ABOUT THE MULTI CRITERIA RANGING PROBLEM IN THE FUZZY ENVIRONMENT

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Abstract: Decision making and technical decision analysis demand computer-aided techniques and therefore more and more support by formal techniques. In recent years fuzzy decision analysis and related techniques gained importance as an efficient method for planning and optimization applications in fields like production planning, financial and economical modeling and forecasting or classification. It is also known, that the hierarchical modeling of the situation is one of the most popular modeling method. It is shown, how to use the fuzzy hierarchical model in complex with other methods of Multiple Criteria Decision Making. We propose a novel approach to overcome the inherent limitations of Hierarchical Methods by exploiting multiple criteria decision making.

Keywords: multiple criteria decision making, fuzzy hierarchical modeling, fuzzy system, alternative choice, unified scale.

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Introduction

Nowadays there are a lot of methods of obtaining and storing information such as OLAP, DBSM and a lot of models of knowledge representation, but the main problem now is how to estimate and analyze obtained data. That means it's necessary to define the set of criteria for estimation of big amount of dynamic changed data and define their relative importance. It must be borne in mind that the storing information can be incomplete, inconsistent and fuzzy. For solving the problem the fuzzy analysis is needed to use.

As the result of the development of Multiple Criteria Decision Making (MCDM) using fuzzy set theory a number of innovations have been made possible. The new approach of MCDM using Fuzzy Hierarchical Modeling is introduced in the paper. It is shown, how to use the fuzzy hierarchical model in complex with other methods of MCDM. We propose a novel approach to overcome the inherent limitations of Hierarchical Methods by exploiting multiple criteria decision making.

Distributed Fuzzy Hierarchical Model

The basis of the model is the hierarchical structure of the factors, which was received as a result of function-structured decomposition of the data domain. The meta-levels of the structure are the following: first level is the level of the global aims, the second level is the level of the rival's aims, and the third level is the level of the measures for the achievement of the global aims and rivals aims removal. The last level is the level of the concrete actions. The links in the hierarchy define the dependency of the upper level element realization from the corresponding underlying level element. Thereby, the realization of possible measures for the achievement of the aim depends on some concrete undertaken actions. This hierarchy allows evaluating the importance of all the elements of the level taking into consideration their contribution in the top levels elements realization. The hierarchical structure analysis model allows to process local factors estimations. These estimations have, as a rule, fuzzy and inconsistent nature, got from sources of different reliability (from expert with different competence level). This hierarchical model also allows to get total global consistent and reliable in the sense of theories of the fuzzy sets estimations. Thereby, each decision will be characterized by its importance taking into consideration its role in the factors structure. But, such a decision characteristic is insufficient for all-round estimation. The additional characteristics of the decision, such as its realization in economic, social, politic etc senses, must be considered. But, the hierarchical model can find the most needed decision in the current situation.

We propose a novel approach to overcome the inherent limitations of Hierarchical Methods by exploiting multiple distributed information repositories. The construction of fuzzy hierarchical model can be distributed between a numbers of experts. They may work to a single domain or to different domains. Also the distributed computational methods are used for making the expert estimation and for receiving the result.

Hierarchy analysis methods

The special role in the complex object analysis plays the analysis of the factors links graph's structure (the graph has the form of ordered hierarchical ranked structure). Directly influenced factors are situated on the graph's last level. The realization of these factors (as a rule they represent the concrete actions), spreading upwards on consecutively located levels of the factors hierarchical structure, will bring into the realization of all above located factors and, finally, - to the achievement of the global aims of the considered complex object development. At the moment, the strict statement of the hierarchy multilevel factors structure building problem doesn't exist. But, it is possible to indicate the principles of its practice construction. These principles are formulated in the form of six necessary conditions, which must satisfy considered hierarchical structures. It is naturally, that real hierarchical structures will satisfy these conditions only in certain measure, which depends on the used methods and algorithms of their formation.

Hierarchical factors structures are built on the base of the profound sense of used factors; the factors in the underlying level reveal the sense of the upper level factors, or the underlying level factors represent the events, which realization promotes the realization of upper level factors.

The realization of some of the factors, lying on the same level, must not influence the realization of the other factors of this level. In other words, the factors of the same level must be independent from each other.

Factors on the considered level directly depends only on the factors of the nearest underlying level of the hierarchy.

Fullness of the factors uncovering: factor on the considered hierarchy level is completely realized, if all the influencing its realization factors of the next underlying level are also realized.

