17:30</b> | <b>A. Cernea</b>      | <b>E. Azizbayov</b>    | <b>M. Bodzioch</b>     |

|                      | <b>218</b>           | <b>306</b>           | <b>315</b>            |
|----------------------|----------------------|----------------------|-----------------------|
| <b>Chairman</b>      | <i>H. Usami</i>      | <i>R. Surmanidze</i> | <i>J. E. Buendia</i>  |
| <b>14:00 - 14:30</b> | <b>H. Kocak</b>      | <b>D. Poltem</b>     | <b>A. Ajlouni</b>     |
| <b>14:30 - 15:00</b> | <b>A. Biswas</b>     | <b>F. Toumi</b>      | <b>A. Kumar</b>       |
| <b>15:00 - 15:30</b> | <b>V. Asthana</b>    | <b>A. Makin</b>      | <b>N. Kereselidze</b> |
| <b>15:30 - 16:00</b> | <b>V. Ergashev</b>   | <b>V. Vlasov</b>     | <b>M. Rajchakit</b>   |
| <b>16:00 - 16:30</b> | <b>Coffee- Break</b> | <b>Coffee- Break</b> | <b>Coffee- Break</b>  |
| <b>Chairman</b>      | <i>C. Serban</i>     | <i>L. Guedda</i>     | <i>A. F. Tojo</i>     |
| <b>16:30 - 17:00</b> | <b>T. Taniguchi</b>  | <b>Z. Kayar</b>      | <b>G. Rajchakit</b>   |
| <b>17:00 - 17:30</b> | <b>H. Mahmoud</b>    | <b>A. Ateiwi</b>     | <b>S. Sevgin</b>      |

Friday, May 22

|                      | Grande Aud.          | Pequeno Aud.         |
|----------------------|----------------------|----------------------|
| <b>Chairman</b>      | <i>I. Gyori</i>      |                      |
| <b>9:00 - 9:45</b>   | <b>J. J. Nieto</b>   |                      |
| <b>Chairman</b>      | <i>O. Dosly</i>      | <i>A. Ashyralyev</i> |
| <b>9:45 - 10:30</b>  | <b>H. Akca</b>       | <b>P. Bates</b>      |
| <b>10:30 - 11:00</b> | <b>Coffee- Break</b> | <b>Coffee- Break</b> |
| <b>11:00 - 11:45</b> | <b>Z. Dosla</b>      | <b>S. Korotov</b>    |
| <b>11:45 - 12:30</b> | <b>F. Sadyrbaev</b>  | <b>A. Sequeira</b>   |
| <b>12:30 - 14:00</b> | <b>Lunch</b>         | <b>Lunch</b>         |

|                      | Pequeno Aud.           | 105                  | 211                   |
|----------------------|------------------------|----------------------|-----------------------|
| <b>Chairman</b>      | <i>D. Hautesserres</i> | <i>V. Vasilyev</i>   | <i>J. Mesquita</i>    |
| <b>14:00 - 14:30</b> | <b>J. Milewski</b>     | <b>H. Ergoren</b>    | <b>M. Kudu</b>        |
| <b>14:30 - 15:00</b> | <b>A. Reinfelds</b>    | <b>H. Zedan</b>      | <b>I. Staliarchuk</b> |
| <b>15:00 - 15:30</b> | <b>L. Saavedra</b>     | <b>Y. Cenesiz</b>    | <b>G. Ram</b>         |
| <b>15:30 - 16:00</b> | <b>S. Bensebaa</b>     | <b>B. Koc</b>        | <b>A. Ainouz</b>      |
| <b>16:00 - 16:30</b> | <b>L. Guedda</b>       |                      | <b>M. D. Aouragh</b>  |
| <b>16:30 - 17:00</b> | <b>Coffee- Break</b>   | <b>Coffee- Break</b> | <b>Coffee- Break</b>  |

|                      | 218                   | 306                   |
|----------------------|-----------------------|-----------------------|
| <b>Chairman</b>      | <i>N. Neumarker</i>   | <i>V. Ergashev</i>    |
| <b>14:00 - 14:30</b> | <b>Al e' damat</b>    | <b>V. Gaiko</b>       |
| <b>14:30 - 15:00</b> | <b>M. Al-Smadi</b>    | <b>Z. El Abidine</b>  |
| <b>15:00 - 15:30</b> | <b>A. Chadha</b>      | <b>M. Tilioua</b>     |
| <b>15:30 - 16:00</b> | <b>C. Serban</b>      | <b>F. Khaireddine</b> |
| <b>16:00 - 16:30</b> | <b>M. Benabdallah</b> |                       |
| <b>16:30 - 17:00</b> | <b>Coffee- Break</b>  | <b>Coffee- Break</b>  |





## 2

# Abstracts

### 2.1 Invited Lectures

#### GLOBAL EXPONENTIAL STABILITY OF NEUTRAL TYPE CELLULAR NEURAL NETWORKS WITH IMPULSES, TIME-VARYING AND DISTRIBUTED DELAYS

**Haydar Akça**, *Abu Dhabi, UAE*

2000 MSC: 34A37, 65Q05, 92B20.

**Abstract:** This is a joint work with Valéry Covachev and Eadah Alzahrani.

From the mathematical point of view, cellular neural networks (CNN) can be characterized by an array of identical nonlinear dynamical systems called cells that are locally interconnected and imposing set of sufficient conditions for ensuring the exponential stability of the considered system. Dynamical systems are often classified into two categories of either continuous-time or discrete-time systems. In addition, many real world evolutionary processes are characterized by an abrupt change at certain time. These changes are so called impulsive phenomena. This third category of dynamical systems displays a combination of characteristic of both the continuous-time and discrete-time systems. Impulsive CNNs with delay and distributed delays are studied by many researchers. We consider following impulsive neutral type cellular neural networks with distributed and time-varying delay dif-

ferential equation

$$\begin{cases} \dot{x}_i = -c_i(x_i(t)) + \sum_{j=1}^n a_{ij}f_j(x_j(t)) + \sum_{j=1}^n b_{ij} \int_0^\infty K_{ij}(s)h_j(x_j(t-s))ds \\ + d_i \dot{x}_i(t - \sigma_i(t)) + I_i, \quad t \neq t_k, \quad t > t_0 \\ \Delta x_i(t_k) = x_i(t_k + 0) - x_i(t_k) = P_{ik}(x_i(t_k)), \quad k = 1, 2, \dots, \\ x_{t_0}(t) = \phi(t), \quad t \in [t_0 - \tau, t_0] \end{cases} \quad (2.1)$$

where  $n \geq 2$  is the number of neurons in the network,  $x_i(t)$  state of the  $i$ th neuron at time  $t$  and  $f_j, h_j$  denote activation function respectively.  $c_i > 0, d_i > 0$  constants and  $a_{ij}, b_{ij}$  represents connection strengths of the  $j$ th neuron on the  $i$ th neuron respectively. The  $P_{ik}$  represents impulsive perturbation of the  $i$ th neuron at time  $t_k$  and always assume that  $x_i(t_k^+) = x_i(t_k), k \in \mathbb{N}$ . When  $P_{ik} = 0$  for all  $x \in \mathbb{R}^n, i = 1, 2, \dots, n, k \in \mathbb{N}$  then model becomes a continuous cellular neural networks. The delay kernel  $K_{ij} : [0, \infty) \rightarrow [0, \infty)$  is a real valued continuous function and satisfies

$$\int_0^\infty e^{\lambda s} K_{ij}(s) ds = \rho_{ij}(\lambda) \text{ and } \sigma = \max_{1 \leq i \leq n} \{\sigma_i\}, \rho = \max\{\tau, \sigma\}, s \in [-\rho, 0]$$

We introduced sufficient conditions for the globally exponential stability of the solution of neutral type CNNs with impulses and improve and generalized some known results in the literature related with neutral type CNNs.

## ON SOURCE IDENTIFICATION PROBLEM FOR A TELEGRAPH PARTIAL DIFFERENTIAL AND DIFFERENCE EQUATIONS

**Allaberen Ashyralyev**, *Istanbul, Turkey*

2000 MSC: 65M12 35L20 35R11

**Abstract:** This is a joint work with F.Çekiç on the source identification problem for a telegraph equation

$$\begin{cases} \frac{d^2 u(t)}{dt^2} + \alpha \frac{du(t)}{dt} + Au(t) = p + f(t) \quad (0 \leq t \leq T), \\ u(0) = \varphi, u'(0) = \psi, u(T) = \xi \end{cases} \quad (2.2)$$

in a Hilbert space  $H$  with the self-adjoint positive definite operator  $A$ . Operator approach permitted us establish stability estimates for the solution of problem (2.2). The first and second order of accuracy difference schemes for the approximate solution of problem (2.2) are presented. Stability estimates for the solution of these difference schemes are established. In applications, three source identification

problems for telegraph equations are investigated. The methods are illustrated by numerical examples.

ON A DISCRETE NUMBER OPERATOR AND ITS  
EIGENVECTORS ASSOCIATED WITH THE 5D  
DISCRETE FOURIER TRANSFORM

**N. M. Atakishiyev**, *Morelos, México*

**Abstract:** This is joint work with M. K. Atakishiyeva and J. Méndez Franco.

We construct an explicit form of a difference analogue of the quantum number operator in terms of the raising and lowering operators that govern eigenvectors of the 5D discrete (finite) Fourier transform. Eigenvalues of this difference operator are represented by distinct nonnegative numbers so that it can be used to systematically classify, in complete analogy with the case of the continuous classical Fourier transform, eigenvectors of the 5D discrete Fourier transform, thus resolving the ambiguity caused by the well-known degeneracy of the eigenvalues of the discrete Fourier transform.

SPECTRAL CONVERGENCE AND BIFURCATION IN NONLOCAL  
DIFFUSION-REACTION EQUATIONS

**Peter Bates**, *Michigan, USA*

**Abstract:** Many physical and biological processes occur with long range interaction, giving rise to equations with nonlocal in space operators in place of the usual Laplacian. These operators are diffusive-like but are bounded rather than unbounded as is the case of the diffusion operator. We study systems which include such nonlocal operators and through a spectral convergence result when a certain scaling parameter becomes small, show that bifurcations occur, including Turing instabilities, leading to stable patterned states.

CONCERNING THE EXISTENCE OF CLASSICAL SOLUTIONS TO THE  
STOKES SYSTEM. ON THE MINIMAL ASSUMPTIONS PROBLEM

**H. Beirão da Veiga**, *Pisa, Italy*

**Abstract:** We say that a solution to a boundary value PDE problem is *classical* if all derivatives appearing in the equations and boundary conditions are continuous up to the boundary. It is well known that, in general, solutions to linear

elliptic boundary value problems (for instance, the Stokes system) are classical if the external forces belong to a Hölder space  $C^{0,\lambda}(\overline{\Omega})$ . It is also well known that, in general, solutions are not classical in the presence of merely continuous external forces. Hence, a challenging problem is to find Banach spaces, strictly containing the Hölder spaces  $C^{0,\lambda}(\overline{\Omega})$ , such that solutions are classical for data in these spaces. We will discuss this matter, and related problems.

### GLOBAL EXPONENTIAL STABILITY FOR NONLINEAR DELAY DIFFERENTIAL SYSTEMS

**L. Berezansky**, *Beer Sheva, Israel*

**Abstract:** One of the main motivations to investigate nonlinear delay differential system is their importance in the study of artificial neural network models and more generally in Mathematical Biology.

In this talk we will discuss global stability problem. Such investigations one can divide by the form of a system: vector or scalar form and also by the method of investigation. The main methods are: constructing of a Lyapunov functional, applications of special matrix such as M-matrix, fixed point approach and using a notion of nonlinear Volterra operator.

In this talk we consider all forms of systems - vector and scalar form and some of methods - applications of M-matrix and matrix measure, and using a Volterra general operator.

### INVERSE PROBLEMS FOR STURM-LIOUVILLE DIFFERENCE EQUATIONS

**Martin Bohner**, *Rolla, MO, USA*

**Abstract:** We consider a discrete Sturm–Liouville problem with Dirichlet boundary conditions. We show that the specification of the eigenvalues and weight numbers uniquely determines the potential. Moreover, we also show that if the potential is symmetric, then it is uniquely determined by the specification of the eigenvalues. The corresponding transformation operator satisfies a partial difference equation of hyperbolic type. These are discrete versions of well-known results for corresponding differential equations.

- [1] R. P. Agarwal, M. Bohner, S. R. Grace, and D. O'Regan: *Discrete Oscillation Theory*, Hindawi Publishing Corporation, 2005.

- [2] M. Bohner and A. C. Peterson: *Dynamic Equations on Time Scales: An Introduction with Applications*, Birkhäuser, 2001.
- [3] M. Bohner and H. Koyunbakan: *Inverse Problems for Sturm–Liouville Difference Equations*, Filomat, in press, 2015.
- [4] G. Freiling and V. Yurko: *Inverse Sturm–Liouville Problems and their Applications*, Nova Science Publishers Inc., 2001.
- [5] W. G. Kelley and A. C. Peterson: *Difference Equations: An Introduction with Applications*, Academic Press, second edition, 2001.

CONSTANT SIGN SOLUTIONS OF DIFFERENTIAL EQUATIONS  
WITH INVOLUTIONS

Alberto Cabada, *Santiago de Compostela, Spain*

2000 MSC: 34B15, 34B18, 34B27

**Abstract:** This is a joint work with Adrián F. Tojo.

This talk is devoted to the study of the following first order functional equation, coupled with periodic boundary value conditions:

$$x'(t) = f(t, x(t), x(\varphi(t))), \text{ for a. e. } t \in J, \quad x(\inf J) = x(\sup J).$$

Here function  $\varphi : J \rightarrow J$  is such that  $\varphi \circ \varphi = \text{Id}$ , and it is called an involution.

It will be studied the constant sign of Green's function related to the linear equation

$$x'(t) + a(t)x(t) + b(t)x(\varphi(t)) = h(t), \text{ for a. e. } t \in J, \quad x(\inf J) = x(\sup J).$$

Moreover, we characterize the oscillation of the solutions of the nonlinear equation

$$x'(t) = f(x(\varphi(t))), \text{ for a. e. } t \in \mathbb{R}.$$

- [1] A. Cabada and A. F. Tojo, Comparison results for first order linear operators with reflection and periodic boundary value conditions. *Nonlinear Anal.* **78** (2013), 32-46.
- [2] A. Cabada and A. F. Tojo, Existence results for a linear equation with reflection, non-constant coefficient and periodic boundary conditions. *J. Math. Anal. Appl.* **412** (2014), 529-546.

# LOCAL AND GLOBAL STABILITY OF 2D NAVIER-STOKES EQUATIONS WITH DELAYS

**Tomás Caraballo**, *Sevilla, Spain*

2000 MSC: 35R10, 35B40, 47H20, 58F39, 73K70

**Abstract:** In this talk we will show several methods to analyze the asymptotic behaviour of solutions to 2D Navier-Stokes models when some hereditary characteristics (constant, distributed or variable delay, memory, etc) appear in the formulation. First the local stability analysis of steady-state solutions is studied by using several methods: the theory of Lyapunov functions, the Razumikhin-Lyapunov technique, by constructing appropriate Lyapunov functionals and finally by using a method based in Gronwall-like inequalities. Then the global asymptotic behavior of solutions can be analyzed by using the theory of attractors. As the delay terms are allowed to be very general, the statement of the problem becomes nonautonomous in general. For this reason, the theory of nonautonomous pullback attractors appears to be appropriate.

- [1] T. Caraballo & J. Real, Asymptotic behaviour of 2D Navier-Stokes equations with delays, *Proc. R. Soc. Lond. A*, 459(2003), 3181-3194.
- [2] T. Caraballo & J. Real, Attractors for 2D Navier-Stokes models with delays, *J. Differential Equations* 205(2004), 271-297.
- [3] T. Caraballo, J. Real & L. Shaikhet, Method of Lyapunov functionals construction in stability of delay evolution equations, *J. Math. Anal. Appl.* 334 (2007), 1130-1145.
- [4] T. Caraballo & X. Han, Stability of stationary solutions to 2D-Navier-Stokes models with delays, *Dyn. Partial Differ. Equ.* 11 (2014), no. 4, 345-359.

# NON HOMOGENEOUS BOUNDARY VALUE PROBLEMS FOR THE STATIONARY NAVIER-STOKES EQUATIONS IN 2-D SYMMETRIC SEMI-INFINITE OUTLETS

**Michel Chipot**, *Zurich, Switzerland*

**Abstract:** This is a joint work with K. Kaulakyte, K. Pileckas and W. Xue.

We would like to present existence results for the stationary non homogeneous Navier-Stokes problem in symmetric domains having a semi-infinite outlet. We assume for this Leray problem the so called general outflow condition.

COEXISTENCE PROBLEM FOR POSITIVE SOLUTIONS OF SECOND  
ORDER DIFFERENTIAL EQUATIONS

**Zuzana Došlá**, *Brno, Czech Republic*

2000 MSC: 34C10

**Abstract:** We resolve the open problem concerning the coexistence on three possible types of nonoscillatory solutions (subdominant, intermediate, and dominant solutions) for the second order quasilinear differential equation

$$(a(t)|x'|^\alpha \operatorname{sgn} x')' + b(t)|x|^\beta \operatorname{sgn} x = 0 \quad (t \geq 0) \quad (2.3)$$

in the super-linear case  $0 < \alpha < \beta$ .

Jointly with this, by using a new approach based on a special energy-type function  $E$ , the existence of intermediate globally positive solutions is examined.

This is a joint research with Mauro Marini, University of Florence.

OSCILLATION THEORY OF EVEN-ORDER LINEAR AND HALF-LINEAR  
DIFFERENTIAL EQUATIONS

**Ondřej Došlý**, *Brno, Czech Republic*

2010 MSC: 34C10

**Abstract:** We investigate oscillatory properties of solutions of the half-linear  $2n$ -order differential equation

$$\sum_{k=0}^n (-1)^k \left( r_k(t) \Phi(y^{(k)}) \right)^{(k)} = 0 \quad (2.4)$$

where  $\Phi(y) = |y|^{p-2}y$ ,  $p > 1$ ,  $r_j$  are continuous,  $j = 0, \dots, n$ ,  $r_n(t) > 0$ , and of its “linear version” corresponding to  $p = 2$ , i.e.,  $\Phi(y) = y$ . Oscillation theory of (2.4) with  $p = 2$  is deeply developed and essentials of this theory can be found e.g. in [1]. We show how linear (non)oscillation criteria extend to (2.4). The presented results were achieved jointly with Vojtěch Růžička and are published in [2].

- [1] I. M. Glazman, *Direct Methods of Qualitative Analysis of Singular Differential Operators*, Davey, Jerusalem 1965.
- [2] O. Došlý, V. Růžička, Nonoscillation of higher order half-linear differential equations, *Electron. J. Qual. Theory Differ. Equ.* **2015** (2015), No. 15, 1–15.

# INVERSE POSITIVITY AND NEGATIVITY PROPERTY OF THE FOURTH ORDER OPERATOR WITH VARIABLE STIFFNESS COEFFICIENT

**Pavel Drabek**, *Pilsen, Czech Republic*

**Abstract:** In this lecture we focus on the positive and negative classical solutions of the fourth order problem  $u^{(4)} + c(x)u = h(x)$ ,  $u(0) = u''(0) = u(1) = u''(1) = 0$ . It models stationary solutions of the supported beam. We consider nonnegative right hand side  $h = h(x)$ , representing the load, and variable stiffness coefficient  $c = c(x)$ . We discuss different conditions on  $c(x)$  under which corresponding problem possesses either strictly positive, or else, strictly negative solution. We show that contrary to the constant stiffness  $c$ , when necessary and sufficient conditions may be formulated, the variable case  $c(x)$  is more complicated. We show that in contrast with the constant case, for variable  $c(x)$  only “its substantial part” must be in prescribed range, while on “small” subsets of  $[0, 1]$ ,  $c(x)$  may assume arbitrarily small or large values.

# ON A POPULATION MODEL EQUATION WITH DELAYED BIRTH AND UNDELAYED INHIBITATION TERM

**István Györi**, *Veszprém, Hungary*

2000 MSC: 34K30

**Abstract:** This is a joint work with Ferenc Hartung and Nahed A. Mohamady. Motivated by simple non-delayed population equations in this work we consider a population model with delay in the birth term and no delay in the self-inhibition term. The form of the delay is based on the works of the authors [3], who argued that the delay should enter in the the birth term instead of the death term. Our main result gives lower and upper estimates for the positive solutions. The results are illustrated by examples. One of the corollaries is a novel global attractivity result which applies to equations with a unique periodic solution and to equations with an asymptotic equilibrium.

- [1] J. Bařtinec, L. Berezansky, J. Diblí k, Z. Šmarda, On a delay population model with a quadratic nonlinearity without positive steady state, *Appl. Math. Comput.* **227** (2014) 622-629.
- [2] T. Faria, A note on permanence of nonautonomous cooperative scalar population models with delays, *Appl. Math. Comput.* **240** (2014) 82-90.



- [3] K. P. Hadeler, G. Bocharov, Where to put delays in population models, in particular in the neutral case, *Can. Appl. Math. Q.* **11** (2003) 159-173.

## RECENT RESULTAS ON STABILITY, VIBRITIONS AND CONTROL OF MECHANICAL STRUCTURES

**Ardeshir Guran**, *Montreal, Canada*

**Abstract:** In this presentation I give an overview of our collaborative research work with i) SVS SVSFEM ANSYS Group, Brno (Czech Republic), ii) Pupin Institute, Belgrade (Serbia) iii) Alexander Dubček University, Púchov (Slovak Republic).

The presentation includes the results of the following ongoing projects:

- Enhancing the capability of walking, running and Jumping using Skyrunners
- Shape Control, and Vibration Suppression of Mechanical Structures using Fluidic Actuators
- Experimental and Numerical Modal Analysis of Elastic Structures Including the Effects of Piezoelectric Sensors and Actuators

## OPERATOR CALCULUS FOR (q,h)-DIFFERENCE EQUATIONS

**Stefan Hilger**, *Eichstätt, Germany*

**Abstract:** We introduce the so called (q,h)-deformed Weyl algebra and study various algebraic features appearing in this algebra. Many properties of (q,h)-difference operators are derived. We shall present explicitly the isomorphisms and limiting behavior between different (q,h)-Weyl algebras. Next we shall show that the q-deformed universal enveloping algebra (quantum group)  $U_q = U_q(sl(2))$  is embedded into the tensor product of two (q,h)-Weyl algebras. Then we will analyze the so called structure ladder of  $U_q$  by means of ladder theory. Finally we will show that we can form the limit  $h = 0/q$  to 1 in the whole theory by exact mathematical reasoning rather than doing it informally.

## CONSTRUCTION OF FOWARD ATTRACTORS IN NONAUTONOMOUS DYNAMICAL SYSTEMS

**Peter Kloeden**, *Wuhan, China*

2000 MSC: 34B45, 37B55, 37C70

**Abstract:** Autonomous systems depend only on the elapsed time, so their attractors and limit sets exist in current time. Similarly, the pullback limit defines a component set of a nonautonomous pullback attractor at each instant of current time. The forward limit defining a nonautonomous forward attractor is different as it is the limit to the asymptotically distant future. In particular, the limiting objects forward in time do not have the same dynamical meaning in current time as in the autonomous or pullback cases. Nevertheless, the pullback limit taken within a positively invariant family of compact subsets allows the component set of a forward attractor to be constructed at each instant of current time. Every forward attractor has such a positively invariant family of compact subsets, which ensures that the component sets of a forward attractor can be constructed in this way. It is, however, only a necessary condition and not sufficient for the constructed family of subsets to be a forward attractor. The analysis here is presented in the state space  $\mathbb{R}^d$  to focus on the dynamical essentials rather than on functional analytical technicalities, in particular those concerning asymptotic compactness properties.

- [1] P.E. Kloeden and M. Rasmussen, *Nonautonomous dynamical systems*, Amer. Math. Soc., Providence, 2011.
- [2] P. E. Kloeden and T. Lorenz, *The construction of non- autonomous forward attractors*. (submitted)

## BISECTION-TYPE ALGORITHMS FOR MESH GENERATION AND ADAPTIVITY

**Sergey Korotov**, *Bergen, Norway*

2000 MSC: 65N50

**Abstract:** In this talk we will present the recently developed bisection-type algorithms which always produce conforming (i.e. face-to-face) finite element (FE) simplicial partitions. The algorithm is also suitable for mesh adaptivity purposes. Various regularity properties of the meshes generated by the algorithms will be discussed and illustrated by numerical tests. Open problems will be presented as well.

## PERIODIC SOLUTIONS OF A DIFFERENTIAL EQUATION WITH A QUEUEING DELAY

**Tibor Krisztin**, *Szeged, Hungary*

2010 MSC: 34K05 34K13

**Abstract:** We consider a differential equation with a state-dependent delay motivated by a queueing process. The time delay is determined by an algebraic equation involving the length of the queue. For the length of the queue a discontinuous differential equation holds. We formulate an appropriate framework to study the problem, and show that the solutions define a Lipschitz continuous semiflow in the phase space. The main result guarantees the existence of slowly oscillating periodic solutions.

This is a joint work with my PhD student, István Balázs.

## FRACTIONAL DIFFERENTIAL EQUATIONS

**Juan J. Nieto**, *Santiago de Compostela, Spain*

We present some recent problems in the area of differential equations of fractional order. New aspects of the theory and practice arise and new methods have to be developed.

We also present some open problems and real world problems modelled by fractional differential equations.

- [1] Agarwal, R.P.; Lakshmikantham, V.; Nieto, J.J. On the concept of solution for fractional differential equations with uncertainty. *Nonlinear Anal.* 72 (2010), 2859-2862.
- [2] Bandyopadhyay, B.; Kamal, S. Stabilization and control of fractional order systems: a sliding mode approach. *Lecture Notes in Electrical Engineering*, 317. Springer, Cham, 2015.
- [3] Debbouche, A., Nieto, J.J. Sobolev type fractional abstract evolution equations with nonlocal conditions and optimal multi-controls. *Applied Mathematics and Computation* 245 (2014), 74-85.
- [4] Herrmann, R. Fractional calculus. An introduction for physicists (Second edition). World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2014.
- [5] Ding, X.-L.; Nieto, J.J. Analytical solutions for the multi-term time-space fractional reactiondiffusion equations on an infinite domain. *Fractional Calculus and Applied Analysis*, To appear, 2015.

