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A METHOD FOR SOLVING A CLASS OF MULTIPLE-CRITERIA ANALYSIS PROBLEMS

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The paper proposes an interactive method solving the multiple criteria choice problem (MCCP) with a large number of discrete alternatives and a small number of quantitative criteria. The decision maker (DM) sets his preferences in terms of desired directions of improving or relaxing of the criteria. On this base the so called reference cone is constructed. A small subset of relatively closed alternatives is defined according to this cone and to the maximal deterioration of the criteria values at each iteration. This subset is evaluated by the DM, who selects the most preferred alternative or enters his/her new preferences.

The method suggested has user-friendly dialog. It enables the DM to explore the set of alternatives comparatively quickly and easy. The method is included in a DSS. It is tested by a number of real multiple criteria choice problems.

Keywords: decision making, discrete multicriteria problems, reference cone.

AMS subject classification: 90C29

1 Introduction

The problems of multicriteria decision making (MCDM), that begin to play a still more important part in operation research area, can be divided in two basic classes [16], [15]: multiple objective mathematical programming problems (MOMP), in which a finite number of explicitly set constraints define implicitly an infinite number of alternatives, the constraints and objective functions being defined by mathematical expressions, and problems of multicritreria decision analysis (MCDA), in which a finite number of explicitly set alternatives is represented into a table form.

The purpose of the interactive MOMP methods is to generate a nondominated (efficient) solution, which satisfies mostly the DM's preferences in the mode of a dialogue between the computing procedure and the DM on the basis of different types of scalarizing techniques. This solution is usually called the best compromise or the best preferred solution. In order to achieve its selection, the DM sets his/her preferences at each iteration and they serve to improve the solution currently found. During the search of the best preferred solution in these methods, it is assumed that the DM optimizes an explicitly or implicitly set value (utility) function or that by learning during the search process he/she tries to satisfy to the highest degree his/her aspirations concerning the values of the separate criteria (the aspiration levels of the criteria). In the two types of methods convergence of the solution process is presumed. Mathematical convergence of the computing process is ensured in the first type of methods called "search methods". In the second type of methods, named "learning oriented methods", behavioral or intuitive convergence of the solution process is expected, which is ensured by the DM. In both types of methods it is assumed that the DM can easily compare two alternatives and in this way estimate whether to prefer one of them or whether the two are equivalent for him/her (indifference). Because of the fact that when solving MOMP problem the purpose is to find the best preferred solution, MOMP problems can be called multicriteria choice problems.

The problems of MCDA can be classified in three main groups. The best preferred nondominated alternative is searched for in the first group of problems (choice problem – MCCP). In the second group of problems, ranking of the nondominated alternatives is established, starting from the best towards the worst one (ranking problem). In the third group of problems the set of alternatives is divided into separate groups (sorting problem). The MCDA problems usually contain a finite number of alternatives and criteria, but they can be of considerable number as well. When the alternative number is large, the DM can hardly perceive these alternatives as a whole, which makes these problems comparatively close to MOMP problems. When the criteria number is large, the DM cannot perceive these criteria as a whole either, particularly when some of them are contradicting. That is why some procedures (techniques) are included in the main MCDA methods, which compare two nondominated alternatives based on preference information set by the DM or more exactly based on DM preference model. Two types of DM preference models are used in the main MCDA methods. The first type of a preference model enables the comparison of every pair of alternatives. This type of a preference model is the utility function. The main methods using this type of DM preference model to different extent are the multiattribute utility theory methods and the analytical hierarchy process methods (see [3], [6], [2] and [13]). The second preference model allows the existence of incomparable alternatives, when the preference information obtained by the DM is insufficient to determine whether one of the alternatives is to be preferred or whether the two alternatives are equal for the DM. Such a preference model is the outranking relation. The methods using this type of a preference model are called outranking methods (see [10], [11] and [1]).

