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AN INTEGRATED APPROACH FOR THE MEASUREMENT OF INEQUALITY, POVERTY, AND RICHNESS

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Abstract: We propose a new and integrated approach to the measurement of inequality in income distribution, poverty, and richness. In the context of the poverty and richness measures, we consider the three dimensions usually analysed – incidence, intensity, and severity. The proposed broad set of indicators is easy to calculate and is based on a neutral income inequality concept. The method also allows an objective interpretation of the values for each measure, a decomposition according to households' characteristics, and an immediate comparison of the results between countries and time periods. We illustrate the application of the measures with data from Portugal.

Key words: income inequality, poverty, richness, measurement.

JEL Codes: D30, D31.

1. Introduction

Income inequality and poverty are well established research fields in the economic literature. Apart from a multiplicity of other empirical and theoretical contributions, several recent books address the state of the art of the research about inequality and poverty, including Wolff (2009), Salverda *et al*. (2011), and Cowell (2011). This analysis can be justified on several grounds. On the one hand is the natural wish to address an issue seen as socially unfair. On the other hand, economic policy concerns have brought the issue of poverty and inequality to the center of public debate, intensifying the research into their determinants. A full knowledge of the real dimension and characterization of these phenomena is thus of widespread interest, seeking the definition of effective socio-economic policies.

The study of top incomes has also recently emerged (Piketty, 2005; Saez and Veall, 2005; Piketty and Saez, 2006; Roine and Waldenström, 2008; Bach *et al*., 2009; Atkinson and Piketty, 2010). According to Atkinson (2007) three reasons justify the analysis of the 'rich': their command over resources, their command over people (income and wealth as sources of power) and their global significance. Atkinson *et al*. (2011) provide a survey on this topic.

Central to all this literature has been the discussion of the procedures and indicators for measuring income inequality, poverty, and richness. The present paper contributes to this line of research by proposing a new methodology that allows an integrated approach for measuring inequality, poverty, and richness.

Our approach starts from an inequality measure which is based on a concept of inequality characterized by its neutrality, seeking to quantify the phenomenon without value judgments on the distribution of inequality. The approach has the following characteristics: (i) simplicity in application; (ii) an objective interpretation of the values obtained for each indicator; (iii) a straightforward comparison of the results between different economic spaces and time

periods; and (iv) decomposability, i.e., the possibility of knowing the contribution of population's sub-groups.

The paper is structured as follows. Section 2 summarizes the main methodological issues and indicators used in the literature. Section 3 presents the approach in which new measures of inequality, poverty, and richness are advanced. Section 4 illustrates the application of the proposed measures using data for Portugal. Section 5 presents some final remarks.

2. Methodological issues and indicators

For the empirical analysis of inequality, poverty, and richness, it is necessary to assume some methodological choices as well as to select the indicator(s) that will be used. In this section, we summarize the main options available.

2.1 Methodological issues

Measuring income inequality, poverty, and richness implies making choices concerning some methodological issues. Four of these issues are common to the analysis of the three phenomena while a fifth is specific to the analysis of poverty and richness. The first group involves choices concerning: (i) the indicator of resources; (ii) the demographic unit; (iii) equivalence scales; and (iv) the weighting of the demographic unit. In order to measure poverty/richness, it is also necessary to define a poverty/richness line.

In relation to the indicator of resources, Cowell (2011) suggests that wealth, lifetime income, and income are, in that order, the most adequate ones, even though none of them covers completely the command over resources for all goods and services in society. The ease of calculation and, mainly, data availability usually justify income as the favoured option.

Regarding the concept of income, the most common option – given the availability of statistical information – is the monetary disposable income. This choice is subject to criticism because of the exclusion of non-monetary forms of income and also of the past accumulation effect through savings and indebtedness.

The second methodological choice relates to the demographic unit, usually between the individual and an aggregate (family or household, the latter also including individuals at the same address who are not part of the nuclear family). The option for households is mainly followed in the literature because of the income sharing phenomenon within the household.

Directly related to the previous option is the issue of comparing unlike units. Households with different compositions and dimensions have distinct needs and thus require different levels of income to achieve similar levels of well-being. The use of equivalence scales allows calculating equivalent adults for each household. A frequently used equivalence scale is the OECD modified scale, which gives a weight of 1 to the first adult, 0.5 to each of the remaining adults, and 0.3 for children under 14 years of age. The income adjusted by the composition and dimension of the household – the adult equivalent income – represents therefore a refinement of the income *per capita*, not neglecting the existence of economies of scale due to the share of housing and expenses.[1]

Concerning the weighting of the demographic unit, the usual choice is to take the number of a household's individuals.

The fifth methodological issue – the poverty/richness line – is exclusive to the analysis of poverty and richness. The main methodological question in this context is the choice between absolute or relative lines. In the first case, the threshold is defined without reference to the standard of living prevailing in society. In the second case, that reference is taken into account.

[1] Disregarding inequality within the household is the main limitation of this concept. As stressed by Haddad and Kanbur (1990), it implies the under-estimation of the actual degree of inequality existing in society.

2.2 Indicators

After considering the methodological questions mentioned above, one must choose the indicators to use.