- [6] Khastan, A.; Nieto, J.J.; Rodriguez-Lopez, R. Schauder fixed-point theorem in semilinear spaces and its application to fractional differential equations with uncertainty. *Fixed Point Theory and Applications*, 2014(21) (2014), 14 pages.
- [7] Losada, J.; Nieto, J.J. Properties of a New Fractional Derivative without Singular Kernel. *Progress in Fractional Differentiation and Applications*, To appear, 2015.

## OSCILLATORY SOLUTIONS OF BOUNDARY VALUE PROBLEMS

**Felix Sadyrbaev**, *Riga, Latvia*

2000 MSC: 34B15

**Abstract:** Basic elements of the theory of the second order nonlinear boundary value problem

$$x'' = f(t, x, x'), \quad x(a) = A, \quad x(b) = B \quad (2.5)$$

are discussed, including existence of solutions, uniqueness, multiplicity, quasi-linear problems, types of solutions etc.

- [1] M. Dobkevich, F. Sadyrbaev, N. Sveikate and I. Yermachenko. On Types of Solutions of the Second Order Nonlinear Boundary Value Problems. *Abstract and Applied Analysis*, Volume 2014 (2014), Article ID 594931.

## TRAVELLING WAVES AND CRITICAL SPEED IN THE PRESENCE OF NONLINEAR DIFFUSION

**Luís Sanchez**, *Lisboa, Portugal*

**Abstract:** We study some features of the parametric boundary value problem

$$(P(u'))' - cu' + g(u) = 0 \quad (2.6)$$

$$u(-\infty) = 0, \quad u(+\infty) = 1. \quad (2.7)$$

(where  $D > 0$  and  $g(0) = g(1) = 0$ ,  $g > 0$  in  $]0, 1[$ ).

The problem arises when one looks for travelling waves to

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( P \left( \frac{\partial u}{\partial x} \right) \right) + g(u) \quad (2.8)$$

$c$  being the wave speed.

Possible models for  $P$  are  $P(v) = \frac{v}{\sqrt{1-v^2}}$ ,  $P(v) = \frac{v}{\sqrt{1+v^2}}$ .

We shall be interested in similarities and differences between those models. Another interesting type of nonlinear diffusion, involving the one-dimensional  $p$ -Laplacian, corresponds to the choice  $P(v) = |v|^{p-2}v$  in (2.6). An advection term may be considered as well.

As in the classical Fisher-Kolmogorov-Petrovskii-Piskounov equations, there exists an interval of admissible speeds  $[c^*, \infty)$  and characterizations of the *critical speed*  $c^*$  can be obtained.

In particular, we present a variational characterization of the critical speed in the  $p$ -Laplacian setting.

The talk is based on papers by Enguiça, Gavioli and Sanchez (2013), and joint work with S. Correia (2012), Isabel Coelho (2014), Maurizio Garrione (2015) and A. Gavioli (2015).

## DYNAMICAL SYSTEMS ON GRAPHS

**Stefan Siegmund**, *Dresden, Germany*

**Abstract:** Motivated by neural networks and the evolution of cooperation in biology, we study dynamical systems on arbitrary networks modeled by directed graphs. We present a new method to find phase-locked solutions and discuss the dynamics of evolutionary games on graphs. This is joint work with Jeremias Epperlein and Petr Stehlik.

## MODELING AND SIMULATIONS OF THE BLOOD COAGULATION PROCESS

**Adélia Sequeira**, *Lisboa, Portugal*

2010 MSC: Primary 92C10, 92C45; Secondary 76M10.

**Abstract:** Blood coagulation is an extremely complex biological process in which blood forms clots (thrombus) to prevent bleeding; it is followed by their dissolution and the subsequent repair of the injured tissue. The process involves different interactions between the plasma, the vessel wall and platelets with a huge impact of the flowing blood on the thrombus growth regularisation.

Recent developments of the phenomenological cell-based models will be explained to demonstrate the current shift from the classical cascade/waterfall models and a

short survey of available mathematical concepts used to describe the blood coagulation process at various spatial scales will be referred.

A new mathematical model and some numerical results for thrombus growth will be presented in this talk. The cascade of biochemical reactions interacting with the platelets, resulting in a fibrin-platelets clot production and the additional blood flow influence on thrombus development will be discussed.

This new reduced model consists of a system of 13 nonlinear convection-reaction-diffusion equations, describing the cascade of biochemical reactions, coupled with a non-Newtonian model for the blood flow. The model includes slip velocity at the vessel wall and the consequent supply of activated platelets in the clot region.

Numerical results showing the capacity of the model to predict different perturbations in the hemostatic system will be presented.

This is a joint work with Antonio Fasano and Jevgenija Pavlova.

- [1] A. Fasano, J. Pavlova, A. Sequeira, A synthetic model for blood coagulation including blood slip in the vessel wall, *Clinical Hemorheology and Microcirculation* (2013) **54**(1): 326-345.
- [2] T. Bodnár, A. Fasano, A. Sequeira, Mathematical models for blood coagulation. In: *Fluid-Structure Interaction and Biomedical Applications, Advances in Mathematical Fluid Mechanics*, Eds: T. Bodnár et al. (2014) 483-569, DOI 10.1007/978-3-0348-0822-4-7.

## ON SOME PERTURBATIONS OF PRODUCT-TYPE DIFFERENCE EQUATIONS AND SYSTEMS OF DIFFERENCE EQUATIONS

**Stevo Stević**, *Beograd, Serbia*

Many of concrete difference equations studied recently are essentially obtained by perturbations of some special cases of the following higher-order product-type difference equation

$$x_n = \prod_{j=1}^k x_{n-j}^{a_j}, \quad n \in \mathbb{N}, \quad (2.9)$$

where  $k \in \mathbb{N}$ , parameters  $a_1, \dots, a_{k-1}$  are real numbers and  $a_k \neq 0$ .

The most frequent cases of the perturbations are obtained by using the maximum operator

$$L_{\max}(y_n) = \max\{a, y_n\}, \quad n \in \mathbb{N}_0,$$

which is considered on the space of real sequences (such an operator obviously cannot act on the space of complex-valued sequences), and by using the translation operator

$$L_{\tau_a}(y_n) = a + y_n, \quad n \in \mathbb{N}_0,$$

which unlike the case of operator  $L_{\max}$  can act on the space of complex-valued sequences. In this talk we present some results on the long-term behavior of solutions of the classes of difference equations and systems which are perturbations of equation (2.9) and related product-type systems obtained by using operators  $L_{\max}$ ,  $L_{\tau_a}$  and related ones.

- [1] G. Papaschinopoulos, C. J. Schinas and G. Stefanidou, On the nonautonomous difference equation  $x_{n+1} = A_n + (x_{n-1}^p/x_n^q)$ , *Appl. Math. Comput.* **217** (2011), 5573-5580.
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- [3] S. Stević, On a nonlinear generalized max-type difference equation, *J. Math. Anal. Appl.* **376** (2011), 317-328.
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## OSCILLATIONS OF DELAY AND DIFFERENCE EQUATIONS WITH SEVERAL VARIABLE COEFFICIENTS AND ARGUMENTS

**Ioannis Stravroulakis**, *Ioannina, Greece*

**Abstract:** Consider the first-order delay differential equation

$$x'(t) + \sum_{i=1}^m p_i(t)x(\tau_i(t)) = 0, \quad t \geq 0,$$

where, for every  $i \in \{1, \dots, m\}$ ,  $p_i$  is a continuous real-valued function in the interval  $[0, \infty)$ , and  $\tau_i$  is a continuous real-valued function on  $[0, \infty)$  such that

$$\tau_i(t) \leq t, \quad t \geq 0, \quad \text{and} \quad \lim_{t \rightarrow \infty} \tau_i(t) = \infty$$

and the discrete analogue difference equation

$$\Delta x(n) + \sum_{i=1}^m p_i(n)x(\tau_i(n)) = 0, \quad n \in \mathbb{N}_0,$$

where  $\mathbb{N} \ni m \geq 2$ ,  $p_i$ ,  $1 \leq i \leq m$ , are real sequences and  $\{\tau_i(n)\}_{n \in \mathbb{N}_0}$ ,  $1 \leq i \leq m$ , are sequences of integers such that

$$\tau_i(n) \leq n - 1, \quad n \in \mathbb{N}_0, \quad \text{and} \quad \lim_{n \rightarrow \infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m$$

Several oscillation conditions for the above equations are presented.

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GROUND STATES FOR PROBLEMS OF BREZIS-NIRENBERG TYPE WITH  
CRITICAL AND SUPERCRITICAL EXPONENT

**Andrzej Szulkin**, *Stockholm, Sweden*

**Abstract:** We consider the elliptic boundary value problem

$$-\Delta u = \lambda u + |u|^{p-2}u \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where  $p = 2(N - k)/(N - k - 2)$  ( $p$  is the so-called  $(k + 1)$ -st critical exponent),  $\Omega \subset \mathbb{R}^{k+1} \times \mathbb{R}^{N-k-1}$  is a bounded domain, invariant with respect to the action of  $O(k+1)$  on  $\mathbb{R}^{k+1}$  and bounded away from  $\mathbb{R}^{N-k-1}$ . We show that this problem has no nontrivial  $O(k+1)$ -invariant solution for any  $\lambda \in (-\infty, \lambda_*)$ , where  $\lambda_* > 0$ , and that such solutions exist in a left neighbourhood of each  $O(k+1)$ -invariant eigenvalue  $\lambda_m^{(k)}$ . Moreover, they are ground states (for solutions with this symmetry) and bifurcate at  $\lambda_m^{(k)}$ .

This problem is related to the question of existence of bifurcating ground states for the anisotropic equation

$$-\operatorname{div}(a(x)\nabla u) = \lambda b(x)u + c(x)|u|^{2^*-2}u \text{ in } \Theta, \quad u = 0 \text{ on } \partial\Theta,$$

where  $\Theta \subset \mathbb{R}^n$  is bounded,  $a, b, c > 0$  and  $2^* = 2n/(n - 2)$ .

This is joint work with Mónica Clapp and Angela Pistoia.

ASYMPTOTIC INTEGRATION OF NONLINEAR  
DIFFERENTIAL EQUATIONS

**Ağacık Zafer**, *Egaila, Kuwait*

**Abstract:** The problem of asymptotic integration of solutions of nonlinear differential equations, mostly second-order, has attracted the attention of several authors for the last decades. In most works sufficient conditions have been obtained for the existence of a solution of  $x'' = f(t, x, x')$  to behave linear like  $at + b$  as  $t \rightarrow \infty$ . It turns out that principle and nonprinciple solutions of associated equations play an important role in studying the asymptotic integration problem.

In this talk we will make an effort to study the asymptotic integration problem in a systematic way, and show how to obtain new results in certain special cases.

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## 2.2 Contributed Talks

### HOMOGENIZATION OF A CONVECTION-DIFFUSION PROBLEM WITH REACTION IN BI-POROUS MEDIA

**Abdelhamid Ainouz**, *Algiers, Algeria*

2000 MSC: 35B27

**Abstract:** Let  $\Omega$  be a bounded and regular domain of  $\mathbb{R}^n$ , underlying the microstructure model for fluid flow. It is assumed that  $\Omega$  has a periodic structure and decomposed as  $\Omega = \Omega_1^\varepsilon \cup \Omega_2^\varepsilon \cup \Sigma^\varepsilon$  where  $\Omega_1^\varepsilon$  and  $\Omega_2^\varepsilon$  are two open subsets of  $\Omega$  separated by the smooth surface  $\Sigma^\varepsilon := \partial\overline{\Omega_1^\varepsilon} \cap \partial\overline{\Omega_2^\varepsilon}$ . We shall be concerned with the homogenization of the following micro-model:

$$\left\{ \begin{array}{ll} -\operatorname{div}(a_i \nabla u_i^\varepsilon) + \varepsilon^{-1} \vec{b}_i \cdot \nabla u_i^\varepsilon + c_i u_i^\varepsilon &= f_i^\varepsilon \quad \text{in } \Omega_i^\varepsilon, \\ a_1 \nabla u_1^\varepsilon \cdot \nu^\varepsilon &= a_2 \nabla u_2^\varepsilon \cdot \nu^\varepsilon \quad \text{on } \Sigma^\varepsilon, \\ a_1 \nabla u_1^\varepsilon \cdot \nu^\varepsilon &= \varepsilon h(u_1^\varepsilon - u_2^\varepsilon) \quad \text{on } \Sigma^\varepsilon, \\ u_1^\varepsilon &= 0 \quad \text{on } \partial\Omega \end{array} \right.$$

where  $a_i, c_i$  and  $h$  are some phenomenological positive parameters and  $\vec{b}_i(y), i = 1, 2$  are such that

$$\operatorname{div}_y \vec{b}_i = 0, \quad \int_Y \vec{b}_i = \vec{0} \quad \text{and} \quad \vec{b}_i \cdot \nu = 0 \quad \text{on } \Sigma.$$

### RADIOACTIVE MATERIALS DISPERSION IN THE ATMOSPHERE

**Abdullah M. S. Ajlouni**, *Irbid AlHuson, Jordan*

**Abstract:** The artificial of radionuclides may result from physical processes involving nuclear fission, nuclear fission and neutronactivation. A variety of systems and processes may introduce radioactivity intoenvironment. The physical and chemical from radionuclides may varydepending on the release and transport conditions in addition to the elements properties. The most serious dispersion ofradioactive materials in the environment is related to escaping of noblegases, halogens and aerosols of non-volatile radioactive materials, from the reactor containment in the event of a sever reactor accident. Here we try to make a mathematical simulation of radionuclidedispersion in the environment by matchingthe mathematical tools and the technical data of the nuclear reactors. This simulation may help in determination of radiation dose may be the public during a sever reactor accident. A demonstration of the international atmosphere is carried out,the demonstration results are compared with real data taken from the field.

ANALYTICAL SOLUTIONS OF TWO-POINT BOUNDARY VALUE  
PROBLEMS: VOLTERRA INTEGRO-DIFFERENTIAL EQUATION

Ayed H. Al e'damat, Ma'an, Jordan

2000 MSC: 45J05, 47B32, 34K28

**Abstract:** In this paper, we applied the reproducing kernel method to approximate solutions of two-point boundary value problems for fourth-order Volterra integrodifferential equations. The analytical solution is represented in the form of convergent series with easily computable components. The solution methodology is based on generating the orthogonal basis from the obtained kernel function in the space  $W_2^5[a, b]$ . After that, the orthonormal basis is constructing in order to formulate and utilize the solution in the same space. The  $n$ -term approximation is obtained and proved to converge to the analytical solution. Moreover, the proposed method has an advantage that it is possible to pick any point in the interval of integration and as well the approximate solutions and its all derivatives will be applicable. Numerical examples are given to demonstrate the computation efficiency of the presented method. Results obtained by the method indicate the method is simple and effective.

The purpose of this paper is to extend the application of the reproducing kernel method in the space  $W_2^5[a, b]$  for obtaining approximate analytical solution for fourth-order boundary value problems of integrodifferential equation of Volterra type. We consider the following equation

$$u^{(iv)}(x) + \gamma u(x) + \int_0^x [g(x)u(x) + h(x)G(u(x))]dx = f(x), \quad (1)$$

subject to the boundary conditions

$$u(a) = \alpha_0, u''(a) = \alpha_2, u(b) = \beta_0, u''(b) = \beta_2, \quad (2)$$

where  $x \in (a, b)$ ,  $\gamma$ ,  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_1$  and  $\beta_2$  are real finite constants,  $G$  is a real nonlinear continuous function,  $f$ ,  $g$  and  $h$  are given and can be approximated by Taylor polynomials, and  $u(x)$  is an unknown analytic function to be determined. We suppose that the boundary value problems (1) and (2) have a unique smooth solution.

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# A RELIABLE ALGORITHM OF THE RKHS METHOD FOR SOLVING LANE-EMDEN TYPE EQUATIONS

**Mohammed H. Al-Smadi**, *Ajloun, Jordan*

2000 MSC: 34A12; 35G20; 47B32

**Abstract:** In this paper, based on the reproducing kernel Hilbert space method, a powerful algorithm is developed for the solution of the Lane-Emden type equations which are nonlinear ordinary differential equations. The solution is represented in the form of series in the space  $W_2^3[a, T]$ . Error estimates are proven that it converge to zero in the sense of the space norm. To illustrate the reliability of the method, some numerical examples are tested. The results obtained by the method indicate the method is simple, effective and it is expected to be further employed to solve similar nonlinear problems. In the present paper, the reproducing kernel Hilbert space (RKHS) method is used to obtain an analytic-numerical solution for the Lane-Emden equations in the following form:

$$u''(x) + \frac{k}{x}u'(x) = F(x, u(x)), a < x < T, \quad (2.10)$$

subject to the initial conditions

$$u(a) = \alpha, u'(a) = \beta, \quad (2.11)$$

where  $k, a, \alpha$ , and  $\beta$  are real finite constants,  $u(x) \in W_2^3[a, T]$  is an unknown function to be determined,  $F(x, u)$  is continuous term in  $W_2^1[a, T]$  as  $u = u(x) \in W_2^3[a, T]$ ,  $a \leq x \leq T$ ,  $-\infty < u < \infty$ , and are depending on the problem discussed, and  $W_2^1[a, T], W_2^3[a, T]$  are two reproducing kernel spaces.

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- [2] F. Geng, Solving singular second order three-point boundary value problems using reproducing kernel Hilbert space method, *Applied Mathematics and Computation* 215 (2009) 2095-2102.
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# NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS IN MODELING AND CONTROL OF INFECTIOUS DISEASE

**Md. H. Ali Biswas**, *Khulna, Bangladesh*

2000 MSC: 93A30; 49K15

**Abstract:** This is joint work with Md. Mohidul Haque and Tahmina Hossain.

Nonlinear phenomena characterize all aspects of global change dynamics, from the Earth's climate system to human physiology [5]. These nonlinear phenomena of rapid change in the human physiological systems can be captured and modeled by the nonlinear ordinary differential equations (NODEs) in the form of mathematical modeling. Since human body is a highly nonlinear, robust, and an adaptive physiological control system, there is a close relationship between control theory and biology [4]. So nonlinearity plays an influential role in describing the mysterious and complex mechanisms of the dynamics of infectious diseases in human body.

In recent years, mathematical models have become the most important tools in analyzing the dynamics of biological and biomedical systems. The processes in biology and medicine can be, in general, described by mathematical models where the nonlinear ordinary differential equations are the key ingredients. The spread of infectious diseases such as HIV [1], NiV [2] or flu [3] may be modeled as a nonlinear system of differential equations. In this paper, we study a HIV immunology model to analyze the nonlinear behavior of the disease dynamics. Optimal control technique is applied to obtain the better immunotherapeutic strategy, special feature of which is the introduction of state constraint. Only numerical treatment is performed to illustrate the results.

- [1] M. H. A. Biswas, On the Evolution of AIDS/HIV Treatment: An Optimal Control Approach. *Current HIV Research*, **12**(1) (2014) 1–12.
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- [4] D. S. Naidu, T. Fernando and K. R. Fister, Optimal control in diabetes, *Optim. Control Appl. Meth.*, **32** (2011) 181–184.
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# ANALYSIS OF DIFFERENCE APPROXIMATIONS TO DELAY PSEUDO-PARABOLIC EQUATIONS

**Gabil Amirali (Amiraliyev)**, *Erzincan, Turkey*

2000 MSC: 65M15, 65M20, 65L05, 65L70

**Abstract:** Finite difference technique is applied to numerical solutions of the initial boundary value problems for the linear and nonlinear delay Sobolev or pseudo-parabolic equations. Equations of this type arise in many areas of mechanics and physics. Such equations are encountered, for example, as a model for two-phase porous media flows when dynamic effects in the capillary pressure are included [1-2]. The discretization in space is based on the method of integral identities with use the appropriate basis functions and interpolating quadrature rules with weight and remainder term in integral form, for the time discretization two level difference schemes are used (see also [3] and references therein, for the cases without delay). The uniform convergence of the discrete problems is proved. The error estimates are obtained in the discrete norm. Numerical results are also presented.

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- [2] Y. Fan, I.S. Pop, Equivalent formulations and numerical schemes for a class of pseudo-parabolic equations, *J. Comput. Appl. Math.* 246 (2013) 86–93.
- [3] G.M. Amiraliyev, H.Duru and I.G.Amiraliyeva, A Parameter-Uniform Numerical Method for a Sobolev Problem with Initial Layer, *Numer. Algorithms* 44 (2007) No2 185-203.

NUMERICAL METHOD FOR A SINGULARLY PERTURBED DELAY  
INTEGRO-DIFFERENTIAL EQUATION

**Ilhame Amirali**, *Duzce, TURKEY*

2000 MSC: 65L11, 65L12, 65L20, 65R05

**Abstract:** This is a joint work with Gabil M. Amiraliyev and Mustafa Kudu. This study is concerned with the singularly perturbed initial value problem for a linear first order Volterra integro-differential equation with delay. Our purpose is to construct and analyse uniform convergent in small parameter numerical method for solving the considered problem. The numerical solution of this problem is discretised using an implicit difference rules for differential part and the composite numerical quadrature rules for integral part. On a layer-adapted mesh error estimations for the approximate solution are established. Numerical examples supporting the theory are presented.

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ON ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO REGULAR AND  
SINGULAR NONLINEAR DIFFERENTIAL EQUATIONS

**Irina Astashova**, *Moscow, Russia*



2000 MSC: 34C

**Abstract:** Consider the equation

$$y^{(n)} = p(x, y, y', \dots, y^{(n-1)})|y|^k \operatorname{sign} y, \quad (2.12)$$

$$n \geq 2, k > 0, k \neq 1, \quad p(x, y_0, \dots, y_{n-1}) \neq 0, \quad p \in C(\mathbf{R}^{n+1}).$$

The asymptotic properties of solutions to equation (2.12) are described. (Cf. [1], [2].) In particular case  $p \equiv p_0 = \text{const} < 0$ , the existence of quasi-periodic solutions is proved. For  $n = 3$  and  $n = 4$  asymptotic classification of all solutions is described. For  $n = 4$ ,  $p \equiv p_0 > 0$ , the existence of periodic solutions is obtained.

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- [2] Astashova I. V. Qualitative properties of solutions to quasilinear ordinary differential equations. In: Astashova I. V. (ed.) Qualitative Properties of Solutions to Differential Equations and Related Topics of Spectral Analysis: scientific edition, M.: UNITY-DANA, 2012, pp. 22–290. (Russian)

## ANALYSIS OF FREE CONVECTION DISSIPATIVE HEAT EFFECT ON MHD FLOW PAST AN INFINITE POROUS VERTICAL PLATE

**Vivek Asthana**, *Lucknow, India*

**Abstract:** This is a joint work with Nand Lal Singh, Jitesh Kumar Singh and Yogesh Sharma. In this paper, we have considered the magnetic field effect with the consideration of Joulean dissipation term in the energy equation. Also, we have assumed variable suction velocity at the plate. Our investigation leads to the conclusion that in the presence of magnetic field, greater viscous dissipative heat leads to a decrease in the mean skin friction of air and an increase in the mean skin friction of water. The values of mean skin friction and mean heat transfer rate with respect to the Grashoff, Eckert and Prandtl parameters are entered in the Tables. It has been observed from the Table that due to greater cooling of the plate by free convection currents, mean skin friction increases always. Further, it leads to an increase in the mean heat transfer rate of air and a decrease in the mean heat transfer rate of water.

SOLVING FIRST-ORDER PERIODIC BVPs  
BASED ON THE REPRODUCING KERNEL METHOD

Ali M. Ateiwi, Ma'an, Jordan

2000 MSC: 34K28; 34K13; 34B15; 47B32

**Abstract:** The aim of this paper is to study an efficient numerical algorithm to obtain an approximate solution of first-order periodic boundary value problems. This algorithm is based on a reproducing kernel Hilbert space method. Its exact solution is calculated in the form of series in reproducing kernel space with easily computable components. In addition, convergence analysis for this method is discussed. In this sense, some numerical examples are given to show the effectiveness of the proposed method. The results reveal that the method is quite accurate, straightforward, and simple. The purpose of this paper is to extend the application of the reproducing kernel Hilbert space method (RKM) to provide approximate solution of a class of first-order periodic BVP of the following form

$$u'(x) + g(x)u(x) = f(x, u(x)), \quad 0 \leq x \leq 1, \quad (2.13)$$

subject to the periodic boundary condition

$$u(0) = u(1), \quad (2.14)$$

where  $g(x)$  is continuous known function,  $f(x, u(x)) \in W_2^1[0, 1]$ ,  $u(x) \in W_2^2[0, 1]$ ,  $\|f(x, u(x)) - f(x, \bar{u}(x))\|_{W_2^1} \leq C \|u(x) - \bar{u}(x)\|_{W_2^1}$  as  $x \in [0, 1]$  and  $f(x, u)$  is linear or nonlinear function of  $u$  depending on the problem discussed and  $u(x)$  is an unknown function to be determined. We assume that Eqs. (2.41) and (2.14) has a unique smooth solution on  $[0, 1]$ .

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- [4] O. Abu Arqub, M. Al-Smadi and S. Momani, Application of reproducing kernel method for solving nonlinear Fredholm-Volterra integro-differential equations, Abstract and Applied Analysis, vol. 2012, Article ID 839836, (2012) 16 pages.

