Electronic Journal of Differential Equations, Vol. 2010(2010), No. 73, pp. 1–9.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

OSCILLATION CRITERIA FOR FORCED SECOND-ORDER MIXED TYPE QUASILINEAR DELAY DIFFERENTIAL EQUATIONS

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Abstract. This article presents new oscillation criteria for the second-order delay differential equation

$$(p(t)(x'(t))^{\alpha})' + q(t)x^{\alpha}(t-\tau) + \sum_{i=1}^{n} q_i(t)x^{\alpha_i}(t-\tau) = e(t)$$

where $\tau \geq 0$, $p(t) \in C^1[0,\infty)$, $q(t), q_i(t), e(t) \in C[0,\infty)$, $p(t) > 0$, $\alpha_1 > \cdots > \alpha_m > \alpha > \alpha_{m+1} > \cdots > \alpha_n > 0$ $(n > m \geq 1)$, $\alpha_1, \dots, \alpha_n$ and α are ratio of odd positive integers. Without assuming that $q(t), q_i(t)$ and $e(t)$ are nonnegative, the results in [6, 8] have been extended and a mistake in the proof of the results in [3] is corrected.

1. Introduction

In this paper, we are concerned with the oscillatory behavior of the quasilinear delay differential equation

$$(p(t)(x'(t))^{\alpha})' + q(t)x^{\alpha}(t-\tau) + \sum_{i=1}^{n} q_i(t)x^{\alpha_i}(t-\tau) = e(t) \tag{1.1}$$

where $\tau \geq 0$, $p(t), q(t), q_i(t) \in C[0,\infty)$, $p(t)$ is positive, nondecreasing and differentiable, $\alpha_1, \dots, \alpha_n, \alpha$ are ratio of odd positive integers, and $\alpha_1 > \cdots > \alpha_m > \alpha > \alpha_{m+1} > \cdots > \alpha_n > 0$.

A solution $x(t)$ of (1.1) is said to be oscillatory if it is defined on some ray $[T,\infty)$ with $T \geq 0$ and has unbounded set of zeros. Equation (1.1) is said to be oscillatory if all solutions extendable throughout $[0,\infty)$ are oscillatory.

For $\tau = 0$ and $\alpha = 1$, the oscillatory behavior of (1.1) has been studied in Sun and Wong [8] and Sun and Meng [6]. When $\alpha = 1$, Chen and Li [3] extended the results established by Sun and Meng [6] to (1.1). A close look into the proof of [3, Theorem 1] reveals that the authors used $x''(t) \leq 0$ for $t \in [a_1-\tau, b_1]$ instead of taking $(p(t)x'(t))' \leq 0$ for $t \in [a_1-\tau, b_1]$. We wish not only to correct the proof of the theorem but also extend the results given in [1, 2, 4, 8] for ordinary and delay differential equations.

2000 *Mathematics Subject Classification.* 34K11, 34C55.
Key words and phrases. Interval oscillation; quasilinear delay differential equation; second order.

Submitted January 10, 2010. Published May 19, 2010.