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A POSTER ABOUT THE OLD HISTORY OF FRACTIONAL CALCULUS

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Abstract

The fractional calculus (FC) is an area of intensive research and development. In a previous paper and poster we tried to exhibit its recent state, surveying the period of 1966-2010. The poster accompanying the present note illustrates the major contributions during the period 1695-1970, the "old history" of FC.

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Preliminary note

Clearly, lists such as those assembled in this paper, can never be complete, and, besides, there must be selective decisions. We do apologize for all omissions. Also, despite of our efforts and quires to many colleagues, the data and photos for some names remained unavailable. We were forced to skip them. Let us emphasize also we decided not to include alive persons, although many of them contributed essentially to the development and applications of FC in the same period of time, see e.g. in [14].

The progress of fractional calculus

Can the order of derivatives and integrals be extended to have meaning with is any number irrational, fractional or complex? Gottfried Leibniz invented that idea in 1695 and prompted Guillaume l'Hôpital about it. This initial spark motivated many mathematicians, physicists and engineers to develop the concept of fractional calculus (FC) both in theoretical aspects and practical implementations.

This note presents a poster (Fig. 1) with the time line of events in this area during 1695-1970. The authors believe that this expose will motivate further research in the area of fractional calculus and will promote it within other scientific areas.

A color A3 format of the poster is enclosed in this journal issue, and also is available online on the journal's websites:

`http://www.math.bas.bg/~fcaa , http://www.diogenes.bg/fcaa.`

In the sequel, we list by alphabetic order the main researchers during the referred period, who are no more between us:

- ABEL, Niels Henrik (5 August 1802 - 6 April 1829), Norwegian mathematician
- AL-BASSAM, M. A., mathematician of Iraque origin
- COSSAR, James (died 24 July 1998), British mathematician
- DAVIS, Harold Thayer (5 October 1892 - 14 November 1974), American mathematician
- DJRBASHJAN, Mkhtar M. (11 September 1918 - 6 May 1994), Armenian mathematician
- ERDÉLYI, Arthur (2 October 1908 - 12 December 1977), Hungarian-born British mathematician
- EULER, Leonhard (15 April 1707 - 18 September 1783), Swiss mathematician and physicist
- FELLER, William (Vilim) (7 July 1906 - 14 January 1970), Croatian-American mathematician

- FOURIER, Jean Baptiste Joseph (21 March 1768 - 16 May 1830), French mathematician and physicist
- GELFAND, Israel (Israïl) Moiseevich (2 Sept. 1913 - 5 Oct. 2009), Russian mathematician
- GEMANT, Andrew (1895 - 1983), American physicist
- GERASIMOV, A. N., Russian physicist
- GRÜNWALD, Anton K. (1838-1920), German mathematician
- HADAMARD, Jacques Salomon (8 December 1865 - 17 October 1963), French mathematician
- HARDY, Godfrey Harold "G. H." (7 Feb. 1877 - 1 Dec. 1947), English mathematician
- HEAVISIDE, Oliver (18 May 1850 - 3 February 1925), English electrical engineer, mathematician, and physicist
- HOLMGREN, Hjalmar J. (1822-1885), Swedish mathematician
- de l'HÔPITAL, Guillaume François Antoine (1661 - 2 February 1704), French mathematician
- KILBAS, Anatoly A. (20 July 1948 - 28 June 2010), Belarusian mathematician
- KOBER, Hermann (1888 - 4 October 1973), mathematician born in Poland, studied and worked in Germany, later in Great Britain
- KRUG, Anton, German mathematician
- LACROIX, Sylvestre François (28 April 1765 - 24 May 1843), French mathematician
- LAGRANGE, Joseph-Louis (25 January 1736 10 April 1813), Italian-French mathematician and astronomer
- LAPLACE, Pierre-Simon (23 March 1749 - 5 March 1827), French mathematician and astronomer