# REPRESENTATION OF THE SOLUTION OF A BIHARMONIC EQUATION WITH A DELAY

**Elvin Azizbayov**, *Baku, Azerbaijan*

**2000 MSC:** 31B30, 35C10.

**Abstract:** This is a joint work with Prof. D. Ya. Khusainov. In the present work we investigate the following biharmonic equation with one delay

$$\frac{\partial^2 u(x, t)}{\partial t^2} = a^2 \frac{\partial^4 u(x, t - \tau)}{\partial x^4}, \quad 0 \leq x \leq l, \quad t \geq 0. \quad (2.15)$$

The initial and boundary conditions are of the form

$$\begin{aligned} u(x, t) = \varphi(x, t), \quad \varphi(0, t) = \varphi(l, t) \equiv 0, \quad \frac{\partial u(x, t)}{\partial t} = \psi(x, t), \quad u(0, t) \equiv 0, \\ u(l, t) \equiv 0, \quad \frac{\partial^2}{\partial x^2} u(0, t) \equiv 0, \quad \frac{\partial^2}{\partial x^2} u(l, t) \equiv 0, \quad t \geq 0. \end{aligned} \quad (2.16)$$

The solution is sought in the form of the product  $u(x, t) = X(x)T(t)$ . After substituting into the equation and separation of variables, we obtain

$$T''(t) - a^2 \mu^4 T(t - \tau) = 0, \quad X^{(4)}(x) - \mu^4 X(x) = 0.$$

The solutions of the second equation satisfying zero conditions, are

$$X_n(x) = \sin \frac{\pi n}{l} x, \quad n = 1, 2, 3, \dots$$

For finding the solution of the first equation

$$T''(t - \tau) - \Omega_n^2 T(t - \tau) = 0, \quad \Omega_n = a \left( \frac{\pi n}{l} \right)^2, \quad n = 1, 2, 3, \dots \quad (2.17)$$

we use the function called a delayed exponential  $\exp_\tau\{\Omega, t\}$  with the exponent  $\Omega$  and delay  $\tau$ .

We introduce two functions representing itself the linear combination of delayed exponentials

$$T_1(t) = \frac{1}{2}[\exp_\tau\{\Omega, t\} + \exp_\tau(-\Omega, t)], \quad T_2(t) = \frac{1}{2\Omega}[\exp_\tau\{\Omega, t\} - \exp_\tau(-\Omega, t)]. \quad (2.18)$$

The solution of the Cauchy problem for the homogeneous equation (3) can be written in the form

$$x(t) = \varphi(-2\tau)T_1(t + \tau) + \varphi'(-2\tau)T_2(t + 2\tau) + \int_{-2\tau}^0 T_2(t - s)\varphi''(s)ds, \quad (2.19)$$

where  $T_1(t)$ ,  $T_2(t)$  are represented in (4),  $\varphi(t)$  is the initial function.

By using the obtained results, the solution of equations (3) may be written in the form

$$\begin{aligned} T_n(t) = & \frac{1}{2}\varphi(-\tau)[\exp_{\tau}\{\Omega_n, t\} + \exp_{\tau}\{-\Omega_n, t\}] + \\ & + \frac{1}{2\Omega_n}\varphi'(-\tau)[\exp_{\tau}\{\Omega_n, t\} - \exp_{\tau}\{-\Omega_n, t\}] + \\ & + \frac{1}{2\Omega_n} \int_{-\tau}^0 [\exp_{\tau}\{\Omega_n, t - \tau - s\} - \exp_{\tau}\{-\Omega_n, t - \tau - s\}]\varphi''(s)ds. \end{aligned}$$

While the solution of the boundary value problem for equation (1) in the form

$$u(x, t) = \sum_{n=1}^{\infty} T_n(t) \sin \frac{\pi n}{l} x.$$

## EXPLICIT COMPUTATION OF SOME RATIONAL AND INTEGRAL COMPLEX GENERA

**Malkhaz Bakuradze**, *Tbilisi, Georgia*

2000 MSC: 55N22, 34C40

**Abstract:** The Krichever-Höhn genus, or the general four variable complex elliptic genus  $\phi_{KH} : MU_* \otimes \mathbb{Q} \rightarrow \mathbb{Q}[q_1, \dots, q_4]$  is a graded  $\mathbb{Q}$ -algebra homomorphism defined by the following property: if one denotes by  $f_{Kr}(x)$  the exponential of the Krichever universal formal group law  $\mathcal{F}_{Kr}$ , then the series  $h(x) := \frac{f'_{Kr}(x)}{f_{Kr}(x)}$  satisfies the differential equation

$$(h'(x))^2 = S(h(x)), \quad (2.20)$$

where  $S(x) = x^4 + q_1x^3 + q_2x^2 + q_3x + q_4$ , the generic monic polynomial of degree 4 with formal parameters  $q_i$  of weights  $|q_i| = 2i$ .

We give a formula to compute  $\phi_{KH}$ .

- [1] M. Bakuradze, Formal group laws by Buchstaber, Krichever and Nadiradze coincide, *Russian Math. Surv.*, **68:3** (2013) 189-190.

- [2] M. Bakuradze, Computing the Krichever genus, *J. of Homotopy and Related Structures*, **9,1** (2014) 85-93.

# CONTROL OF SOME DEGENERATE DIFFERENTIAL SYSTEMS IN HILBERT SPACES

**Mehdi Benabdallah**, *Oran, Algeria*

**Abstract:** The aim of this research is to generalize the famous General Theorem of Lyapounov of the stability to the Degenerate Differential systems of the form :

$$Ax'(t) = Bx(t), \quad t \geq 0$$

where  $A$  and  $B$  are bounded operators in Hilbert spaces, using the spectral theory of the corresponding pencil of operators  $\lambda A - B$ .

The archived results can be applied to the stability and stabilization for certain degenerate controlled systems.

# MULTIPLE POSITIVE SOLUTIONS FOR A FRACTIONAL BOUNDARY VALUE PROBLEM

**Salima Bensebaa**, *Annaba, Algeria*

2000 MSC: 34B10, 26A33, 34B15

**Abstract:** This is a joint work with A. Guezane-Lakoud. In this paper, using the fixed-point index theorem, we study the existence of at least one or two positive solutions to a boundary value problem of nonlinear fractional differential equation. As an application, we also give some examples to demonstrate our results.

- [1] R. P. Agarwal, M. Benchohra, S. Hamani, A Survey on Existence Results for Boundary Value Problems of Nonlinear Fractional Differential Equations and Inclusions, *Acta Appl Math* (2010) 109: 973–1033.
- [2] B. Ahmad, J. Nieto, Existence results for nonlinear boundary value problems of fractional integro differential equations with integral boundary conditions, *Boundary Value Problems* Vol. 2009 (2009), Article ID 708576, 11 pages.
- [3] A. Guezane-Lakoud and R. Khaldi, Positive Solution to a Fractional Boundary Value Problem, *International Journal of Differential Equations*, Vol 2011, Article ID 763456, 19 pages.

- [4] J. Xu and Z. Yang, Multiple Positive Solutions of a Singular Fractional Boundary Value Problem, *Applied Mathematics E-Notes*, 10(2010), 259-267.

# FIBONACCI TYPE COLLOCATION APPROACH FOR SOLVING SYSTEMS OF HIGH-ORDER LINEAR DIFFERENTIAL EQUATIONS

**Ayşe Betül Koç**, *Konya, Turkey*

2000 MSC: 33E20, 34A30, 65L60

**Abstract:** In this study, Fibonacci type collocation method that gives successful results for the solution of ordinary differential [1], pantograph [2] and Telegraph [3] equations is applied to solve the systems of high-order linear differential equations. For this, required operational matrices are derived. Using these matrices and the collocation points, the problem is transformed to matrix equations. By solving the algebraic system, the unknown coefficients are determined so that the truncated Fibonacci series approaches are easily obtained. Accuracy of the proposed method is tested on the numerical examples.

- [1] A. B. Koc, M. Cakmak, A. Kurnaz, K. Uslu, A new Fibonacci type collocation procedure for boundary value problems, *Advances in Difference Equations* **262: 2013** (2013).
- [2] A. B. Koc, M. Cakmak, A. Kurnaz, A matrix method based on the Fibonacci polynomials to the generalized pantograph equations with functional arguments, *Advances in Mathematical Physics* Article ID: **694580** (2014).
- [3] A. B. Koc, A. Kurnaz, An efficient approach for solving telegraph equation, *AIP Conf. Proc.*, **1648**, **370006** (2015).

# BEHAVIOR OF WEAK SOLUTIONS TO THE OBLIQUE DERIVATIVE PROBLEM FOR ELLIPTIC WEAK QUASI-LINEAR EQUATIONS IN A NEIGHBORHOOD OF A CONICAL BOUNDARY POINT

**Mariusz Bodzioch**, *Olsztyn, Poland*

2000 MSC: 35J20, 35J25, 35J60, 35J70

**Abstract:** This is joint work with Mikhail Borsuk.

We study the behavior of weak solutions to the oblique derivative problem for weak quasilinear second-order elliptic equations in a neighborhood of a conical

boundary point of an  $n$ -dimensional bounded domain. We establish an exponent of the solution's decreasing rate near the conical boundary point, i.e. we show that  $|u(x)| = O(|x|^\lambda)$  with an exact exponent  $\lambda$ .

## EXISTENCE RESULTS FOR A NONLINEAR BOUNDARY LAYER PROBLEM

**Gabriella Bognar**, *Miskolc, Hungary*

2000 MSC: 34B40, 35G45

**Abstract:** Boundary layer flow induced by the uniform motion of a continuous plate in a Newtonian fluid has been first analytically studied by Sakiadis. The flow of an incompressible fluid over a stretching surface has numerous industrial applications in aerodynamic extrusion of plastic sheets, glass fiber and paper production or cooling of metallic sheets and many others.

The problem considered here is the steady boundary layer flow due to a moving flat surface in an otherwise quiescent Newtonian fluid medium moving at a speed of  $U_w(x)$ . The laminar boundary layer equations expressing conservation of mass and the momentum boundary layer equations for an incompressible fluid are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}. \quad (2.21)$$

where  $x$  and  $y$  denote the Cartesian coordinates along the sheet and normal to the sheet, respectively; while  $u$  and  $v$  are the velocity components of the fluid in the  $x$  and  $y$  directions,  $\nu$  stands for the kinematic viscosity. We consider the boundary-layer flow induced by a continuous surface stretching with velocity  $U_w(x) = Ax^m$ , where  $m$  is a positive integer. The corresponding boundary conditions on the surface are  $u(x, 0) = U_w(x)$  and  $v(x, 0) = 0$ , far from the surface  $\lim_{y \rightarrow \infty} u(x, y) = 0$ . Our aim is to show the existence results of a non-negative classical solution to the boundary layer problem of (2.21).

- [1] I. J. Crane, Flow past a stretching plate, *Z. Angew. Math. Phys.* **21** (1970) 645-647.

## SINGULAR LINEAR BVPs WITH UNSMOOTH DATA

**Jana Burkotová**, *Olomouc, Czech Republic*

2000 MSC: 34A12, 34B05, 65L10

**Abstract:** In the recent joint work with professor Irena Rachůnková (UP Olomouc) and professor Ewa B. Weinmüller (TU Wien) we investigate analytical and numerical properties of systems of linear ordinary differential equations with unsmooth inhomogeneities and a time singularity of the first kind:

$$y'(t) = \frac{M(t)}{t}y(t) + \frac{f(t)}{t}, \quad B_0y(0) + B_1y(1) = \beta.$$

We specify conditions guaranteeing the existence and uniqueness of a solution  $y \in C[0, 1]$ . Moreover, we study the convergence behaviour of collocation schemes applied to solve the problem numerically.

- [1] J. Burkotová, I. Rachůnková, S. Staněk and E. B. Weinmüller, On linear ODEs with a time singularity of the first kind and unsmooth inhomogeneity, Bound. Value Probl. **2014:183**, (2014).
- [2] J. Burkotová, I. Rachůnková and E. B. Weinmüller, On singular BVPs with unsmooth data. Part 1: Analysis of the linear case with variable coefficient matrix, work in progress.
- [3] J. Burkotová, I. Rachůnková and E. B. Weinmüller, On singular BVPs with unsmooth data. Part 2: Convergence of the collocation schemes, work in progress.

## FRACTIONAL COMPLEX TRANSFORM FOR CONFORMABLE FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS

**Yücel Çenesiz**, *Konya, TURKEY*

2000 MSC: 26A33, 35R11

**Abstract:** This is a joint work with Ali Kurt. In this paper fractional complex transform is introduced to convert fractional partial differential equations to differential equations. The new definition of the fractional derivative, conformal fractional derivative, is used into solution procedure. With this transformation analytical methods in advanced calculus can be used to solve these type equations. Fractional advection diffusion equation and a fractional partial differential equation is solved for the illustration of the method.

- [1] K. Oldham and J. Spanier, The Fractional Calculus, Theory and Applications of Differentiation and Integration of Arbitrary Order, Academic Press, 1974.



- [2] K.S. Miller, B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, A Wiley-Interscience Publication, 1993.
- [3] I. Podlubny, Fractional Differential Equations, Academic Press, 1999.

STABILITY REGIONS FOR LINEAR FRACTIONAL  
DIFFERENTIAL SYSTEMS WITH A CONSTANT DELAY

**Jan Čermák**, *Brno, Czech Republic*

2010 MSC: 34K37

**Abstract:** We discuss stability and asymptotic properties of the linear fractional differential system

$$D^\alpha y(t) = Ay(t - \tau), \quad t \in (0, \infty) \quad (2.22)$$

where  $D^\alpha$  is the Caputo derivative of a real order  $0 < \alpha < 1$ ,  $A \in \mathbb{R}^{d \times d}$  is a constant real  $d \times d$  matrix and  $\tau > 0$  is a constant delay. As the main result, necessary and sufficient stability conditions are formulated via eigenvalues of the system matrix and their location in a specific area of the complex plane. These conditions represent a direct extension of the Matignon's stability criterion for fractional differential systems with respect to the inclusion of a delay. Also, we show that our conditions generalize existing stability criteria for the corresponding first order delay differential system (when  $D^\alpha y(t)$  with  $\alpha = 1$  is reduced to  $y'(t)$ ). In particular, we are going to discuss the influence of the non-integer derivative order  $\alpha$  and delay  $\tau$  on stability and asymptotic behaviour of (2.22).

ON THE EXISTENCE OF SOLUTIONS FOR SOME NONLINEAR  
q-DIFFERENCE INCLUSIONS

**Aurelian Cernea**, *Bucharest, Romania*

2000 MSC: 34A60, 39A13

**Abstract:** We consider the following problems

$$D_q^3 x(t) \in F(t, x(t)), \quad t \in J, \quad x(0) = 0, \quad D_q x(0) = 0, \quad x(1) = 0,$$

$$D_q^2 x(t) \in F(t, x(t)), \quad t \in J, \quad x(0) = \eta x(1), \quad D_q x(0) = \eta D_q x(1),$$

where  $D_q, D_q^2, D_q^3$  denotes the first, the second, respectively, the third order  $q$ -derivative,  $I = [0, 1]$ ,  $J = \{q^n, \quad n \in \mathbf{N}\} \cup \{0, 1\}$ ,  $q \in (0, 1)$ ,  $\eta \neq 1$  and  $F :$

$I \times \mathbf{R} \rightarrow \mathcal{P}(\mathbf{R})$  is a set-valued map. We establish Filippov type existence results for the problems considered in the case of nonconvex set-valued maps. In this way we improve some results existing in the literature.

EXISTENCE OF THE MILD SOLUTION FOR NONLOCAL FRACTIONAL  
DIFFERENTIAL EQUATION OF SOBOLEV TYPE  
WITH ITERATED DEVIATING ARGUMENTS

Alka Chadha, Roorkee, India

2000 MSC:26A33, 34K37, 34K40, 34K45, 35R11, 45J05, 45K05.

**Abstract:** This is joint work with Dwijendra N Pandey. In this work, we investigate the existence and uniqueness of the mild solution of nonlocal Sobolev type fractional differential equation with iterated deviating arguments in the Banach space illustrated by

$${}^c D_t^\beta [EB y(t)] = Ly(t) + F(t, y(t), y(h_1(t, y(t)))), \quad t \in J = [0, T], \quad (2.23)$$

$$y(0) = y_0 + h(y), \quad y_0 \in \mathbb{X} \quad (2.24)$$

where  $h_1(t, y(t)) = b_1(t, u(b_2(t, \dots, y(b_\delta(t, y(t)))) \dots))$ ,  ${}^c D_t^\beta$  is the fractional derivative in Caputo derivative of order  $\beta$ ,  $\beta \in (0, 1]$  and  $T \in (0, \infty)$ . In (2.23), we assume that the operator  $L : D(L) \subset \mathbb{X} \rightarrow \mathbb{Z}$ ,  $B : D(B) \subset \mathbb{X} \rightarrow \mathbb{Y}$  and  $E : D(E) \subset \mathbb{Y} \rightarrow \mathbb{Z}$  are closed operators, where  $\mathbb{X}, \mathbb{Y}$  and  $\mathbb{Z}$  are the Hilbert spaces such that  $\mathbb{Z}$  is continuously and densely embedded in  $\mathbb{X}$ , the state  $y(\cdot)$  takes its values in  $\mathbb{X}$ . The sufficient condition for providing the existence of mild solution to the nonlocal Sobolev type fractional differential equation with iterated deviating arguments is obtained via techniques of fixed point theorems and analytic semigroup. Finally, an example is given for explaining the applicability of the obtained abstract result.

- [1] A. Debbouche, J. J. Nieto, Sobolev type fractional abstract evolution equations with nonlocal conditions and optimal multi-controls, *Applied Math. Comp.*, 245 (2014), 74-85.
- [2] A. Pazy, *Semi-groups of Linear operator and Applications of Partial Differential Equations*, Springer Verlag (1983).
- [3] S. Stevic. Solutions converging to zero of some systems of nonlinear functional differential equations with iterated deviating arguments, *Applied Mathematics and Computation*, 219 (2012): 4031-4035.

- [4] S. Stevic. Globally bounded solutions of a system of nonlinear functional differential equations with iterated deviating argument, *Applied Mathematics and Computation*, 219 (2012): 2180-2185.
- [5] I. Podlubny, *Fractional differential equations*, Mathematics in Science and Engineering, vol. 198. Academic Press, San Diego (1999).
- [6] L. Byszewski, Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem, *J. Math. Anal. Appl.*, 162 (1991), 497-505.
- [7] L. Byszewski and V. Lakshmikantham, Theorem about the existence and uniqueness of a solution of a nonlocal abstract Cauchy problem in a Banach space, *Applied Analysis*, 40 (1990), 11-19.

## SMOOTH SOLUTIONS OF LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

**Valery Cherepennikov**, *Irkutsk, Russia*

2000 MSC: 34K06, 34K10.

**Abstract:** The paper considers an initial problem with an initial function for a linear functional differential equation of neutral type

$$\dot{x}(t) + p(t)\dot{x}(t-1) = a(t)x(t) + b(t)x(t/q), \quad t \in R, \quad q > 1, \quad (1)$$

where the coefficients of equations are represented in the form of polynomials. The study investigates the problem of finding such an initial function that the generated solution to the initial problem has the required smoothness at the points divisible by the constant delay time. This problem is solved with the use of the method of polynomial quasisolutions [1], based on the representation of an unknown function in the form of a polynomial of some degree. When it is substituted in equation (1), the incorrectness appears in the polynomial dimension, compensated for by the introduction of a discrepancy into the equation, which has an accurate analytical formula characterizing the extent of disturbance of the initial problem. The paper shows that if for the studied initial problem we choose the polynomial quasisolution of degree  $N$  as an initial function, the generated solution will have smoothness no lower than  $N$  at the joining points.

The results of the numerical experiment are presented.

- [1] V. B. Cherepennikov and P. G. Ermolaeva, Polynomial quasisolutions of linear differential difference equations, *Opuscula Mathematica* **V** (2006) 47 - 57.

FINITE DIFFERENCE FORMULATION FOR THE MODEL OF A  
COMPRESSIBLE VISCOUS AND HEAT-CONDUCTING FLUID WITH  
SPHERICAL SYMMETRY

Nelida Črnjarić-Žic, *Rijeka, Croatia*

2000 MSC: 35Q35, 76M20, 65M06, 76N99

**Abstract:** This is a joint work with Prof Nermina Mujaković. We consider the nonstationary 3D flow of a compressible viscous heat-conducting micropolar fluid in the domain to be a subset of  $\mathbf{R}^3$ , bounded with two concentric spheres. In the thermodynamical sense the fluid is perfect and polytropic. The homogeneous boundary conditions for velocity, microrotation, heat flux and spherical symmetry of the initial data are proposed. This spherically symmetric problem in Eulerian coordinates is transformed to the 1D problem in Lagrangian coordinates in the domain that is a segment. We define then the finite difference approximate equations system and construct the sequence of approximate solutions to our problem. By investigating the properties of these approximate solutions, we establish their convergence to the generalized solution of our problem globally in time. Numerical experiments are performed by solving the proposed finite difference formulation. We compare the numerical results obtained by using the finite difference and the Faedo-Galerkin approach and investigate the convergence to the stationary solution.

ON THE TRAJECTORIES OF THE LINEAR SWINGING MODEL

Laszlo Csizmadia, *Szeged/Kecskemet, Hungary*

2000 MSC: 34D20, 70J40

**Abstract:** This is a joint work with Prof Laszlo Hatvani. The equation

$$x'' + a^2(t)x = 0,$$

$$a(t) := \begin{cases} \sqrt{\frac{g}{l - \varepsilon}} & \text{if } 2kT \leq t < (2k + 1)T, \\ \sqrt{\frac{g}{l + \varepsilon}} & \text{if } (2k + 1)T \leq t < (2k + 2)T, \quad (k = 0, 1, \dots) \end{cases}$$

is considered, where  $g$  and  $l$  denote the constant of gravity and the length of the pendulum, respectively;  $\varepsilon > 0$  is a parameter measuring the intensity of swinging.

Concepts of solutions going away from the origin and approaching to the origin are introduced. Necessary and sufficient conditions are given in terms of  $T$  and  $\varepsilon$  for the existence of solutions of these types, which yield conditions for the existence of  $2T$ -periodic and  $4T$ -periodic solutions as special cases. The domain of instability, i.e. the Arnold tongues of parametric resonance are deduced from these results.

- [1] Hatvani, L. An elementary method for the study of Meissner's equation and its application to proving the Oscillation Theorem. *Acta Sci. Math.* **79**(2013), no. 1–2, 87–105.
- [2] Hochstadt, H. A special Hill's equation with discontinuous coefficients. *Amer. Math. Monthly* **70**(1963), 18–26.

EXISTENCE OF SOLUTION AND APPROXIMATE CONTROLLABILITY OF  
A SECOND-ORDER NEUTRAL STOCHASTIC DIFFERENTIAL EQUATION  
WITH STAT DEPENDENT DELAY AND DEVIATED ARGUMENT

**Sanjukta Das**, *Roorkee, India*

2000 MSC: 35R15, 35R60, 93B05, 93E03

**Abstract:** This is a joint work with D. N. Pandey.

This paper has two sections which deals with a second order stochastic neutral partial differential equation with state dependent delay. In the first section the existence and uniqueness of mild solution is obtained by use of measure of non-compactness. In the second section the conditions for approximate controllability are investigated for the distributed second order neutral stochastic differential system with respect to the approximate controllability of the corresponding linear system in a Hilbert space. Thereby, we remove the need to assume the invertibility of a controllability operator used by authors in [1], which fails to exist in infinite dimensional spaces if the associated semigroup is compact. Our approach also removes the need to check the invertibility of the controllability Gramian operator and associated limit condition used by the authors in [2], which are practically difficult to verify and apply. An example is provided to illustrate the presented theory.

Specifically we study the following second order equations modelled in the form

$$\begin{aligned}
 d(x'(t) + g(t, x_t)) &= [Ax(t) + f(t, x_{\rho(t, x_t)}) + Bu(t)]dt \\
 &+ G(t, x(a(t)), x_t)dW(t), \text{ a.e. on } t \in J = [0, a] \\
 x_0 = \phi \in \mathfrak{B}, \quad x'(0) &= \psi \in X
 \end{aligned} \tag{2.25}$$

where  $A$  is the infinitesimal generator of a strongly continuous cosine family  $\{C(t) : t \in \mathbb{R}\}$  of bounded linear operators on a Hilbert space  $X$ .

- [1] P. Balasubramaniam, J. P. Dauer, Controllability of semilinear stochastic delay evolution equations in hilbert spaces, *International Journal of Mathematics and Mathematical Sciences*, Hindawi Publishing Corporation 31(3) (2002) 157–166.
- [2] N. Mahmudov, M. McKibben, Approximate controllability of second-order neutral stochastic evolution equations, *Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications and Algorithms* 13 (2006) 619–634.
- [3] H. Bao, J. Cao, Existence and uniqueness of solutions to neutral stochastic functional differential equations with infinite delay, *Appl. Math. Comput.* 215 (2009) 1732–1743.

## SINGULAR LINEAR SYSTEMS OF FRACTIONAL NABLA DIFFERENCE EQUATIONS

**Ioannis Dassios**, *Dublin, Ireland*

2000 MSC: 37N35; 37N40; 65F05.

**Abstract:** In this talk we will study the initial value problem of a class of non-homogeneous singular linear systems of fractional nabla difference equations whose coefficients are constant matrices. By taking into consideration the cases that the matrices are square with the leading coefficient singular, non-square and square with a matrix pencil which has an identically zero determinant, we will provide necessary and sufficient conditions for the existence and uniqueness of solutions. More analytically we will study the conditions under which the system has unique, infinite and no solutions. For the case of uniqueness we will derive a formula that provides the unique solution and for the other cases we will provide optimal solutions. Finally, we study the Kalman filter for singular non-homogeneous linear control systems of fractional nabla difference equations. Numerical examples will be given to justify our theory.

- [1] I.K. Dassios, Optimal solutions for non-consistent singular linear systems of fractional nabla difference equations, *Circuits systems and signal processing*, Springer (2014).

- [2] I.K. Dassios, D. Baleanu, G. Kalogeropoulos, On non-homogeneous singular systems of fractional nabla difference equations, *Appl. Math. Comput.* **227** (2014), 112–131.

