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Aequationes Mathematicae

Oscillations of difference equations with several deviated arguments

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Abstract. We establish sufficient conditions for the oscillation of all solutions to the retarded difference equation

$$\Delta x(n) + \sum_{i=1}^m p_i(n)x(\tau_i(n)) = 0, \quad n \geq 0,$$

and the (dual) advanced difference equation

$$\nabla x(n) - \sum_{i=1}^m p_i(n)x(\sigma_i(n)) = 0, \quad n \geq 1,$$

where $(p_i(n)), 1 \leq i \leq m$ are sequences of nonnegative real numbers, $(\tau_i(n)), 1 \leq i \leq m$ are sequences of integers such that

$$\tau_i(n) \leq n - 1 \quad \forall n \geq 0, \quad \text{and} \quad \lim_{n \rightarrow \infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m,$$

$(\sigma_i(n)), 1 \leq i \leq m$ are sequences of integers such that

$$\sigma_i(n) \geq n + 1 \quad \forall n \geq 1, \quad 1 \leq i \leq m,$$

Δ denotes the forward difference operator $\Delta x(n) = x(n+1) - x(n)$ and ∇ denotes the backward difference operator $\nabla x(n) = x(n) - x(n-1)$. Examples illustrating the results are also given.


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Keywords. Difference equations, retarded argument, advanced argument, oscillatory solutions, nonoscillatory solutions.

1. Introduction

In this paper we study the oscillation of all solutions of the difference equation with several variable retarded arguments of the form

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