ABOUT LEARNING DISCRETE MATHEMATICS AT BULGARIAN SCHOOL

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Abstract. *The present work is dedicated to the learning discrete mathematics at Bulgarian school. A review of syllabuses and standards has been made. A project of learning discrete mathematics elements from first to twelve class is proposed.*

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Introduction

In contemporary life the methods of mathematics are used widely not only in the traditional areas of practice such as physics, chemistry and technics, but they also enter actively scientific fields, about which, years ago, it was concerned that mathematics could not take any place in, for example medicine, biology, linguistics. Today it is impossible to imagine the development not only of the technical progress, but also of economics, planning, management and many other fields of the human practice without mathematics. The working out of the contemporary problems of science and technique is no longer able to be done by the traditional mathematical way – by hand, with a sheet of paper and a pencil. Today the main role in the mathematical calculations is taken by the contemporary computers. For this reason, the working out of computer models, corresponding with the mathematical models of the examined objects and phenomena is an essential activity of the contemporary mathematicians and computer specialists.

The computers use a digital presentation of the information regardless of its kind (sequences of zeros and ones). In the same way, the information is presented in the means of electronic computer communication – phone, TV and radio transmission, satellite connections, etc. The digital signal is discrete, it means that it is interrupted signal and this is the main difference from the conventional means of transmission of 19 and 20 century. In conventional means of transmission, the communication is based on the, so called, “analogue” (continuous) signals. The mathematics of the previous centuries serves mostly continuous processes. Along with the wide entering of the computers in the practice, today the discrete mathematics is current. The following definition is widely spread: **The Discrete mathematics** is a part of the mathematics, which examines countable

mathematical sets. On one hand, it includes the fundamental bases of the mathematics – theory of set, mathematical logic, theory of algorithms, and on the other hand it appears as a basic mathematical apparatus of the informatics and computer technics. In contrast to the traditional mathematics (mathematical analysis, linear algebra etc.), the methods and the constructions of which have numerical interpretation, the discrete mathematics works with objects of non-numerical character: mathematical set, logical expressions, algorithms, graphs. Due to this circumstance, the discrete mathematics allows the spread of the mathematical methods in areas, which before that were far from the mathematics. The knowledge of the theory of set, algebra, mathematical logic and the theory of graphs is absolutely necessary for the precise formulation of concepts and treatments of different applied tasks, their formalization and computerization, and also for mastering and development of contemporary informational technologies. The concepts and the methods of the theory of algorithms and the Boolean algebra are in the basis of the contemporary theory and practice of the programming.

Because of its currency and importance of the subject, “Discrete mathematics” is a basic subject for the university courses of mathematics and computer sciences. We concern that it is time for this subject to “go down” to the high school syllabus.

A review of syllabuses and standards for learning discrete mathematics at the Bulgarian school

Bulgaria is one of the few countries in Europe in which learning “Discrete mathematics” is provided at the high school. Known are attempts to study this discipline in the U.S. as described in [1] and [2]. In [3] described curriculum for university courses as part of it can be used in secondary education. In 2000, in our country a syllabus was initiated, according to which the subjects Informatics and Informational Technologies (IT) are compulsory for learning at the high school. The learning is provided on two levels. The first level is compulsory for all students, and the second level is provided for students, who have chosen a specialized schooling in informatics. In the developed by the Ministry of Education, state requirement form (standards) for educational content and syllabuses for the second level of Informatics in 10 class, a "Discrete mathematics" learning is provided [4].

Educational standards for the module “Discrete mathematics” are:

Standard: The student is able to calculate integers, presented in different positional number systems.

Standard: The student knows the basic Boolean functions (conjunction, disjunctions, collection of module 2, negative) and presentation of Boolean functions with disjunctive normal forms.

Standard: The student understands the syntax of the language specified by some formalism.

Standard: The student knows the basic properties of the relations over finite set – equivalence, regulation, graph, tree.

Standard: The student knows the basic linear and nonlinear abstract types and their presentation in structures of data.

Standard: The student realizes the basic linear and nonlinear structures of data in certain environment for programming.

According to the state requirement form as a result of the learning discrete mathematics, the student has to:

Know the most important mathematical concepts of the theoretical informatics.

Understand the specificity of the theories of the discrete mathematics.

Realize the constructive of the discrete theories and their sheer connection with the algorithm of the certain life problems.

Be able to mold certain life problems with discrete mathematics structures and practice their knowledge to find out useful features.

According to the syllabus of "Discrete mathematics" module, 7 educational topics are learned.

In **TOPIC 1. Sets,** the student has to get familiar with the term set, to learn to do basic operations with finite sets union, section, addition, Cartesian product, finding Boolean, to know the principle of the mathematical induction and to be able to practice it to prove features of inductively defined sets.