ASYMPTOTIC REPRESENTATION OF THE SOLUTIONS FOR A SECOND ORDER NONLINEAR IMPULSIVE DIFFERENTIAL EQUATION

Sibel Dođru Akgöl, *Ankara, TURKEY*

2000 MSC: 34A37

**Abstract:** This is a joint work with Prof. Dr. Ağacık Zafer. We obtain asymptotic representation for the solutions of the second order impulsive differential equation

$$\begin{cases} x'' = f(t, x), & t \neq \theta_i, \\ \Delta x' + q_i x = \tilde{f}_i(x), & t = \theta_i \end{cases}$$

where  $f \in C([t_0, \infty) \times \mathbb{R}, \mathbb{R})$ ,  $\tilde{f}_i \in C(\mathbb{R}, \mathbb{R})$ ,  $t_0 > 0$  and  $\Delta x(t) = x(\theta_i+) - x(\theta_i-)$  with  $x(\theta_i\pm) = \lim_{t \rightarrow \theta_i\pm} x(t)$ .

- [1] O. Lipovan, On the asymptotic behaviour of the solutions to a class of second order nonlinear differential equations, *Glasgow Math. J.* 45, (2003), 179-187.
- [2] T. Ertem, A. Zafer, Asymptotic integration of second-order nonlinear differential equations via principal and nonprincipal solutions, *Appl. Math. Comput.*, 219, (2013) 5876-5886.

3-D FLOW OF A COMPRESSIBLE VISCOUS MICROPOLAR FLUID  
WITH SPHERICAL SYMMETRY: STABILIZATION AND  
REGULARITY OF THE SOLUTION

Ivan Dražić, *Rijeka, Croatia*

2000 MSC: 76N10, 35Q35

**Abstract:** This is a joint work with Prof Nermina Mujaković. We consider nonstationary 3-D flow of a compressible viscous heat-conducting micropolar fluid in the domain to be the subset of  $\mathbf{R}^3$  bounded with two concentric spheres that present the solid thermoinsulated walls. In thermodynamical sense fluid is perfect

and polytropic. Assuming that the initial density and temperature are strictly positive we know that for smooth enough spherically symmetric initial data there exists a unique spherically symmetric generalized solution globally in time. In this talk we will present some new results concerning the described model, particularly the stabilization theorem (large time behavior of the solution) and regularity theorem.

#### UNIFORM STABILIZATION OF A HYBRID SYSTEM OF ELASTICITY

**Moulay Driss Aouragh** , *Errachidia, Morocco*

2000 MSC: 93C20, 93D15, 35D35, 35P10.

**Abstract:** The purpose of this work is to study the boundary feedback stabilization of the well-known SCOLE model. Consisting of an elastic beam, linked to a rigid antenna, this dynamical system can be described by the following system:

$$\begin{aligned} \partial_{tt}y + \partial_{xxxx}y &= 0, \quad 0 < x < 1, \quad t > 0 \\ M\partial_{tt}y(1, t) - \partial_{xxx}y(1, t) &= 0, \quad t > 0 \\ J\partial_{xtt}y(1, t) + \partial_{xx}y(1, t) &= 0, \quad t > 0 \end{aligned} \tag{2.26}$$

Here,  $t$  is the time variable and  $x$  the space coordinate along the beam, in its equilibrium position. The function  $y$  is the transverse displacement of the beam.  $M$  the mass of the antenna and  $J$  the moment of inertia associated with the antenna. Our goal is to choose suitable boundary damping at the end  $x = 0$  such that the hybrid system can be stabilized uniformly.

- [1] W.Littman, and L. Markus, Exact boundary controllability of a hybrid system of elasticity, *Arch. Rational Mech. Anal.* **103** (1988) 193-236.
- [2] B. Z. Guo, Riesz basis approach to the stabilization of a flexible beam with a tip mass, *SIAM J. Control and Optimization.* **39** (2001) 1736-1747

#### ON THE CONVERGENCE OF SUCCESSIVE APPROXIMATIONS FOR A FRACTIONAL DIFFERENTIAL EQUATION IN BANACH SPACES

**Aldona Dutkiewicz**, *Poznan, Poland*



2000 MSC: 34G20

**Abstract:** We consider the Cauchy problem

$$D^\beta x = f(t, x), \quad x(0) = x_0,$$

where  $0 < \beta < 1$  and  $D^\beta$  denotes the fractional derivative of order  $\beta$  in the Caputo sense. Assume that  $E$  is a Banach space,  $B$  is a ball in  $E$  and  $f : [0, a] \times B \rightarrow E$  is a bounded continuous function. It is our object in this talk to establish a convergence theorem for the successive approximations for this nonlinear fractional initial value problem under the generalized Osgood type condition. Also an example is given to illustrate our result.

- [1] A. Dutkiewicz, On the convergence of successive approximations for a fractional differential equation in Banach spaces, *Zeitschrift für Analysis und ihre Anwendungen*, **33** (2014) 305-310.
- [2] W. Mydlarczyk, The existence of nontrivial solutions of Volterra equations, *Math. Scand.* **68** (1991) 83-88.

## CONTINUOUS DEPENDENCE OF THE MINIMUM OF THE BOLZA TYPE FUNCTIONAL ON THE PARAMETERS IN THE OPTIMAL CONTROL PROBLEM WITH DISTRIBUTED DELAY

**Pridon Dvalishvili**, *Tbilisi, Georgia*

2010 MSC: 34K35, 34K27, 49J21

**Abstract:** As a rule, various small values are ignored in the numerical solutions of optimal problems, therefore it is important to establish the connection between initial and perturbed problems. For an optimal control problem it is studied continuous dependence of the functional minimum to the initial data. The continuity problems of the functional minimum with respect to perturbations for various classes of optimal problems are given in [1-3].

- [1] G.L Kharatishvili, T.A. Tadumadze, Regular perturbation in optimal control problems with variable delays and free right end. *Soviet mat dokl.* **42** (1991) 399-403.
- [2] T. A. Tadumadze, Some problems in the qualitative theory of optimal control. (Russian) *Tbilis. Gos. Univ.*, Tbilisi, (1983)

- [3] P.Dvalishvili, On the well-posedness of a class of the optimal control problem with distributed delay. Seminar of I. Vekua Institute of Applied Mathematics, REPORTS. **39** (2013) 21-26

# OSCILLATION OF FOURTH-ORDER TRINOMIAL DELAY DIFFERENTIAL EQUATIONS

**Jozef Dzurina**, *Kosice, Slovakia*

2000 MSC: 34C10

**Abstract:** This is joint work with Blanka Baculikova.

The objective of this paper is to study oscillatory properties of the fourth-order linear trinomial delay differential equation

$$y^{(4)}(t) + p(t)y'(t) + q(t)y(\tau(t)) = 0. \quad (E)$$

Applying suitable comparison principles, we present new criteria for oscillation of Eq.(??). In contrast with the existing results, we establish oscillation of all solutions of (??). Obtained results essentially simplify the examination of the studied equations. An example is included to illustrate the importance of results obtained.

- [1] J.Dzurina, B. Baculikova, I. Jadlovská, Oscillation of Fourth-Order Trinomial Delay Differential Equations, EJDE, Vol. 2015 (2015), No. 70, pp. 1-10.

# COMBINED EFFECTS IN A SEMILINEAR FRACTIONAL TWO-POINT BOUNDARY VALUE PROBLEMS

**Zagharide Z. El Abidine**, *Tunis, Tunisia*

2000 MSC: 34A08, 34B15, 34B18, 34B27.

**Abstract:** This is a joint work with Prof Faten Toumi.

We investigate the existence, uniqueness and the asymptotic behavior of positive continuous solutions to the following semilinear fractional boundary value problem

$$\begin{cases} D^\alpha u(x) = p_1(x)u^{\sigma_1} + p_2(x)u^{\sigma_2}, & x \in (0, 1), \\ u(0) = u(1) = D^{\alpha-3}u(0) = u'(1) = 0, \end{cases}$$

where  $3 < \alpha \leq 4$ ,  $D^\alpha$  is the standard Riemann-Liouville fractional derivative,  $\sigma_1, \sigma_2 \in (-1, 1)$  and  $p_1, p_2$  are two nonnegative continuous functions on  $(0, 1)$  satisfying some appropriate assumptions related to Karamata regular variation theory.

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- [3] R. Seneta, Regular varying functions, Lectures Notes in Math. 508 Springer-Verlag, Berlin, 1976.

## LOCAL GEOMETRY OF TRAJECTORIES OF PARABOLIC TYPE SEMIGROUPS

**Mark Elin**, *Karmiel, Israel*

2010 MSC: 47H20, 30C45

**Abstract:** It is well known that the geometric nature of semigroup trajectories essentially depends on the semigroup type. In this work, we concentrate on parabolic type semigroups of holomorphic self-mappings of the open unit disk and the right half-plane and study the asymptotic behavior of semigroups and the structure of their trajectories near the attractive fixed point.

In particular, we find the limit contact order and the limit curvature of trajectories. This enable us to establish a new rigidity property for semigroups of parabolic type.

## NONOSCILLATION CRITERIA FOR DISCRETE TRIGONOMETRIC SYSTEMS

**Julia Elyseeva**, *Moscow, Russia*

2000 MSC: 39A12; 39A21; 39A22

**Abstract:** This is a joint work with Prof Ondřej Došlý. We investigate oscillation properties of discrete trigonometric systems [1] whose coefficients matrices are simultaneously symplectic and orthogonal. The main result generalizes a necessary and sufficient condition of nonoscillation of trigonometric systems proved by M. Bohner and O. Došlý [2] in the case when the block in the upper right corner

of the coefficient matrix is symmetric and positive definite. Now we present this oscillation criterion for an arbitrary trigonometric system basing on the so-called symplectic SVD decomposition [3] and the comparative index theory [4], [5]. The obtained results are applied to formulate a discrete analog of a necessary and sufficient condition for nonoscillation of even-order Sturm-Liouville differential equations [6].

- [1] D. Anderson, Discrete trigonometric matrix functions, *Panamer. Math. J.*, 7 (1997), 39–54.
- [2] M. Bohner, O. Došlý, Trigonometric transformations of symplectic difference systems, *J. Differential Equations*, 163 (2000), 113–129.
- [3] C. Paige, C. Van Loan, A Schur decomposition for Hamiltonian matrices, *Linear Algebra and Appl.*, 41(1981), 11–32.
- [4] Yu. V. Eliseeva, Comparative index for solutions of symplectic difference systems, *Differential Equations* 45 (2009), 445–457.
- [5] J. V. Elyseeva, Transformations and the number of focal points for conjoined bases of symplectic difference systems, *J. Difference Equ. Appl.* 15(11–12) (2009), 1055–1066.
- [6] O. Došlý, On some problems in the oscillation theory of selfadjoint linear differential equations. *Math. Slovaca* 41 (1991), 101–111.

## ON THE NUMERICAL SOLUTION OF SINGULARLY PERTURBED REACTION DIFFUSION EQUATIONS

**Fevzi Erdogan**, *Van, Turkey*

2000 MSC: 34D15, 33F05

**Abstract:** This is joint work with M.Giyas Sakar, Onur Saldır.

A linear reaction-diffusion boundary value problem is considered. Its second-order derivative is multiplied by a small positive parameter, which induces boundary layers. An exponentially fitted difference scheme is constructed in an equidistant mesh, which gives first order uniform convergence in the discrete maximum norm. The method is shown to uniformly convergent with respect to the perturbation parameter. A numerical experiment illustrate in practice the result of convergence proved theoretically.

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- [2] H.G. Roos, M. Stynes and L. Tobiska, Numerical Methods for Singularly Perturbed Differential Equations, Convection-Diffusion and Flow Problems, Springer Verlag, Berlin, 1996.
- [3] G.M. Amiraliyev and H. Duru, A uniformly convergent finite difference method for a initial value problem, *Applied Mathematics and Mechanics* 20,4(1999) 363-370.
- [4] N. Kopteva, M. Stynes, Numerical analysis of a singularly perturbed nonlinear reaction-diffusion problem with multiple solutions, *Applied Numerical Mathematics* 51 (2004) 273-288.

# BIFURCATION STUDY OF 3-RD PRADATOR - PREY MODEL WITH SATURATION AND COMPETITION AFFECTS IN THE PRADATORS POPULATION

**Vafokul E. Ergashev**, *Samarkand, Uzbekistan*

**Abstract:** This is a joint work with T. E. Buriyev and Ya Muxtarov. The purpose of the present work is to study model of dynamics of three populations (two predator one prey) interacting by a principle a predator - prey with the additional count of affects of intraspecific competition and saturation affect in predators populations, under stationary and periodic environmental fluctuations.

Results of analytic investigations and computer experiments shows that depending of values of parameters the following dynamics regimes of behaviour are possible in a system:

- 1) exist the values of number of prey populations that capacity prey does not provide fixed number of prey population (produced) sufficient for existence even one of predators,
- 2) steady coexistence of all three types populations realized in steady regime (stable equilibrium points),
- 3) steady coexistence of all three types of populations realized in Self-Oscillating regime(three dimensional limit cycle),

4) exist value of parameters the system behavior does not depend on initial conditions: a prey population in always unrestrictedly increased (unstable equilibrium points),

5) under periodic environmental fluctuations exist values of parameters of system which correspond to the existence of stochastic regimes of behaviours.

The investigations has been carried out qualitatively based on the bifurcation and perturbations theory of ODE , as well as by means of a computer experiments.

# IMPULSIVE DELAY FRACTIONAL DIFFERENTIAL EQUATIONS WITH VARIABLE MOMENTS

**Hilmi Ergoren**, *Van, TURKEY*

2000 MSC: 26A33, 34A08, 34A37, 34K37.

**Abstract:** It is known that impulsive functional differential equations of integer order with fixed and variable moments and the ones of fractional order fixed moments appear in the related literature several times (see for instance [1], [2] and [3]. To the best of our knowledge, the ones of fractional order with variable moments have not been considered yet. This study is concerned with establishing some existence results for solutions of impulsive delay differential equations of fractional order with variable [1], [2] and [3] moments.

- [1] Benchohra, M., Henderson J., Ntouyas, S. K, Ouahab, A., Impulsive functional differential equations with variable times, *Comput. Math. Appl.* 47: 1659 – 1665, 2004.
- [2] Anguraj, A., Ranjini, M. C., Existence results for fractional impulsive neutral functional differential equations, *JFCA* **3**(4): 1–12, 2012.
- [3] Benchohra, M., Ouahab, A., Impulsive neutral functional differential equations with variable times, *Nonlinear Anal.* 55: 679 – 693, 2003.

# DYNAMICAL ANALYSIS OF THE HAAVELMO GROWTH CYCLE MODEL WITH FRACTIONAL DERIVATIVE

**Vedat Ertürk**, *Samsun, Turkey*

2000 MSC: 26A33,91B62

**Abstract:** This is a joint work with Prof Syed Abbas. In this conference contribution we study the fractional counterpart of Haavelmo growth cycle model[1]. First we study the stability of equilibrium points. Further we give some sufficient conditions ensuring the existence and uniqueness of integral solution. Finally, we perform several numerical simulations to validate our analytical findings.

- [1] M.Guzowska, Non-standard method of discretization on the example of Haavelmo Growth Cycle Model, *Folia Oeconomica Stetinensia* 7 (2008) 45-55.

# ATTRACTING FIXED POINTS FOR THE BUCHNER-ZEBROWSKI EQUATION: THE ROLE OF SCHWARZIAN DERIVATIVE

**José G. Espín Buendía**, *Murcia, Spain*

2000 MSC: 39A30

**Abstract:** This is a joint work (still in progress) with Prof. V. Jiménez and E. Pareño (University of Murcia). Roughly speaking, if  $I \subset \mathbb{R}$  is an interval, we say that a  $C^3$  map  $h : I \rightarrow I$  belongs to the class  $S$  if it is unimodal, has a unique fixed point and its Schwarzian derivative is negative. The well-known Singer-Allwright theorem states that maps belonging to the class  $S$  have a locally attracting fixed point only if this attractor is global.

If, given a map  $h$  as above, the one-dimensional system  $x_{n+1} = h(x_n)$  presents a repelling fixed point  $u$ , one can perturb the system to a multi-dimensional one

$$x_{n+1} = (1 - \alpha)h(x_n) + \alpha x_{n-k} \quad (2.27)$$

where  $0 < \alpha < 1$  (the so-called Buchner-Zebrowski equation) with, when  $k$  is even, (possibly)  $u$  being a local attractor.

In this context the following question seems natural: does local attraction for the perturbed system (2.2) imply global attraction? We present our investigation in this regard: a negative answer to the question (in general) is given. Furthermore, we shall discuss the role that the Schwarzian derivative plays in this research and why it allows us to conjecture that a positive answer to the question is plausible for large values of  $k$ .

# INTERNAL REGULARITY OF SOLUTIONS TO CERTAIN QUASILINEAR DISPERSIVE EVOLUTION EQUATIONS

**Andrei Faminskii**, *Moscow, Russia*

2000 MSC: 35Q53

**Abstract:** This is a joint work with A.P. Antonova and M.A. Opritova.

The most well-known quasilinear dispersive evolution equation is Korteweg–de Vries equation (KdV)  $u_t + u_{xxx} + uu_x = 0$ . Besides this one we consider its generalizations such as Kawahara equation  $u_t - u_{xxxxx} + bu_{xxx} + uu_x = 0$  and Zakharov–Kuznetsov equation (ZK) on the plane  $u_t + u_{xxx} + u_{xyy} + uu_x = 0$ .

Consider for these equations the initial value problem with non-regular initial functions  $u_0 \in L_2$ . Existence and uniqueness results for global weak solutions to these problems were established earlier. In the case of KdV equation it was also known that if  $u_0$  possessed certain additional decay when  $x \rightarrow +\infty$ , then the corresponding solution became more regular for positive  $t$ .

Similar results on internal regularity of weak solutions are established now for Kawahara and ZK equations. We prove existence both of Sobolev and continuous derivatives of any prescribed order depending on the power decay rate of  $u_0$  when  $x \rightarrow +\infty$ . For continuous derivatives estimates in Hölder norms are also obtained.

Besides the initial value problems we consider initial-boundary value problems in the domains  $x > 0$  and establish the same internal regularity.

## PROBLEMS WITH NONLOCAL NEUMANN CONDITIONS: A TOPOLOGICAL APPROACH

**F. Adrián Fernández Tojo**, *Santiago de Compostela, Spain*

2000 MSC: 34B10 (Primary), 34B18 (Secondary), 34B27, 47H30

**Abstract:** This talk is based on a joint work with Professors Gennaro Infante and Paola Pietramala [1]. We prove new results on the existence, non-existence, localization and multiplicity of nontrivial solutions for perturbed Hammerstein integral equations. Our approach is topological and relies on the classical fixed point index. Some of the criteria involve a comparison with the spectral radius of some related linear operators. We apply our results to some boundary value problems with local and nonlocal boundary conditions of Neumann type.

- [1] G. Infante, P. Pietramala and F. A. F. Tojo, Nontrivial solutions of local and nonlocal Neumann boundary value problems. *Proc. Edinb. Math. Sect. A.* (To appear).



A PRIORI BOUNDS & EXISTENCE RESULTS FOR SINGULAR BVPS WITH  
AN APPLICATION TO THOMAS–FERMI EQUATIONS WHEN THE ATOM  
IS NEUTRAL

Nicholas Fewster-Young, *Sydney, Australia*

**Abstract:** In 1927, L. H. Thomas and E. Fermi derived a nonlinear differential equation that models the electrical potential in an atom under varying conditions. This talk investigates the instance when the atom is neutral; presenting *a priori bounds* and novel existence results concerning the theory and a numerical approximation to a solution. The results complement and extend on the work of R. Agarwal & D. O'Regan & P. Palamides in Singular Differential Equations. Also, the work aligns with the computation methods derived by C. Chan and Y. Hon for a numerical solution in 2004. The approach uses new singular differential inequalities to yield a priori bounds on possible solutions and topological methods to prove the existence results.

- [1] Agarwal, Ravi P.; O'Regan, Donal. An upper and lower solution approach for a generalized Thomas-Fermi theory of neutral atoms. *Math. Probl. Eng.* 8 (2002), no. 2, 135–142.
- [2] Agarwal, Ravi P.; O'Regan, Donal; Palamides, Panos K. The generalized Thomas-Fermi singular boundary value problems for neutral atoms. *Math. Methods Appl. Sci.* 29 (2006), no. 1, 49–66.
- [3] Chan, C. Y.; Hon, Y. C. Computational methods for generalized Thomas-Fermi models of neutral atoms. *Quart. Appl. Math.* 46 (1988), no. 4, 711–726
- [4] Wong, S. M.; Hon, Y. C. Numerical approximations for Thomas-Fermi model using radial basis functions. *Dynamic systems and applications*. Vol. 4, 175–182, Dynamic, Atlanta, GA, 2004.
- [5] Fewster-Young, Nicholas. A priori bounds & the existence of solutions to non-linear, second order, singular boundary value problems with Bohr conditions. *Nonlinear Analysis: Theory, Methods & Applications*. Submitted (2015).

LIMIT CYCLE BIFURCATIONS OF POLYNOMIAL  
DIFFERENTIAL AND DIFFERENCE EQUATIONS

Valery Gaiko, *Minsk, Belarus*

2000 MSC: 34C05; 34C23; 37D45; 37G15; 37G35

**Abstract:** The global qualitative analysis of polynomial differential and difference equations and their corresponding systems is carried out. Using new bifurcational and topological methods, we solve first Hilbert's Sixteenth Problem on limit cycles for the general 2D Liénard polynomial system with an arbitrary number of singular points. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles. We discuss also how to apply our approach for studying global limit cycle bifurcations of discrete polynomial (and rational) dynamical systems which model the population dynamics in biomedical and ecological systems.

This work was partially supported by the Simons Foundation of the International Mathematical Union and the Department of Mathematics and Statistics of the Missouri University of Science and Technology.

## BLOW-UP SITUATION FOR SOME DIFFERENTIAL INEQUALITIES WITH SHIFTED ARGUMENT

**Evgeny Galakhov** and **Olga Salieva** , *Moscow, Russia*

2000 MSC: 34C10

**Abstract:** This is a joint work with Olga Salieva and Liudmila Uvarova. Let  $q > 1$ . Consider the problem of finding a function  $y(t)$ , which satisfies the first order differential inequality with advanced argument

$$\frac{dy(t)}{dt} \geq |y(t + \tau)|^q \quad (t > 0) \quad (2.28)$$

and the initial condition

$$y(0) = y_0 > 0. \quad (2.29)$$

We obtain upper estimates for the blow-up time of solutions of problem (2.28)–(2.29).

Note that blow-up phenomena for differential equations with a shifted argument occur in heat transfer processes (see [1]).

This work is supported by the Russian Foundation for Fundamental Research (projects 13-01-12460-ofi-m and 14-01-00736), by the grant NS 4479.2014.1 of the President of Russian Federation, and by the Ministry of Education and Science of Russia in the framework of a state order in the sphere of scientific activities (order No. 2014/105, project No. 1441).

- [1] Deviaterikova E., Galakhov E., Salieva O., Uvarova L., Blow-up time for a problem of heat transfer with coefficients depending on their formation mechanisms, Proceedings of the XIIth International Conference on Numerical Analysis and Applied Mathematics, Rhodes, Greece, September 22-27, 2014 (to appear in 2015).

# A NEW METHOD FOR SOLVING TIME-FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS

Mehmet Gıyas Sakar, Van, Turkey

2000 MSC: 26A33

**Abstract:** This is joint work with Fevzi Erdogan, Onur Saldır

In this paper, we proposed a new approach based on variational iteration method with an auxiliary parameter for solving time-fractional nonlinear partial differential equations. The convergence of method is showed by using a special case of Banach fixed point theorem. Maximum error bound is given. The fractional derivatives are described in the Caputo sense. Numerical illustrations that include the time-fractional fifth order KdV, time-fractional Fornberg-Whitham equation and time-fractional Klein-Gordon equation are investigated to show the appropriate features of the technique. The results reveal that a new approach is very effective and convenient.

- [1] K. Diethelm, The analysis of fractional differential equations, Berlin Heidelberg, Springer-Verlag, 2010.
- [2] M. G. Sakar, H. Ergören, Alternative Variational iteration method for solving the time-fractional Fornberg-Whitham equation, *Appl. Math. Modelling*, (in press).
- [3] Z. M. Odibat, A study on the convergence of variational iteration method, *Math. Comp. Modelling*, 51 (2010) 1181-1192.
- [4] H. Ghanei, M. M. Hosseini, S. Y. Mohyud-Din, Modified variational iteration method for solving a neutral functional-differential equation with proportional delays, *International Journal of Numerical Methods for Heat & Fluid Flow* 22 (8) (2012) 1086-1095.

ON THE STRUCTURE OF THE SOLUTION SET OF ABSTRACT  
INCLUSIONS WITH INFINITE DELAY IN A BANACH SPACE

**Lahcene Guedda**, *Tiaret, Algeria*

2000 MSC: 34A60; 47H09

**Abstract:** We study the topological structure of the solution set of abstract inclusions, not necessarily linear, in a Banach space with infinite delay on an abstract Banach space defined axiomatically. By using the techniques of the theory of condensing maps and multivalued analysis tools, we prove that the solution set is a compact  $R_\delta$  set. As applications, we describe some concrete situations where our result is applicable.

- [1] G. Conti, V. Obukhovski, and P. Zecca, On the topological structure of the solution set for a semilinear functional-differential inclusion in a Banach space, *Topology in Nonlinear Analysis*. 35 (1996) 159-169.
- [2] L. Górniewicz, Topological structure of solution sets: current results, *Arch. Math. (Brno)* 36 (2000) 343-382.

ANALYTICAL INTEGRATION OF THE OSCULATING LAGRANGE  
PLANETARY EQUATIONS IN THE ELLIPTIC ORBITAL MOTION - J2, 3RD  
BODY, SRP, + ATMOSPHERIC DRAG - SOFTWARE NADIA

**Denis Hautesserres**, *Toulouse, France*

2000 MSC: 70F05, 70F07, 70F15

**Abstract:** In the field of orbital motion perturbation methods a half century of works has produced a lot of analytical theories. These theories are either based on hamiltonian developments and series expansions of the perturbing functions (Brouwer), or use iterative approximation algorithms (Kozai). Generally speaking, analytical theories have difficulties to deal with high eccentric satellite orbits. They have also a great difficulty to deal with satellite orbits at the critical inclination. The present work comes back to the original Lagrange Planetary Equations using Kosai's method [1]. The goal is to solve the restricted 3-body problem with an analytical method accurate enough to deal with any satellite orbit having a large eccentricity, around an oblateness central body, perturbed by a third body and the SRP, plus the atmospheric drag if it's necessary (using GVOP). It's a simple method

efficient to analyse each effect of the Sun and of the Moon over one orbital period of the satellite. Moreover, a long duration simulation allows to observe the long period and the secular effects on the orbital parameters. The results of the method on GTO and HEO are provided. As a conclusion, the high eccentric satellite orbit has been solved in an analytical way. Because of the osculating parameters maybe that the software NADIA could be useful for technology.