2.2.1 Inequality indicators

Four main groups of inequality indicators can be considered. The first refers to measures that compare the income share of the top *x*% of the income distribution with that of the bottom *x*%. Frequent values for *x* are 5, 10, and 20. The main advantage of this type of indicator – and the reason for its strong support (at least as a preliminary indicator) – is the ease of calculation and interpretation. However, evaluating inequality through these measures is limited because the income distribution inside each income group is not considered (Haughton and Khandker, 2009).

The most widely used measure of income inequality is the well-known Gini coefficient, which varies between 0 (total equality) and 1 (maximum inequality). However, this index is not (easily) decomposable, which is one of its main limitations.

A third way to measure inequality is the index proposed by Atkinson (1970). Its most important characteristic is making the value judgments involved in the measurement of inequality explicit, by taking into account a parameter that captures the degree of inequality aversion. That parameter can vary between 0 (inequality indifference) and $+\infty$ (corresponding to the Rawlsian criterion that values only the income of the poorest).

A last group of inequality indicators corresponds to the Generalized Entropy (GE) measures, including the Theil indices and the mean log deviation measure (Cowell and Kuga, 1981a,b).

Similar to the Atkinson index, GE measures clearly assume the incorporated value judgments through a parameter representing the weight attributed to income differences in different parts of the distribution. The most common values for that parameter are 0, 1, and 2. The inexistence of inequality implies that GE measures assume a value of 0. The increase of the value of such indicators corresponds to an increase in inequality. GE measures are additively decomposable, a crucial property for the evaluation of inequality determinants.

2.2.2 Poverty indicators

Several poverty measures are available in the literature, capturing the different dimensions of this phenomenon (incidence, intensity, and severity). The headcount index (P_0) captures the first dimension, measuring the proportion of individuals classified as poor (i.e., with an income lower than the poverty line) in the total population. The main merit of this measure is the simplicity of calculation and interpretation. However, an important weakness of P_0 is the fact that it is only an accounting of the poor, with no sensibility regarding the magnitude of the problem.

In its turn, the poverty gap index (P_1) measures the mean deviation of income from the poverty line, capturing the intensity of poverty. Thus P_1 overcomes the main limitation of P_0.

The poverty severity index (P_2) is a third poverty indicator, which measures the inequality among the poor by calculating the sum of poverty gaps weighted by the gaps themselves (Haughton and Khandker, 2009). Thus P_2 is especially affected by extreme poverty situations.

A particularly appealing way to present the three above measures of poverty is through the class of poverty measures proposed by Foster *et al*. (1984):

$$P_{\alpha} = \frac{\sum_{i=1}^{N} \left(\frac{G_i}{Z} \right)^{\alpha}}{N}, \qquad (1)$$

in which N is the total number of individuals in the population, Z the poverty line, and G_i the poverty gap associated with individual *i*. G_i will be zero if the income of *i* (Y_i) is greater than or equal to Z and ($Z - Y_i$) in the opposite case (i.e., when *i* is poor). The parameter α ($\alpha \geq 0$) represents the sensitivity of the index to poverty. When α is 0, 1, and 2, one obtains the poverty measures mentioned above, that is, the headcount index, the poverty gap index, and the poverty severity index, respectively. Decomposability is an interesting property of P_{α}.

On the contrary, the index proposed by Sen (1976) – also attempting to capture in a single measure the three above dimensions – does not satisfy that property. As shown by Blackwood and Lynch (1994), the Sen index is more sensitive to a reduction in the headcount index compared to a decrease in the poverty gap or in the inequality among the poor. Therefore, the Sen index is somewhat biased toward policies that reduce the number of poor (Blackwood and Lynch, 1994).

2.2.3 Richness indicators

While the methodologies used to analyse inequality and poverty are well consolidated in the literature, this is not so for the evaluation of richness (Peichl *et al*., 2010). In that context, the most commonly applied measures are the income share of the top *x*% of the income distribution and headcount measures. As stated above, both measures have however serious limitations and thus give only a partial indication of the richness phenomenon. An important

contribution is given by Peichl *et al*. (2010) who have suggested a class of richness measures analogous to the existent for the poverty measures.

3. An integrated approach for the measurement of inequality, poverty, and richness

3.1 On the concept(s) of income inequality

In the previous section we synthesized the most common methodological options for the measurement of inequality, poverty, and richness, as well as the main indicators available, stressing their specificities and (implicit or explicit) value judgments.

In this section we propose a new and integrated approach for measuring these phenomena. We start by considering a new measure of income inequality. We then derive poverty and richness measures, capturing their different dimensions (incidence, intensity, and severity).

However, 'before trying to quantify anything one must first be clear about the concept to be measured' (Ravallion, 2003, p.740). We therefore start the analysis by discussing the concept of inequality underlying the indicator that serves as our point of departure.

As argued by Bellù and Liberati (2006), 'inequality is not a self-defining concept, as its definition may depend on economic interpretations as well as ideological and intellectual positions' (Bellù and Liberati, 2006, p.2). Although an explicit discussion about the different concepts of inequality underlying the measures available does not exist in the literature, one can distinguish four main concepts.

