# PRIMARY AND SECONDARY EMPIRICAL VALUES IN NETWORK REDIMENSIONING 

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#### Abstract

A model of an overall telecommunication network with virtual circuits switching, in stationary state, with Bernoulli-Poisson-Pascal (BPP) input flow, repeated calls, limited number of homogeneous terminals and 8 types of losses is considered. One of the main problems of network redimensioning is estimation of the traffic offered in the network because it reflects on finding of necessary number of equivalent switching lines on the basis of the consideration of detailed users behavior and target Quality of Service (QoS).

The aim of this paper is to find a new solution of Network Redimensioning Task (NRDT) [4], taking into account the inconvenience of necessary measurements, not considered in the previous research [5].

The results are applicable for redimensioning of every (virtual) circuit switching telecommunication system, both for wireline and wireless systems (GSM, PSTN, ISDN and BISDN). For packet - switching networks proposed approach may be used as a comparison basis and when they work in circuit switching mode (e.g. VoIP).


Keywords: Overall Network Traffic, Offered Traffic, Virtual Circuits Switching.
ACM Classification Keywords: C.2.1 Network Architecture and Design; C.2.3 Network Operations; C. 4 Performance of Systems.

## Introduction

The task of Teletraffic engineering, often considered in real telecommunication system, is to find dependencies between three basic quantities: traffic demand, quality of services ( QOS ) and technical parameters of the servicing system [6]. An estimation of some teletraffic parameters ( offered traffic intensity, incoming rate of the first call attempts, etc.) in the network is one of the main problems of network redimensioning [2]. Network redimensioning [3] is necessary for medium term traffic management in an advance determined level. Based on the ITU definitions 4.1, 4.2, 2.8 and 2.11 in [1], of QoS parameters, we use the following two parameters, dependable from the network macro-state (Yab - traffic of all network terminals): probability Pbs (blocked switching) due to lack of resources, and probability Pbr of finding B-terminals busy. We denote the target value of blocked switching by trg.Pbs.
In this paper we consider detailed conceptual and its corresponded analytical traffic model [4] of telecommunication system with channel switching, in stationary state, with generalized BPP input flow, repeated calls, limited number of homogeneous terminals and losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned conversation.
A system of equations based on the conceptual model and some dependencies between parameters of the researched telecommunication system, is derived. An analytical solution of a network redimensioning task (NRDT) and the necessary conditions for it are researched.
The results are useful for finding of suitable method for estimation of the necessary number of equivalent internal switching lines ( $N s$ s) in dimensioning and redimensioning tasks.

## Conceptual model and analytical models

The conceptual model (shown on Fig. 1) [4] of the telecommunication system includes the paths of the calls, generated from (and occupying) the A-terminals in the proposed network traffic model and its environment.
The names of the devices are constructed according to their position in the model.

### 2.1. The comprising virtual devices

The following important comprising virtual devices are shown on Fig.1:
$a=$ comprises all the A-terminals (calling) in the system (shown with continuous line box).
$b=$ comprises all the B-terminals (called) in the system (box with dashed line).
$a b=$ comprises all the terminals (calling and called) in the system (not shown on Fig.1);
$s=$ virtual device corresponding to the switching system. It is shown with dashed line box into the $a$-device.
Ns stand for the capacity (number of equivalent internal switching lines) of the switching system.


STAGE: dialling; switching; ringing; communication.


Virtual Device Name $=<$ BRANCH EXIT $><$ BRANCH $><$ STAGE $>$
Fig. 1. Normalized conceptual model of the telecommunication system and its environment and the paths of the calls, occupying A-terminals ( $a$-device), switching system ( $s$ - device) and B-terminals ( $b$-device); base virtual device types, with their names and graphic notation.

### 2.2. Stages and branches in the conceptual model:

Considered service stages: dialing, switching, ringing and communication.
Every service stage has branches: enter, abandoned, blocked, interrupted, not available, carried (correspondingly to the modeled possible cases of ends of the calls' service in the branch considered).
Every branch has two exits: repeated, terminated (which show what happens with the calls after they leave the telecommunication system). Users may make a new bid (repeated call), or to stop attempts (terminated call).

### 2.3. Parameters and its notations in the conceptual model:

$F=$ the calling rate (frequency) of the bids' flow [calls/sec.], $P=$ probability for directing the calls of the external flow to the device considered, $T=$ mean service time, in the device [sec.], $Y=$ intensity of the device traffic [Erl], $N=$ number of service places (lines, servers) in the virtual device (capacity of the device). In the normalized models [4], used in this paper, every base virtual device, except the switch, has no more than one entrance and/or one exit. Switches have one entrance and two exits. For characterizing the calling rate of the flow, we are using the following notation: inc.F for incoming flow, dem.F, ofr.F and rep.F for demand, offered and repeated flows respectively [4]. The same characterization is used for traffic intensity ( $Y$ ).
Fo is the demand calling rate of first call attempts of one idle terminal; inc.Fa = Fa is calling rate of incoming flow; dem. Fa is the calling rate of all demand calls, $M$ is modifier of incoming flow.
For creating a simple analytical model, a system of fourteen assumptions is made [4].