- [1] Y. Kozai, *The motion of a close Earth satellite*, The Astronomical Journal, (1959) pp. 367-377.

## BIFURCATIONS FROM POSITIVE STATIONARY SOLUTIONS IN SPECIAL SUSPENSION BRIDGE MODELS

**Gabriela Holubová**, *Pilsen, Czech Republic*

2000 MSC: 35B10, 58E07,

**Abstract:** We consider a modified version of a suspension bridge model originally introduced by Lazer and McKenna:

$$\begin{cases} u_{tt} + u_{xxxx} + br(x)u^+ = h(x) & \text{in } (0, 1) \times \mathbb{R}, \\ u(0, t) = u(1, t) = u_{xx}(0, t) = u_{xx}(1, t) = 0, \\ u(x, t) = u(x, -t) = u(x, t + 2\pi). \end{cases}$$

Here, the term  $br(x)u^+$  represents the nonlinear restoring force due to the suspension bridge cables with the stiffness  $b$  and density function  $r$ . The original model considered  $r(x) \equiv 1$ . Letting  $0 \leq r(x) \leq 1$ , we can model the “distinct distribution” of the cables. We study the qualitative and quantitative properties of the model and compare the cases of constant and non-constant density  $r(x)$ . In particular, we focus on the bifurcation and on the existence of multiple solutions.

## SPLITTING METHODS FOR OPTION PRICING IN A GENERALISED BACK-SCHOLES MODEL

**Anwar Hussein**, *UAE University, UAE*

MSC2010: 35K99

**Abstract:** This is a joint work with Youssef El Khatib.

We present an operator splitting method to compute the pricing of a European option in markets under stress which is described by a generalized Black-Scholes

partial differential equation (PDE). The idea here is to split the model into a set of simpler sub-problems. Each sub-problem is then solved using an appropriate numerical scheme. The overall integrator for the system is formed by piecing these individual solutions together by operator splitting. We illustrate the performance of the method using numerical experiments.

### INTERPOLATION METHOD OF SHALVA MIKELADZE FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

**Liana Karalashvili**, *Tbilisi, Georgia*

2000 MSC: 34K10 **Abstract:** This report presents an interpolation method of Shalva Mikeladze. Accuracy of the method depends on the number of interpolation points. This is a method without saturation. The method was designed to find a numerical solution of ordinary differential equations and was constructed on the basis of Shalva Mikeladze interpolation formula to solve numerically linear and nonlinear ordinary differential equations of any order as well as systems of such equations. Using different versions of interpolation formula it became possible to solve boundary value problems, eigenvalue problems and Cauchy problem. Later this method in combination with the method of lines was applied to solve boundary value problem for partial differential equations of elliptic type. The Dirichlet problem for the Poisson equation in the symmetric rectangle was considered as a model for application. This application created a semi-discrete difference scheme with matrices of central symmetry having certain properties.

### EXISTENCE AND UNIQUENESS OF SOLUTIONS OF INHOMOGENOUS BOUNDARY VALUE PROBLEM (BVP) FOR LINEAR IMPULSIVE FRACTIONAL DIFFERENTIAL EQUATIONS

**Zeynep Kayar**, *Van, Turkey*

2000 MSC: 34B37, 26A33

**Abstract:** We study inhomogenous BVP for linear impulsive fractional differential equations involving the Caputo fractional derivative of order  $\alpha$  ( $1 < \alpha \leq 2$ ) and present an existence and uniqueness theorem for these equations by using Lyapunov type inequality obtained for the corresponding homogeneous BVP. To the best of our knowledge, the connection between BVP and Lyapunov type inequality for linear impulsive fractional differential equations, which has not been noted even for the nonimpulsive case, is given for the first time.

- [1] R. A. C. Ferreira, On Lyapunov-type inequality and the zeros of a certain Mittag-Leffler function, *Journal of Mathematical Analysis and Applications* **412** (2014) 1058-1063.
- [2] Z. Kayar and A. Zafer, Impulsive Boundary Value Problems for Planar Hamiltonian Systems, *Abstract and Applied Analysis* **2013** (2013), 6 pages.

# PIECEWISE-DEFINED DIFFERENCE EQUATIONS: OPEN PROBLEM

**Candace M. Kent**, *Richmond, Virginia USA*

2000 MSC: 39A20

**Abstract:** We consider autonomous and nonautonomous piecewise-defined difference equations of the form

$$x_{n+1} = f(x_n, x_{n-1}, \dots, x_{n-k}), n = 0, 1, \dots, \quad (2.30)$$

and

$$x_{n+1} = f(n, x_n, x_{n-1}, \dots, x_{n-k}), n = 0, 1, \dots, \quad (2.31)$$

respectively, where  $k \in 0, 1, \dots$  and  $f : D^{k+1} \rightarrow D$ ,  $D \subset \mathbf{R}$ , whose behavior of solutions is such that *every solution is eventually periodic*. There exist numerous examples of difference equations that are both piecewise-defined and characterized by this behavior. We divide these examples into four cases and then briefly describe them. However, not all piecewise-defined difference equations have solutions with this behavior, and we point out some of these exceptions within the four cases. Under each of the four cases, we attempt to extract some properties (perhaps more descriptive than mathematical at times) that our sampling of eventually periodic piecewise-defined difference equations have in common. We follow up, after the four cases, with a request for a rigorous explanation of why many piecewise-defined difference equations have every solution eventually periodic (and why some do not). We give a highly annotated bibliography.

- [1] A. Al-Amleh, E.A. Grove, C.M. Kent, and G. Ladas , *On some difference equations with eventually periodic solutions*, *J. Math. Anal.* 233 (1998), pp. 196-215.
- [2] Y. Chen, *All solutions of a class of difference equations truncated periodic*, *Appl. Math. Lett.* 15 (2002), pp. 975-979.

- [3] E.A. Grove and G. Ladas, *Periodicities in Nonlinear Difference Equations*, Chapman & Hall/CRC Press, Baton Rouge (2005).
- [4] J.C. Lagarias, H.A. Porta, K.B. Stolarsky, *Asymmetric tent map expansions. I. Eventually periodic points*, J. Lond. Math. Soc. 47(2) (1993), pp. 542-556.

ABOUT ON A FEATURE OF THE OPTIMAL CONTROL PROBLEM IN  
MATHEMATICAL AND COMPUTER MODELS OF THE INFORMATION  
WARFARE

**Nugzar Kereselidze**, *Sukhumi-Tbilisi, Georgia*

2000 MSC: 49J15

**Abstract:** Catching up on the mathematical modeling of a direction of the information warfare [1], there was a problem of optimal control, which has several of peculiarities, and that we offer to call the task *ChilKer*:

$$B_0 = \int_0^{\max(t^*, t^{**})} (u_1^2(t) + u_2^2(t)) dt \rightarrow \inf \quad (2.32)$$

$$\begin{aligned} \frac{dx(t)}{dt} &= \alpha_1 x(t) \left( 1 - \frac{x(t)}{I_1} \right) - \beta_1 z(t), \\ \frac{dy(t)}{dt} &= \alpha_2 y(t) \left( 1 - \frac{y(t)}{I_2} \right) - \beta_2 z(t), \end{aligned} \quad (2.33)$$

$$\begin{aligned} \frac{dz(t)}{dt} &= u_2(t) (x(t) + y(t)) \left( 1 - \frac{z(t)}{u_1(t)} \right) \\ x(0) &= x_0, y(0) = y_0, z(0) = z_0, x(t^*) = 0, y(t^{**}) = 0 \end{aligned} \quad (2.34)$$

Where, in (2.33), (2.34)  $x(t), y(t)$  - the amount of information at a time  $t$ , disseminate relevant antagonistic parties to achieve information superiority in information warfare under consideration;  $z(t)$  - the amount of information peacekeeping party in a time point  $t$ , containing appeals to the antagonistic parties to finish distributions of misinformations, i.e. to stop information warfare;. In the model  $x(t), y(t), z(t)$  functions defined on the segment  $[0, T]$ ;  $\alpha_1, \alpha_2$  - indexes of aggressiveness appropriate antagonistic parties,  $\beta_1, \beta_2$  - the index of the peacekeeping readiness appropriate antagonistic parties,  $u_1(t), u_2(t)$  - control parameters.  $u_2(t)$  the indexes of the peacekeeping activities of third parties.  $I_1, I_2, u_1(t)$ , - a kind of "equilibrium" amount of information relevant part, determined by the level of development of their own IT, financial, or other unauthorized use of IT capabilities. We believe



that  $x_0, y_0 > 0, z_0 \geq 0; \Delta = [0, T]; t^*, t^{**} \in \text{int}\Delta$ ; or  $t^* = t^{**}$ , or  $t^* > t^{**}$ , or  $t^* < t^{**}$ ;  $x(t), y(t), z(t) \in C^1(\Delta); u_1(t), u_2(t) \in C(\Delta)$ . Note, that we do not know in advance which of the relations  $t^* = t^{**}, t^* > t^{**}, t^* < t^{**}$  performed,  $t^*$  and  $t^{**}$  are not fixed in the  $\Delta$ !

With the aid of computer modelling have shown the fundamental possibility of control system (2.33), (2.34) [2]. The paper proposes an approach to solve the problems of the type *ChilKer* (2.32)- (2.34).

- [1] Chilachava T., Kereselidze N. Optimizing Problem of the Mathematical Model of Preventive Information Warfare. Informational and Communication Technologies – Theory and Practice: *Proceedings of the International Scientific Conference ICTMC - 2010 USA*, Imprint: Nova. p. 525 -529.
- [2] Kereselidze N. About relations of levels of Information Technology sides in one of the Mathematical Model of the Information Warfare. *Book of abstracts IV international conference of the Georgian Mathematical Union*. Tbilisi – Batumi, 9-15 September 2013. p. 168 -169.

## A NUMERICAL METHOD FOR SOLVING LINEAR FREDHOLM BY THE RATIONALIZED HAAR (RH) FUNCTIONS

**Fernane Khaireddine**, *Guelma, Algeria*

2010 MSC: 65R20, 33D45, 45D05, 45F05.

**Abstract:** In this paper, we introduce a numerical method for solving linear Fredholm Integro-differential equations of the first order. To solve these equations, we consider the equation solution approximately from rationalized Haar (RH) functions.

The numerical solution of a linear integro-differential equation is reduced to solving a linear system of algebraic equations.

Also, Some numerical examples is presented to show the efficiency of the method.

- [1] H. Brunner, Collocation Methods for Volterra Integral and Related Functional Equations, Cambridge University Press, 2004.
- [2] H. Brunner, A. Pedas and G. Vainikko, Piecewise polynomial collocation methods for linear Volterra integro-differential equations with weakly singular kernels, SIAM J. Numer. Anal., 39 (2001), pp. 957–982.

- [3] S.M. Hosseini, S. Shahmorad, Numerical solution of a class of integro-differential equations with the Tau method with an error estimation, *Appl. Math. Comput.* 136 (2003) 559–570.
- [4] S.M. Hosseini, S. Shahmorad, Tau numerical solution of Fredholm integro-differential equations with arbitrary polynomial bases, *Appl. Math. Model* 27 (2003) 145–154.
- [5] S.M. Hosseini, S. Shahmorad, A matrix formulation of the tau for the Fredholm and Volterra linear integrodifferential equations, *Korean J. Comput. Appl. Math.* 9 (2) (2002) 497–507.
- [6] S. Shahmorad, Numerical solution of a class of integro-differential equations by the Tau method, Ph.D Thesis, Tarbiat Modarres University, Tehran, 2002.

## JUPITER’S BELTS, OUR OZONE HOLES, AND DEGENERATE TORI

**Hüseyin Koçak**, *Miami, USA*

2000 MSC: 34, 37, 86

**Abstract:** The celebrated Kolmogorov-Arnold-Moser (KAM) theorem and its many variants establish that under some sort of non-degeneracy assumption most invariant tori of integrable Hamiltonian systems survive under small perturbations. Recent studies, however, have shown that degenerate tori can have remarkable stability properties. We will present theoretical and numerical work on the stability and bifurcations of degenerate tori. We will demonstrate how such tori may arise as transport barriers in geophysical flows. In particular, we will provide a theoretical explanation of the persistence of the transport barriers that trap ozone-depleted air inside the the annually recurring Antarctic ozone hole, the unusual 2011 Arctic ozone hole, and the remarkable stability of the belts and zones of the atmosphere of Jupiter.

## FIRST INTEGRALS OF ODES VIA $\lambda$ -SYMMETRIES

**Roman Kozlov**, *Bergen, Norway*

2000 MSC: 34C14

**Abstract:** A method for finding first integrals of ODEs which admit  $\lambda$ -symmetries is presented. It can be used for ODEs which do not possess a variational formulation.

The method has two approaches. The first approach is based on the standard variational operators. The second approach uses modified variational operators, which are called  $\lambda$ -variational operators. Each approach is based on a newly established identity which links  $\lambda$ -symmetries of the underlying ODE, solutions of the adjoint equation and first integrals.

- [1] N. Ibragimov, *Nonlinear self-adjointness in constructing conservation laws*, Archives of ALGA **7/8**, Karlskrona, Sweden, 2010–2011.
- [2] C. Muriel and J.L. Romero (2001) New methods of reduction for ordinary differential equations *IMA J. Appl. Math.* **66** 111–125.

## MINIMAL AND MAXIMAL SOLUTIONS OF FOURTH-ORDER NONLINEAR DIFFERENCE EQUATIONS

**Jana Krejčová**, *Prague, Czech Republic*

2000 MSC: 39A22

**Abstract:** This is a joint work with Zuzana Došlá from Masaryk University (Brno, Czech Republic) and Mauro Marini from University of Florence (Florence, Italy).

We investigate positive solutions of a fourth-order nonlinear difference equation of the form

$$\Delta a_n \left( \Delta b_n (\Delta c_n (\Delta x_n)^\gamma)^\beta \right)^\alpha + d_n x_{n+\tau}^\lambda = 0 \quad (2.35)$$

where  $\alpha, \beta, \gamma, \lambda$  are the ratios of odd positive integers,  $\tau \in \mathbb{Z}$  is a deviating argument and  $\{a_n\}, \{b_n\}, \{c_n\}, \{d_n\}$  are positive real sequences defined for  $n \in \mathbb{N}_0 = \{n_0, n_0 + 1, \dots\}$ ,  $n_0$  is a positive integer, and  $\Delta$  is the forward difference operator given by  $\Delta x_n = x_{n+1} - x_n$ .

We present a classification of nonoscillatory solutions which is based on their asymptotic behavior and we give necessary and sufficient conditions for the existence of the so-called minimal and maximal solutions.

- [1] Zuzana Došlá, Jana Krejčová, Mauro Marini, Minimal and maximal solutions of fourth-order nonlinear difference equations, *Journal of Difference Equations and Applications*, Vol. **21** Iss. 3 (2015).

TOPOLOGICAL TURBULENCE AND OTHER PROPERTIES IN  
ONE-DIMENSIONAL MODEL OF PERCUSSION DRILLING

**Sergey Kryzhevich**, *Saint-Petersburg, Russia*

2000 MSC: 37E05

**Abstract:** This is a joint work with Prof. Victor Avrutin from University of Stuttgart. We consider a discontinuous 1D map, describing a dynamical system with dry friction. Physically this model corresponds to percussion drilling. For this implicitly defined map we develop an analytic approach that allows us to describe discontinuity points of the map and expanding rates corresponding to reduction of the map to segments where it is continuous. This allows us to find periodic points of the considered map and (if applicable) invariant SRB-measures. Similar ideas i.e. reduction of dimension together with analysis of the obtained low-dimensional map can be used to study piecewise smooth systems of different origins.

ASYMPTOTIC ESTIMATES FOR SINGULARLY PERTURBED BOUNDARY  
VALUE PROBLEMS DEPENDING ON A PARAMETER

**Mustafa Kudu**, *Erzincan, Turkey*

2000 MSC: 34K10, 34K26, 34B08

**Abstract:** This is a joint work with Gabil Amirali(Amiraliyev). In this study we consider various singularly perturbed second order boundary value problems depending on a parameter. Asymptotic estimates for the solution and its derivatives have been established. The theoretical estimates have been justified by concrete examples.

ANALYSIS OF TWO-DIMENSIONAL TRANSPORT OF NUTRITION  
THROUGH HETEROGENEOUS POROUS MEDIUM WITH SPATIALLY  
RETARDATION FACTOR

**Atul Kumar**, *Utter Pradesh, India*

**Abstract:** An analysis of analytical solutions of two-dimensional transport of nutrient through heterogeneous porous media. In the present problems is developed for conservative nutrient transport in semi-infinite porous media. The nutrient dispersion parameter is considered uniform while the seepage flow velocity is considered spatially or positional variable. Retardation factor is considered inversely

proportional to square of the flow velocity. The seepage flow velocity is considered inversely proportional to the positional or spatially variable function. The present problems is derived for two cases: first is for uniform pulse type input point source and second is for varying pulse type input point source. The first input condition is considered initially and second is the far end of the domain. It is considered flux type of homogeneous nature. Both the cases domain is considered initially not nutrient free. In both the cases Laplace Transform Technique is used to get the analytical solutions of the present problems. In the process, a new space or positional variable is introduced. The effects of heterogeneity of the medium, on the nutrient transport behavior, in the presence of the source and in the absence of the source, are also studied. The solutions are graphically interpreted through graphically (Mathematica 7.0).

#### ON THE SECOND ORDER IMPULSIVE PERIODIC PROBLEM AT RESONANCE

**Martina Langerová**, *Plzen, Czech Republic*

2000 MSC: 34A37, 34B37

**Abstract:** This is a joint work with Prof. Pavel Drábek. We consider the periodic problem for the second order equation with impulses in the derivative at fixed times. We study the resonance problems and formulate general sufficient condition for the existence of a solution in terms of the asymptotic properties of both nonlinear restoring force and nonlinear impulses which generalizes the classical Landesman-Lazer condition. Our condition also implies the existence results for some open problems with vanishing and oscillating nonlinearities.

#### CONVERGENCE OF SOLUTIONS OF DYNAMIC EQUATIONS ON TIME SCALES

**Bonita Lawrence**, *Huntington, West Virginia, USA*

2000 MSC: 34C10

**Abstract:** The focus of our study is a collection of dynamic equations defined on a sequence of time scales,  $\mathbb{T}_n$ , of the form  $\mathbb{T}_n = [t_0, t_1] \cup [t_2, t_3]$ . Our study includes dynamic initial value problems,

$$u^\Delta = u, \quad u(t_0) = u_0, \quad t \in \mathbb{T}_n, \quad (2.36)$$

$$u^{\Delta\Delta} = -u, \quad u(t_0) = u_0, t \in \mathbb{T}_n, \quad (2.37)$$

and

$$u^\Delta = (1 \ominus \frac{1}{2}u)u, \quad u(t_0) = u_0, t \in \mathbb{T}_n, \quad (2.38)$$

We are interested in the behavior of solutions as the sequence  $\mathbb{T}_n$  converges to a closed interval of the real line, denoted by  $\mathbb{T}$ . We will offer analytical as well as experimental results collected using the Marshall Differential Analyzer.

- [1] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Applications*, Birkhäuser (2001).
- [2] J. Crank, *The Differential Analyzer*. Longman's, London (1947).
- [3] D.R. Hartree and A. Porter, The construction and operation of a model differential analyzer, *Memoirs and Proceedings of the Manchester Literary and Philosophical Society* 79 (1935) 51-74.

## A CORRECTOR FOR THE WAVE EQUATION IN A BOUNDED DOMAIN

**Faustino Maestre**, *Seville, Spain*

2000 MSC: 35B27, 35L20

**Abstract:** In this work we study the asymptotic behavior of the following wave equation in the one dimensional interval  $(\alpha, \beta)$ , when a parameter  $\epsilon$  tends to zero:

$$\begin{cases} \partial_t(\rho_\epsilon \partial_t u_\epsilon) - \partial_x(a_\epsilon \partial_x u_\epsilon) + B_\epsilon \cdot \nabla_{t,x} u_\epsilon = f_\epsilon \text{ in } (0, T) \times (\alpha, \beta) \\ (-c_\alpha a_\epsilon \partial_x u_\epsilon + d_\alpha u_\epsilon)|_{x=\alpha} = 0, \quad (c_\beta a_\epsilon \partial_x u_\epsilon + d_\beta u_\epsilon)|_{x=\beta} = 0 \text{ in } (0, T) \\ u_\epsilon|_{t=0} = u_\epsilon^0, \quad \partial_t u_\epsilon|_{t=0} = \vartheta_\epsilon \text{ in } (\alpha, \beta) \\ u_\epsilon \in L^\infty(0, T; H^1(\alpha, \beta)), \quad \partial_t u_\epsilon \in L^\infty(0, T; L^2((\alpha, \beta))). \end{cases} \quad (2.39)$$

Assuming for the oscillating variables, periodicity in the space variable and almost-periodicity in the time one (see [3] for more details) and using the two-scale convergence (see [1], [2]) we prove the existence of functions  $u_0$  and  $u_1$  such that  $u_0(t, x) + \epsilon u_1(t, x, \frac{t}{\epsilon}, \frac{x}{\epsilon})$  is a corrector for  $u_\epsilon$  solution of (2.39). The main result of this work corresponds with the determination of this corrector and the analysis of the boundaries conditions (because the waves travel and shock with the walls) which influence the behavior of  $u_\epsilon$  at the interior of  $(\alpha, \beta)$ .

- [1] G. Allaire. Homogenization and two-scale convergence. *SIAM J. Math. Analysis* 23 (1992), 1482-1518.

- [2] J. Casado-Díaz, J. Couce-Calvo, F. Maestre, J.D. Martín-Gómez. Homogenization and correctors for the wave equation with periodic coefficients, *Mathematical Models and Methods in Applied Sciences*, 24, (2014), 1343-1388.
- [3] J. Casado-Díaz, J. Couce-Calvo, F. Maestre, J.D. Martín-Gómez. A corrector for a wave problem with periodic coefficients in a 1D bounded domain, *ESAIM: Control, Optimisation and Calculus of Variation*. 21(2), (2015) pp.

ON EXISTENCE RESULTS FOR NONLINEAR FRACTIONAL  
DIFFERENTIAL EQUATIONS WITH FOUR-POINT BOUNDARY VALUE  
PROBLEM

**Helal Mahmoud**, *Famagusta, Turkey*

2000 MSC: 34C10

**Abstract:** We discuss the existence and uniqueness of solution for nonlinear fractional differential equation with nonlocal four-point boundary condition for the following equation:

$$\begin{cases} {}^C D_{0+}^{\alpha} x(t) = f(t, x(t)), & 0 \leq t \leq T, \quad 1 < \alpha < 2, \\ x(0) + \mu_0 x(T) = \sigma_0 x(\eta_0), & 0 < \eta_0 < \eta_1 < T, \\ {}^C D_{0+}^{\alpha} x(0) + \mu_1 {}^C D_{0+}^{\alpha} x(T) = \sigma_1 x(\eta_1), \end{cases}$$

where  ${}^C D_{0+}^{\alpha}$  is a Caputo derivative of order  $\alpha$ .

Our results are based on the Banach fixed point theorem and Schauder's fixed point theorems.

- [1] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*. In: *North-Holland Mathematics studies*, vol.204 (2006) Elsevier, Amsterdam.
- [2] B. Ahmad and J. J. Nieto, Riemann-Liouville fractional integro-differential equations with fractional non-local integral boundary conditions. *Bound. Value Prob.* 1.36 (2011) 1-9.

SPECTRAL ANALYSIS FOR THE STURM-LIOUVILLE OPERATOR  
WITH DEGENERATE BOUNDARY CONDITIONS

**Alexander Makin**, *Moscow, Russia*

2000 MSC: 34L40

**Abstract:** We consider the Sturm-Liouville equation with degenerate [1] boundary conditions

$$u'' - q(x)u + \lambda u = 0, \quad u'(0) + du'(\pi) = 0, \quad u(0) - du(\pi) = 0, \quad (1)$$

where  $d \neq 0$  and  $q(x)$  is an arbitrary complex-valued function of the space  $L_1(0, \pi)$ .

We study the structure of the spectrum of problem (1), corresponding inverse problem, and also study the completeness property and the basis property of the root function system.

- [1] V.A. Marchenko, Sturm-Liouville Operators and Their Applications. Kiev, 1977 (in Russian); English transl.: Birkhäuser, Basel, 1986.

#### ON SPECTRUM OF MAGNETIC GRAPHS

**Stepan Manko**, *Prague, Czech Republic*

2000 MSC: 81Q35

**Abstract:** We analyze spectral properties of a quantum graph with a  $\delta$  coupling in the vertices exposed to a homogeneous magnetic field perpendicular to the graph plane. We find the band spectrum in the case when the chain exhibits a translational symmetry and study the discrete spectrum in the gaps resulting from compactly supported coupling, magnetic or geometric perturbations (see [?]). The method we employ is based on translating the spectral problem for the differential equation in question into suitable difference equations.

- [1] P. Exner and S. Manko, Spectra of magnetic chain graphs: coupling constant perturbations, *J. Phys. A: Math. Theor.* **48** (2015) 125302.

#### EXISTENCE OF SOLUTION RESULT FOR MEASURE DIFFERENTIAL EQUATIONS WITH KURZWEIL-HENSTOCK INTEGRABLE RIGHT-HAND SIDES

**Rafael Marques**, *São Carlos, Brasil*

34K05, 34K45

**Abstract:** This is a joint work with Prof. Marcia Federson and Prof. Eduard Toon. We recover Ralph Henstock results on the decomposition of the right-hand



side of an ODE of type  $\dot{x} = f(x, t)$ , where  $f$  satisfies Henstock's conditions and the existence theorem it implies. We investigate this approach and get similar results for generalized ODEs. In the present paper, we also consider measure functional differential equations and measure differential equations and apply the results obtained through the correspondence of solutions between these types of equations and GODEs.