A first – the simplest – associates inequality with the difference, in absolute terms, between the current distribution and the egalitarian one (*concept of inequality 1*). In turn, the second and third concepts of inequality are relative measures of inequality. While the first one – *concept of inequality 2* – measures the deviation from the benchmark distribution in relative

terms, the *concept of inequality 3* also takes into account the distribution of inequality among the different receiving units. The last concept – *concept of inequality 4* – explicitly incorporates welfare considerations in the measurement of inequality (e.g., Sen, 1976), arguing that the level of income should also be considered in the assessment of this phenomenon. A distribution *B* obtained from a distribution *A*, for example, by doubling all incomes corresponds to a better situation since it ensures a higher level of welfare.

This conceptual discussion is critical because the inequality indices used for empirical purposes should reflect the specific concept considered. The measures that correspond to each of the concepts presented above can be distinguished by their compliance with the principles proposed by Fields and Fei (1978), which are usually used in the analysis of inequality indicators, namely: (i) symmetry; (ii) population size independence; (iii) mean independence; and (iv) Pigou-Dalton transfers principle.

The principle of symmetry requires that any measure of inequality should be invariant to swaps of income among individuals. In turn, according to the principle of population size independence, the inequality measure should not change in response to a replication of the original population. The principle of mean independence requires that the index must not vary when all the original incomes are multiplied by a constant. Finally, according to the Pigou-Dalton transfers principle, any transfer of income from a richer to a poorer individual which does not reverse their positions, reduces inequality.

Table 1 synthesizes the principles verified by the measures included in each of the four concepts of inequality.

[Table 1]

Given the notable differences between the various concepts of inequality it is important: (i) to discuss which ones seem more appropriate for a proper measurement of the phenomenon; and (ii) to justify the option assumed in the present study.

In the context of *concept 1*, a distribution *B* derived from a distribution *A* by multiplying all incomes by a positive constant reveals a greater degree of inequality than the original distribution. This breach of the mean independence principle makes this concept difficult to accept as an adequate basis for empirical measures of inequality.

According to the *concept of inequality 4*, the comparison of distributions *A* and *B* leads to the opposite conclusion, i.e., *B* is preferable because it provides a higher level of welfare. This concept is subject to criticism, however, because it captures more than what the concept of inequality is intended to measure.

In this study we argue that the *concepts of inequality 2* and *3* are the most interesting to measure inequality. It is important to note however that these concepts capture different perspectives of the phenomenon and should not therefore be seen as alternatives but rather as complements.

Concept 3 is the framework of the inequality measures most frequently used in the literature, such as those generally discussed in Section 1. This concept assumes the simultaneous verification of the four principles suggested by Fields and Fei (1978). The Pigou-Dalton principle is not, however, immune to criticism, as noted, for example, by Chateauneuf and Moyes (2006) in a study entitled 'measuring inequality without the Pigou-Dalton condition'. Discussing the inclusion of this principle in most measures of inequality, the authors argue that 'one may however raise doubts about the ability of such a condition to capture the very idea of inequality in general' (Chateauneuf and Moyes, 2006, p.2).[2]

[2] On this topic see Magdalou and Moyes (2009) and Magdalou and Nock (2011). Some experimental studies also confirm that the principle of transfers is not universally accepted, as shown, for instance, by Amiel and Cowell (1992), Harrison and Seidl (1994), and Gaertner and Namezie (2003).

In this study we assume the *concept of inequality 2*. As a result, our inequality index does not verify the Pigou-Dalton principle. The reason for this choice stems directly from the main distinguishing feature of this concept in comparison to *concept 3* – its neutrality, i.e., the exclusion of any kind of distribution sensitivity. The measures based on this concept seek only the quantification of the phenomenon. Following this perspective, the measure that we suggest below aims to quantify the distance between the current distribution and the egalitarian one, without any value judgment on the distribution of inequality. This index measures the proportion of the total income that would be necessary to redistribute in order to obtain equality. As suggested by Schutz (1951), 'equality of income distribution is found when every income-receiving unit receives its proportional share of the total income' (Schutz, 1951, p. 107).

In addition to the neutrality that characterizes the measure of inequality and the full range of indicators of poverty and richness derived from it, the approach developed throughout this section has other appealing features: (i) it is an integrated perspective of inequality, poverty, and richness; (ii) the simplicity of calculation of the suggested indicators; (iii) their decomposability, allowing the identification of the contribution of population's sub-groups; and (iv) the specific economic interpretation of the values obtained, as opposed to most available measures, whose values can only be interpreted by comparison with figures for other countries or periods. The latter is an important feature to the extent that the several indices provide useful information for the definition of policy interventions.

Regarding the methodological questions presented in the previous section, we assume the most common choices concerning the second and third issues – households as recipient units of income and an equivalence scale (namely the OECD modified scale) to account for the existence of economies of scale – while following a different approach in relation to the

fourth and fifth questions previously mentioned. The first option is unnecessary for the present section. We will return to that subject in Section 3.

3.2 Income inequality

Taking into account the discussion of the previous section, our inequality index (*I*) is defined as:

$$I = \chi \sum_{i=1}^{N} \left| \psi_i - \lambda_i \right|, \tag{2}$$

in which:

$$\psi_i = \frac{Y_i}{\sum_{i=1}^{N} Y_i} \tag{3}$$

and

$$\lambda_i = \frac{D_i}{\sum_{i=1}^{N} D_i} \ . \tag{4}$$

N is the total number of households, Y_i represents the total income of household *i*, and D_i expresses the number of equivalent adults in that household. Thus, ψ_i is the income weight of household *i* and λ_i its weight in terms of equivalent adults. There will be an equality situation

in the income distribution when all households have an income share equal to their share in terms of equivalent adults, that is, when $\forall i, \psi_i = \lambda_i$.

