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Nonoscillations in Retarded Equations

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1 Introduction

This note regards the existence of nonoscillatory solutions of the linear difference retarded functional equation

$$x(t) = \int_{-1}^{0} x(t - r(\theta))\, dq(\theta), \tag{1}$$

where $x(t) \in \mathbb{R}$, $r(\theta)$ is a positive real continuous function on $[-1, 0]$ and $q(\theta)$ is a function of bounded variation on $[-1, 0]$, normalized in a manner such that $q(-1) = 0$.

In the case where $q(\theta)$ is a step function, with a number p of jump points, we obtain the important class of delay difference equations

$$x(t) = \sum_{j=1}^{p} a_j x(t - r_j), \tag{2}$$

where the a_j are nonzero real numbers and each r_j is a positive real number $(j = 1, \ldots, p)$.

Considering the value $\|r\| = \max\{r(\theta) : -1 \leqslant \theta \leqslant 0\}$, by a *solution* of (1) we mean a continuous function $x : [-\|r\|, +\infty[\to \mathbb{R}$, which satisfies (1) for every $t \geq 0$. A solution is said to be *oscillatory* whenever it has an infinite number of zeros; otherwise it will be said to be *nonoscillatory*. When all solutions are oscillatory, the equation (1) is called *oscillatory*. If (1) has at least one nonoscillatory solution, then the equation will be said to be *nonoscillatory*.

We will say that a function $\phi : [-1, 0] \to \mathbb{R}$ is increasing (decreasing) on $J \subset [-1, 0]$, if ϕ is nonconstant on J and for every $\theta_1, \theta_2 \in J$ such that $\theta_1 < \theta_2$, one has $\phi(\theta_1) \leq \phi(\theta_2)$ (respectively, $\phi(\theta_2) \leq \phi(\theta_1)$). For a given $\theta \in [-1, 0]$, if for every $\varepsilon > 0$, sufficiently small, ϕ is increasing (decreasing) in $[\theta - \varepsilon, \theta + \varepsilon]$ ($[-\varepsilon, 0]$ if $\theta = 0$,