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DESIGN, MANUFACTURING & ANALYSIS OF AN ABS DIFFERENTIAL GEAR SYSTEM MODEL

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ABSTRACT

This work presents the assessment of Von Mises stresses in straight-tooth bevel differential gear system of a pedagogical model. This model was designed and the straight-tooth gears were formed using a 3D printing rapid prototyping. Two different percentages of densities (60% and 100%) of printed gears with ABS (Acrylonitrile butadiene styrene) were analysed and compared. Von Mises stresses in the teeth of the gears were obtained using finite element analysis (FEA) with *PTC Creo Parametric 2.0*. Results showed that the manufactured gears by the method of 60% of ABS density are capable to withstand the applied forces, i.e. the allowable stress of the material is not achieved.

Keywords: differential, straight-tooth bevel gears, ABS, rapid prototyping.

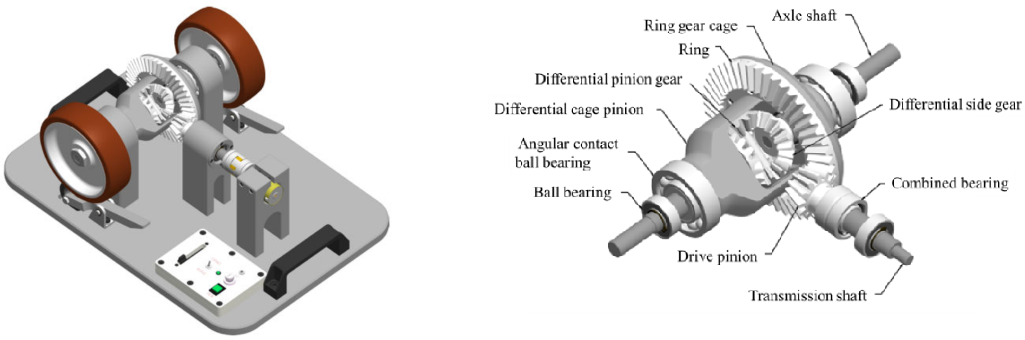
INTRODUCTION

The differential gear mechanism is located between the axle shafts of a vehicle and permits the same speed of wheel rotation when in straight moving, also allows different rotational speeds when in curved path. Its origin dates back to ancient times and its progress was made through the use of modern technology. Some sources indicate that the origin of the differential can go as far as 2634BC, as the South Pointing Chariot [1]. The study of automobile differential gear system is academically studied in Mechanical Engineering higher education [2-5] and more practical experience is needed. This way a differential gear set was designed and constructed as a pedagogical model to be as light as possible for manual handling.

Weight reduction can be achieved primarily by the introduction of lighter material, design optimization or better manufacturing processes. Gears are mainly manufactured from metallic or non–metallic materials. The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machine ability and ease of producing complicated shapes by casting method [6]. The non–metallic materials like Nylon, Acrylic, (PC) Polycarbonate or (ABS) Acrylonitrile Butadiene Styrene are used for gears, especially for reducing weight and noise. Plastic gears have corrosion resistance and are easily formed into complex shapes by the use of 3D printing rapid prototyping [7-10].

EDUCATIONAL MODEL

The proposed model was composed by several components that are included in a real rear drive differential in a smaller but similar scale size, including two friction wheels in order to simulate the behaviour of the differential when car describes a curvilinear path, rolling bearings and an DC electric motor to activate the drive controlled by an electronic panel to adjust the gears rotation. It was also included an elastic shaft coupling to absorb vibrations from the motor and several strait bevel gears were fabricated via 3D rapid prototyping. Fig. 1a) presents the pedagogical model and Fig. 1b) shows the straight-tooth bevel differential gear system used.



a) b)

Fig. 1 - Differential gear system model: a) pedagogical model; b) gear system of differential

The electric DC motor transmits rotation to the elastic coupling, which is fixed to the transmission shaft. The coupling absorbs vibrations caused by the motor and allows the elimination of possible axial misalignment. A ball bearing and a combined one place the transmission shaft. At the other edge of the shaft the drive pinion transmits the rotation to the ring (crown), which by the ring gear cage conveys the movement to the differential cage pinion. If the satellites (differential pinion gear) do not find resistance from differential side gear, the rotation passes equally to both axle shafts (Fig. 1b). The axle shafts are also supported by ball bearings. The crown like the rest of the gears was strait bevel gear instead of a spiral bevel gear, which is used in real car differentials. This consideration was made because the strait bevel is specially used for small velocities also the noise is not an important consideration and is less expensive to produce.