If we set $\chi = 0.5$, the possible values for I are in the range [0,1[. This value for χ allows a more intuitive interpretation of the results and is thus more adequate than alternatives such as $\chi = 1$, according to which I would vary between 0 and 2. The open range at right is due to the fact that the value of 1 corresponds to a situation where the full amount of income is held by households of a zero dimension, an impossible case.

Taking into account the inequality measure, I, we can deepen the analysis, proposing poverty and richness measures. The first step is to set criteria to define if household i is poor (P), rich (R) or if it is in an intermediate situation, what we will call middle class (MC).[3] These criteria are based on the comparison between what the household has in terms of income with what it should have, considering its dimension and composition, in order to obtain an equal distribution of resources:

$$S_i = \begin{cases} \text{R if } \dfrac{\psi_i}{\lambda_i} > \upsilon \\ \text{MC if } \dfrac{1}{\beta} \leq \dfrac{\psi_i}{\lambda_i} \leq \upsilon \\ \text{P if } \dfrac{\psi_i}{\lambda_i} < \dfrac{1}{\beta} \end{cases}, \tag{5}$$

in which $\beta, \upsilon \geq 1$.

Once we classify each household according to its position in the income distribution, we can obtain aggregated measures of poverty and richness.

[3] A similar classification is adopted, for instance, by Eisenhauer (2011).

3.3 Poverty

As seen above, a detailed analysis of poverty should take into account three dimensions: incidence, intensity, and severity. Following the approach presented in the previous section, we now propose poverty measures that focus on each of these dimensions. Additionally, we discuss the case of the near poor.

We start by defining a measure of poverty incidence, *POV*. Defining H_i as the number of individuals of household *i*, then:

$$POV = \frac{\sum_{i=1}^{N} H_i}{\sum_{i=1}^{N} H_i}_{(S_i=P)}. \quad (6)$$

POV is a headcount index, indicating the percentage of individuals that belong to poor households in relation to the total number of individuals.

Following, we define an index of poverty intensity (*POV'*). Let us start by calculating θ_i, which expresses the percentage of the total income in the economy that household *i* would have to receive to become non-poor:

$$\theta_i = \frac{\lambda_i}{\beta} - \psi_i. \quad (7)$$

Thus, *POV'* corresponds to the percentage of the total income in the economy that needs to be transferred from the non-poor to the poor in order to eradicate poverty:

$$POV' = \sum_{\substack{i=1 \\ (S_i=P)}}^{N} \theta_i \ . \tag{8}$$

If we divide *POV'* by the number of poor households, we obtain an indicator of the average intensity of poverty.

The third dimension of poverty that needs to be taken into account is its severity. To capture this dimension, we consider a set of indicators that aim to reflect different aspects of the phenomenon. In this context, the first step of our analysis is the definition of a new poverty threshold reflecting a higher degree of resource privation. Therefore, a situation of extreme poverty is defined as:

$$S_i = SP \ \text{if} \ \frac{\psi_i}{\lambda_i} < \frac{1}{\zeta\beta}, \tag{9}$$

in which $\zeta > 1$.

With reference to this line of extreme poverty, we can quantify the incidence and intensity of severe poverty. The incidence of severe poverty can be defined in relation to either the total population or the poor population, being expressed, respectively, as follows:

$$S - POV(1) = \frac{\sum_{\substack{i=1 \\ (S_i=SP)}}^{N} H_i}{\sum_{i=1}^{N} H_i} \tag{10}$$

and

$$S-POV(2)=\frac{\sum_{\substack{i=1\\(S_i=SP)}}^{N} H_i}{\sum_{\substack{i=1\\(S_i=P)}}^{N} H_i}. \qquad (11)$$

In a similar vein, the intensity of severe poverty can be calculated by reference to either the poverty line or the severe poverty line, being measured, respectively, as:

$$S-POV'(1)=\sum_{\substack{i=1\\(S_i=SP)}}^{N}\theta_i \qquad (12)$$

and

$$S-POV'(2)=\sum_{\substack{i=1\\(S_i=SP)}}^{N}\omega_i\,, \qquad (13)$$

in which θ_i corresponds to its expression in (7) and:

$$\omega_i=\frac{\lambda_i}{\zeta\beta}-\psi_i\,. \qquad (14)$$

The measures of severe poverty intensity express the percentage of the total income in the economy that would be necessary to transfer to the extreme poor in order to take them out of poverty (in the case of *S-POV'(1)*) or severe poverty (in the case of *S-POV'(2)*).

To complement the analysis of the severity of poverty, we can calculate an inequality index among poor (I_P):

$$I_P = k \sum_{\substack{i=1 \\ (S_i = P)}}^{N} \left| \eta_i - \rho_i \right|, \tag{15}$$

in which:

$$\eta_i = \frac{Y_i}{\sum_{\substack{i=1 \\ (S_i = P)}}^{N} Y_i}, \tag{16}$$

and

$$\rho_i = \frac{D_i}{\sum_{\substack{i=1 \\ (S_i = P)}}^{N} D_i}. \tag{17}$$

This indicator quantifies the percentage of the total income of poor households that has to be re-affected among them in order to obtain an equal intensity of poverty. An increase in I_P reflects higher levels of poverty severity.