## Analytical model

## Some general equations

For the proposed conceptual model we have derived the following system of equations [4]:
$Y a b=F a\left[S_{1}-S_{2}(1-P b s) P b r-S_{3} P b s\right]$
$F a=$ dem. $\mathrm{Fa}+$ rep.Fa
$d e m . F a=F o(N a b+M Y a b)$
rep. $\mathrm{Fa}=\mathrm{Fa}\left[R_{1}+R_{2} \operatorname{Pbr}(1-\mathrm{Pbs})+R_{3} \mathrm{Pbs}\right]$
Pbr $=\left\{\begin{array}{lll}\frac{Y a b-1}{N a b-1} & \text { in case of } 1 \leq Y a b \leq N a b, \\ 0 & \text { in case of } & 0 \leq Y a b<1 .\end{array}\right.$
$T s=S_{1 z}-S_{2 Z} P b r$
$o f r . F s=F a(1-$ Pad $)(1-$ Pid $)$
$o f r . Y s=o f r . F s T s$
$E r l_{-} b(N s, o f r . Y s)=\frac{\frac{(o f r . Y s)^{N s}}{N s!}}{\sum_{j=0}^{N s} \frac{(o f r . Y s)^{j}}{j!}}$
crr. $Y s=(1-P b s)$ ofr.Ys
The following notations are used:
$S_{1}=$ Ted + Pad Tad $+(1-$ Pad $)[$ Pid Tid $+(1-$ Pid $)[$ Tcd + PisTis +
(1-Pis)[PnsTns $+(1-$ Pns $)[T c s+2 T b]]]]$
$S_{2}=(1-P a d)(1-P i d)(1-P i s)(1-P n s)[2 T b-T b r]$
$S_{3}=(1-$ Pad $)(1-$ Pid $)[$ PisTis - Tbs $+(1-$ Pis $)[P n s T n s+(1-P n s)[T c s+2 T b]]]$
$S_{1 z}=$ PisTis $+(1-$ Pis $)[$ PnsTns $+(1-P n s)(T b+T c s)]$
$S_{2 z}=(1-P i s)(1-P n s)(T b-T b r)$
$R_{1}=$ Pad Prad $+(1-$ Pad $)($ Pid Pr id $+(1-$ Pid $)[$ Pis Pr is +
$(1-\operatorname{Pis})($ Pns Pr ns $+(1-$ Pns $)) Q])$
$R_{2}=(1-$ Pad $)(1-$ Pid $)(1-$ Pis $)(1-$ Pns $)(\operatorname{Pr} b r-Q)$
$R_{3}=(1-\operatorname{Pad})(1-\operatorname{Pid})\{\operatorname{Pr} b s-[$ Pis Pr is $+(1-$ Pis $)[P n s \operatorname{Pr} n s+(1-P n s) Q]]\}$
$Q=$ Par Prar $+(1-$ Par $)[$ Pac Prac $+(1-$ Pac $)$ Prcc $]$

## Right Teletraffic Tasks Parameters

## Full parameters' set

In the conceptual model presented, we have 31 base and $4(a, b, a b$ and $s)$ comprising virtual devices. Since every device has 5 parameters ( $\mathrm{P}, \mathrm{F}, \mathrm{T}, \mathrm{Y}, \mathrm{N}$ ), the total sum of parameters in our model is 175.

## Base parameters' set

There are many obvious dependencies in a system tuple [4], corresponding to the Full Parameters' Set of the Conceptual Model. For example, the sum of probabilities of outgoing transitions in every virtual switch devices has value one; in stationary state Little's formula $(Y=F T)$ is in force for every virtual device; we assume most of devices with infinite capacity. As a result, there are sets of base parameters (sub-tuples), with the following property: If we know the values of the base parameters, we may calculate the values of all other parameters of the same system tuple. Several different base parameters' sets may exist. After careful analysis [4] we have chosen a base parameters' set with 41 parameters.

