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OSCILLATION CRITERIA FOR CERTAIN FOURTH ORDER NONLINEAR DIFFERENCE EQUATIONS

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ABSTRACT. We shall establish some new criteria for the oscillation of the fourth order nonlinear difference equation

$$\Delta^2\left(a(k)\left(\Delta^2 x(k)\right)^\alpha\right)+q(k)\,f(x(g(k)))=0$$

via comparison with some difference equations of less order whose oscillatory characters are known.

1. INTRODUCTION

Consider the fourth order nonlinear difference equation

$$\Delta^2\left(a(k)\left(\Delta^2 x(k)\right)^\alpha\right)+q(k)\,f(x(g(k)))=0 \tag{1}$$

where Δ is the forward difference operator defined by $\Delta x(k)=x(k+1)-x(k)$ and α is the ratio of positive odd integers. We shall assume that $g,a:\mathbb{N}(k)\to\mathbb{R}^+=(0,\infty)$ for some $k\in\mathbb{N}=\{0,1,\dots\}$ and $\mathbb{N}(n_0)=\{n_0,n_0+1,\dots\}$ where $n_0\in\mathbb{N}$, $g\in\bar{G}:=\{g:\mathbb{N}(k)\to\mathbb{N}$ for some $k\in\mathbb{N}: g(k)\leqslant k$, $g(k)$ is non-decreasing and $\lim_{k\to\infty}g(k)=\infty\}$, and $f:\mathbb{R}\to\mathbb{R}$ is continuous satisfying $xf(x)>0$ for $x\neq 0$ and f is non-decreasing.

By a solution of equation (1), we mean a nontrivial sequence $\{x(k)\}$ satisfying equation (1) for all $k\in\mathbb{N}(K)$ where K is some nonnegative integer. A solution $\{x(k)\}$ is said to be oscillatory if it is neither eventually positive nor eventually negative and it is nonoscillatory otherwise. Equation (1) is said to be oscillatory if all its solutions are oscillatory. Determining oscillation criteria for difference equations has received a great deal of attention in the last two decades, see for example the Monographs of Agarwal et. al. [1]–[3]. This interest is motivated by the importance

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