Finally, let us consider the case of the near poor. An effective poverty policy cannot focus only on the poor, but should, in line with the analysis of poverty vulnerability (Pritchett *et al*., 2000; Guimarães, 2007; Zhang and Wan, 2009; Dutta *et al*., 2011), also give special attention to those who are very near of being poor in order to avoid new poverty cases. Accordingly, we propose measures to capture the importance of this phenomenon. We define:

$$S_i = P^+ \text{ if } \frac{1}{\beta} \leq \frac{\psi_i}{\lambda_i} < \varepsilon, \tag{18}$$

in which $\frac{1}{\beta} \leq \varepsilon < 1$.

Near-poverty incidence, representing the percentage of total individuals that belong to near-poor households, is given by:

$$POV^{+} = \frac{\sum_{i=1}^{N} H_i}{\sum_{i=1}^{N} H_i} \quad (S_i = P^{+}) \,. \tag{19}$$

In this context, it is also interesting to know the safety net of the near-poor population. For household *i*, that safety margin is given by the symmetric of θ_i. In overall terms, we quantify this as:

$$POV'^{+} = \sum_{\substack{i=1 \\ (S_i = P^{+})}}^{N} \left(\psi_i - \frac{\lambda_i}{\beta} \right) = \sum_{\substack{i=1 \\ (S_i = P^{+})}}^{N} (-\theta_i) \,, \tag{20}$$

expressing the percentage of the total income in the economy by which the near-poor are above the poverty line. The average safety margin of near-poor can be obtained dividing POV'^{+} by the number of near-poor households.

3.4 Richness

The indicators used in the analysis of poverty can be adapted for the measurement of the corresponding richness dimensions. We thus conceive incidence, intensity, and severity

measures of richness.[4] For terminological reasons, we opt to designate the last case as 'richness depth'.

We define *RICH* as the ratio between the number of individuals in rich households and the total number of individuals:

$$RICH = \frac{\sum_{\substack{i=1 \\ (S_i=R)}}^{N} H_i}{\sum_{i=1}^{N} H_i}. \qquad (21)$$

To obtain a measure of richness intensity, we define:

$$\delta_i = \psi_i - \upsilon\lambda_i. \qquad (22)$$

Then, richness intensity is given by:

$$RICH' = \sum_{\substack{i=1 \\ (S_i=R)}}^{N} \delta_i, \qquad (23)$$

representing the percentage of the total income in the economy according to which the rich are above the richness line. Dividing *RICH'* by the number of rich households we can obtain the average intensity of richness.

Finally, we attend to richness depth. We do so using the same approach we have applied in the poverty case. As a first step, we define an extreme richness line, above which households are classified as extremely rich:

[4] On the definition of the richness line, see Medeiros (2006).

$$S_i = ER \text{ if } \frac{\psi_i}{\lambda_i} > \sigma\upsilon, \tag{24}$$

in which $\sigma>1$.

The incidence of extreme richness can be expressed in relation to either the total population or the rich population. In each case we have, respectively:

$$E-RICH(1) = \frac{\sum_{\substack{i=1 \\ (S_i=ER)}}^{N} H_i}{\sum_{i=1}^{N} H_i} \tag{25}$$

and

$$E-RICH(2) = \frac{\sum_{\substack{i=1 \\ (S_i=ER)}}^{N} H_i}{\sum_{\substack{i=1 \\ (S_i=R)}}^{N} H_i}. \tag{26}$$

In turn, taking as reference either the richness line or the extreme richness line, the intensity of extreme richness can be defined respectively as:

$$E-RICH'(1) = \sum_{\substack{i=1 \\ (S_i=ER)}}^{N} \delta_i \tag{27}$$

and

$$E-RICH'(2) = \sum_{\substack{i=1 \\ (S_i=ER)}}^{N} \varphi_i, \tag{28}$$

where δ_i is expressed in (22) and:

$$\varphi_i = \psi_i - \sigma\upsilon\lambda_i . \tag{29}$$

As in the case of the severity of poverty, we can also calculate an inequality measure applied exclusively to the rich population, aiming to determine the amount of income that it is necessary to redistribute among the rich in order to equalize the distance of each rich individual to the richness line:

$$I_R = \phi \sum_{\substack{i=1 \\ (S_i=R)}}^{N} \left| \varpi_i - \mu_i \right| , \tag{30}$$

in which:

$$\varpi_i = \frac{Y_i}{\sum_{\substack{i=1 \\ (S_i=R)}}^{N} Y_i} , \tag{31}$$

and

$$\mu_i = \frac{D_i}{\sum_{\substack{i=1 \\ (S_i=R)}}^{N} D_i} . \tag{32}$$

3.5 Middle class inequality

As mentioned above, the evaluation of richness has recently joined the well-established analyses of inequality and poverty. Least explored has been the study of income distribution in the middle class. However, this is also a relevant issue since the degree of inequality in this income group is an important indicator of countries' economic and social cohesion.[5]