## Parameters' classification based on characterized entities

The 41 parameters of the chosen base parameters' set may be classified, according characterized entities, in the five following groups, corresponding to:

1. Human Behaviour Parameters are 21: Fo, Nab, Prad, Tid, Prid, Pris, Tis, Pns, Tns , Prns, Tbs, Prbs, Tbr, Prbr, Par, Tar, Prar, Tcr, Prac, Tcc, Prcc;
2. Technical Characteristics Parameters are 4: Pid, Pis, Tcs, Ns;
3. Mix Factors' Parameters are 6: Ted, Pad, Tad, Tcd, Pac, Tac;
4. Modeller Chosen Values Parameter (1): $M$;
5. Derived Parameters from the previous four groups are 9: Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fs, Ts, ofr. Ys.

## Parameters' classification based on their values' determination and its notations

We consider

1. Administratively determined values with

- Target parameters' values: denoted by prefix trg., e.g. trg.Pbs is a target values of blocking probability due to lack of sufficiency switching lines;

2. Parameters empirical evaluated are called

Primary:

- If empirical parameters' values are received after direct measurements, denoted by prefix emp., e.g. emp.crr. Ys is empirical values of carried traffic intensity. $_{\text {s }}$


## Secondary:

- Parameters with empirical values, received from primary after calculation: For example, emp.Fo, emp.Yab, emp.crr. Ys;
- Designed parameters and their values: denoted by dsn., e.g. dsn.ofr.Ys, dsn Ts;
- Thresholds values of parameters are received as restriction in the dependencies, denoted by thr, i.e. thr.Fo.


## Static and Dynamic Parameters' Classification

In this paper, we propose a short term classification of the chosen base parameters' set with 31 static and 10 dynamic parameters.
For the static parameters we assume that their values don't depend on the state of the system and correspondingly on the intensity of the input flow. They may depend on other factors, e.g. the time of the day; seasons, human temperament, Telecom Administration, Gross Domestic Product and so on, but for the observed and modelled time interval we consider them as constants.
The 10 dynamic parameters, with mutually dependent values are: Fo Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofd.Fs, Ts, ofd. Ys.

## Finding Terminal Teletraffic Parameters

## Task Formulation:

We consider the overall telecommunication system conceptual model, presented in Fig. 1 and described in Section 2. Parameters with known values are all the $P$ (probability for call direction) and $T$ (holding time) parameters of the base virtual devices, plus values of the intensity of incoming calls flow (Fa) and traffic carried (crr.Ys).

## Known parameters' values:

Parameters with empirical values:
Primary: emp.crr.Ys = crr.Ys, emp.inc. $F a=F a, e m p . T b=T b$
Secondary: $S_{1}, S_{2}, S_{3}, R_{1}, R_{2}, R_{3}, S_{1 z}, S_{27}$, Pad, Pid, Pbs
Aim: Based on previous experience, to determine the incoming input flow, generated from one idle terminal and probability of finding $B$ - terminal busy in the real system;
Unknown parameters' values: emp.Fo, emp.Pbr
We assume the values of parameters emp.Fo, $S_{1}, S_{2}, S_{3}, R_{1}, R_{2}, R_{3}, S_{12}, S_{2 z}, P a d$, Pid, Pbs as mutually independent and having the same values before and after changing the number of switching lines.

### 5.2. Analytical Solution:

Theorem 1: Empirical values of emp.Pbr fulfills the dependence:

$$
\begin{equation*}
e m p . P b r=\frac{S_{1 z}}{S_{2 z}}-\frac{c r r . Y s}{S_{2 z} F a(1-P a d)(1-P i d)(1-P b s)} \tag{5.4}
\end{equation*}
$$

Proof: From equations (3.7), (3.8), (3.10), if ofr.Ys $<S_{1 z} F a(1-\mathrm{Pad})(1-\mathrm{Pid})$ follows

$$
\begin{equation*}
c r r . Y s=F a(1-P a d)(1-P i d)(1-P b s) T s \tag{5.5}
\end{equation*}
$$

We receive from equations (3.9) (3.6) and (5.5), regarding emp.Pbr ,the expression (5.4).
Theorem 2: The follow expression regarding emp.Fo exist

$$
\begin{align*}
& \text { emp.Fo }=\frac{F a(1-P b s)\left\{\left(1-R_{1}-R_{3} P b s\right) S_{2 z}-R_{2} \Omega\right\}}{S_{2 z}(N a b+M)(1-P b s)+M(N a b-1) \Omega} \text {, where }  \tag{5.6}\\
& \qquad \Omega=S_{1 z}(1-P b s)-\frac{c r r . Y s}{F a(1-\text { Pad })(1-\text { Pid })} .
\end{align*}
$$

Proof: If $S_{2 z} \neq 0$ then from equations (3.2) and (3.4) follows

$$
\begin{equation*}
\text { dem.Fa }=F a\left\{1-R_{1}-R_{3} P b s-R_{2}(1-P b s) e m p . P b r\right\} \tag{5.7}
\end{equation*}
$$