In spite of the continuous development of the MCDA methods discussed, including their interactive variants, there still remain some serious problems, two of them being the following:

– difficulties connected with the design of procedures, user-friendly to the DM. The DM is responsible for the obtaining of the final solution and he must well comprehend these procedures in order to be sure in the quality of the final solution;

– difficulties connected with the possibilities for DM training with respect to the problems solved, especially problems with a large number of alternatives and criteria. This refers to the relative importance of the criteria, the correlation among the criteria, the possibilities for compensation, scaling effects, amplitude of deviations among the criteria values and others.

In order to solve multicriteria choice problems with a large number of alternatives and a small number of quantitative criteria, which can be regarded by the DM as problems close to MOMP problems, the so called "optimizationally motivated" interactive methods, inspired by MOMP methods, have been suggested. On account of the higher pressure and engagement of the DM, some of the difficulties above described are overcome in these methods (see [4], [7], [9], [5], [8] and [14]).

The paper offers learning oriented interactive method designed to solve multicriteria choice problems (MCCP) with a large number of alternatives and a small number of quantitative criteria, which is similar to the above given optimizationally motivated methods,but in this method the DM interference is considerably decreased. This is achieved introducing two main alterations. The first one consists in the fact, that to improve the alternative currently found, the DM is obliged to give the desired directions of the criteria alteration only. The second change is that the DM is provided with a few (sometimes only two) close or comparable alternatives for comparison. In this way he can take into consideration more definitely and realistically the criteria importance, their correlation, the possibilities for compensation among them, as well as to estimate better the criteria values in the alternatives being compared. Besides this, taking into account the current preferred solutions at each iteration, the DM can compare the distributed alternatives also. In this way the interactive method suggested enables also the combination of efficient search and DM learning in the set of the nondominated alternatives.

The paper is organized in the following way. Some notations and definitions are introduced in the next section. A general description of the method is given in Section 3, and its algorithmic scheme is shown in Section 4. Section 5 represents one illustrative example. The advantages of the method are summarized in the conclusion.

2 Preliminary considerations

The multiple criteria choice problem can be defined as follows:
A set I of n (> 1) of deterministic alternatives and a set J of k (≥ 2) criteria be given which define a $n \times k$ decision matrix A.

By criterion we mean any concept used by the DM to compare the available alternatives. A criterion is quantitative when its magnitude can be described on a numerical scale and the preference or utility function of the DM is monotonous on this scale.

The element a_{ij} of the matrix A denotes the evaluation of the alternatives $i \in I$ with respect to the criterion $j \in J$. The evaluation of the alternative $i \in I$ with respect to all the criteria in the set J is given by the vector $(a_{i1}, a_{i2}, \ldots, a_{ik})$.

The assessment of all the alternatives in the set I for the criterion $j \in J$ is given by the column vector $(a_{1j}, a_{2j}, \ldots, a_{nj})$.

A unique solution of the above problem in the criteria space R^k rarely exists because of conflicting nature of criteria in most cases. Therefore, a conception of satisfactory (best compromise and so on) solution is used in solving MCCP. This solution is searched through the set of non-dominated alternatives.

The alternative $i \in I$ is called non-dominated if there is no other alternative $r \in I$ for which $a_{rj} \geq a_{ij}$ for all $j \in J$ and $a_{rj} > a_{ij}$ for at least one $j \in J$.

Further, the following definitions are introduced.

A current preferred alternative is a non-dominated alternative the DM chooses as the best one from the set presented at the current iteration with respect to the set of the criteria.

The alternative preferred best (a best compromise alternative, a final alternative) is a preferred alternative that satisfies the DM to the greatest degree.

Desired directions of change for the criteria at each iteration are the directions (of improving or relaxing), along which the criteria values are acceptable for the DM.

A reference cone is a convex cone, in which k generators are defined by the desired directions of change for the criteria in a current preferred alternative. A reference cone contains of all the alternatives, for which the sign of the differences between the values for each criterion and the preference alternative coincide with the changes of the last one desired by the DM.