To conceive such an indicator, we start by focusing on households in which S_i = MC, calculating:

$$I_{MC} = \tau \sum_{\substack{i=1 \\ (S_i=MC)}}^{N} \left| \vartheta_i - o_i \right|, \tag{33}$$

in which:

$$\vartheta_i = \frac{Y_i}{\sum_{\substack{i=1 \\ (S_i=MC)}}^{N} Y_i}, \tag{34}$$

and

$$o_i = \frac{D_i}{\sum_{\substack{i=1 \\ (S_i=MC)}}^{N} D_i} \quad . \tag{35}$$

[5] On this topic see Gigliarano and Mosler (2009) or Winkelmann and Winkelmann (2010). For a discussion on the quantitative limits and socio-economic characteristics of the middle class see also Atkinson and Brandolini (2011) and Eisenhauer (2011).

I_{MC} is an income inequality measure for the middle class, indicating the percentage of the total income of the middle class that, if adequately redistributed among middle class households, would eliminate the inequality in this income group.

4. Inequality, poverty, and richness – an application with evidence from Portugal

4.1 Data and empirical evidence

In order to illustrate the application of the set of measures presented in the previous section, we consider data from Portugal, since it is among the European countries with the highest levels of inequality and poverty. According to the European Union Statistics on Income and Living Conditions (EU-SILC), in 2008, Portugal was the fourth country in the EU-27 with the highest level of inequality and the fifth country in the EU-15 with the highest level of poverty (10^{th} position considering the EU-27).

We use micro-data on the income and structure of households living in Portugal from the Office of National Statistics (INE)'s Household Budget Survey (IDEF).[6] We use the last available wave of that survey, of 2005/2006.[7] The results are based on a representative sample of the Portuguese economy with 10,403 households and a total of 28,359 individuals. The IDEF is a large-dimension survey associated with a questionnaire filled in by households with detailed information on the whole set of collective and individual expenditures. It also includes demographic data, income data, and data on non-frequently consumed goods and services.

[6] Statistics on household budgets is information followed at an European level – Household Budget Survey.
[7] This is the seventh wave of this type of survey in Portugal. The first goes back to the 1967-68 period. The following waves have enlarged the range of covered questions, allowing a more detailed analysis of the population's living conditions.

The subsequent analysis takes into account not only monetary income but also total income. The comparison of the results is particularly important for two reasons: (i) the relative weight of non-monetary income (approximately 19% of total income); and (ii) the asymmetry in the non-monetary income distribution.

Table 2 presents the results of the application of the proposed indicators taking as a reference the following values for the parameters: χ=0.5, β=2, υ=2, ζ=2, κ=0.5, ε=0.6, σ=2, ϕ=0.5, and τ=0.5.

[Table 2]

Focusing on the results based on total income, we find the need to redistribute 23.78% of the total income in the economy to reach a situation of equality in income distribution. It is important to note, however, that such an overall value presupposes an adequate redistribution of income, that is, one that does not waste resources.

Regarding the distribution of individuals by income groups, we conclude that 17.78% are poor, 7.03% are rich, and the remaining 75.19% are from the middle class. Concentrating on the bottom of the income distribution, we see that 10.41% of the poor (corresponding to 1.85% of the total population) face a situation of severe poverty. Additionally, individuals that can be classified as near-poor comprise 10.52% of the total population. Finally, when focusing on the top of the income distribution, we identify 10.98% of the rich (0.77% of the total population) exhibiting an extreme richness situation.

The analysis of poverty intensity allows us to conclude that a value equivalent to 2.09% of the total income in the economy is necessary to eliminate it. That amount includes a fraction of 0.54% of the total income in the economy corresponding to what is necessary in order to eliminate severe poverty situations and thus raising those households to the poverty line level.

Only 0.09% of the total income in the economy would be needed to improve the situation of these households to the level of the severe poverty line. A complementary way of analyzing the level of inequality among the poor population is to apply an inequality measure exclusively to the poor. In that case, we observe a need to re-affect (at least) 9.58% of the poor income to remove that inequality and thus have the different poor households at the same distance from the poverty line. In addition, the near-poor possess, as a whole, a safety net equivalent to 0.54% of the total income.

Concerning the evaluation of richness, the income surplus from the richness line equals 7.18% of the total income. A value equivalent to 3.17% of that total income is the amount needed to reduce the income of the extremely rich to the richness line level. That income reduction to the level of the extreme richness line implies the movement of 1.59% of the total income in the economy. The measurement of richness inequality indicates the need to re-affect 13.88% of the total income of the rich population in order to eliminate that inequality.

Finally, looking at the middle class, we find that 14.97% of the income in middle-class households would have to be redistributed among them to ensure total income equality for the middle-class.

The concrete results obtained naturally depend on the values assumed for the different parameters. However, there are no valid reasons to unequivocally support certain values for those parameters, namely the ones that are a reference for the definition of the income groups. They are explicitly and subjectively defined by the researcher and thus sensitivity analyses based on alternative values are welcomed. A preliminary analysis of the kind is presented in annex Table A.1 considering other values for β, υ, and ε.[8]

[8] A more in-depth analysis of the issue should also consider alternative values for ζ and σ.