We receive from equations (3.3) and (3.5)

$$
\begin{equation*}
\text { dem.Fa }=e m p . F o\{N a b+M+M(N a b-1) e m p \cdot P b r\} \tag{5.8}
\end{equation*}
$$

Based on (5.7), (5.8) and the proved dependence in Theorem 1 (5.4), we determine (5.6).
Conclusion: The evaluation of, calls flow generated from one idle terminal (emp.Fo), is based on carried traffic intensity crr.Ys and calling rate of the input flow Fa . These parameters are easily measurable. Others parameters as $S_{1}, S_{2}, S_{3}, R_{1}, R_{2}, R_{3}$ are measurable with difficulty, in principle, but they are considered as independent of the input flow and steady approximately, i.e. as constants.
Therefore, based on previous experience and the above specifications, parameters' evaluation with one measurement of some primary parameter values in a short time interval is possible. It is make a network redimensioning task solution easier.

## Network Redimensioning Task

Based on previous experience, determining the volume of telecommunication resources that is sufficient to serve a given input flow, with prescribed characteristics of QoS, is one of the main problems that often have to be solved by network operators. It includes the following sub-tasks:

1. Redimensioning a network means to be found of number of equivalent internal switching lines necessary to satisfy a level of QoS that has been administratively pre-determined.
2. Finding the values of the designed parameters, describing the designed system state, based on known and target parameters' values. For example, a system parameter, describing offered traffic intensity of the switching system (dsn.ofr.Ys), designed probability to find B terminal "busy" (dsn.Pbr), etc...

## Analytical Solution of a NRDT on Basis of Easy Measurable Empirical Values

On the basis of equation (5.6) from this paper and equations (7.2.4) and (7.2.10) from [5], the Network redimensioning task solution, using easy measurable parameters, we obtain the following two equations:

The expression Erl_b (Ns, dsn.ofr.Ys) is the famous Erlang B - formulae.
It is proved in Theorem 7.4 in [5] that only one solution of $N s$ exists, fulfilling the equation (5.9) and corresponding to the determined administratively in advance value of the blocking probability trg.Pbs $\in(0 ; 1]$.

## Numerical Results

Computer program on the basis of empirical values, received from the new parameters dependencies (5.6), (5.8) and (5.9) is worked out.
Verification of the new parameters dependencies is made and the maximal absolute difference between the real and calculated values is less than $1.561 \times 10^{-17}$ in the whole admissible interval.

## Conclusions

New dependencies between empirical values of carried traffic intensity emp.crr.Ys and calling rate emp.Fa, from one hand, and the demand rate of calls of one idle terminal emp.Fo , from other hand, are derived.

Based on the new dependencies, network redimensioning task is solved using easy measurable values of empirical parameters.
The new parameters dependencies are numerical verified in the whole admissible interval.
The received results make the network dimensioning/redimensioning, based on QoS requirements easier. The described approach is applicable for every (virtual) circuit switching telecommunication system (like GSM and PSTN) and may help considerably for ISDN, BISDN and most of core and access networks dimensioning. For packet switching systems, proposed approach may be used when they work in circuit switching mode.

## Bibliography

1. ITU-T Recommendation E.600: Terms and Definitions of Traffic Engineering. Melbourne, 1988; revised at Helsinki, 1993
2. ITU-T Recommendation E.501: Estimation of traffic offered in the network. (revised 26. May 1997);
3. ITU-T Recommendation E. 734 Methods for allocating and dimensioning Intelligent Network (IN) resources, October 1996
4. S. A. Poryazov, E. T. Saranova., 2006. Some General Terminal and Network Teletraffic Equations in Virtual Circuit Switching Systems. Chapter in: A. Nejat Ince, Ercan Topuz (Editors). "Modelling and Simulation Tools for Emerging Telecommunications Networks: Needs, Trends, Challenges, Solutions", Springer Sciences+Business Media, LLC 2006, pp. 471-505. Printed in USA, Library of Congress Control Number: 2006924687. ISBN-13: 978-0387-32921-5 (HB)
5. Saranova E. T., 2006. Redimensioning of Telecommunication Network based on ITU definition of Quality of Services Concept, In: Proceedings of the International Workshop "Distributed Computer and Communication Networks", Sofia, Bulgaria, 2006, Editors: V. Vishnevski and Hr. Daskalova, Technosphera publisher, Moscow, Russia, 2006, ISBN 5-85638-111-4 pp. 165-179;
6. Цанков Б. ,2006. Телекомуникации фиксирани, мобилни и IP, изд. Нови знания, ТУ - София, 2006.ISBN -10: 954-9315-58-4, ISBN - 13: 978-954-9315-58-5

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