## PERIODIC AVERAGING THEOREM FOR QUANTUM CALCULUS

**Jaqueline G. Mesquita**, *Ribeirão Preto, Brazil*

**Abstract:** This is a joint work with Prof. Martin Bohner. The theory of averaging plays an important role for applications, since it can be used to study perturbation theory, control theory, stability of solutions, bifurcation, among others. In this paper, we prove a periodic averaging principle for  $q$ -difference equations and present some examples to illustrate our result.

- [1] M. Bohner, R. Chieochan, The Beverton-Holt  $q$ -difference equation, *Journal of Biological Dynamics* (2013) vol.7, no. 1, 86-95.
- [2] M. Bohner, R. Chieochan, Floquet theory for  $q$ -difference equations. *Sarajevo J. Math.*, (2012) 8(21)(2): 1-12.
- [3] M. Bohner, J. G. Mesquita, Periodic averaging principle for quantum calculus, submitted.
- [4] J. G. Mesquita, A. Slavík, Periodic averaging theorems for various types of equations, *J. Math. Anal. Appl.* 387 (2012), 862-877.

## PROPERTIES OF SOLUTIONS OF HIGHER-ORDER NEUTRAL DIFFERENCE EQUATIONS

**Małgorzata Migda**, *Poznań, Poland*

2000 MSC: 39A10

**Abstract:** We consider the higher-order neutral difference equations of the following form

$$\Delta^m(x_n + p_n x_{n-\tau}) + f(n, x_n, x_{n-\sigma}) = 0.$$

We present sufficient conditions for the existence of solutions which tend to a given constant. We derive also conditions under which all nonoscillatory solutions are asymptotically polynomial. Some oscillation criteria are also given.

AN APPLICATION OF CHEBYSHEV POLYNOMIALS TO  
DIAGONALISATION OF THE HEISENBERG HAMILTONIAN  
FOR TWO SPIN DEVIATIONS

**Jan Milewski**, *Poznań, Poland*

2000 MSC: 33C45, 42C40, 81Q80

**Abstract:** The characteristic polynomial of the Heisenberg Hamiltonian for the ring with number of nodes  $N$  deviations is expressed by Chebyshev polynomials. Four types of Chebyshev polynomials depending on parity  $N$  and quasimomenta  $k$  are considered. Bethe parameters are derived by means of Inverse Bethe Ansatz.

REMARKS ON CONTINUOUS DEPENDENCE OF SOLUTION OF  
ABSTRACT GENERALIZED DIFFERENTIAL EQUATIONS

**Giselle A. Monteiro**, *Prague, Czech Republic*

2000 MSC: 34A30

**Abstract:** This is a joint work with Prof M. Tvrdý. In this work, we discuss continuous dependence results for generalized differential equations with a particular interest in the linear case. More precisely, we investigate integral equations of the form

$$x(t) = \tilde{x}_k + \int_a^t d[A_k]x + f_k(t) - f_k(a), \quad t \in [a, b], \quad k \in \mathbb{N},$$

where  $A_k : [a, b] \rightarrow L(X)$  have bounded variations on  $[a, b]$ ,  $f_k : [a, b] \rightarrow X$  are regulated on  $[a, b]$  and  $\tilde{x}_k \in X$ , with  $X$  being a Banach space. Herein we pay special attention to the results found in [2], where we extend Theorem 4.2 from [1] to the non-homogeneous case. An example showing that the obtained conditions are somehow optimal is provided as well.

- [1] G. A. Monteiro and M. Tvrdý, Generalized linear differential equations in a Banach space: Continuous dependence on a parameter, *Discrete Contin. Dyn. Syst.* **33** (1) (2013) 283-303.
- [2] G. A. Monteiro and M. Tvrdý, Continuous dependence of solutions of abstract generalized linear differential equations with potential converging uniformly with a weight, *Boundary Value Problems* **71** (2014) 1-18.

# ON STABILITY REGION FOR CERTAIN FRACTIONAL DIFFERENCE SYSTEM

**Luděk Nechvátal**, *Brno, Czech Republic*

2010 MSC: 39A30, 34A08, 39A12

**Abstract:** This is a joint work with J. Čermák, I. Győri and T. Kisela. The contribution discusses the problem of stability of a fractional difference system in the form

$$\Delta^\alpha y(n) = Ay(n), \quad n = 0, 1, \dots \quad (2.40)$$

where  $\Delta^\alpha$  is the forward Caputo difference operator of order  $0 < \alpha < 1$  and  $A$  is a  $d \times d$  real matrix. Alongside an explicit description of the asymptotic stability region for (2.40), a decay rate of the solutions is provided. Some comparisons with the backward case (including also the Riemann-Liouville operator) are presented as well. The results are based on the recent articles [1] and [2].

- [1] J. Čermák, I. Győri and L. Nechvátal, On explicit stability conditions for a linear fractional difference system, *Fractional Calculus & Applied Analysis*, to appear.
- [2] J. Čermák, T. Kisela and L. Nechvátal, Stability regions for linear fractional differential systems and their discretizations, *Applied Mathematics and Computation* **219** (2013) 7012-7022.

# RATIONAL MAPS AND DIFFERENCE EQUATIONS OVER FINITE FIELDS

**Natascha Neumärker**, *Opava, Czech Republic*

2000 MSC: 37P25 (37P35)

**Abstract:** This is joint work with Prof. John Roberts (UNSW) and Prof. Franco Vivaldi (QMUL).

In this talk, I will give an account of the reduction of piecewise linear and rational maps modulo a prime  $p$ , many of which were originally studied in the shape of difference equations. I will discuss how certain quantities related to the period lengths these maps admit modulo  $p$  are related to structural properties of the map and will also address computational aspects of these studies.

- [1] N. Neumärker, J.A.G. Roberts, F. Vivaldi, *Distribution of periodic orbits for the Casati-Prosen map on rational lattices*, *Phys.D*, **241** (2012), 360–371

- [2] D. Jogia, J.A.G. Roberts, F. Vivaldi, *An algebraic geometric approach to integrable maps of the plane*, J. Phys. A: Math. Gen. **39** (2006) 1133-1149
- [3] J.A.G. Roberts, F. Vivaldi, *Signature of time-reversal symmetry in polynomial automorphisms over finite fields*, Nonlinearity **18** (2005) 2171-2192

CHARACTERIZATION OF ROOTS OF EXTENDED BLASCHKE  
PRODUCTS VIA ITERATION  
**David C. Ni**, *Taipei, Taiwan*

2010 MSC: 39A33, 39A45, 39A60

**Abstract:** We explore root solving via iteration on an extended Blaschke product (EBP) in terms of difference equations, and have mapped the convergent domain to a disconnected set of roots, whose count violates the statement of Fundamental Theorem of Algebra (FTA). The results imply new views and characteristics of the complex polynomials. We define EBP as  $f = z^q \prod_i C_i$ , where  $z$  is a complex variable,  $q$  is an integer, and  $C_i$  has following form.

$$C_i = \exp(g_i(z)) [(a_i - z) / (1 - \bar{a}_i z)]$$

Further, we classify geometrically the fractal subsets of the convergent domain to the corresponding mapped root sets. The transitions among the different root count are examined and remarked in conjunction with FTA violation.

- [1] W. Blaschke, Eine Erweiterung des Satzes von Vitali über Folgen analytischer Funktionen, *Berichte Math.-Phys. Kl., Sachs. Gesell. der Wiss. Leipzig*, 67 (1915), 194-200.
- [2] D. C. Ni, A Counter Example of Fundamental Theorem of Algebra: Extended Blaschke Mapping, *ICM 2014*, (August 13 to 21, 2014) SC09-11-02, Seoul, South Korea.

ON OSCILLATION AND NONOSCILLATION TO CERTAIN  
TWO-DIMENSIONAL SYSTEMS OF NONLINEAR  
DIFFERENTIAL EQUATIONS  
**Zdeněk Opluštil**, *Brno, Czech Republic*

2000 MSC: 34C10

**Abstract:** On the half-line  $\mathbb{R}_+ = [0, +\infty[$ , we consider two-dimensional system of non-linear ordinary differential equations

$$\begin{aligned} u' &= g(t)|v|^{\frac{1}{\alpha}} \operatorname{sgn} v, \\ v' &= -p(t)|u|^{\alpha} \operatorname{sgn} u, \end{aligned} \tag{2.41}$$

where  $\alpha > 0$ ,  $p : [0, +\infty[ \rightarrow \mathbb{R}$  and  $g : [0, +\infty[ \rightarrow [0, +\infty[$  are locally Lebesgue integrable functions. New oscillation and nonoscillation criteria of the system (2.41) are established in the case  $\int^{+\infty} g(s)ds = +\infty$ .

- [1] M. Dosoudilova, A. Lomtadze, J. Šremr Oscillatory properties of solutions to certain two-dimensional systems of non-linear ordinary differential equations, *Nonlinear Analysis* **120** (2015) 57-75.

## ON A ONE EQUATION TURBULENT MODEL OF RANS TYPE WITH STRONG NONLINEAR FEEDBACKS

**Ana Paiva**, *Faro, Portugal*

**Abstract:** In this talk we consider a one equation model that describes steady flows of turbulent fluids of Reynolds averaged Navier-Stokes (RANS) type. The mathematical problem is posed by the coupling between the RANS equations and the equation for the turbulent kinetics energy (TKE). For the associated boundary-value problem, we prove the existence of weak solutions in the sense of Leray-Hopf. Uniqueness in 2 dimensions and interior higher regularity of the weak solutions is also proved. The novelty of our work relies in the consideration of a strong nonlinearity on the rate of dissipation of TKE. We consider also the case of a feedback forces field with the same kind of nonlinearity and that can be potentially used to model some geophysical flows where the Coriolis force is important, or to describe the flow through porous media to take into account the Brinkman-Forchheimer terms. This is a joint work with H.B. de Oliveira (holivei@ualg.pt).

## CHAOS IN A MODEL FOR MASTING

**Kenneth Palmer**, *Taipei, Taiwan*

2000 MSC: 37E05, 37D45, 37G35

**Abstract:** This is joint work with Kaijen Cheng. Isagi et al introduced a model for masting, that is, the intermittent production of flowers and fruit by trees. A tree produces flowers and fruit only when the stored energy exceeds a certain threshold value. If flowers and fruit are not produced, the stored energy increases by a certain fixed amount; if flowers and fruit are produced, the energy is depleted by an amount proportional to the excess stored energy. Thus a one-dimensional model is derived for the amount of stored energy. When the ratio of the amount of energy used for flowering and fruit production in a reproductive year to the excess amount of stored energy before that year is small, the stored energy approaches a constant value as time passes. However when this ratio is large, the amount of stored energy varies unpredictably and as the ratio increases the range of possible values for the stored energy increases also. In this talk this chaotic behavior is described precisely with complete proofs.

# ASYMPTOTIC FORMULAS FOR THE SOLUTIONS OF A LINEAR DELAY DIFFERENTIAL EQUATION

Mihály Pituk, Veszprém, Hungary

2000 MSC: 34K25

**Abstract:** This is a joint work with István Győri (University of Pannonia, Hungary) and Gergely Röst (University of Szeged, Hungary). Consider the linear delay differential equation

$$x'(t) = p(t)x(t - r),$$

where  $r > 0$  and  $p : [t_0, \infty) \rightarrow \mathbb{R}$  is a continuous function which tends to zero as  $t \rightarrow \infty$ . In a recent paper [1], we have shown that every solution of the above equation satisfies the asymptotic relation

$$x(t) = \frac{1}{y(t)}(c + o(1)), \quad t \rightarrow \infty,$$

where  $y$  is an eventually positive solution of the associated formal adjoint equation with bounded growth and  $c$  is a constant depending on  $x$ . In this talk we give a description of the special solution  $y$  of the formal adjoint equation which yields explicit asymptotic formulas for the solutions of the delay differential equation.

- [1] M. Pituk and G. Röst, Large time behavior of a linear delay differential equation with asymptotically small coefficient, *Boundary Value Problems* **2014**, 2014:114

MODIFICATION OF ADOMAIN DECOMPOSITION METHOD FOR  
N-ORDER NONLINEAR DIFFERENTIAL EQUATIONS

**Duangkamol Poltem**, *Chonburi, Thailand*

2000 MSC: 34B15

**Abstract:** This is a joint work with Assist. Prof. Sineenart Srimongkol, Piyada Totassa and Araya Wiwatwanich. In this work, a modification of Adomian Decomposition method is illustrated by studying suitable forms of nonlinear differential equation as follow

$$y^{(n)} + P(x)y^{(n-1)} + N(x, y, y', y'', \dots, y^{(n-2)}) = g(x),$$

$$y(0) = a_0, y'(0) = a_1, \dots, y^{(n-1)}(0) = a_{n-1}.$$

We extend earlier works [1] and [2] to obtain a new modification of Adomian Decomposition method. Some examples are demonstrated the advantages of the proposed method.

- [1] M. M. Hosseini and H. Nasabzadeh, Modified Adomian Decomposition Method for Specific Second Order Ordinary Differential Equations, *Appl. Math. Comput.* **186** (2007) 117-123.
- [2] M. M. Hosseini and M. Jafari, A Note on the use of Adomian Decomposition Method for Higher-Order and System of Nonlinear Differential Equations, *Commun. Nonlinear. Sci.* **14** (2009) 1952-1957.

FINITE-DIMENSIONAL REDUCTION OF DISCRETE DYNAMICAL  
SYSTEMS

**Yarema Prykarpatsky**, *Krakow, Poland*

2000 MSC: 37K05, 37K10, 37J15

**Abstract:** Consider an infinite dimensional discrete manifold  $M \subset l_2(\mathbb{Z}; \mathbb{C}^m)$  for some integer  $m \in \mathbb{Z}_+$  and a general nonlinear dynamical system on it in the form

$$dw/dt = K[w], \tag{2.42}$$

where  $w \in M$  and  $K : M \rightarrow T(M)$  is a Fréchet smooth nonlinear local functional on  $M$  and  $t \in \mathbb{R}$  is the evolution parameter. Assume that a given nonlinear dynamical

system (2.42) on the manifold  $M$  is Lax type integrable, i.e. it possesses a related Lax type representation in the following generic form:

$$\Delta f_n := f_{n+1} = l_n[w; \lambda] f_n, \quad (2.43)$$

where  $f := \{f_n \in \mathbb{C}^r : n \in \mathbb{Z}\} \subset l_2(\mathbb{Z}; \mathbb{C}^r)$  for some integer  $r \in \mathbb{Z}_+$  and matrices  $l_n[w; \lambda] \in \text{End} \mathbb{C}^r$ ,  $n \in \mathbb{Z}$ , in (2.43) are local matrix-valued functionals on  $M$ , depending on the “spectral” parameter  $\lambda \in \mathbb{C}$ , invariant with respect to the dynamical system (2.42). It was shown that the reduced via the Bogoyavlensky-Novikov approach integrable Hamiltonian dynamical system (2.42) on the corresponding invariant periodic submanifolds generates finite dimensional Liouville integrable Hamiltonian system with respect to the canonical Gelfand-Dikii type symplectic structures.

## BOUNDARY VALUE PROBLEMS WITH IMPULSES AT STATE-DEPENDENT MOMENTS

**Irena Rachůnková**, *Olomouc, Czech Republic*

2000 MSC: 34B37; 34B15

**Abstract:** We investigate the solvability of the impulsive problem

$$x'(t) = f(t, x(t)), \text{ a.e. } t \in [a, b], \quad \ell(x) = c \in R^n, \quad (2.44)$$

$$x(t+) - x(t) = J(t, x(t)) \text{ for } g(t, x(t)) = 0. \quad (2.45)$$

The impulse instants  $t \in (a, b)$  in (2.45) are not known before and they are determined as solutions of the equation  $g(t, x(t)) = 0$ . The operator  $\ell$  is an arbitrary linear and bounded operator. We provide conditions which allow to realize a construction of a solution of problem (2.44)–(2.45).

- [1] I. Rachůnková and J. Tomeček, Existence principle for BVPs with state-dependent impulses, *Topol. Methods in Nonlin. Analysis* 44 (2014) 349-368.
- [2] I. Rachůnková, A. Rontó, L. Rachůnek and M. Rontó, Constructive method for investigation of solutions to state-dependent impulsive boundary value problems, submitted.

## OPTIMAL GUARANTEED COST CONTROL FOR UNCERTAIN NEUTRAL NETWORKS

**Grienggrai Rajchakit**, *Chiang Mai, Thailand*



2000 MSC: 15A09

**Abstract:** This paper studies the problem of optimal guaranteed cost control for a class of uncertain delayed neural networks. The time delay is a continuous function belonging to a given interval, but not necessary to be differentiable. A cost function is considered as a nonlinear performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of augmented Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, a guaranteed cost controller is designed via memoryless state feedback control and new sufficient conditions for the existence of the guaranteed cost state-feedback for the system are given in terms of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the obtained result.

- [1] Hopfield J.J., “Neural networks and physical systems with emergent collective computational abilities,” *Proc. Natl. Acad. Sci. USA*, **79**(1982), 2554-2558.
- [2] Kevin G., *An Introduction to Neural Networks*, CRC Press, 1997.
- [3] Wu M., He Y., She J.H., *Stability Analysis and Robust Control of Time-Delay Systems*, Springer, 2010.
- [4] Arik S., An improved global stability result for delayed cellular neural networks, *IEEE Trans. Circ. Syst.* **499**(2002), 1211-1218.

## OPTIMAL GUARANTEED COST CONTROL FOR STOCHASTIC NEURAL NETWORKS

**Manlika Rajchakit**, *Chiang Mai, Thailand*

2000 MSC: 52A10

**Abstract:** This paper studies the problem of optimal guaranteed cost control for a class of stochastic delayed neural networks. The time delay is a continuous function belonging to a given interval, but not necessary to be differentiable. A cost function is considered as a nonlinear performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some mean square exponential stability constraints on the closed-loop poles. By constructing a set of augmented Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, a guaranteed cost controller is designed via memoryless state feedback control

and new sufficient conditions for the existence of the guaranteed cost state-feedback for the system are given in terms of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the obtained result.

- [1] Xu S., Lam J., A survey of linear matrix inequality techniques in stability analysis of delay systems. *Int. J. Syst. Sci.*, **39**(2008), 12, 1095–1113.
- [2] Xie J. S., Fan B. Q., Young S.L., Yang J., Guaranteed cost controller design of networked control systems with state delay, *Acta Automatica Sinica*, **33**(2007), 170-174.
- [3] Yu L., Gao F., Optimal guaranteed cost control of discrete-time uncertain systems with both state and input delays, *Journal of the Franklin Institute*, **338**(2001), 101–110.

#### EXISTENCE OF MILD SOLUTIONS FOR IMPULSIVE FRACTIONAL FUNCTIONAL DIFFERENTIAL EQUATIONS OF ORDER $\alpha \in (1, 2)$

**Ganga Ram Gautam**, *IIT Roorkee, India.*

2000 MSC: 26A33,34K37,34A12.

**Abstract:** This is a joint work with Jaydev Dabas. This paper investigates the existence and uniqueness result of mild solutions for fractional order functional differential equations subject to not instantaneous impulsive condition by applying the classical fixed point technique. At last, an example involving partial derivatives is presented to verify the uniqueness result.

#### SPECTRAL ANALYSIS OF INTEGRO-DIFFERENTIAL EQUATIONS ARISING IN VISCOELASTICITY THEORY

**Nadezhda Rautian**, *Moscow, Russia*

2000 MSC: 45K05

**Abstract:** We study integro-differential equations with unbounded operator coefficients in Hilbert spaces. The principal part of the equation is an abstract hyperbolic equation perturbed by summands with Volterra integral operators. These equations represent an abstract form of the integro-partial differential equation arising in viscoelasticity theory, heat conduction theory in media with memory, etc.

A spectral analysis of the operator-valued functions, which are the symbols of considered integro-differential equations is provided.

- [1] V. V. Vlasov, N. A. Rautian and A. S. Shamaev Spectral analysis and correct solvability of abstract integrodifferential equations arising in thermophysics and acoustics, *Journal of Mathematical Sciences* **190:1** (2013) 34-65.
- [2] N. A. Rautian, On the structure and properties of solutions of integro-differential equations arising in thermal physics and acoustics, *Mathematical Notes* **90:3-4** (2011) 455-459.

## STABILITY OF DYNAMIC SYSTEMS ON TIME SCALE

Andrejs Reinfelds, *Rīga, Latvia*

2010 MSC: 34N05, 39A10, 39A30

**Abstract:** This is a joint work with Sandra Janovska.

We consider the dynamic system in a Banach space on unbounded above time scale:

$$\begin{cases} x^\Delta &= A(t)x + f(t, x, y), \\ y^\Delta &= B(t)y + g(t, x, y). \end{cases} \quad (2.46)$$

This system satisfies the conditions of integral separation with the separation constant  $\nu$ ; nonlinear terms are  $\varepsilon$ -Lipshitz, and the system has a trivial solution.

We also consider the reduced dynamic system

$$z^\Delta = A(t)z + f(t, z, u(t, z)) \quad (2.47)$$

with the integral contraction constant  $\mu$ .

**Theorem 1** *If  $4\varepsilon\nu < 1$ , then there exists a unique map  $u$  satisfying the following properties:*

- (i)  $u(t, x(t, s, x, u(s, x))) = y(t, s, x, u(s, x))$  for  $t \geq s$ ;
- (ii)  $|u(t, x) - u(t, x')| \leq k|x - x'|$ ;
- (iii)  $u(t, 0) = 0$ .

**Theorem 2** *Let  $4\varepsilon\nu < 1$  and  $2\varepsilon\mu < 1 + \sqrt{1 - 4\varepsilon\nu}$ . The trivial solution of the dynamic system (2.2) is integrable stable, integrable asymptotically stable or integrable nonstable if and only if when the trivial solution of the dynamic system (2.21) is integrable stable, integrable asymptotically stable or integrable nonstable.*

THE EIGENVALUE CHARACTERIZATION FOR DISCONJUGACY OF  $n$   
ORDER LINEAR DIFFERENTIAL EQUATIONS

**Lorena Saavedra**, *Santiago de Compostela, Spain*

2000 MSC: 34B27

**Abstract:** This is a joint work with Prof Alberto Cabada. It is well known that the disconjugacy of the  $n$ th order linear differential equation

$$T_n[M] u(t) \equiv u^{(n)}(t) + a_1(t) u^{(n-1)}(t) + \cdots + a_{n-1}(t) u'(t) + (a_n(t) + M) u(t) = 0, \quad (2.48)$$

in a given interval, implies the constant sign of the Green's function related to the so-called  $(k, n - k)$  boundary value problem, see [2].

So, in order to ensure the constant sign of the considered Green's function, we need to make a description of the interval of parameters  $M$  for which equation (2.48) is disconjugate.

In this talk we present a characterization of such interval by means of spectral theory. Some results about parameter set of constant sign Green's functions, given in [1], are applied.

- [1] A. Cabada, *Green's Functions in the Theory of Ordinary Differential Equations*, Springer Briefs in Mathematics, 2014.
- [2] W. A. Coppel, *Disconjugacy. Lecture Notes in Mathematics*, Vol. 220. Springer-Verlag, Berlin-New York, 1971.

A NOVEL APPROACH TO SOLUTION OF SINGULARLY PERTURBED  
CONVECTION-DIFFUSION PROBLEM

**Onur Saldır**, *Van, Turkey*

2000 MSC: 34D15, 41A60

**Abstract:** This is joint work with Mehmet Gıyas Sakar, Fevzi Erdogan.

In this research, a new approach is introduced for solving singularly perturbed convection-diffusion problems. We used the asymptotic expansion and the variational iteration method (VIM) with an auxiliary parameter for solution of problems. Requirements of the method, firstly asymptotic expansion formed on inner region and then reduced terminal value problem solved via variational iteration method with auxiliary parameter on outer solution. Some problems are solved by using the presented method. The numerical results show that the presented method is reliable and very effective for singularly perturbed convection-diffusion problems.

- [1] Doolan, E.P., Miller, J.J.H. and Schilders, W.H.A. (1996), Uniform Numerical Methods for Singular-Perturbation Problems: Error Estimates in the Maximum Norm for Linear Problems in One and Two Dimensions, World Scientific Publishing Company, Singapore.
- [2] Kadalbajoo, M.K. and Kumar, D., Initial value technique for singularly perturbed two point boundary value problems using an exponentially fitted finite difference scheme, *Comput. Math. Appl.*, 57 (2009) 1147-1156.
- [3] Geng, F., Qian, S., Li, S., Numerical solutions of singularly perturbed convection-diffusion problems, *International Journal of Numerical Methods for Heat & Fluid Flow*, 24 (2014) 1268-1274

PRINCIPAL AND ANTIPRINCIPAL SOLUTIONS  
OF LINEAR HAMILTONIAN SYSTEMS

**Peter Šepitka**, *Brno, Czech Republic*

2000 MSC: 34C10

**Abstract:** This is a joint work with Prof. Roman Šimon Hilscher. We study the existence and properties of principal and antiprincipal solutions at infinity for possibly abnormal linear Hamiltonian systems. We show that the principal and antiprincipal solutions can be classified according to the rank of their first component and that they exist for any rank in the range between explicitly given minimal and maximal values. We also establish a limit characterization of the principal solutions. These are generalizations of the results of W. T. Reid, P. Hartman, or W. A. Coppel for controllable linear Hamiltonian systems.

- [1] W. T. Reid, Principal solutions of nonoscillatory linear differential systems, *J. Math. Anal. Appl.* 9 (1964) 397-423.
- [2] P. Šepitka, R. Šimon Hilscher, Principal solutions at infinity of given ranks for nonoscillatory linear Hamiltonian systems, *J. Dynam. Differential Equations* 27 (2015) no. 1 137-175.
- [3] P. Šepitka, R. Šimon Hilscher, Principal and antiprincipal solutions of linear Hamiltonian systems, submitted (2014).