In **TOPIC 2 Relations,** the student has to get familiar with the features of the relations over finite sets (reflexivity, symmetry and transitive) and to be able to find them in certain relations. The student has to know the concepts equivalence and regulation and to be able to check if one finite relation is an equivalence or a regulation.

In **TOPIC 3. Functions,** the student has to get familiar with the concept mutually unambiguous function and to be able to check if a certain function is of this kind. The student has to know the features of Boolean functions conjunction, disjunctions and negative and to be able to present Boolean function with disjunctive normal form (DNF) and a scheme of functional elements.

In **TOPIC 4. Numbering systems,** the student has to get familiar with the presentation of the integers in positional number systems (PNS) and to be able to transform numbers from one PNS to another.

In **TOPIC 5. Combinatorics,** the student has to get familiar with the principles of the numeral combinatorics and the formulas of the number of the basic combinatorial configurations. The student has to learn to apply the principles and the formulas for finding the number of the elements of the finite set.

In **TOPIC 6, Graphs,** the student has to learn how to distinguish the basic types of graphs and trees and their characteristics and to be able to mould real situations with a graph or a tree.

In **TOPIC 7, Formal languages,** the student has to get familiar with the concept formal grammar. He or she has to be able to distinguish the grammar of the cutting and of the context-free languages, to understand the meaning of the abstract mathematical machines and their role in distinguishing formal languages

and to be able to construct an automation, which to be able to distinguish a certain cutting language.

The analysis of the upper educational content shows that it has a sufficient volume for the students of the upper class of the Bulgarian school and it provides a solid foundation for further expansion of the knowledge in discrete mathematics. Unfortunately a very limited contingent of students learns the module of Discrete mathematics – they are mostly students from the Math schools and Math classes, who have chosen to learn Informatics. We consider, that it would be reasonable to propose the idea of learning some elements of the Discrete mathematics during the whole period of education 1-12 class at High schools. It has to be learnt not as a particular discipline, but as a part of the Math course at school.

A plan of learning elements of the discrete mathematics from first to twelve class

Here we propose a short plan of continuous learning of elements of the discrete mathematics from first to twelve class. Although, the concept "discrete mathematics" may seem unknown to the students from first to eight class, a lot of elements from the educational content of this kind of mathematics appears in the classroom. In each case of counting objects, arranging elements of a given set in a particular order, following instructions for solving a particular kind of problems, the teachers are obliged to teach elements of the discrete mathematics. Of course, at early school age, there should not be given strict definitions, and the knowledge is supposed to be taught as a form of entertaining tasks, puzzles, games etc.

Elementary knowledge of discrete mathematics can be taught even at the kinder garden. With the help of pictures, drawings and other images, elements of finite sets can be presented. This is the beginning of the preparation for the assimilation of combinatorial knowledge. Children solve particular tasks, for example: In the kinder garden there are two swings, and Ljuben, Maria and Kalin want to swing. In how many different ways they can swing, if two of the kids are swinging, and the other kid is watching them, and then they change their places? After counting the combinations, in order to show the generality of the task, we letter the children with A, B, and C, and solve the task again. This is how the bases of the modeling are laid – the important characteristics of the task are defined, and the minor characteristics as sex, hair-color, relations between the kids etc. are ignored. The descriptions of the different correspondences prepare the children to assimilate the concept "function".

In table 1 are shown elements of the discrete mathematics, which can be taught to the preschoolers.

Knowledge of:	**Preparation for:**	**Development of consideration skills in:**	**Habituation to:**
Unit Many/Much Integer Counting Comparison Addition and subtraction of small integers	Creating the simplest models Realizing the concept "function" Relation Graphs	Using models in describing situations and solving tasks. Full combination of opportunities and options.	Accuracy and thoroughness of thinking Consideration Combinativeness

Table 1

It is known, that with a decision of Bulgarian Ministry of Education, during the school 2011-2012 year will start a compulsory pre-school for five years old children. This is a suitable situation to propose the introduction of the studying elements of the discrete mathematics in pre-school period

At primary school (1-4 class), solving tasks for finding connections between elements of sets has to continue, which later will allow students to realize the concept "power of set", and to be able to compare sets according to their power. Even from first class, algorithms to solve problems using graphs can be proposed. At first, these are examples of implementation of arithmetic or comparison of numbers. Numbers are written down at the top of the graph, and the operations of these numbers are put on the ribs of graphs. Later the algorithm has to become more complex. For examples: in 4 class, instead of specific numbers, there can be written variables at the top of the graph, which can take different values. In this way, the results of the operations will change accordingly to the initial conditions, and the students realize that the algorithm can solve a massive task, instead of particular one.

