

YOKOHAMA MATHEMATICAL
JOURNAL VOL. 60, 2014

OSCILLATION CRITERIA OF DIFFERENCE EQUATIONS WITH SEVERAL DEVIATING ARGUMENTS

By

G. E. CHATZARAKIS, J. MANOJLOVIC, S. PINELAS,
AND I. P. STAVROULAKIS

(Received June 13, 2013; Revised May 5, 2014)

Abstract. New sufficient conditions for the oscillation of all solutions of difference equations with several deviating arguments are presented. Corresponding difference equations of both retarded and advanced type are studied. The significance of the conditions established are demonstrated by comparing with known oscillation conditions. Examples illustrating the results are also given.

1. Introduction

In this paper we study the oscillation of all solutions of the difference equation with several variable retarded arguments of the form

$$\Delta x(n) + \sum_{i=1}^m p_i(n)x(\tau_i(n)) = 0, \quad n \in \mathbb{N}_0, \quad m \in \mathbb{N} \quad (\text{E}_R)$$

and the (dual) difference equation with several variable advanced arguments of the form

$$\nabla x(n) - \sum_{i=1}^m p_i(n)x(\sigma_i(n)) = 0, \quad n \in \mathbb{N}, \quad m \in \mathbb{N}, \quad (\text{E}_A)$$

where $(p_i(n))$, $1 \leq i \leq m$ are sequences of nonnegative real numbers, $(\tau_i(n))$, $1 \leq i \leq m$ are sequences of integers such that

$$\tau_i(n) \leq n - 1 \quad \forall n \in \mathbb{N}_0, \quad \text{and} \quad \lim_{n \rightarrow \infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m \quad (1.1)$$

$(\sigma_i(n))$, $1 \leq i \leq m$ are sequences of integers such that

$$\sigma_i(n) \geq n + 1 \quad \forall n \in \mathbb{N}, \quad 1 \leq i \leq m. \quad (1.2)$$

2010 Mathematics Subject Classification: 39A10, 39A21

Key words and phrases: difference equations, retarded argument, advanced argument, oscillatory solutions, nonoscillatory solutions

This paper was supported by CINAMIL - Centro de Investigação, Desenvolvimento e Inovação da Academia Militar