4.2 Decomposition by households' characteristics – an example

As stressed above, the measures proposed in Section 2 allow their decomposition by any household's characteristic, such as type of household (dimension and composition), region of residence, or variables associated with the individual of reference of that household (i.e., the individual with the largest proportion of the annual net total income of the household), such as age, gender, educational level, labour market state, amongst others. We have conducted a decomposition by region of residence to illustrate that possibility. This exercise allows focusing on the existence of regional inequalities in Portugal for the dimensions analysed in this paper.

[Table 3]

Table 3 illustrates the decomposition by region of all the measures calculated in Table 2. Additionally, the last row presents evidence in relation to $\sum_{i=1}^{N}(\psi_i - \lambda_i)$, allowing us to emphasize the regions where households' weight in terms of income exceeds their respective weight in terms of dimension.

The reading of both incidence and intensity indicators is immediate. The value corresponding to each region should be interpreted in the same way as the overall indicator, though applied exclusively to the given region. Let us consider the poverty indicators as examples. Regarding *POV*, we found a poverty incidence at the national level of 17.78%. A disaggregation by regions reveals that 3.99% of the individuals from the sample are poor living in the *Norte* region, 2.60% in the *Centro*, 1.47% in the region of *Lisboa*, etc. Adding up the values of the different regions we obtain the incidence of poverty at the national level. We should note

however that this concept is different from the measurement of poverty incidence within the context of each given region.

In the same vein, regarding *POV'*, we can say, for instance, that the amount necessary to eradicate poverty in the *Algarve* corresponds to (at least) 0.25% of the total income in the economy, while for *Madeira* that value is equivalent to 0.40% of the total income in the economy. In the national total, as has been seen, a mobilization of 2.09% of the total income in the economy is needed to overcome poverty. The interpretation made for the values of *POV* and *POV'* is also valid for the other incidence or intensity indicators – *S-POV(1)*, *S-POV(2)*, *S-POV'(1)*, *S-POV'(2*), POV^{+}, POV'^{+}, *RICH*, *RICH'*, *E-RICH(1)*, *E-RICH(2)*, *E-RICH'(1)*, and *E-RICH'(2)*.

Concerning the inequality indicators (I, I_P, I_R, and I_{MC}), the value for each region expresses half of the deviation assigned to households of that region in relation to an egalitarian situation (having the same weight in terms of income and equivalent adults). So, for instance, taking into account the overall indicator of inequality (I), the deviation from the egalitarian situation of households living in the *Norte* region equals 8.82% of the total income in the economy.

The percentage of the total income in the economy needed to eradicate inequality within each region cannot, in this case, be identified, because there are inter-regional transfers of income apart from intra-regional transfers. The transfers between regions are net positive amounts transferred to other regions when $\sum_{i=1}^{N}(\psi_i - \lambda_i) > 0$ and net positive amounts received from other regions in the opposite case.

Finally, looking at the last row of Table 3, we can identify three regions (*Lisboa, Algarve, and Açores*) in which the weight in overall income is greater than the corresponding weight in equivalent adults. *Lisboa* – the most developed region in the country – has the greatest difference. On the contrary, *Madeira* shows the most significant negative deviation.

5. Final remarks

The main contribution of this paper is the proposal of an integrated approach for the measurement of inequality, poverty, and richness. We have proposed a set of indicators characterized by their simplicity in application, neutrality, and decomposability. Another important characteristic of the measures proposed in this study is the fact that they allow a concrete economic interpretation of the results, thereby contributing to a more adequate definition of social policies.

The proposed measures were applied, for illustrative purposes, to the Portuguese economy. Taking total income as a reference, that application has identified 17.78% of individuals in poor households, 7.03% in rich households, and the remaining 75.19% in the middle class. A severe poverty situation was found in 1.85% of the individuals analysed (10.41% of the poor). Particularly important in quantitative terms is the near-poverty phenomenon, accounting for 10.52% of the population. Concerning inequality, we have calculated the need to re-affect (at least) 23.78% of the total income in the economy to reach a full equality situation. With a focus on poverty intensity, we conclude that 2.09% of the total income in the economy is the amount needed to be transferred from the non-poor to the poor in order to eradicate poverty. Additionally, the comparison between the results obtained with monetary income and total income stresses significant differences, highlighting the importance of taking into account this last concept of income.

The proposed measures can be decomposed with reference to a given characteristic of the household. We have illustrated that property by considering a regional decomposition that includes the seven Portuguese NUTS II regions. In that analysis we found the region of

Lisboa (the most developed region in the country) to be the most favourable in terms of poverty and richness.

Regarding the topics developed in this paper, important research avenues remain. In methodological terms, the main challenge resides in testing the robustness of the results based on alternative values for the parameters in order to check the sensitivity of these results. This is especially important for the parameters that distinguish the main income categories (β, υ, ζ, σ, and ε), that is, poor, rich, middle class, severe poor, extremely rich, and near-poor.