3 Description of the proposed method

The idea of the presented here method is to generate at each iteration *iter* a ranked set $M = \{m_1, m_2, \ldots, m_p\}$ of alternatives, the first alternative being the current preferred alternative. There p is the number of generated alternatives he/she is willing or is able to evaluate at the each iteration The DM has to estimate the relatively close alternatives of this set and to choose one of them either as a current preferred or as the best preferred alternative. In the second case the discrete multiple criteria choice problem is solved. In the first case on the basis of the preferred alternative selected the DM sets the desired changes of the criteria (in terms of desired directions for improving/relaxing) in order to search for a new better alternative with respect to all the criteria.

Let denotes

L_h – the set of indices $j \in J$ of the criteria, for which the DM wishes to increase their values in comparison with the values in the current preferred alternative, its index being assigned to h.

E_h – the set of indices $j \in J$ of the criteria for which the DM is inclined to deteriorate their values in comparison with the values in the current preferred alternative h.

$$L_h \cup E_h = J.$$

Let a_i denotes the point (vector) of the criteria space R^k, which corresponds to the alternative with an index $i \in I$. When the DM sets the desired directions of change of the criteria values, the defining of the set M by alternatives neighboring to the current preferred alternative h can be determined in space R^k on the basis of the alternatives allocation with respect to a convex cone with a vertex in the current preferred alternative. This cone is called a reference cone. The generators of the reference cone are defined on the basis of the directions desired by the DM for change of the criteria values they have in the current preferred alternative. The reference cone $V(h)$ has $q \leq k$ generators $v^1, \dots, v^s, \dots, v^q$ and may be defined as follows:

$$V(h) = \left\{ v \in R^k \quad v = a_h + \sum_{s \in J} \beta_s v^s, \quad \beta_s \geq 0 \right\},$$

The components v_j^s of the generator v^s are defined according to the relations:

$$v_j^s = \begin{cases} 0, & \text{if} \quad j \neq s; \\ 1, & \text{if} \quad j = s \quad \text{and} \quad s \in L_h; \\ -1, & \text{if} \quad j = s \quad \text{and} \quad s \in E_h. \end{cases}$$

Further, for every alternative $i \in I$, $i \neq h$, a vector μ^i is entered, whose components are defined as follows:

$$\mu_j^i = \begin{cases} 1, & \text{if} \quad a_{ij} \geq a_{hj} \\ -1, & \text{otherwise.} \end{cases}, \ j \in J$$

Let the distance $d(V(h), i)$ between the reference cone $V(h)$ and every alternative $i \in I/i \neq h$ be defined as:

$$d(V(h), i) = \sum_{j \in J} |v_j^i - \mu_j^i|/2.$$

From mathematical point of view, $d(V(h), i)$ shows the number of directions of change for the criteria by which the alternative $i \in I/i \neq h$ differ from every alternative that belongs to the cone $V(h)$. It is obvious that these alternatives have a distance equal to zero. From a view point of the multicriteria choice problem total number of the directions along which the alternative with the index $i \in I/i \neq h$ differs from each alternative belonging to the reference cone $V(h)$ is not so important as the number of directions, where these two alternatives differ, having in mind the criteria the values of which the DM wants to improve. This number is given by the distance $d'(V(h), i)$, defined as follows:

$$d'(V(h), i) = \sum_{j \in L_h} |v_j^i - \mu_j^i|/2, \quad \text{for } i \in I$$

If the alternative i belongs to the cone $V(h)$, then $d'(V(h), i) = 0$. The following procedure can be used for generating the set M at iteration *iter*: At first, rank the alternatives into no more than $|L_h| + 1$ groups G_l, $l = 1, \dots, |L_h| + 1$, according to their distance $d'(V(h), i)$ and choose the first nonempty one. Here $|L_h|$ denotes the number of elements in the set L_h. Let $G_l(h)$ is this group. Further, rank the alternatives from $G_l(h)$ in ascending order according to the value of the parameter $t(i, h)$, defined as follows:

$$t(i, h) = \max_{j \in J} (a_{hj} - a_{ij})^+ / w_j,$$

where
$w_j = a_{\max j} - a_{\min j}$, $j = 1, 2, \ldots, k$;
$a_{\max j}$ is the maximal element in the j-th column of input matrix A;
$a_{\min j}$ is the minimum element in the j-th column of input matrix A;
$(a_{hj} - a_{ij})^+ = \max(a_{hj} - a_{ij}, 0)$.

