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OSCILLATION OF NONLINEAR DIFFERENCE EQUATIONS WITH DELAYED ARGUMENT

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ABSTRACT. The following difference equation with delayed argument

$$\Delta^2 u(k) + F(k, u(\tau(k))) = 0$$

is considered, where $F : N \times R \to R$, $\tau : N \to N$, $\tau(k) \leq k$ for $k \in N$, $\lim_{k\to+\infty} \tau(k) = +\infty$ and $\Delta u(k) = u(k+1) - u(k)$, $\Delta^2 = \Delta \circ \Delta$. In the paper sufficient (necessary and sufficient) conditions are established for all proper solutions of the above equation to be oscillatory.

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1. INTRODUCTION

Consider the difference equation

$$\Delta^2 u(k) + F(k, u(\tau(k))) = 0, \tag{1.1}$$

where $F : N \times R \to R$, $\tau : N \to N$, $\Delta u(k) = u(k+1) - u(k)$ and $\Delta^2 = \Delta \circ \Delta$. Everywhere it will be assumed that

$$F(k, x) \operatorname{sign} x \geq 0 \quad \text{for} \quad k \in N \quad \text{and} \quad x \in R, \tag{1.2}$$

$$\tau(k) \leq k \quad \text{for} \quad k \in N, \quad \lim_{k\to+\infty} \tau(k) = +\infty \tag{1.3}$$

and

$$\sup\big\{|F(i, x)| : i \geq k\big\} > 0 \quad \text{for} \quad k \in N, \quad x \neq 0. \tag{1.4}$$

For any $n \in N$, denote $N_n = \{n, n+1, \dots\}$.

Definition 1.1. For $n \in N$ put $n_0 = \min\{\tau(k) : k \in N_n\}$. A function $u : N_{n_0} \to R$ is said to be a proper solution of (1.1), if it satisfies (1.1) on N_n and

$$\sup\big\{|u(i)| : i \geq k\big\} > 0 \quad \text{for} \quad k \in N.$$

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