In applied terms, cross-country comparative studies also enable raising the knowledge on the phenomena under examination for a wide range of countries with distinct characteristics. The same comparative analysis could be conducted at the regional level, emphasizing the regional inequalities prevailing within a given country

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Table 1: Inequality concepts and the principles proposed by Fields and Fei (1978)

Principles / Inequality concept	Symmetry	Population size independence	Mean independence	Pigou-Dalton transfers principle
Concept 1	Yes	Yes	No	No
Concept 2	Yes	Yes	Yes	No
Concept 3	Yes	Yes	Yes	Yes
Concept 4	Yes	Yes	No	Yes

Table 2: Inequality, poverty, and richness indicators for Portugal (%)

	Monetary income	Total income
Inequality		
I	26.14	23.78
Poverty		
POV	21.85	17.78
POV'	2.99	2.09
S-POV(1)	3.13	1.85
S-POV(2)	14.31	10.41
S-POV'(1)	1.00	0.54
S-POV'(2)	0.20	0.09
I_P	11.04	9.58
POV+	10.42	10.52
POV'+	0.52	0.54
Richness		
RICH	7.94	7.03
RICH'	9.15	7.18
E-RICH(1)	1.09	0.77
E-RICH(2)	13.77	10.98
E-RICH'(1)	4.56	3.17
E-RICH'(2)	2.33	1.59
I_R	15.14	13.88
Middle class - inequality		
I_{MC}	15.19	14.97

Source: own calculations based on IDEF

Table 3: Regional decomposition of inequality, poverty, and richness indicators

Region Index	*Norte*	*Centro*	*Lisboa*	*Alentejo*	*Algarve*	*Açores*	*Madeira*	∑
Inequality								
I	4.41	3.45	4.09	3.04	3.40	2.44	2.97	23.78
Poverty								
POV	3.99	2.60	1.47	2.44	2.02	2.02	3.23	17.78
POV'	0.42	0.33	0.19	0.30	0.25	0.20	0.40	2.09
S-POV(1)	0.45	0.21	0.16	0.20	0.21	0.14	0.48	1.85
S-POV(2)	2.54	1.19	0.87	1.13	1.17	0.79	2.72	10.41
S-POV'(1)	0.13	0.07	0.05	0.07	0.06	0.04	0.13	0.54
S-POV'(2)	0.021	0.011	0.011	0.013	0.010	0.005	0.021	0.09
I_P	2.13	1.44	0.78	1.44	1.08	0.91	1.80	9.58
POV^+	2.44	1.76	0.56	1.79	1.21	1.04	1.71	10.52
POV'^+	0.13	0.09	0.03	0.10	0.06	0.05	0.08	0.54
Richness								
RICH	1.13	0.84	1.87	0.69	1.16	0.81	0.52	7.03
RICH'	0.97	1.00	2.54	0.47	0.99	0.85	0.38	7.18
E-RICH(1)	0.13	0.09	0.31	0.04	0.09	0.09	0.03	0.77
E-RICH(2)	1.81	1.30	4.36	0.50	1.30	1.30	0.40	10.98
E-RICH'(1)	0.41	0.48	1.32	0.11	0.30	0.46	0.08	3.17
E-RICH'(2)	0.16	0.27	0.69	0.04	0.11	0.28	0.03	1.59
I_R	2.10	1.81	4.33	1.05	1.96	1.90	0.73	13.88
Middle class - inequality								
I_{MC}	2.85	2.21	1.96	2.21	2.29	1.40	2.05	14.97
$\sum_{i=1}^{N}(\psi_i - \lambda_i)$	-1.74	-0.72	4.37	-1.22	0.96	0.34	-1.99	0

Source: own calculations based on IDEF.

Annex

Table A.1: Inequality, poverty, and richness indicators for Portugal using both monetary and total income – sensitivity tests

	$\beta = 2.1; \upsilon = 2.1; \varepsilon = 0.6$		$\beta = 1.9; \upsilon = 1.9; \varepsilon = 0.6$		$\beta = 2.1; \upsilon = 1.9; \varepsilon = 0.55$	
	Monetary income	Total income	Monetary income	Total income	Monetary income	Total income
Inequality						
I	26.14	23.78	26.14	23.78	26.14	23.78
Poverty						
POV	19.43	15.46	24.54	20.45	19.43	15.46
POV'	2.49	1.69	3.62	2.59	2.49	1.69
S-POV(1)	2.69	1.60	3.58	2.22	2.69	1.60
S-POV(2)	13.87	10.33	14.58	10.85	13.87	10.33
S-POV'(1)	0.82	0.43	1.21	0.68	0.82	0.43
S-POV'(2)	0.16	0.07	0.24	0.12	0.16	0.07
I_P	10.96	9.50	11.11	9.66	10.96	9.50
POV+	12.84	12.84	7.73	7.85	2.45	2.40
POV'+	0.80	0.82	0.29	0.30	0.16	0.15
Richness						
RICH	7.18	6.22	8.96	8.08	8.96	8.08
RICH'	8.39	6.51	9.99	7.94	9.99	7.94
E-RICH(1)	0.94	0.67	1.29	0.92	1.29	0.92
R-RICH(2)	13.07	10.82	14.36	11.35	14.36	11.35
E-RICH'(1)	4.12	2.90	5.08	3.55	5.08	3.55
E-RICH'(2)	2.13	1.45	2.57	1.77	2.57	1.77
I_R	15.01	13.77	15.35	14.05	15.35	14.05
Middle class - inequality						
I_{MC}	15.98	15.72	14.27	14.10	15.16	14.90