The fixation of the wheels was via a conical sleeve and a lever under each main wheel was used for the simulation of different rotation between them. The lever is attached to a lever support and a friction wheel actuates to control rotation. Also two polyamide reinforced glass fibre handles were used for model transportation. The material used in the several components that were manufactured are presented in Table 1 likewise in Table 2 is shown the several components that were acquired.

Table 1 -Material and Manufacturing method of various components

Component	Material	Manufacturing method
Ring gear cage	Aluminium alloy 2030	CNC
Differential cage pinion	Aluminium alloy 2030	CNC
Axle shafts	Aluminium alloy 2030	Lathe
Journal bearings	Aluminium alloy 5083	CNC
Lever and Base	Aluminium alloy 5083	CNC
Electronic panel	Polyethylene PE1000	CNC
Wheels bushing	Polyethylene PE1000	CNC
Electric motor bushing	Polyurethane SIKABLOCK M440	CNC
Gears	ABS Plus	3D printing rapid prototyping

Table 2 - Acquired Components

Component	Characteristics	Manufacturer
Electric DC motor	$T_{max} = 1.22Nm$ $\omega = 180rpm$	Canon Precision
Elastic coupling	$T_{max} = 6Nm$	R+W
Combined bearing (roller + ball)	KIA 5904	INA
Ball bearings angular contact	7206 BEP	SKF
Conical sleeve	$T_{max} = 99Nm$	Halder
Polyamide reinforced glass fibre handles	$\sigma_{max} = 2200N$	Elesa
Wheels	$F_{max} = 300kg$	Mecanarte
Lever support	$\sigma_{max} = 188N$	Festo
Friction wheels	$F_{max} = 35kg$	Mecanarte

GEARS DESIGN AND CONSTRUCTION

The selection of all gears of the pedagogical model took into account the uniform wear criteria i.e. for a two-gear pair and odd number of teeth was used. The criterion of additional tooth was also taken into account [2].

It was considered 3mm module for all the gears. In the conical group, the drive pinion had 12 teeth and the ring (crown) had 47 teeth; in the differential group, satellites had 12 teeth and planetary had 17 teeth. The bevel gears must be addressed as sets: in the conical group, the first gear was the pinion and the second gear was the wheel ring; in the differential group, the wheel 1 was a satellite and the wheel 2 was a planetary (Fig. 2).

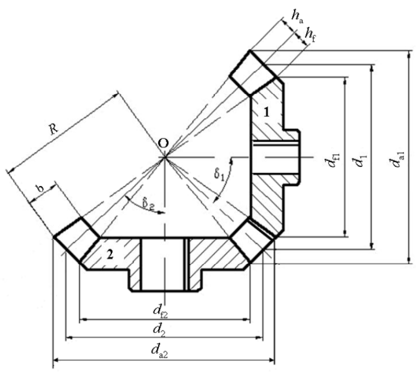


Fig. 2 - Geometry of a pair of gears [11]

The two sets of gears forms between them 90 degree angle and the chosen contact angle α was 20 degree. Since gears with different number of teeth constitute the sets, there are two-transmission ratios i that can be obtained by eq. (1), where Z_1 is the number of teeth of gear 1 and Z_2 is the number of teeth of gear 2.

$$i = \frac{Z_2}{Z_1} \quad (1)$$

From Fig. 2, d_i is the pitch diameter of the wheel and can be determined by eq. (2), where m is the module and Z_i the number of teeth.

$$d_i = m \times Z_i \qquad (2)$$

The δ_1 primitive cone angle can be found by eq. (3) where Σ is the angle between the two axes and δ_2 can be obtained from eq. 4.

$$\delta_1 = \tan^{-1}\left(\frac{\sin\Sigma}{\frac{Z_2}{Z_1} + \cos\Sigma}\right) \qquad (3)$$

$$\delta_2 = \Sigma - \delta_1 \qquad (4)$$

The distance from primitive cone R was determined by eq. (5) using either wheel 1 or wheel 2.

$$R = \frac{d_1}{2 \times \sin\delta_1} \qquad (5)$$

The tooth height, h_a, is equal to the modulus, the tooth foot height, h_f, was obtained from eq. (6) and the outer diameter, d_a, of any wheel can be obtained using the eq. (7):

$$h_f = 1{,}25 \times \mathrm{m} \qquad (6)$$

$$d_a = d + 2 \times h_a \times \cos\delta \qquad (7)$$

and the inner diameter of the root of the tooth, d_f, of any wheel is obtained by:

$$d_f = d - 2 \times h_f \times \cos\delta \qquad (8)$$

Table 3 summarises some of the geometric details of the strait bevel gears that compose the differential.