# SOLVING HEAT EQUATIONS FOR REDUCED DIFFERENTIAL TRANSFORM METHOD WITH FIXED GRID SIZE

**Sema Servi**, *Konya, Turkey*

2000 MSC: 35K05

**Abstract:** This is a joint work with Assoc. Prof. Dr. Yildiray KESKIN and Prof. Dr. Galip OTURANÇ. In this study, fixed grid size is applied to reduced differential transform method (RDTM) used to find the approximate solution of partial differential equations for the first time and thus it is provided that calculation of the approximate solutions with fewer errors [1], [2] and [3]. An example of homogenous heat equation existed in the literature is solved to described method [4].

- [1] S.Servi, Phd Thesis, Selcuk University, 2014 (in Turkish), Konya (to appear)
- [2] O. Özkan and Y. Keskin, An Application of the differential transformation method to the boundary value problems of the system of integro-differential equations, Selçuk Journal of Applied Mathematics 6.1. (2005) 43-53.
- [3] M. Jang , C. Chen , Y. Liy , On solving the initial-value problems using the differential transformation method, Applied Mathematics and Computation, 115 (2000) 145-160.
- [4] H. Çağlar, M. Özer, N. Çağlar, The numerical solution of the one-dimensional heat equation by using third degree B-spline functions, Chaos, Solitons & Fractals, 38 (2008) 1197-1201

# ULAM STABILITY OF SOME VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

**Sebaheddin Şevgin**, *Van, Turkey*

2000 MSC: 45J05, 47H10, 45M10

**Abstract:** In [1], S.M. Jung applied the fixed point method for proving the Hyers-Ulam-Rassias stability and the Hyers-Ulam stability of a Volterra integral equation of the second kind. In 2012, the Hyers-Ulam-Rassias stability of a linear Volterra integro-differential equation investigated by Jung, Şevgin and Şevli [2].

In this study, we will establish the Hyers-Ulam stability of some Volterra integro-differential equation.

- [1] S.-M. Jung, A fixed point approach to the stability of a Volterra integral equation, *Fixed Point Theory and Applications*, Vol. 2007 (2007), Article ID 57064, 9 page.
- [2] S.-M. Jung, S. Şevgin, and H. Şevli, On the perturbation of Volterra integro-differential equations, *Applied Mathematics Letters*, 26, (2013), 665-669.

HOMOGENIZATION OF A VARIATIONAL INEQUALITY FOR THE  
P-LAPLACE OPERATOR  
WITH NONLINEAR RESTRICTION ON THE PERFORATED PART  
OF THE BOUNDARY IN CRITICAL CASE

**Tatiana A. Shaposhnikova**, *Moscow, Russia*

**Abstract:** We study the asymptotic behavior of the solution of variational inequality for the p-Laplace operator in domains periodically perforated by balls ( $n \geq 3$ ) with radius  $C_0\varepsilon^\alpha$ ,  $C_0 > 0$ ,  $\alpha \in (1, \frac{n}{n-p}]$ ,  $p \in [2, n)$ , and distributed over whole domain with period  $\varepsilon$ . The following nonlinear restrictions  $u_\varepsilon \geq 0$ ,  $\partial_\nu u_\varepsilon \geq -\varepsilon^{-\gamma}\sigma(x, u_\varepsilon)$ ,  $u_\varepsilon \left( \partial_\nu u_\varepsilon + \varepsilon^{-\gamma}\sigma(x, u_\varepsilon) \right) = 0$ ,  $\gamma = \alpha(n-1) - n$  are specified on the boundary of the balls. We construct the effective equations and prove the weak convergence of the solutions  $u_\varepsilon$  as  $\varepsilon \rightarrow 0$  to a solution of the homogenized problem. In the critical case  $\alpha = \frac{n}{n-p}$ ,  $\gamma = \frac{n(p-1)}{n-p}$  the effective equation contains a new nonlinear term which has to be determined as a solution of a functional equation.

THEORY OF RECESSIVE SOLUTIONS AT INFINITY FOR  
NONOSCILLATORY SYMPLECTIC DIFFERENCE SYSTEMS

**Roman Šimon Hilscher**, *Brno, czech Republic*

2000 MSC: 39A21, 39A12

**Abstract:** This is a joint work with Dr. Peter Šepitka. In this talk we will discuss new concept of a recessive solution at infinity for discrete symplectic systems, which does not require any eventual controllability assumption on the system. We show that the existence of a recessive solution at infinity is equivalent with the nonoscillation of the system and that recessive solutions can have any rank between explicitly given lower and upper bounds. The smallest rank corresponds to the minimal recessive solution, which is unique up to a right nonsingular multiple, while the largest rank yields the traditional maximal (invertible) recessive solution.

- [1] P. Šepitka, R. Šimon Hilscher, Recessive solutions for nonoscillatory discrete symplectic systems, *Linear Algebra Appl.* 469 (2015), 243–275.

## PARABOLIC OBSTACLE PROBLEM IN GENERALIZED MORREY SPACES

**Lubomira Softova**, *Naples, Italy*

2000 MSC: 35K87; 35R05; 35B65; 46E30

**Abstract:** This is a joint work with Sun-Sig Byun. We establish *Calderón-Zygmund type* estimate for the weak solutions of variational inequalities for divergence form parabolic systems with discontinuous data in non-smooth domains. The *obstacle* is constrained in the frameworks of the *generalized Morrey spaces*  $M^{p,\varphi}$ ,  $p > 1$  under various conditions on the weight  $\varphi$ . The coefficients of the operator supposed to be *only measurable* in one of the space variables and to have *small mean oscillation* in the others. Regarding the non-smooth domain we suppose that its boundary is *flat in the sense of Reifenberg*.

- [1] S. Byun, L. Softova, Parabolic obstacle problems with measurable coefficients in generalized Morrey spaces (preprint).
- [2] C. Scheven, *Existence and gradient estimates in nonlinear problems with irregular obstacles*, Habilitationsschrift, Universität Erlangen. (2011)

## MIXED PROBLEM FOR KLEIN-GORDON-FOCK-EQUATION WITH CURVE DERIVATIVES ON BOUNDS

**Ivan Staliarchuk**, *Minsk, Belarus*

2000 MSC: 35L20

**Abstract:** This is a joint work with Prof Korzyuk Victor Ivanovich.

Let's consider the problem in area  $Q = \{(t, x) | t \in [0; \infty), x \in [0, l]\}$

$$\partial_{tt}u - a^2\partial_{xx}u - \lambda(t, x)u = f(t, x), \quad (2.49)$$

with initial

$$u(0, x) = \varphi(x), \quad u_t(0, x) = \psi'(x), \quad x \in [0, l] \quad (2.50)$$



and boundary conditions

$$r_1^{(i)}(t)u_t(t, 0) + r_2^{(i)}(t)u_x(t, 0) + r_3^{(i)}(t)u(t, 0) = \mu^{(i)}(t), t \in [0, \infty), i \in \{0, l\}. \quad (2.51)$$

**Theorem 1.** Assuming that functions  $\mu^{(i)} \in C^{(2)}([0; \infty))$ ,  $\varphi \in C^{(3)}([0, l])$ ,  $\psi \in C^{(2)}([0, l])$ ,  $\lambda \in C^{(1,1)}(Q)$  then solution of problem (2.49) – (2.51) will exist and be unique in class  $C^{(2)}(Q)$  if and only if homogeneous matching conditions are met. Solution of this equation is reduced to solution of Volterra's equations.

- [1] Cheb H.S., Karpechina A.A., Korzyuk V.I. Second-order hyperbolic equation in case of two independent variables, News of NAS of Belarus, physical-mathematical section 1(2013), 71–80.
- [1] Hapaev A.M., Volodin B.A. Exact solution of Klein-Gordon relativistic wave equation. Tyhe journal of computing mathematics and mathematical physics (6)1990, 877–886.

## VARIATIONAL METHODS AND DISCRETE REACTION-DIFFUSION EQUATION

**Petr Stehlík**, *Pilsen, Czech Republic*

2000 MSC: 34A33, 35K57, 39A12

**Abstract:** This is a joint work with Jonáš Volek.

The standard discrete reaction-diffusion obtained by an explicit (Euler) discretization of

$$\partial_t u = k \partial_{xx} u + \lambda f(u) \quad (2.52)$$

exhibits certain interesting features (maximum principles, existence of travelling waves, ...) but other aspects (like existence of solutions) are rather trivial. Once we consider implicit discretization of (2.52), i.e.,

$$\frac{u(x, t+h) - u(x, t)}{h} = ku(x-1, t+h) - 2ku(x, t+h) + ku(x+1, t+h) + \lambda f(u(x, t+h)). \quad (2.53)$$

with  $h > 0, x \in \mathbb{Z}, t \in \{0, h, 2h, \dots\}$ , the existence of solutions is no longer straightforward. In this talk, we use variational methods to provide sufficient conditions for the existence of solutions of (2.53).

## A DISCRETE DIRAC-KÄHLER EQUATION ON A DOUBLE COMPLEX

**Volodymyr Sushch**, *Koszalin, Poland*

2000 MSC: 39A12, 81Q05

**Abstract:** This work is a direct continuation of the paper [1] in which we constructed a new discrete analogue of the Dirac-Kähler equation. We propose a discretization scheme based on the language of differential forms. Let  $\Omega$  be a complex-valued inhomogeneous form on Minkowski space-time. The Dirac-Kähler equation on exterior forms is given by

$$i(d + \delta)\Omega = m\Omega, \quad (2.54)$$

where  $d$  denotes the exterior differential,  $\delta = *d*$  is the co-differential and  $*$  is the Hodge star operator. Here,  $i$  is the usual complex unit and  $m$  is a real nonnegative constant. We adapt a double complex construction [2] to Minkowski space and define a discrete Hodge star operator in such way that  $** = \pm 1$ . The algebraic relations between the operators  $d$  and  $\delta$  are captured by their discrete analogues. We show that, just as in the continuum case, a discrete massless counterpart of equation (2.54) admits the chiral invariance.

- [1] V. Sushch, A discrete model of the Dirac-Kähler equation, *Rep. Math. Phys.* **73** (2014), no. 1, 109–125.
- [2] V. Sushch, Self-dual and anti-self-dual solutions of discrete Yang-Mills equations on a double complex, *Cubo* **12** (2010), no. 3, 99–120.

## ASYMPTOTIC ANALYSIS OF STRONGLY MONOTONE SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS

**Tomoyuki Tanigawa**, *Kumamoto, Japan*

2010 MSC: 34C11, 26A12

**Abstract:** Two types of nonlinear differential systems

$$(A) \quad x' + p(t)y^\alpha = 0, \quad *2emy' + q(t)x^\beta = 0;$$

$$(B) \quad x' - p(t)y^\alpha = 0, \quad *2emy' - q(t)x^\beta = 0$$

are considered under the assumption that  $\alpha$  and  $\beta$  are positive constants such that  $\alpha\beta < 1$  and  $p(t)$  and  $q(t)$  are continuous regularly varying functions on a neighborhood of infinity. An attempt is made to obtain precise information on the existence

and asymptotic behavior of strongly monotone regularly varying solutions  $(x(t), y(t))$  of (A) and (B) whose  $x$ -components or  $y$ -components are slowly varying. It is shown that the results thus obtained are applied to the generalized Thomas-Fermi equations of the form  $(p(t)|x'|^{\alpha-1}x')' = q(t)|x|^{\beta-1}x$  to provide new useful knowledge of their strongly monotone solutions. The present paper is designed to supplement the pioneering results on the asymptotic analysis of (A) and (B) by means of regular variation developed in the paper [1].

- [1] J. Jaroš and T. Kusano, Existence and precise asymptotic behavior of strongly monotone solutions of systems of nonlinear differential equations, *Differ. Equ. Appl.* 5 (2013), 185–204.

# EXISTENCE AND ASYMPTOTIC BEHAVIOR OF WEAK SOLUTIONS TO NONLINEAR WAVE EQUATIONS WITH NONLINEAR BOUNDARY CONDITION

**Takeshi Taniguchi**, *Fukuoka, Japan*

**Abstract:** Let  $D \subset R^d$  be a open bounded domain in the  $d$ -dimensional Euclidian space  $R^d$  with smooth boundary  $\Gamma = \partial D$ . In this talk we discuss the existence, uniqueness and exponential stability on the domain  $D^*$  near the origin of weak solutions to the following wave equation with nonlinear boundary condition:

$$\left\{ \begin{array}{l} u_{tt}(t) - \Delta u(t) - \alpha |u(t)|^q u(t) = 0, \alpha > 0 \\ u(t) = 0 \text{ on } \Gamma_0 \times (0, T) \\ \frac{\partial u(t)}{\partial \nu} + \gamma(u_t(t)) = 0 \text{ on } \Gamma_1 \times (0, T) , \\ u(0) = u_0, u_t(0) = u_1, \end{array} \right. \quad (2.55)$$

where  $\Gamma = \Gamma_0 \cup \Gamma_1$  and  $\Gamma_0 = \bar{\Gamma}_0 \cap \bar{\Gamma}_1 = \emptyset$ . The function  $\gamma$  is a monotone nondecreasing function and  $q$  satisfies  $0 < q \leq \min \left\{ \frac{4}{d-2}, \frac{2}{d-4} \right\}$ .

- [1] M.M. Cavalcanti, V.N.D. Cavalcanti and P. Martinez, Existence and decay rate estimates for the wave equation with nonlinear boundary damping and source term, *JDE*, 203(2004), 119-158.
- [2] M.M. Cavalcanti, V.N.D. Cavalcanti and J.A. Soriano, On existence and asymptotic stability of solutions of the degenerate wave equation with nonlinear boundary conditions, *JMAA*, 281(2003), 108-124.

# ISOPERIMETRIC PROBLEMS WITH DEPENDENCE ON FRACTIONAL DERIVATIVES

**Dina Tavares**, Aveiro, Portugal

2000 MSC: 49K05 26A33

**Abstract:** This is joint work with Ricardo Almeida and Delfim F. M. Torres.

In this work we present two fractional isoperimetric problems where the Lagrangian depends on a combined Caputo derivative of variable fractional order.

We establish necessary optimality conditions in order to determine the minimizers of the fractional isoperimetric problem. In the considered problem, the terminal time and the terminal state are free and thus transversality conditions are deduced.

- [1] R. Almeida, R.A.C. Ferreira, D.F.M. Torres, Isoperimetric problems of the calculus of variations with fractional derivatives. *Acta Math. Sci. Ser. B Engl. Ed.* 32 (2012), no 2, 619–630.
- [2] D. Tavares, R. Almeida and D.F.M. Torres, Optimality conditions for fractional variational problems with dependence on a combined Caputo derivative of variable order. *Optimization* 64 (2015), no. 6, 1381–1391.
- [3] B. van Brunt, The calculus of variations. New York: Springer, 2004

# ON NECESSARY AND SUFFICIENT ASYMPTOTIC STABILITY CONDITIONS FOR LINEAR DIFFERENCE EQUATIONS

**Petr Tomášek**, Brno, Czech Republic

2000 MSC: 39A06, 39A30

**Abstract:** We introduce an efficient form of necessary and sufficient conditions for asymptotic stability of special two-parametric full-term difference equation

$$y(n+k) + a \sum_{j=1}^{k-1} (-1)^j y(n+k-j) + by(n) = 0,$$

where  $a, b \in \mathbb{R}$ ,  $k \in \mathbb{N}$  and  $n \in \mathbb{N}_0$ . The stability region in  $(a, b)$  plane of this equation will be constructed and discussed with respect to some related linear difference equations.

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COMPARATIVE STUDY OF NUMERICAL METHODS FOR THE  
TIME-FRACTIONAL DIFFUSION EQUATION

**Magda Rebelo**, *Lisboa, Portugal*

2000 MSC: 35R11, 34A08, 26A33, 65N80

**Abstract:** This is joint work with Nuno F. Martins and Maria Luísa Morgado. This work is concerned with the numerical solution of the one-dimensional time-fractional diffusion equation

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} = D_\alpha \frac{\partial^2 u(x, t)}{\partial x^2} + f(x, t), \quad 0 < t \leq T, \quad 0 < x < L, \quad (2.56)$$

subject to the Dirichlet boundary conditions

$$u(0, t) = u_0(t), \quad u(L, t) = u_L(t), \quad 0 < t \leq T,$$

and initial condition

$$u(x, 0) = g(x), \quad x \in (0, L),$$

where  $D_\alpha$  is a sort of fractional diffusion coefficient, where the fractional derivative is given in the Caputo sense:

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - s)^{-\alpha} \frac{\partial u(x, s)}{\partial s} ds, \quad 0 < \alpha < 1.$$

In [3] we proposed a scheme based on a combination of a recently proposed non-polynomial collocation method for fractional ordinary differential equations (c.f. [2]) and the method of lines. On the other hand, recently in [2] we provide a meshfree method based on fundamental solutions basis functions for the time-fractional diffusion equation (2.56). In this work we intend to provide a comparison between these methods and finite difference methods existing in the literature in terms of robustness, accuracy and computational cost.

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- [3] N. F. M. Martins, M. L. Morgado, M. Rebelo, A meshfree numerical method for the time-fractional diffusion equation, *Proceedings of the 14th International Conference on Computational and Mathematical Methods in Science and Engineering, CMMSE, Cadiz, Spain, Vol. III*, (2014) 892–904.

INITIAL VALUE PROBLEM FOR PARTIAL DIFFERENTIAL EQUATIONS:  
CONVERGENCE ANALYSIS OF AN ITERATIVE SCHEME

**Josef Rebenda**, *Brno, Czech Republic*

2000 MSC: 35A35, 35A02, 65M12, 65M15

**Abstract:** This is a joint work with Professor Zdeněk Šmarda. Existence and uniqueness of solutions of initial value problem for certain classes of multidimensional linear and nonlinear partial differential equations are proved using Banach fixed-point theorem. An iterative scheme is derived and rigorous convergence analysis of presented technique and an error estimate are included as well as several numerical examples.

**Acknowledgment:** The author was supported by the project CZ.1.07/2.3.00/30.0039 of Brno University of Technology. This support is gratefully acknowledged.

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DISCONTINUOUS PERTURBATIONS OF SINGULAR  $\phi$ -LAPLACIAN  
OPERATOR

**Călin-Constantin Șerban**, *Timișoara, Romania*

2000 MSC: 34A60, 47H14, 49J53.

**Abstract:** Systems of differential inclusions of the form

$$-(\phi(u'))' \in \partial F(t, u), \quad t \in [0, T],$$

where  $\phi = \nabla \Phi$ , with  $\Phi$  strictly convex, is a homeomorphism of the ball  $B_a \subset \mathbb{R}^N$  onto  $\mathbb{R}^N$ , are considered under Dirichlet, periodic and Neumann boundary conditions. Here,  $\partial F(t, x)$  stands for the generalized Clarke gradient of  $F(t, \cdot)$  at  $x \in \mathbb{R}^N$ . Using nonsmooth critical point theory, we obtain existence results under some appropriate conditions on the potential  $F$ . The talk is based on joint work with Petru Jebelean and Jean Mawhin.

**Acknowledgements:** The work of the speaker was supported by the grant POS-DRU/159/1.5/S/137750, “Project Doctoral and Postdoctoral programs support for increased competitiveness in Exact Sciences research”.

## FIXED POINTS AND COMPLEX DYNAMICS

**David Shoikhet**, *Karmiel, Israel*

2010 MSC: 37F45, 47H10

**Abstract:** The laws of dynamics are usually presented as equations of motion which are written in the abstract form of a dynamical system:  $\frac{dx}{dt} + f(x) = 0$ , where  $x$  is a variable describing the state of the system under study, and  $f$  is a vector-function of  $x$ . The study of such systems when  $f$  is a monotone or an accretive (generally nonlinear) operator on the underlying space has been recently the subject of much research by analysts working on quite a variety of interesting topics, including boundary value problems, integral equations and evolution problems.

In this talk we give a brief description of the classical statements and their modern interpretations for discrete and continuous semigroups of hyperbolically nonexpansive mappings in Hilbert spaces. Also we present backward flow-invariance conditions for holomorphic and hyperbolically monotone mappings.

Finally we give some applications of complex dynamical systems to geometry of domains in complex spaces and operator theory.

## FOURTH ORDER DYNAMICAL INVARIANTS IN ONE-DIMENSION FOR CLASSICAL AND QUANTUM DYNAMICAL SYSTEMS

**Jasvinder Singh Virdi**, *Chandigarh, India*

**Abstract:** We present the possibility of fourth order dynamical invariant. To achieve this we use rational-ization method to study two-dimensional complex dynamical systems. Such invariants play an important role in the analysis of trajectories of classical and quantum dynamical systems. Role and scope of obtained invariant is pointed out.

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- [3] C.R.Holt, J. Math. Phys. 23, 1037 (1982)

# WELL-POSEDNESS RESULTS FOR ABSTRACT GENERALIZED DIFFERENTIAL EQUATIONS AND MEASURE FUNCTIONAL DIFFERENTIAL EQUATIONS

**Antonín Slavík**, *Prague, Czech Republic*

2000 MSC: 34G20, 34A12, 34A36, 34K05, 34K45

**Abstract:** In 1898, W. F. Osgood discovered an existence-uniqueness theorem for ordinary differential equations, which is weaker than Picard's theorem and does not require Lipschitz continuity of the right-hand side. The theorem remains valid for equations in Banach spaces, but the original proof has to be replaced by a more sophisticated argument.

In this talk (which is based on the recent paper [3]), we focus on Osgood-type theorems for abstract generalized ordinary differential equations in J. Kurzweil's sense [1], whose solutions need not be differentiable or even continuous. As a corollary, we obtain results concerning the existence, uniqueness and continuous dependence of solutions to measure functional differential equations, which were introduced in [2].

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- [3] A. Slavík, *Well-posedness results for abstract generalized differential equations and measure functional differential equations*, J. Differential Equations **259** (2015), 666-707.

# APPLICABLE METHOD FOR SOLVING THE SYSTEM OF FRACTIONAL DIFFERENTIAL EQUATIONS

**Samaneh Soradi Zeid**, *Mashhad, Iran*

2000 MSC: 34C10

**Abstract:** Recently, fractional differential operators are indisputably found to play a fundamental role in the modeling of a considerable number of phenomena. Because of the nonlocal property of fractional derivative, they can utilize for modeling of memory dependent phenomena and complex media such as porous media and anomalous diffusion. In the last decade or so, extensive research has been carried out on the development of numerical methods for fractional partial differential equations, including finite difference method, finite element methods, and spectral methods, variational iteration method and the Adomian decomposition method. Nowadays, fractional calculus are used to model various different phenomena in nature, but due to the non-local property of the fractional derivative, it still remains a lot of improvements in the present numerical approaches. In this paper, some new numerical approaches based on linear programming and minimizing the total error and some new improved approaches based on the AVK method for the system of fractional differential equations are proposed. Finally, numerical examples are presented to illustrate the efficiency and accuracy of the proposed method.

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- [2] Lin. Y., Xu, C.: Finite difference/spectral approximations for the time-fractional diffusion equation. Journal of Computational Physics, 255 (2007) 1533-1552.

# SMALL VALENCE INVARIANT TENSOR FIELDS ON SOME HOMOGENEOUS RIEMANN SPACES AND SHUR'S POLYNOMIALS

**Ruslan Surmanidze**, *Tbilisi, Georgia*

2000 MSC: 53C30

**Abstract:** We consider one class of isotropic irreducible homogeneous Riemann spaces  $M = \mathcal{G}/\hbar$ , where  $\hbar$  is a linear group of the type simple Lie algebra  $A_n$ , given by the transformation group with the higher weight [1]. We study the Shur's polynomials for tensor squared of isotropic irreducible representation defined by given spaces [2],[3].

- [1] O. V. Manturov, Homogeneous Riemannian spaces with an irreducible rotation group, *Trudy Sem. Vektor. Tenzor. Anal.* **13** (1966) 68-145.
- [2] W. Fulton, Young Tableaux with Applications to Representation Theory and Geometry, *Cambridge University Press*, 1997
- [3] R. M. Surmanidze, Tensor invariants and homogeneous Riemann spaces, *Journal of Mathematical Sciences*, **195,2** (2013) 245-257.

# AN INTERVAL VERSION OF FINITE DIFFERENCE METHOD FOR THE WAVE EQUATION

**Barbara Szyszka**, *Poznan, Poland*

2000 MSC: 65G40, 65M06

**Abstract:** We consider the wave equation

$$v^2 \frac{\partial^2 u(x, t)}{\partial x^2} - \frac{\partial^2 u(x, t)}{\partial t^2} = 0, \quad (2.57)$$

where the function  $u(x, t)$ , for  $0 < x < L$  and  $t > 0$ , satisfies the following boundary-initial conditions:

$$\begin{aligned} u(0, t) &= u(L, t) = 0, \\ u(x, 0) &= \varphi(x), \\ \left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} &= \psi(x), \end{aligned} \quad (2.58)$$

for given functions  $\varphi(x)$  and  $\psi(x)$ .

We study an interval version [1] of finite difference method (CTCS) [2] for solving the problem (2.57)-(2.58) in floating-point interval arithmetic.

- [1] Moore R.E., Kearfott R.B., Cloud M.J.: Introduction to Interval Analysis. SIAM, (2009).

- [2] Davis L. J.: Finite Difference Methods in Dynamics of Continuous Media. Macmillan Publishing Company, New York, (1986).

## ON THE LARGE TIME BEHAVIOUR OF A FERROELECTRIC SYSTEM

**Mouhcine Tilioua**, *Errachidia, Morocco*

2000 MSC: 35L10, 35K05

**Abstract:** We consider the mathematical model for the ferroelectric materials investigated in [1]. It consists on a Maxwell system for electromagnetic field coupled with a second-order time-dependent equation for polarization. We study the large time behaviour of weak solutions and prove that all points of the  $\omega$ -limit set of any trajectories are solutions of the stationary model.

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- [2] G. Carbou and P. Fabrie. Time Average in Micromagnetism, *J. Differential Equations*, 147 (1998) 383–409.
- [3] J. M. Greenberg, R. C. MacCamy and C. V. Coffman, On the long-time behavior of ferroelectric systems. *Phys. D*, 134 **3** (1999) 362–383.