- LAURENT, Paul Matthieu Hermann (2 Sept. 1841 - 19 Feb. 1908), French mathematician
- LEIBNIZ, Gottfried Wilhelm (1 July 1646 - 14 November 1716), German mathematician and philosopher
- LETNIKOV, Aleksey V. (1 January 1837 - 27 February 1888), Russian mathematician
- LÉVY, Paul Pierre (15 September 1886 - 15 December 1971), French mathematician
- LIOUVILLE, Joseph (24 March 1809 - 8 September 1882), French mathematician
- LITTLEWOOD, John Edensor (9 June 1885 - 6 September 1977), British mathematician
- LOVE, Eric Russel (31 March 1912 - 7 Aug. 2001), British-Australian mathematician
- MARCHAUD, André (born 1887 - n.a.), French mathematician
- MIKLÓS, Mikolás, 1st half of 20th century, Hungarian mathematician
- MITTAG-LEFFLER, Magnus Gustaf (Gösta) (16 March 1846 - 7 July 1927), Swedish mathematician
- MONTEL, Paul Antoine Aristide (29 April 1876 - 22 January 1975), French mathematician
- NAGY, Bela Szökefalvi (29 July 1913 - 22 December 1998), Hungarian mathematician
- NEKRASOV, Pavel Alekseevich (13 Feb. 1853 - 20 Dec. 1924), Russian mathematician
- NEWTON, (Sir) Isaac (4 January 1643 - 31 March 1727), English physicist, mathematician, astronomer, natural philosopher, alchemist, and theologian
- PINCHERLE, Salvatore (11 March 1853 - 10 July 1936), Italian mathematician

- POST, Emil Leon (11 February 1897 - 21 April 1954), Polish-American mathematician
- RIEMANN, Georg Friedrich Bernhard (17 Sept. 1826 - 20 July 1866), German mathematician
- RABOTNOV, Yury Nikolaevich (24 Feb. 1914 - 15 May 1985), Russian mechanical scientist
- RIESZ, Marcel (16 November 1886 - 4 September 1969), Hungarian mathematician
- ROSS, Bertram (1917 - 27 Oct. 1993), American mathematician
- SCOTT-BLAIR, George William (1902-1987), British chemist
- SHILOV, Georgi Evgen'evich (3 February 1917 - 17 January 1975), Russian mathematician
- SNEDDON, Ian Naismith (8 December 1919 - 4 Nov. 2000), Scottish mathematician
- SONINE, Nikolay Ya. (22 Feb. 1849 - 27 Feb. 1915), Russian mathematician
- TARDY, Placido (23 October 1816 - 2 November 1914), Italian mathematician
- WALLIS, John (23 November 1616 - 28 October 1703), English mathematician
- WEIERSTRASS, Karl Theodor Wilhelm (31 October 1815 - 19 February 1897), German mathematician
- WEYL, Hermann Klaus Hugo (9 November 1885 - 8 Dec. 1955), German mathematician
- WIDDER, David Vernon (25 March 1898 – 8 July 1990), American mathematician
- ZYGMUND, Antoni (25 December 1900 - 30 May 1992), Polish-born American mathematician

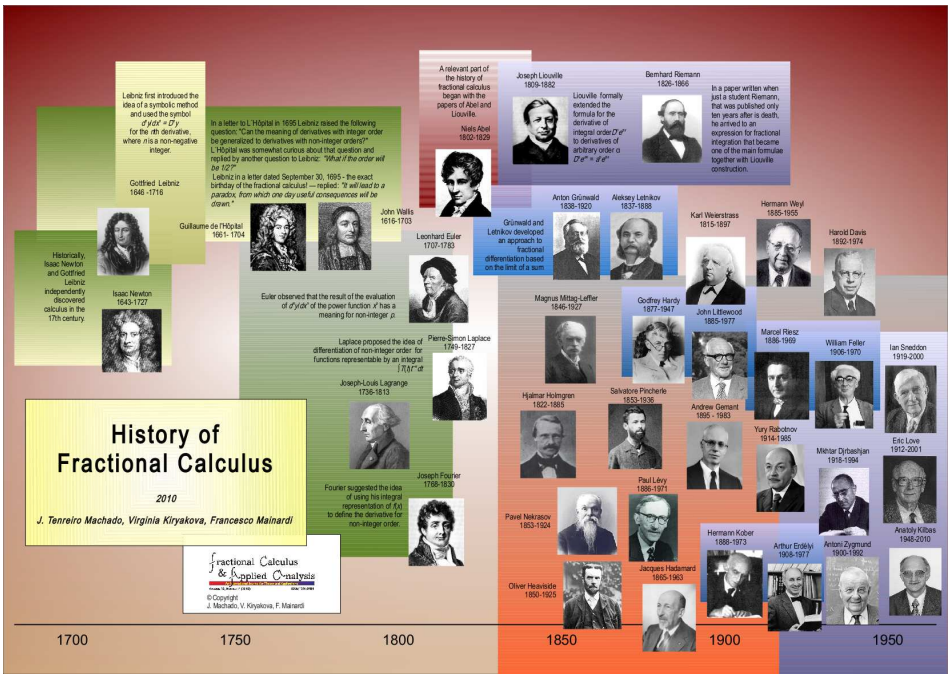

Figure 1: The time line of fractional calculus during the period 1695-1970.

As References, we list also some of the previous surveys and books, containing information on the history of fractional calculus.

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