The set M contains the first $p-1$ alternatives of the group $G_l(h)$. These alternatives form the set M if their number is less than $p-1$.

Note that:

1. The parameter $t(i,h)$ shows the maximal deterioration that the alternative with an index i has with respect to the current alternative with an index h according to all of the criteria.

2. $w_j > 0$ always. If $w_j = 0$ for some j, then the j criterion is rejected in the initial phase of the algorithm.

4 The algorithmic scheme

On the basis of the distance function $d'(V(h), i)$, and parameter $t(i,h)$ an algorithmic scheme can be designed for solving the multiple criteria choice problem. With the help of this algorithm the DM has the possibility, setting desired directions of change of the criteria values, to estimate iteratively a small subset of alternatives. The alternatives of these subsets are to some extent close to the current preferred alternative. Thus in the learning process and after that the DM can take into mind such factors that can be hardly formalized. In order to assist the DM in the estimation of the alternatives from the set M it is useful to give additional parameters for each alternative from this set, such as the values of the function $d'(V(h), i)$ and parameter $t(i,h)$, the maximal deterioration of the criteria, which the DM agrees to be weakened, the criteria levels reached upto that moment, the maximal and minimal feasible values of the criteria and others. The visualization of the alternatives is particularly appropriate (bar graphs and so on).

The main steps of the algorithm proposed are as follows:

Step 1. Reject all the dominated alternatives and define the decision matrix A. Set $iter = 1$ and ask the DM to choose an initial preferred alternative and assign it to h. Check if $w_j = 0$ for some $j \in J$ and reject the corresponding criterion in this case.

Step 2. If the DM wants to store the current preferred alternative h – check if it has been saved before and if it has not been saved – add h to $LIST$ – a set of stored preferred alternatives; otherwise – inform the DM and if this is the final choice – **Stop.**

Step 3. Ask the DM to define the desired directions for change of the values of the criteria according to h. The sets L_h and E_h are formed. L_h includes the indices of the criteria which the DM wishes to improve and E_h includes the indices of the criteria that the DM agrees to be weakened. Ask the DM to specify a parameter p – the number of generated alternatives he/she is willing or is able to evaluate at the next iteration.

Step 4. Rank the alternatives into the groups $G_l, l = 1, \ldots, |L_h + 1|$ according to their distance $d'(V(h), i)$ and choose the first non-empty one. Let $G_l(h)$ be this group.

Step 5. Rank the alternatives from $G_l(h)$ in ascending order according to $t(i, h)$ – a parameter, indicating the maximal deterioration of these alternatives with respect to all the criteria.

Step 6. Select the first $p - 1$ of them or all of them if their number is less than $p - 1$ and form the set M by adding the current alternative at the first place.

Step 7. Present the set M to the DM for evaluation.

Step 8. If the best compromise alternative has been found – **Stop;** otherwise – update $iter = iter + 1$.

Step 9. If the DM selects the new current preferred alternative – assign it to h and go to Step 2;

Step 10. If the DM wants to return to one of the stored alternatives – assign the selected stored alternative to h and go to Step 3;

Step 11. If the DM wants to update the preferential information not changing the current preferred alternative – go to Step 3.

Remark 1. Any alternative can be selected as an initial preferred alternative in Step 1. One acceptable initial preferred alternative can be found optimizing one criterion.

Remark 2. The rejecting of dominated alternatives is done once in the initial phase of the algorithm (Step 1). A number of algorithms are known – [14]. Their complexity is measured by $O(kn^2)$.