## A GENERAL DELTA-NABLA CALCULUS OF VARIATIONS ON TIME SCALES WITH APPLICATION TO ECONOMICS

**Delfim F. M. Torres**, *Aveiro, Portugal*

2010 MSC: 34N05; 49K05; 91B02; 91B62.

**Abstract:** This is a joint work with Monika Dryl [1]. We consider a general problem of the calculus of variations on time scales with a cost functional that is the composition of a certain scalar function with delta and nabla integrals of a vector valued field. Euler–Lagrange delta-nabla differential equations are proved, which lead to important insights in the process of discretization. Application of the obtained results to a firm that wants to program its production and investment policies to reach a given production rate and to maximize its future market competitiveness is discussed. This work was supported by Portuguese funds through the *Center for Research and Development in Mathematics and Applications* (CIDMA) and FCT, within project UID/MAT/04106/2013.

- [1] Monika Dryl and Delfim F. M. Torres, A General Delta-Nabla Calculus of Variations on Time Scales with Application to Economics, *Int. J. Dyn. Syst. Differ. Equ.*, **5** (2014), no. 1, 42-71.

# MONOTONE ITERATIVE TECHNIQUE FOR SYSTEMS OF NONLINEAR CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS

**Faten Toumi**, *Tunis, Tunisia.*

2000 MSC: 34B15, 34B27, 34B34.

**Abstract:** In this work, we deal with the existence of extremal quasisolutions for the following finite system of nonlinear fractional differential equations

$${}^C D^q u(t) + f(t, u(t)) = 0 \text{ in } (0, 1)$$

$$u(0) - \alpha u'(0) = \lambda, u(1) + \beta u'(1) = \mu,$$

where  $1 < q < 2, \alpha, \beta \in (\mathbb{R}^+)^n, \lambda, \mu \in \mathbb{R}^n$  and  $f \in C([0, 1] \times \mathbb{R}^n, \mathbb{R}^n)$  and  ${}^C D^q$  is the Caputo fractional derivative of order  $q$ . We shall prove constructive existence results for a class of nonlinear equations by the use of iterative method technique combined with upper and lower quasisolutions. We construct a pair of sequences of coupled lower and upper quasisolutions which converge uniformly to extremal quasisolutions. Then, a uniqueness result is given under additional conditions on the nonlinearity  $f$ .

# NEW OSCILLATION CRITERIA FOR NONLINEAR FOURTH ORDER DELAY DIFFERENCE EQUATIONS

**A. K. Tripathy**, *Sambalpur, India*

2000 MSC: 39A10, 39A12

**Abstract:** In this work, oscillatory behaviour of solutions of a class of fourth order neutral functional difference equations of the form

$$\Delta^2(r(n)\Delta^2(y(n) + p(n)y(n-m))) + q(n)G(y(n-k)) = 0$$

is studied under the assumption

$$\sum_{n=0}^{\infty} \frac{n}{r(n)} < \infty.$$

New oscillation criteria have been established which generalizes some of the existing results in the literature.

## HOMOGENIZATION OF A COMPRESSIBLE CAVITATION MODEL

**Afonso F. Tsandzana**, *Lulea, Sweden*

2000 MSC: 35B27

**Abstract:** We present a mathematical model in hydrodynamic lubrication that takes into account three phenomena: cavitation, surface roughness and compressibility of the fluid. The model is mass preserving. We compute the homogenized coefficients in the case of unidirectional roughness. A one-dimensional problem is also solved explicitly.

If we assume that the fluid is Newtonian with viscosity  $\mu$ , then

$$\vec{q}_\epsilon = -\frac{\rho_\epsilon h_\epsilon^3}{12\mu} \nabla p_\epsilon + \frac{\rho_\epsilon h_\epsilon}{2} \vec{U}, \quad (2.59)$$

where both the density  $\rho_\epsilon = \rho_\epsilon(x)$ , and the pressure  $p_\epsilon = p_\epsilon(x)$ , are unknown. By requiring conservation of mass the classical Reynolds equation is obtained

$$\nabla \cdot \vec{q}_\epsilon = 0. \quad (2.60)$$

- [1] A. Almqvist, J. Fabricius, R. Larsson and P. Wall, A new approach for studying cavitation in lubrication, *J. Tribol.* 136 (2014), no. 1, 1011706, 6 pp. doi:10.1115/1.4025875
- [2] G. Bayada and M. Chambat, The transition between the Stokes equations and the Reynolds equation: A mathematical proof, *Appl. Math. Optim.* 14 (1986), no. 1, 73–93.
- [3] D. Cioranescu and P. Donato, *An Introduction to Homogenization*, Oxford University Press. Oxford, 1999.

## EXISTENCE OF POSITIVE SOLUTIONS FOR A SYSTEM OF FRACTIONAL BOUNDARY VALUE PROBLEMS

**Rodica Luca Tudorache**, *Iasi, Romania*

2000 MSC: 34A08, 45G15

**Abstract:** This is a joint work with Prof. Johnny Henderson (Baylor University, Waco, Texas, USA) and stud. Alexandru Tudorache (Gh. Asachi Technical University, Iasi, Romania).

We investigate the existence, multiplicity and nonexistence of positive solutions for some systems of nonlinear Riemann-Liouville fractional differential equations subject to Riemann-Stieltjes integral boundary conditions. In the proof of our main results, we use some theorems from the fixed point index theory, the Guo-Krasnosel'skii fixed point theorem and the Schauder fixed point theorem.

## PERIODIC OSCILLATIONS RELATED TO THE LIEBAU PHENOMENA

**Milan Tvrđý**, *Prague, Czech Republic*

2000 MSC: 34B16, 34B18, 34D20.

**Abstract:** The contribution is based on the recent joint research with J. Á. Cid, G. Infante, G. Propst and M. Zima (cf. [1] and [2]). We will present results on the existence and asymptotic stability of positive  $T$ -periodic solutions to singular differential equations of the form  $u'' + a u' = \frac{1}{u} (e(t) - f(u')) - c$ , where  $a \geq 0$ ,  $c > 0$ ,  $e$  is  $T$ -periodic and  $f(x)$  behaves like  $x^2$ .

- [1] J. Á. Cid, G. Propst and M. Tvrđý: On the pumping effect in a pipe/tank flow configuration with friction. *Phys. D* **273-274** (2014) 28–33.
- [2] J. Á. Cid, G. Infante, M. Tvrđý and M. Zima: Topological approach to periodic oscillations related to the Liebau phenomenon. *J. Math. Anal. Appl.* **423** (2015), 1546-1556.

## STABILITY IN DELAY DYNAMIC EQUATIONS BY FIXED POINT THEORY: NONLINEAR CASE

**Mehmet Ünal**, *Sinop, TURKEY*

**Abstract:** Stability plays an important role in the theory of dynamic equations on time scales. In this talk, we study stability properties of Nonlinear delay dynamic equations by means of fixed point theory

$$x^\Delta(t) = -a(t)g(x(\delta(t)))\delta^\Delta(t), \quad t \in [t_0, \infty)_{\mathbb{T}} \quad (2.61)$$

on an arbitrary time scale  $\mathbb{T}$  which is unbounded above.

- [1] T. A. Burton, *Stability and fixed points: Addition of terms*, Dynamic Systems and Applications **13** (2004), 459-478.
- [2] M. Unal and Y. Raffoul, *Qualitative Analysis of Solutions of Nonlinear Delay Dynamic Equations*, International Journal of Differential Equations, Volume 2013, doi:10.1155/2013/764389.

## ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS OF A KIND OF LANCHESTER-TYPE SYSTEM

**Hiroyuki Usami**, *Gifu, Japan*

2000 MSC: 34D05

**Abstract:** Let us consider the system

$$x' = -a(t)xy, \quad y' = -b(t)xy, \tag{S}$$

where  $a, b \in C[0, \infty)$  are positive functions satisfying the hypotheses (H):  $0 < \inf_{t \geq 0} a(t) \leq \sup_{t \geq 0} a(t) < \infty$  and  $0 < \inf_{t \geq 0} b(t) \leq \sup_{t \geq 0} b(t) < \infty$ .

If  $x(0), y(0)$  are positive, then the solution  $(x(t), y(t))$  of (S) exists on  $[0, \infty)$ , and remains positive there. So  $\lim_{t \rightarrow \infty} x(t)$  and  $\lim_{t \rightarrow \infty} y(t)$  exist in  $[0, \infty)$ . Denote the solution of (S) with  $(x(0), y(0)) = (\alpha, \beta)$ ,  $\alpha, \beta > 0$ , by  $(x(t; \alpha, \beta), y(t; \alpha, \beta))$ . When we fix  $\alpha$  and move  $\beta > 0$ , we want to examine how  $\lim_{t \rightarrow \infty} x(t; \alpha, \beta)$  and  $\lim_{t \rightarrow \infty} y(t; \alpha, \beta)$  vary according to  $\beta$ .

**Theorem 1.** *There are constants  $\beta_1 = \beta_1(\alpha)$  and  $\beta_2 = \beta_2(\alpha)$  ( $0 < \beta_1 \leq \beta_2$ ) such that: (i) if  $\beta < \beta_1$ , then  $\lim_{t \rightarrow \infty} x(t; \alpha, \beta) > 0$ ,  $\lim_{t \rightarrow \infty} y(t; \alpha, \beta) = 0$ ; (ii) if  $\beta_1 \leq \beta \leq \beta_2$ , then  $\lim_{t \rightarrow \infty} x(t; \alpha, \beta) = \lim_{t \rightarrow \infty} y(t; \alpha, \beta) = 0$ ; (iii) if  $\beta > \beta_2$ , then  $\lim_{t \rightarrow \infty} x(t; \alpha, \beta) = 0$ ,  $\lim_{t \rightarrow \infty} y(t; \alpha, \beta) > 0$ .*

## ON EXTENSTIONS OF SERRIN'S CONDITION FOR THE NAVIER-STOKES EQUATIONS

**Werner Varnhorn**, *Kassel, Germany*

2000 MSC: 35Q30, 76D05

**Abstract:** In this joint work with Reinhard Farwig and Hermann Sohr we consider a smooth bounded domain  $\Omega \subseteq \mathbb{R}^3$ , a time interval  $[0, T)$ ,  $0 < T \leq \infty$ , and a weak solution  $u$  of the Navier-Stokes system. Our aim is to develop several new sufficient conditions on  $u$  yielding uniqueness and/or regularity. Based on semigroup

properties of the Stokes operator we obtain that the local left-hand Serrin condition for each  $t \in (0, T)$  is sufficient for the regularity of  $u$ , see [1]. Somehow optimal conditions are obtained in terms of Besov spaces [2]. In particular we obtain such properties under the limiting Serrin condition  $u \in L_{\text{loc}}^\infty([0, T]; L^3(\Omega))$ . The complete regularity under this condition has been shown recently for bounded domains using some additional assumptions in particular on the pressure. Our result avoids such assumptions but yields global uniqueness and the right-hand regularity at each time when  $u \in L_{\text{loc}}^\infty([0, T]; L^3(\Omega))$  or when  $u(t) \in L^3(\Omega)$  pointwise and  $u$  satisfies the energy equality.

- [1] R. Farwig, H. Sohr, W. Varnhorn: *On optimal initial value conditions for local strong solutions of the Navier-Stokes equations*, Ann. Univ. Ferrara Sez. VII Sci. Mat. 55 (2009), 89–110.
- [2] R. Farwig, H. Sohr, W. Varnhorn: *Besov space regularity conditions for weak solutions of the Navier-Stokes equations*, J. Math. Fluid. Mech. 16 (2014) 307–320

## ON SOME BOUNDARY VALUE PROBLEMS FOR DIFFERENCE EQUATIONS

**Vladimir Vasilyev**, *Lipetsk, Russia*

2000 MSC: 47B39

**Abstract:** One considers the equation

$$(Au)(x) = v(x), \tag{2.62}$$

where  $A$  is a difference operator of the type

$$(Au)(x) = \sum_{|k|=0}^{+\infty} a_k(x)u(x + \alpha_k), \quad x \in \mathbf{R}_+^m, \quad \{\alpha_k\} \subset \mathbf{R}_+^m,$$

where  $\mathbf{R}_+^m = \{x \in \mathbf{R}^m : x_m > 0\}$ ,  $k$  is a multi-index.

Here we establish some results on a solvability of the equation (2.62) in the Sobolev–Slobodetsky spaces  $H^s(\mathbf{R}_+^m)$  and show that in general for unique solvability of a difference equation we need some additional boundary conditions.

Some results related to discrete equations of type (2.62) were obtained in [1].

This research is a joint work with Alexander V. Vasilyev, and it was partially supported by RFBR, project No. 14-41-03595-a.



- [1] A.V. Vasilyev, V.B. Vasilyev, Discrete singular operators and equations in a half-space, *Azerbaijan Journal of Mathematics* **3** (2013) 84-93.

# ON THE STABILIZATION FOR A SCHRÖDINGER EQUATION WITH DOUBLE POWER NONLINEARITY

**Octavio Vera**, *Concepción, Chile*

2000 MSC: 35Q53, 47J353

**Abstract:** In this paper we are interested to obtain the decay rates of the solutions for an Schrödinger equation with double power nonlinearity in  $L^\infty(\mathbb{R})$ -norm and  $L^p(\mathbb{R})$ -norm for  $2 < p \leq +\infty$ .

We consider the nonlinear Schrödinger equation with double power nonlinearity

$$i u_t + u_{xx} + a |u|^{p-1} u - b |u|^{q-1} u = 0, \quad (x, t) \in \mathbb{R} \times \mathbb{R}, \quad (2.63)$$

where  $a$  and  $b$  are positive constants,  $1 < p < q < +\infty$ .  $u = u(x, t)$  is a complex unknown function. For instance, the cubic-quintic nonlinear Schrödinger equation  $i \mathbf{u}_t + \mathbf{u}_{xx} + \delta |\mathbf{u}|^2 \mathbf{u} - \varepsilon |\mathbf{u}|^4 \mathbf{u}$  arise in a number of independent physics field: nuclear hydrodynamic with Skyrme [2], the optical pulse propagations in dielectric media of non-Kerr type [3]. Also, it is used to describe the boson gas with two and three body interaction [1].

- [1] I.V. Barashenkov, A.D. Gocheva, V.G. Makhankov and I.V. Puzinin, Soliton-like bubbles in the system of interacting bosons. *Phys. Lett. A.* 128(1988)52-56.
- [2] V.G. Kartavenko. Soliton-like solutions in nuclear hydrodynamics, *Sov. J. Nucl. Phys.* 40(1984)240-246.
- [3] A. Kumar, S.N. Sarkar and A.K. Ghatak. Effect of fifth-order non-linearity in refractive index on Gaussian pulse propagation in lossy optical fiber, *Opt. Lett.* 11(1986)321-323.

# SHARP ESTIMATES FOR SOLUTIONS OF SYSTEMS WITH AFTEREFFECT

**Victor Vlasov**, *Moscow, Russia*

2000 MSC: 34K12

**Abstract:** We study differential-difference equations with matrix coefficients. Sharp estimates are established for strong solutions of systems of differential-difference equations of both neutral and retarded type.

The approach is based on the study of the resolvent corresponding to the generator of the semigroup of shifts along the trajectories of a dynamical system. In the case of neutral type equations, the Riesz basis property of the subsystem of exponential solutions is used.

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- [2] V. V. Vlasov and D. A. Medvedev, Functional-differential equations in Sobolev spaces and related problems of spectral theory, *Journal of Mathematical Sciences* **164:5** (2010) 659-841.

## MAXIMUM PRINCIPLES FOR DISCRETE REACTION-DIFFUSION EQUATIONS

**Jonáš Volek**, *Pilsen, Czech Republic*

2000 MSC: 34A33, 35B50, 35K57, 39A12

**Abstract:** This is a joint work with Petr Stehlík. We study reaction-diffusion equations with a general reaction function  $f$  on one dimensional lattices with continuous or discrete time

$$u'_x \text{ (or } \Delta_t u_x) = k(u_{x-1} - 2u_x + u_{x+1}) + f(u_x), \quad x \in \mathbb{Z}.$$

We prove weak and strong maximum and minimum principles for corresponding initial-boundary value problems. Whereas the maximum principles in the semidiscrete case (continuous time) exhibit similar features to those of fully continuous reaction-diffusion model, in the discrete case the weak maximum principle holds for a smaller class of functions and the strong maximum principle is valid in a weaker sense. We describe into detail how the validity of maximum principles depends on the nonlinearity and the time step. We illustrate our results on the Nagumo equation with the bistable nonlinearity.

- [1] P. Stehlík, J. Volek, Maximum principles for discrete and semidiscrete reaction-diffusion equation, *in preparation*.

- [2] J. Volek, Maximum and minimum principles for nonlinear transport equations on discrete-space domains, *Electron. J. Diff. Equ.* 2014 (2014), no. 78, 1–13.

## SYMMETRY PROPERTIES FOR NONLINEAR SYSTEM OF FRACTIONAL DIFFERENTIAL EQUATIONS

**Hassan A. Zedan**, *Jeddah , Egypt*

**Abstract:** This article is a straightforward introduction to symmetry methods to solve system of fractional differential equations. The nonlinear system of fractional differential equations is used as an example to illustrate the effectiveness of the Lie group method and exact solution is obtained.

- [1] Hassan Zedan, Exact solutions for the generalized KdV equation by using Backlund transformations, *Journal of the Franklin Institute* 348 (2011), 17511768.
- [2] R. Sahadevan and T. Bakkyaraj, Invariant analysis of time fractional generalized Burgers and Kortewegde Vries equations, *J. Math. Anal. Appl.*, 393 (2012) 341347.
- [3] G. Wang, X. Liu and Y. Zhang, Lie symmetry analysis to the time fractional generalized fth- order KdV equation, *Commun Nonlinear Sci Numer Simulat*, 2012.

## SYMPLECTIC SYSTEMS WITH ANALYTIC DEPENDENCE ON SPECTRAL PARAMETER

**Petr Zemánek**, *Brno, Czech Republic*

2000 MSC: 39A12, 34B20.

**Abstract:** The talk is based on results of a joint research with Prof. Roman Šimon Hilscher, see [1, 2, 3]. We consider discrete symplectic systems with analytic (or polynomial) dependence on the spectral parameter. It is a generalization of the discrete symplectic system

$$z_{k+1}(\lambda) = (\mathcal{S}_k + \lambda \mathcal{V}_k) z_k(\lambda), \quad k \in [0, \infty) \cap \mathbb{Z},$$

where  $\lambda \in \mathbb{C}$  is the spectral parameter,  $\mathcal{S}_k, \mathcal{V}_k$  are  $2n \times 2n$  complex-valued matrices such that  $\mathcal{S}_k^* \mathcal{J} \mathcal{S}_k = \mathcal{J}$ , i.e.,  $\mathcal{S}_k$  is conjugate symplectic,  $\mathcal{S}_k^* \mathcal{J} \mathcal{V}_k$  is Hermitian, and

$\mathcal{V}_k^* \mathcal{J} \mathcal{V}_k = 0$  with  $\mathcal{J} := \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ . In particular, we focus on square summable solutions and their number. We show that the maximal number of linearly independent square integrable solutions (i.e., the limit circle case) is not any more invariant under  $\lambda \in \mathbb{C}$ . We also discuss a continuous counterpart of this system, i.e., linear Hamiltonian differential system.

- [1] R. Šimon Hilscher and P. Zemánek, *Weyl–Titchmarsh theory for discrete symplectic systems with general linear dependence on spectral parameter*, J. Difference Equ. Appl. **20** (2014), no. 1, 84–117.
- [2] R. Šimon Hilscher and P. Zemánek, *Time scale symplectic systems with analytic dependence on spectral parameter*, J. Difference Equ. Appl. **21** (2015), no. 3, 209–239.
- [3] R. Šimon Hilscher and P. Zemánek, *Limit circle invariance for two differential systems on time scales*, Math. Nachr. **288** (2015), no. 5-6, 696–709.

## RINGS AND FIELDS OF CONSTANTS OF CYCLIC FACTORIZABLE DERIVATIONS

**Janusz Zieliński**, *Toruń, Poland*

2010 MSC: 34A34, 13N15, 12H05, 92D25

**Abstract:** We give characteristics of rings and fields of constants of some families of cyclic factorizable derivations. Thereby, we determine also all polynomial and rational first integrals of their corresponding systems of differential equations. Examples of cyclic factorizable derivations are Volterra and Lotka-Volterra derivations, which play a significant role in population biology, laser physics and plasma physics. Factorizable derivations are important in derivation theory, because we may associate the factorizable derivation with any given derivation of a polynomial ring and that construction helps to determine constants of arbitrary derivations. Besides, we describe the cofactors of strict Darboux polynomials of derivations in question. All considerations are over an arbitrary field of characteristic zero.

## ON POSITIVE SOLUTIONS OF SECOND-ORDER NONLOCAL SINGULAR BOUNDARY VALUE PROBLEM

**Mirosława Zima**, *Rzeszów, Poland*

2000 MSC: 34B16

**Abstract:** We discuss the existence of positive solutions for the following second-order boundary value problem

$$\begin{cases} u''(t) + f(t, u(t), u'(t)) = 0, & t \in [0, 1], \\ au(0) - bu'(0) = \alpha[u], \\ u'(1) = \beta[u]. \end{cases}$$

The nonlinearity  $f$  may be singular at the value 0 of its space variables and the boundary conditions are given by Riemann-Stieltjes integrals, that is,

$$\alpha[u] = \int_0^1 u(s) dA(s) \text{ and } \beta[u] = \int_0^1 u(s) dB(s),$$

where  $A$  and  $B$  are of bounded variation, and  $dA$  and  $dB$  are positive measures. Our approach is based on the Krasnoselskii-Guo fixed point theorem on cone expansion and compression.

- [1] M. Zima, Positive solutions of second-order non-local boundary value problem with singularities in space variables, *Boundary Value Problems* 2014 2014:200.

## SOLVABILITY OF TERMOVISCOELASTICITY PROBLEM FOR VOIGT MODEL

**Andrey Zvyagin**, *Voronezh, Russia*

2000 MSC: 76A05

**Abstract:** This is a joint work with Vladimir Orlov.

In this report we investigate the existence of a weak solution for initial boundary-value problem of thermo-visco-elasticity in the mathematical model describing a motion of linearly elastic-delayed Voigt fluid:

$$\partial v / \partial t + v_i \partial v / \partial x_i - \nu_0 \Delta v - 2 \operatorname{Div} (\nu(\theta) \mathcal{E}(v)) - \kappa \partial \Delta v / \partial t + \operatorname{grad} p = f; \quad (2.64)$$

$$\operatorname{div} v = 0 \text{ in } Q_T; \quad v|_{t=0} = v_0 \text{ in } \Omega; \quad v|_{[0,T] \times \partial \Omega} = 0; \quad (2.65)$$

$$\partial \theta / \partial t + v_i \partial \theta / \partial x_i - \chi \Delta \theta = 2 \tilde{\nu}(\theta) \mathcal{E}(v) : \mathcal{E}(v) + 2 \kappa \partial \mathcal{E}(v) / \partial t : \mathcal{E}(v) + g; \quad (2.66)$$

$$\theta|_{t=0} = \theta_0 \text{ in } \Omega; \quad \theta|_{[0,T] \times \partial \Omega} = 0. \quad (2.67)$$

Here  $\Omega \subset \mathbb{R}^n$ ,  $n = 2, 3$ , be a bounded domain;  $v$ ,  $\theta$  and  $p$  are the velocity vector-function, the functions of temperature and medium pressure,  $f$  is the density of

external forces,  $g$  is external heat source,  $\varkappa > 0$  is retardation time,  $\chi > 0$  is the coefficient of thermal conductivity,  $\nu_0 > 0$  is initial viscosity of the fluid,  $\nu(s)$  is viscosity of the fluid;  $\tilde{\nu}(s) = \nu_0 + \nu(s)$ ;  $\mathcal{E}(v)$  is strain rate tensor.

The work was supported by Russian Foundation for Basic Research (13-01-00041 and 14-01-31228) and Russian Science Foundation (14-21-00066).

ON A WEAK SOLVABILITY OF A SYSTEM OF  
THERMOVISCOELASTICITY OF THE OLDROID TYPE

**Victor Zvyagin**, *Voronezh, Russia*

2000 MSC: 34C10

**Abstract:** This is a joint work with Vladimir Orlov.

Consider in  $Q_T = \Omega \times [0, T]$ ,  $\Omega \subset R^n$ ,  $n = 2, 3$ , the problem:

$$\partial u / \partial t + u_i \partial u / \partial x_i + \nabla p = \text{Div } \sigma + f; \quad \text{div } u = 0; \quad (2.68)$$

$$\sigma + \lambda (\partial \sigma / \partial t + u_i \partial \sigma / \partial x_i) = 2\eta(\theta) D(u) + 2\varkappa (\partial D(u) / \partial t + u_i \partial D(u) / \partial x_i); \quad (2.69)$$

$$\partial \theta / \partial t + u_i \partial \theta / \partial x_i - \chi \Delta \theta = g + \sigma : D(u); \quad (2.70)$$

$$u|_{\partial \Omega} = 0, \quad u|_{t=0} = u_0; \quad \sigma|_{t=0} = \sigma_0; \quad \theta|_{t=0} = \theta_0, \quad \theta|_{\partial \Omega} = 0. \quad (2.71)$$

Here  $v$ ,  $p$ ,  $\theta$ , and  $\sigma$  are the velocity, the pressure, the temperature and the stress deviator  $\sigma$ , respectively.  $D(v)$  is the strain velocity tensor,  $\lambda, \varkappa > 0$ ,  $0 < d_1 \leq \eta(s) \leq d_2$ ,  $\eta(s) \geq \varkappa/\lambda$ ,  $s \in (-\infty, +\infty)$ . The nonlocal weak solvability of problem (2.68)–(2.71) is established.

The work was supported by Russian Foundation for Basic Research (13-01-00041) and Russian Science Foundation (14-21-00066).

- [1] V. G. Zvyagin and V. P. Orlov, On certain mathematical models in continuum thermomechanics, *Journal of Fixed Point Theory and Applications* **15** 1 (2014) 3-47.

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