Table 3 - Geometric details of strait bevel gears

	Conical group		**Differential group**	
Variable	**Pinion (1)**	**Ring Crown (2)**	**Satellites (1)**	**Planetary (2)**
$\Sigma\ (deg.)$	90			
Z	12	47	12	17
i	3,917		1,417	
$m\ (mm)$	3			
$d\ (mm)$	36	141	36	51
$d_a\ (mm)$	41,814	142,484	40,902	54,460
$d_f\ (mm)$	28,733	139,145	29,873	46,675
$h_a\ (mm)$	3		3	
$h_f\ (mm)$	3,75		3,75	
$b\ (mm)$	22	22	12	12
$\emptyset\ (deg.)$	20			
$\delta\ (deg.)$	14,323	75,677	35,218	54,782
$R\ (mm)$	72,760	72,760	31,213	31,213

After geometric dimension of the bevel gears, these were sketched using *PTC Creo Parametric 2.0* CAD software (Fig. 3) to obtain the 3D printing rapid prototyping gears and also to simulate the finite element analysis [11,12]

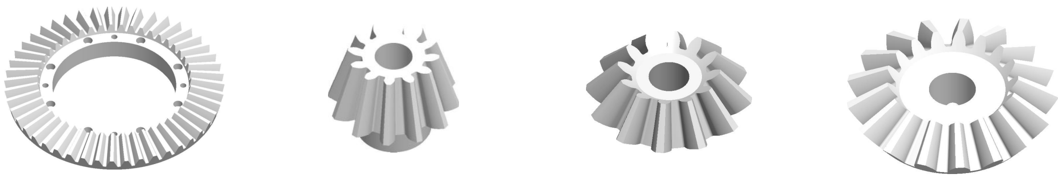


Fig. 3 - Gears sketched in *PTC Creo*: a) ring (crown); b) pinion; c) satellite; d) planetary

The differential gears were shaped in ABS Plus via *Dimension®* *uPrint* 3D printing rapid prototyping by an economic issue; the print strategy used was a low density i.e. 60% of full density. Fig. 4 shows the resulted gears. Depositing molten material in layers of 0.1mm performed the printing.

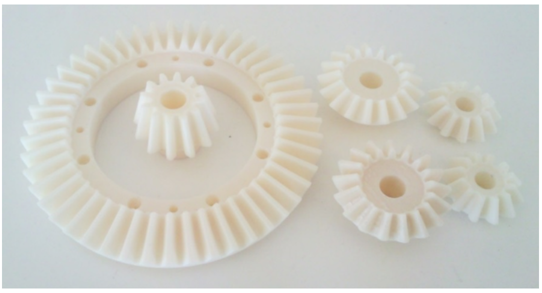


Fig. 4 - ABS Gears obtained by 3D Printing

GEARS ANALYSIS AND DISCUSSION

Analytical

Since it should not be considered metallic gears equations such as AGMA, the resistance of ABS tooth of gears considers a maximum tangential load F_t according to manufacturer sprockets *SDP/SI®* [13] and the plastics manufacturer *DuPont*™ [14]. The differential resistance was calculated through eq. (9), where y is a geometric factor, σ_b the allowable bending stress and K_v the velocity factor.

$$F_t = m \times \frac{R-b}{R} \times y \times b \times \sigma_b \times K_V \quad (9)$$

The geometrical factor, the allowable bending stress and the velocity factor were obtained from *DuPont*™ manufacturer (Table 4) considering that the gears are 100% density of ABS, i.e. high density (HD). Due to budgetary constraints, always looking for the optimization of consumables, it was analysed the tooth gears using high and low densities (100% - HD and 60% - LD) in order to understand the viability of low density strategy for the proposed model.

Table 4 ABS gears properties from *DuPont*™

y	$\sigma_b\ [kgf/mm^2]$	K_v
0.415	1.35	1

In table 5 is presented the obtained load capacity and gears velocity for both low and high density ABS. Also allowable stress is presented in Table 5 for both high and low density found from *DuPont*™.