In table 2 are shown elements of the discrete mathematics, which can be studied at primary school.

At junior high school (5-8 class), another possibility to study elements of the discrete mathematics appears – the discipline "Information Technology". At this age, the solving to find links between elements of sets, the formation of a Cartesian product of sets has to be continued. Studying elements of the Boolean algebra starts, as logical operations can be realized in a specific information technology – the spreadsheets. The knowledge of algorithms is widening – algorithms of sorting, search and replace are examined. They can take a practical place in the information technology - text processing and spreadsheets. The preparation of graph – schemes for solving computational tasks continues, as on the ribs, logical operations are put, so that the graph - schemes turn into flowcharts of algorithms. Many of the Math and IT lessons can be illustrated with appropriate flowcharts. For example: In 5-6 class, the students has to get familiar with the establishment of timetables and schedules – for example, preparation of traffic routes to the school bus, mail

delivery, schedule for cleaning the school yard snow, etc. These problems have to be shaped by using graphs and to be solved by finding appropriate ways in these graphs.

Knowledge of:	**Preparation for:**	**Development of consideration skills in:**	**Habituation to:**
Arithmetic operations with small and large integers. Measuring astronomical time in the face of a clock. Measurement of sections. Understanding measures of weight.	Establishments of simple models. Understanding the concept of graph. Establishment of the algorithms. Understanding the concept function. Relation. Decomposition of the decision task in simple tasks.	Using models to describe situations and solve problems. Full combinations of opportunities and options. Switching from one measure to another. Logical thinking.	Accuracy and precision in the setting of tasks. Ability to decomposition of a task. Accuracy and thoroughness of thinking. Consideration Combinativeness.

Table 2

The students in 7 and 8 class have to be able to recognize and work with repetitive and recursive processes and to reveal their previous knowledge by exploring recurring patterns and procedures. Problems, which were solved iteratively before, now have to be examined recursively. This can be realized by solving tasks, using Fibonacci Numbers or solving the task of the Hanoi towers.

In this present work, we have shown an approach to study Discrete mathematics at high school. We consider, that there can be formed a discipline with the same name. In table 3 are described topics from the educational content of this discipline and expected results.

Knowledge In Maths:	**Knowledge in Informatics:**	**Skills in:**	**Preparation for:**
Integers – Divisibility, signs of divisibility, prime factorizations, Rational and real numbers, representation of real numbers with rational. Numeric sequences, progressions. Discrete functions. Algorithm. Euclidean algorithm. Horner`s rule. Summary theorem Viet. Dirichlet principle. Elements of Combinatorics. Elements of mathematical logic. Boolean functions. Discrete probability. Elements of Coding Theory. Elements of Graph Theory. Elements of Theory of schedules. Linear Programming. Transportation problem. Dynamic programming. Method of branches and borders. Matrices and their use. Simple Groups (Group of	Mathematical model. Numbering systems. Algorithms of the integer arithmetic. Data Structures. Sorting and Searching. Recursive formulas and algorithms. Algorithms for generating combinatorial objects. Algorithms for graphs – crawling in breadth and depth, topological sorting, shortest paths in the graph, stream algorithms, construction of minimal covering tree. Total exhaustion with return. Programming simplex method and the transport task. Programming operations on matrices . Study of discrete models with applications	Using graphs as a language for describing and compiling models. Descriptions of matches and discrete functions. Illustrate bijection, injection and surjection. Presentation of algorithms, flowcharts. Using the method of mathematical induction. Using the “necessary and sufficient” conditions. Illustrate the creation of the data structures and operations with them. Description of any options under the method of complete exhaustion. Description of the Turing machines and Post.	Working with models. Building models of real objects and systems using columns. Study of the complexity of the algorithms. Assessments of the most commonly used algorithms. Using the concepts of equivalence and isomorphism.

permutations, group of transformations). Concepts of field and ring. Automata. Turing machines and Post.	packages.		

Table 3

We consider, that introducing elements of the discrete mathematics in the high school syllabus will expand the possibilities for variegation of studying mathematics by using practical examples and tasks.

Reference

[1] Discrete Mathematics Across the Curriculum K-12. 1991 Yearbook of the National Council of Teachers of Mathematics.
[2] Discrete Mathematics in the Schools, DIMACS Series in Discrete Mathematics and Theoretical Computer Science Volume 36**.**
[3] K. Manev, Introduction to discrete mathematics ,KLMN Sofia, 2003
[4] MES**,** Science **„**Policy Directorate in general education", Curricula part of Area III - IX, X, XI and XII class, Sofia, 2003.

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