Positive relationship between the upper level factors and underlying level factors: the realization of the underlying level factors must not provide the reduction of the realization possibility of the upper level factors.

Linearity of the functional links between the adjacent levels factors.

Analysis of a hierarchy with fuzzy estimations

First of all, we should build the hierarchy. On the objects set Z = {1, 2, …, *N*} is defined the oriented graph G_r = (Z, W) without cycles with the vertexes set coinciding with the objects set, and the arcs set W. The presence of the arc (i, j) $\in$ W means that the weight z_i of the object (vertex) i directly depends on the weight z_j of the object j. The graph G_r has the structure of the purposes and tasks graph of some complex system, if all the vertexes of this graph can be located on non crossing levels $V_1, \ldots, V_M$ in such a way, that the graph's arcs connect only the vertexes of the adjacent levels and these arcs lead from top to bottom, from the level V_i to the level V_{i+1}, i = 1, …, M-1; the vertexes, from which arcs don't leave, are located on the level V_M; all the vertexes, in which arcs do not enter, are located on the level V_1. The construction of the hierarchy is one of the most difficult stages because of the difficult formalization of the used objects, such as aims, rivals aims etc. After hierarchy construction, the elementary estimations should be made by experts. The elementary estimation consists on the getting for certain vertex i $\in V_m$ paired estimations $^{(i)}_{jk}$ of the arcs weights (i, j) $\in$ W,

$j \in \Gamma_i = \{k \mid (i, k) \in W\}$. Paired estimations show, in how many times the contribution of the object j is more than the contribution of the object k in the achievement of the object i aim; j, $k \in \Gamma_i$. These estimations can be exact ($r_{jk}^{(i)} \in R_+$ - nonnegative numbers), interval ($r_{jk}^{(i)} = [a_{jk}^{(i)}, b_{jk}^{(i)}] \subset R$ - intervals) or fuzzy numbers ($r_{jk}^{(i)} = \{(t, \mu_{jk}^{(i)}(t)) \mid t \in R_+\}$ – closed convex fuzzy sets on R_+). The last case includes the linguistic estimates and two previous cases. Thereby, we get as a result of an elementary estimation an weighted binary relation $R^{(i)} = \{((j,k), r_{jk}^{(i)}) \mid j, k \in \Gamma_i\}$ on the objects set Γ_i, which gives the intensity of the objects superiority. After getting the estimations, we must average them. In each of the elementary estimations can participate several experts, so for some pairs (j, k) of the objects j, $k \in \Gamma_i$ different experts s can assign different estimations $r_{jk}^{(i)s}$ (s – expert's number). The procedure of the expert estimation averaging consists in the determination of the mean geometric estimation.

Hierarchic structure arcs weights determination

The result of the pairs estimations average in the i elementary estimation – exact, interval or fuzzy relation $R_{(i)}$ – is used in the determination of the weights $y_{i,j}$, of all the arcs $(i, j) \in W$, coming out of the vertex i. The arcs weights satisfy the following condition:

$$\sum_{j \in \Gamma_i} y_{ij} = 1; \quad y_{ij} \geq 0, \quad \forall\, i \in \Gamma_i .$$

If there are several objects on the first level y_1, then the “zero” elementary estimation is made, it means, that the pair comparison of the objects importance coefficients must be made. As a result of the “zero” estimation, the importance coefficients of the first level objects are determined.

The importance coefficients determination

After the elementary estimations results processing, the importance coefficients z_j of the objects $j \in V_1$ of the first level of the hierarchic structure are determined. And also the weights y_{ji} of all the arcs $(i, j) \in W$ are determined (the coefficients of the relative importance of the vertex $Y_{ji}^{(s)}$ for the vertex $Y_i^{(s-1)}$ of the nearest upper level, where s – is a level number. The weights of the underlying level objects are determined by the recurrence from top to bottom recalculation of the objects weights (objects importance coefficients):

$$z_i = \sum_{j \in \Gamma_i^{-1}} y_{ji} z_j , \; i \in V_2 ,$$

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$$z_i = \sum_{j \in \Gamma_i^{-1}} y_{ji} z_j , \; i \in V_M$$

($\Gamma_i^{-1} = \{j \mid (j, i) \in W\}$).