Table 5 -ABS gears load capacity

	LD		*HD*	
	Pinion Conical group	**Satellite Differential group**	**Pinion Conical group**	**Satellite Differential group**
$F_t(N)$	55.56	27.78	253.06	121.791
$\omega\ (rpm)$	500	255.319	500	255.319
$v_t\ (m/s)$	0.942	0.481	0.942	0.481
$\sigma_y\ [MPa]$	21.60		36.00	
$\sigma_{all}\ [MPa]$	10.8		18	

Finite Element

Finite element analyses (FEA) were taken in order to compare with the analytical results and to understand the viability of the low-density strategy used in the 3D gears printing.

FEA by means of *Von Mises* stresses were undertaken after sketched gears (Fig. 3) using *PTC Creo Parametric 2.0* CAD software. In order to obtain the *Von Mises* stresses in both conical and differential gears groups, low-density and high-density maximum allowable tangential loads of the plastic material characteristics (Table 5) were used.

Von Mises Results

Figure 5 and Figure 6 shows the low-density FEA of the conical and the satellite group respectively, showing the analysis of the loaded tooth.

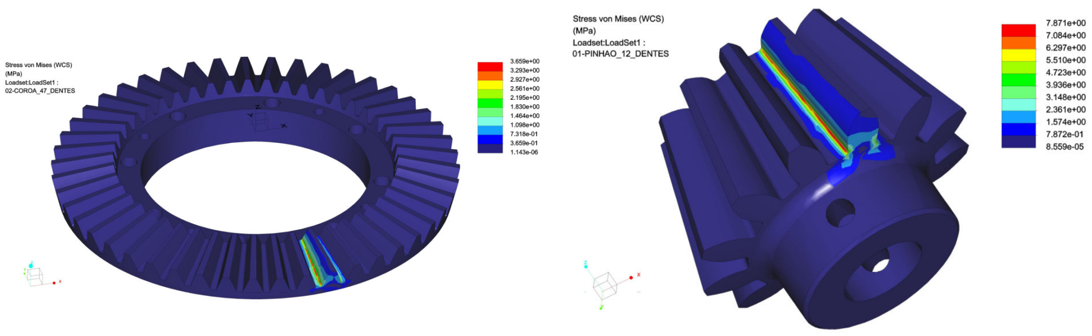


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Fig. 5 - Pinion conical group LD FEA: a) ring (crown); b) pinion