5 An illustrative example

A simple example will be considered in order to illustrate the algorithm proposed for solving the multiple choice problem. It is about evaluation and choice of a set of 10 industrial enterprises from the "Cellulose – Paper Industry" branch in the process of privatization in Bulgaria.

The four most characteristic ratios evaluated are: Assets turnover ratio, Liquidity ratio, Profitability on net sales and Gearing ratio. The ratios are scaled for convenience of computations. The alternatives (the choice of separate enterprise) are nondominated. The decision matrix A for this multiple attribute choice problem has the following form:

$i \backslash j$	1	2	3	4
1	134	125	60	-60
2*	618	107	7	-14
3**	348	122	20	-13
4	120	180	24	-28
5****	300	116	58	-13
6***	330	180	33	-36
7	240	90	25	-47
8	425	75	8	-28
9	129	173	5	-38
10	145	165	17	-15
w_j	498	136	55	47

Since the fourth criterion is minimized, the column A_4 is multiplied by (-1). Let us suppose that the second alternative is selected as the initial preferred alternative based on maximizing the first criterion. Set $iter = 1$ and $h = 2$.

Also, let the DM wants to improve the second and the third criteria and to relax the others criteria and he/she wants to evaluate 3 alternatives ($p = 3$) in the next iteration.

The sets $L_2 = \{2, 3\}$, $E_2 = \{1, 4\}$ are defined. The distances $d'(V(h), i)$ are computed for every alternative.

i	1	3	4	5	6	7	8	9	10
$d'(V(h), i)$	0	0	0	0	0	1	1	1	0
$t(i, h)$	0.98	0.542	1	0.638	0.578				0.949

In accordance with the values of $d'(V(h), i)$ the alternatives (Step 4) are devided into two groups: $G_1 = \{1, 3, 4, 5, 6, 10\}$ and $G_2 = \{7, 8, 9\}$ The alternatives of the first non-empty group G_1 are arranged in ascending order with respect to the values computed of the deviation parameter $t(i, h)$. $G_1(1) = \{3, 6, 5, 10, 1, 4\}$ The set M consists of the first two alternatives and the initial preferred alternative, i.e. – $M = \{2, 3, 6\}$. This set is presented to the DM at Step 7.

Let us suppose that the DM cannot make his/her final decision from this set M. At the second iteration $iter = 2$ he/she chooses the third alternative for the current preferred alternative – $h = 3$ and go to Step 2. The DM stores this alternative to $LIST = \{3\}$.

Then he/she sets the new preferences: to improve the first, second and third criteria and to deteriorate the fourth. The new sets $L_3 = \{1, 2, 3\}$, $E_3 = \{4\}$ are defined and $p = 4$ at this iteration.

Again, the distances $d'(V(h), i)$ are computed for every alternative.

i	1	2	4	5	6	7	8	9	10
$d'(V(h)i, i)$	1	2	1	2	1	2	2	2	2
$t(i, h)$	1		0.458		0.489				

The set G_l are formed: $G_1 = \{\emptyset\}$, $G_2 = \{1, 4, 6\}$ $G_3 = \{2, 5, 7, 8, 9, 10\}$. G_2 is ranked according to $t(i, h)$ – $G_2(2) = \{4, 6, 1\}$ and the set M is defined as $M = \{3, 4, 6, 1\}$.

At iteration $iter = 3$ the DM chooses a new current preferred alternative $h = 6$, adds this alternative to $LIST = \{3, 6\}$ and enters his/her new preferences – $L_6 = \{1, 3, 4\}$, $E_6 = \{2\}$ and $p = 3$. $d'(V(h), i)$ is again computed for each of the alternatives and the first non-empty group G_2 is ranked with respect to the values of $t(i, h)$.

i	1	2	3	4	5	7	8	9	10
$d'(V(h)i, i)$	2	1	1	2	1	3	1	3	2
$t(i, h)$		0.54	0.43		0.47		0.77		

$G_1 = \{\emptyset\}$, $G_2 = \{2, 3, 5, 8\}$, $G_3 = \{1, 4, 10\}$ and $G_4 = \{7, 9\}$. $G_2(3) = \{3, 5, 2, 8\}$ is ranked. The set M consist of the alternatives $\{6, 3, 5\}$.