The Different Experts Estimations Consensus Analysis

The coefficients importance validity is determined by the elementary estimations results validity. In the case, then the initial pairs estimations are fuzzy or mixed, the results validity is equal to the consensus degree of the initial fuzzy relation R$^{(i)}$ and the resulting over transitive matrix, which is determined as a result of a special estimations approximation problem solution. The solution of the estimation approximation problem is made, using a modified method of Makeev and Shahnov. In the case, then the estimations are exact or interval, the results validity is characterized by the degree of the intervals bounds changes, which are assigned by the experts.

Model of Fuzzy Multiple Criteria Decision Making

Various character of source data used in alternative ranking problem cause difference of the problem statement. At the research the main source data are alternative estimations by each criterion (from some limited set of criteria). Feature of the expert obtained information is expert's dispensing from giving accurate estimations. The alternative estimations by each criterion can be fuzzy linguistic data defined by their distribution on criterion scales. The experts can make estimations of alternative by different criteria.

There are two fundamental issues. First, it's choosing the way of handle with multiple criteria. At the paper the conception of unified scale is introduced. The unified scale is made from criteria scales by merging. The use of the unified scale is correct if the criteria used for alternative estimation don't depend from each other by value. It isn't necessary for making the unified scale to have concrete value of gradation importance.

Second, it's necessary to define the way of comparing alternatives by one criterion in case of fuzzy alternative estimations. At the paper the combination of Jake-Lagrez method and fuzzy relation approximation by fuzzy reversible quasi-series method is used.

Let the ranging alternatives form the set $X = \{1,\ 2,\ \dots\ , n\}$. Each alternative is estimated by criteria and every criterion ξ is defined on the ordinal scale $E^{\xi} = \left\{ e_k^{\xi} \mid k = \overline{1,\ m^{\xi}} \right\}$ with discrete gradations e_k^{ξ}, m^{ξ} is the number of scale's E^{ξ} gradations.

For each alternative $i \in X$ on each scale E^{ξ} experts make fuzzy estimations $\gamma^{\xi}(i)$ as the distribution $\gamma^{\xi}(i) = \left\{ P_{i1}^{\xi}, \dots, P_{ik}^{\xi}, \dots, P_{im^{\xi}}^{\xi} \right\}$.

The value P_{ik}^{ξ} is interpreted as assurance that the alternative's *i* evaluation on the scale E^{ξ} is e_k^{ξ}.

One way of determination evaluation $\gamma^{\xi}(i)$ with alternatives group valuation is to take value P_{ik}^{ξ} proportionally to number of expert votes (or equal a fraction of number of expert votes), believing that alternative estimation *i* on the scale E^{ξ} is e_k^{ξ}.

Besides, the values can be interpreted as value of fuzzy estimation $\gamma^{\xi}(i)$ membership function defined over the base set E^{ξ}.

We shall assume that the estimations $\gamma^{\xi}(i)$ are distributed values, that means value $\sum_{k=1}^{m^{\xi}} P_{ik}^{\xi}$ doesn't depend on i and ξ, without loss of generality we can consider that $\sum_{k=1}^{m^{\xi}} P_{ik}^{\xi} = 1$, $\forall\, i \in X$, $\xi = \overline{1, \alpha}$. (1.1)

Hence, each alternative i is characterized by α distributed values $\{\gamma^{1}(i), \gamma^{2}(i), \dots, \gamma^{\alpha}(i)\}$.

It seems natural that relative importance of different scales and gradations play a part in the comparison of alternatives i and j as well as the estimations $\gamma^{\xi}(i)$ and $\gamma^{\xi}(j)$ on all scales E^{ξ} play a part in the comparison too. We assume that the set $\left\{ e_k^{\xi} \mid k = \overline{1,\ m^{\xi}},\ \xi = \overline{1, \alpha} \right\}$ of all scales gradations can be ordered, that means the set can be decomposed into equal valued gradations classes $C_1,\ \dots\ , C_m$ and the classes can be strict ordered $C_1 > C_2 > \ldots > C_m$. (1.2)

Where ">" - strong preference relation.

Thus, if the gradations $e_j^{\xi_1}$ and $e_t^{\xi_2}$ have the equal significance than they belong the same class C_p. If the first gradation is preferred than the second it means that $e_j^{\xi_1} \in C_q$, $e_t^{\xi_2} \in C_r$ and $q < r$.

The existence of the decomposition into classes can be guarantied by axiomatic approach developed in the utility theory.

