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COMPARISON AND OSCILLATION THEOREM FOR SECOND-ORDER NONLINEAR NEUTRAL DIFFERENCE EQUATIONS OF MIXED TYPE

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ABSTRACT. In this paper, we establish some comparison theorems for the oscillation of second order neutral difference equations of mixed type

$$\Delta\left(a_n \Delta\left(x_n + b_n x_{n-\sigma_1} + c_n x_{n+\sigma_2}\right)^\alpha\right) + q_n x^\beta_{n-\tau_1} + p_n x^\beta_{n+\tau_2} = 0,$$

where α and β are ratio of odd positive integers, σ_1, σ_2, τ_1 and τ_2 are positive integers. Our results are new even if $p_n = c_n = 0$. Examples are provided to illustrate the results.

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1. INTRODUCTION

In this paper, we shall study the oscillatory behavior of the second order nonlinear neutral difference equation of mixed type

$$\Delta\left(a_n \Delta\left(x_n + b_n x_{n-\sigma_1} + c_n x_{n+\sigma_2}\right)^\alpha\right) + q_n x^\beta_{n-\tau_1} + p_n x^\beta_{n+\tau_2} = 0, \tag{1.1}$$

where $n \geq n_0 \in \mathbb{N}$, subject to the following conditions:

(H1) $\{a_n\}$ is a positive sequence for all $n \geq n_0$ and $\sum\limits_{n=n_0}^{\infty} \frac{1}{a_n} = \infty$;

(H2) $\{b_n\}$ and $\{c_n\}$ are nonnegative sequences such that $0 \leq b_n \leq b$ and $0 \leq c_n \leq c$, where b and c are constants;

(H3) $\{p_n\}$ and $\{q_n\}$ are nonnegative real sequences and not eventually zero for many values of n;

(H4) σ_1, σ_2, τ_1 and τ_2 are nonnegative integers and α and β are ratio of odd positive integers.

We put $z_n = (x_n + b_n x_{n-\sigma_1} + c_n x_{n+\sigma_2})^\alpha$. By a solution of equation (1.1), we mean a real sequence $\{x_n\}$ defined for all $n \geq n_0 - \max\{\sigma_1, \tau_1\}$, and satisfies equation (1.1) for

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