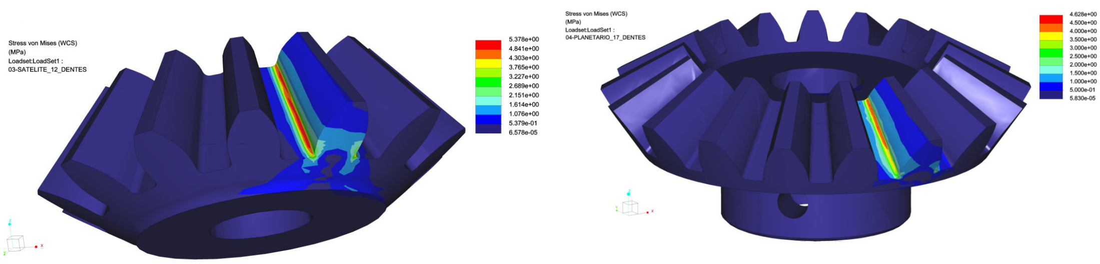

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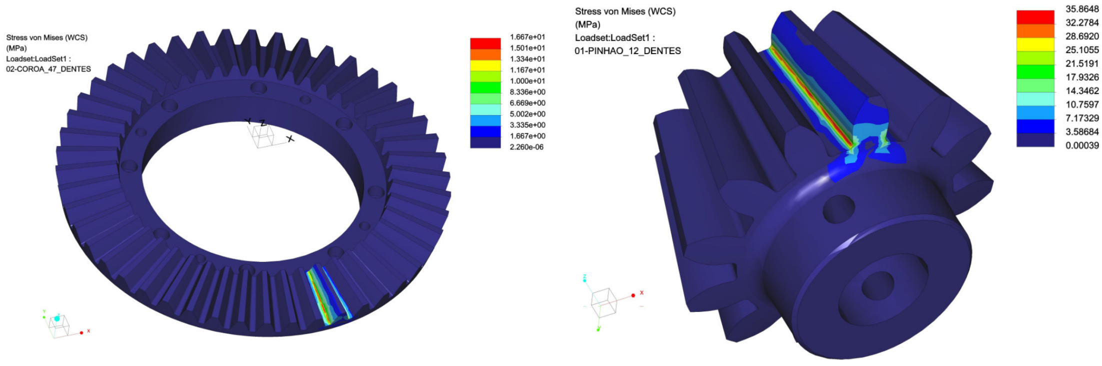
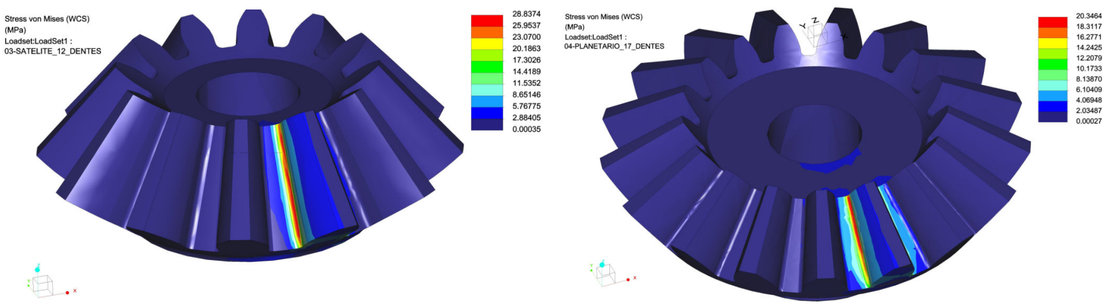
Fig. 6 - Satellite differential group LD FEA: a) satellite; b) planetary

Figure 7 and Figure 8 shows the high-density FEA of the conical and the satellite group respectively.



a) b)

Fig. 7 - Pinion conical group HD FEA: a) ring (crown); b) pinion



a) b)

Fig. 8 - Satellite differential group HD FEA: a) satellite; b) planetary

From both LD and HD analysis, maximum allowable stresses were obtained and are present in Table 6.

Table 6 -Maximum *Von Mises* stresses on tooth gears

	σ_{max}^{LD} $[MPa]$	σ_{max}^{HD} $[MPa]$
conical pinion	7.87	35.86
conical ring	3.66	16.67
differential satellite	5.38	28.84
differential planetary	4.63	20.35

Results from the FEA shown in Table 6 for both LD and HD ABS gears demonstrates that the maximum *Von Mises* stresses obtained in both cases do not achieve the allowable stress specified by *DuPont*™ for ABS gears.

Even for the higher stress gear (conical pinion) the HD FEA *Von Mises* stresses presents lower deviation from *DuPont*™ manufacturer maximum stresses (Table 7). All other gears have a large deviation showing that the maximum stress is lower than that indicated by the ABS manufacturer, demonstrating that the LD strategy can be applied to the specific case.

Table 7 -LD and HD ABS maximum allowable stress deviation

	LD [%]	HD [%]
conical pinion	63.56	0.39
conical ring	83.06	53.69
differential satellite	75.09	19.89
differential planetary	78.56	43.47

ROTATION ANALYSIS AND DISCUSSION

After assembly of the entire gear system in the pedagogical model (Fig. 9), experimental input and output rotation speed of the electric motor such as the conical and satellite group were obtained via a *Telemecanique*® XUB-HO02323 accelerometer and compared with analytical results.
The input rotation on the gear system is the rotation obtained from the electric DC motor (Table 2) and the output fallouts of the theoretical and experimental results are present in Table 8.

Fig. 9 - Pedagogical model

Table 8 - Analytical and experimental output rotation of gears

	Conical group	Differential group
Analytical [rpm]	46.0	92.0
Experimental [rpm]	45.5	84.0
Deviation	1.1%	8.7%

The output rotation of the differential group for the unblocked wheel is the simulation of a strait moving. The output rotation of the conical group is the simulation of a blocked wheel and is admitted that the free wheel will run twice as much as the crown.
It is quite normal losses between gears, not only by the contact friction between teeth but also from bearings. In the case of this model, one should also consider that available misalignments could introduce losses. However the results of deviations are almost insignificant and can be considered dispersible.

CONCLUSION

Two different percentages of low (LD) and high-densities (HD) (60% and 100%) of 3D printed gears with ABS (Acrylonitrile Butadiene Styrene) were analysed and compared using analytical and finite element analysis using *PTC Creo Parametric 2.0* software.

FEA and analytical results for both LD and HD showed that ABS 3D printed gears are capable to support *Von Mises* stresses proving themselves through experimental and theoretical output rotation.

This way, the present pedagogical model using low-density strategy by an economic issue is a viable consideration and an added value for the mechanical engineering higher education when differential gear system is academically studied.

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