At this iteration the DM selects the fifth alternative as the best compromise one.

Conclusion

An "optimization – motivated" method for solving a class of multiple criteria choice problems with a large number of alternatives and a small number of quantitative criteria, that are explicitly given is proposed in the paper. The method enables the DM to screen consecutively and systematically the set of the alternatives.

In comparison with the existing methods solving this class of MCCP, the method proposed has the following advantages:

– user-understandable by the DM, which gives him confidence about the decision suggested;

– a comparison of quite close alternatives at each iteration, which does not engage the DM too much, but enables the more realistic account of his preferences;

– a possibility for a relatively easy learning of the DM about the problem solved;

– possibility for evaluation of distributed alternatives, stored in the proccess of solution.

The method has been included in a software system for estimation of the efficiency, the financial stability and economic potential of some industrial enterprises from different branches and their rating on the basis of the estimates obtained. The experts of several Bulgarian investment funds have used this system in the purchase of various state enterprises during the process of privatization in the country.

REFERENCES

[1] J. Braus, B. Mareschal. The Promethee methods for MCDM: the promcale, Gaia and Bankad viser software. In: Readings in Multiple Criteria Decision Aid, (eds. A. Carlos, Bana Costa). Springer, 1990, 216-252.

[2] P. Farquhar. Utility assessment methods. *Management Sci.* **30** (1984), 1283-1300.
[3] P. C. Fishburn. Utility Theory for Decision Making. Wiley, New York, 1970.
[4] P. Korhonen, J. Wallenius, S. Zionts. Solving discrete multiple criteria problems using convex cones. *Management Sci.* **30** (1984), 1336-1345.
[5] P. Korhonen. A visual reference direction approach to solving discrete multiple criteria problems. *European J. Oper. Res.* **34** (1988), 152-159.
[6] R. Keeney, H. Raiffa. Decisions with Multiple Objectives. Wiley, New York, 1976.
[7] M. Koksalan, M. H. Karvan, S. Zionts. An improved method for solving multiple criteria problems involving discrete alternatives. *IEEE Trans. Systems Man Cybernet.* **44** (1984), 24-34.
[8] V. Lotfi, T. J. Stewart, S. Zionts. An aspiration level interactive model for multiple criteria decision making. *Comput. Oper. Res.* **19** (1992), 671-681.
[9] O. Marcotte, R. Soland. An interactive branch and bound algorithm for multiple criteria optimization. *Management Sci.* **32** (1986), 61-75.
[10] M. Roubens. Preference relations on actions and criteria in multicriteria decision making. *European J. Oper. Res.* **10** (1981), 51-55.
[11] B. Roy. Decision aid and decision making. *European J. Oper. Res.* **45** (1990), 324-331.
[12] B. Roy. The outranking approach and the foundations of ELECTRE methods. In: Readings in Multiple Criteria Decision Aid, (eds. A. Carlos, Bana Costa). Springer, 1990, 156-183.
[13] T. Saaty. The Analytical Hierarchy Process. McGraw-Hill, New York, 1980.
[14] M. Sun, R. Steuer. InterQuad: An interactive quad free based procedure for solving the discrete alternative multiple criteria problem. *European J. Oper. Res.* **89** (1996), 462-472.
[15] P. Vincke. Multicriteria Decision Making. Wiley, New York, 1992.
[16] S. Zionts. Multiple Criteria Mathematical Programming: An overview and several approaches. In: Multiple Criteria Decision Methods and Applications, (eds. G. Fandel, J. Spronk). Springer-Verlag, Heidelberg, 1985, 85-128.

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