Dynamic Systems and Applications 23 (2014) 63-74

OSCILLATION RESULTS FOR ODD-ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS OF MIXED TYPE

E. THANDAPANI[a], S. PADMAVATHI[b], AND S. PINELAS[c]

[a,b]Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai 600 005, India
E-mail: ethandapani@yahoo.co.in
[c]Academia Militar, Departamento de Ciências Exactas e Naturais, Av. Conde Castro Guimarães, 2720-113 Amadora, Portugal
E-mail: sandra.pinelas@gmail.com

ABSTRACT. In this paper the authors establish some new comparison theorems and Philos-type criteria for oscillation of solutions to the odd order neutral mixed type differential equation

$$(x(t) + ax(t - \tau_1) + bx(t + \tau_2))^{(n)} + p(t)x^{\alpha}(t - \sigma_1) + q(t)x^{\beta}(t + \sigma_2) = 0, \quad t \geq t_0,$$

where $n \geq 3$ is an odd integer, $\alpha \geq 1$ and $\beta \geq 1$, are ratio of odd positive integers. Examples are provided to illustrate the main results.

AMS (MOS) Subject Classification. 34C15.

1. PRELIMINARIES

This paper is concerned with the oscillation and asymptotic behavior of solutions of odd order nonlinear neutral mixed type differential equation of the form

$$(x(t) + ax(t - \tau_1) + bx(t + \tau_2))^{(n)} + p(t)x^{\alpha}(t - \sigma_1) + q(t)x^{\beta}(t + \sigma_2) = 0, \ t \geq t_0, \tag{1.1}$$

where $n \geq 3$ is an odd integer, $\alpha \geq 1$ and $\beta \geq 1$ are the ratios of odd positive integers, $p(t)$ and $q(t)$ are continuous and positive functions for all $t \geq t_0$, and $a, b, \tau_1, \tau_2, \sigma_1, \sigma_2$ are non-negative constants. We set $z(t) = x(t) + ax(t - \tau_1) + bx(t + \tau_2)$. By a solution of equation (1.1), we mean a function $x(t) \in C([T_x, \infty), \mathbb{R})$, $T_x \geq t_0$, which has the property $z(t) \in C^n([T_x, \infty), \mathbb{R})$ and satisfies equation (1.1) on $[T_x, \infty)$. We consider only those solutions $x(t)$ of equation (1.1) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \geq T_x$. We assume that equation (1.1) possesses such a solution. A solution of equation (1.1) is called oscillatory if it has infinitely large zeros in $[T_x, \infty)$ and otherwise, it is said to be nonoscillatory. Equation (1.1) is said to be almost oscillatory if all its solutions are either oscillatory or convergent to zero asymptotically.

Received April 29, 2013