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Nonautonomous oscillators and coefficient evaluation for orthogonal function systems

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Dedicated to Professor Saber Elaydi on the occasion of his 65th birthday

The analytic behaviour of classical and difference versions of Hermite polynomials is investigated from two different viewpoints: first using oscillation theory which is important in quantum oscillator systems and second using a factorization method which may serve as a stable numerical investigation of the function systems under investigation. The resulting polynomial systems are basically of type unbounded orthogonal polynomial systems.

Keywords: orthogonal polynomials; oscillations; coefficient evaluation; quantum oscillators

1. Introduction

In this article, we deal with the oscillatory behaviour of the Hermite polynomials and some of their generalizations. These oscillations are essentially for the understanding of quantum oscillator systems and give insight into the energetic behaviour of the underlying dynamical systems in quantum theory. In the first sections, these prominent function systems are investigated in the light of oscillation theory. This is part of a major project, namely applying successively oscillation theory methods to the understanding of orthogonal function systems of discrete quantum oscillators. The current work is a first step into this direction. Evaluating the Hermite polynomials and their generalizations successively, through the three-term recurrence relations they satisfy. However, this turns out to be from the numerical viewpoint not stable. An alternative would be to use more stable evaluation algorithms like the Miller algorithm. However, we elucidate a different way: Developing a difference ladder operator formalism, we can approach the expansion coefficients of the Hermite polynomials directly. This will – for later purpose – serve to evaluate the polynomials numerically using the simple, safe and superstable method of Horner. The numerical aspects of the function systems we consider will be the subject of a different article. For physical motivations of the function systems we consider, we recommend the interested reader to consider references [1–12].

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