The following is correct for the defined classes.

$$\left(e_s^{\xi'}, e_t^{\xi''} \in C_i\right) \Leftrightarrow \left(U_{\xi'}\left(e_s^{\xi'}\right) = U_{\xi''}\left(e_t^{\xi''}\right)\right)$$

$$\left(e_s^{\xi'} \in C_i\,, e_t^{\xi''} \in C_j\,, i(j) \Leftrightarrow \left(U_{\xi'}\left(e_s^{\xi'}\right)\right)\ U_{\xi''}\left(e_t^{\xi''}\right)\right).$$

Essential property is that for making relation of strict preference > under the preceding condition

$$e_s^{\xi'} >_e e_t^{\xi''} \Leftrightarrow U_{\xi'}\left(e_s^{\xi'}\right) > U_{\xi''}\left(e_t^{\xi''}\right)$$

It is required from the system analyst the information only about the decomposition of all gradations $\left\{ e_k^{\xi} \mid k = \overline{1, m^{\xi}},\ \xi = \overline{1, \alpha} \right\}$ into classes $C_1, \ \dots, \ C_m$. Value of the utility function *U* doesn't needed. That "simplified" information is supposed to obtain from system analyst.

So, initial information for alternatives ranging is:

$X = \{1, \dots, n\}$ – the set of alternatives;

$\gamma^{\xi}(i)$ – every alternative $i \in X$ fuzzy estimations on each scale E^{ξ};

$C_1, \ \dots, \ C_m$ – ordered classes scale E^{ξ} gradations $\xi = \overline{1, \alpha}$ which have the equal significance.

The goal of ranging is to make using mentioned initial information such decomposition of set *X* into equal valued alternatives classes $K_1, \dots, K_i, \dots, K_l$ where alternatives from class K_i are strict preferred the alternatives from classes $K_{i+1}, \dots, K_l$ for each $i = 1, 2, \dots, l-1$.

Let's discuss the method proposed for given ranging problem solution.

First, at any multi criteria making decision problem there is the problem of making generalized criteria. There are many generalized criteria creation methods.

Notice that at many practical problems there isn't reason to suggest that one criterion is more important than other. At the proposed method system analyst must make ranging by (1.2) gradation classes $C_1, \ \dots, \ C_m$ with the equal significance. It turned out first goal of research: with the knowledge of classes $C_1, \ \dots, \ C_m$ and their order we need to define concept of unified scale and create the method of evaluation of each alternative estimation $i \in X$ on the scale (generalized estimations $\gamma(i)$) based on particular estimations $\gamma^{\xi}(i)\ \ \xi = \overline{1, \alpha}$ .Second, it's necessary to propose a method of pair comparison for each two alternatives *i* and *j* from the set *X* based on the knowledge of their generalized estimations $\gamma(i)$ and $\gamma(j)$ and determine as a result of the comparison the value r_{ij} of alternative *i* preference on *j.*

Making the comparison for all alternatives pares $i, j \in X$ we construct $n \times n$ matrix $R = (r_{ij})$ determining some binary fuzzy relation *R* over *X.* The Matrix consists of either the generalized preference of system analyst, either the preference of experts group.

Third, It is necessary to range the objects using the fuzzy relation R. The problem reduces to the approximation of R by fuzzy reversible quasi-series.

Thus, the given method follows. Using information given above we make unified scale, calculate alternatives generalized estimations, based on the estimations we build binary fuzzy preference relation *R* and than looking for nearest to *R* reversible quasi-series.

Method's algorithm

Decomposition in Hierarchic Model is made until the level which contains factors with qualitative or quantitative scale of values. To apply the MCDM the construction of every scale reflection to [0,1] is needed. It means that it necessary to create membership function that will convert each value from the scale to the real number from [0,1]. The number is interpreted as preference of selected factor value for the main hierarchical goal (factor of the upper level) achievement. "Zero" is interpreted as index of minimum preference than "One" is interpreted as index of maximum preference.

On the basis of relative importance weights it is possible to construct unified scale for the scale gradations of the last level factors. We use the last level factors as the criteria for MCDM. Using MCDM we get the preference coefficient of each alternative, it means the preference coefficients of last level factors value collection.

Conclusion

Fuzzy structured models require a reliable knowledge on the underlying rules of the systems and can not be easily changed in a new situation. Decision making in the field often requires the analysis of large amount of data and complex relations. Very often, it is difficult to the expert to give the exact estimation of some objects of the field or some relations between the objects. In such cases, the analysis of a mathematical model can support rational decision making. The hierarchic structure is one of the most demonstrative models and it can really help the analyst to see all the aspects of the considered problem. The possibility to make the fuzzy elementary estimations gives to the expert the possibility to operate with the natural language concepts.

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