REPRESENTING DEFAULT LOGIC IN MODAL LOGIC

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***Abstract**: The nonmonotonic logic called Default Logic is shown to be representable in a monotonic Modal Quantificational Logic whose modal laws are stronger than S5. Specifically, it is proven that a set of sentences of First Order Logic is a fixed-point of the fixed-point equation of Default Logic with an initial set of axioms and defaults if and only if the meaning or rather disquotation of that set of sentences is logically equivalent to a particular modal functor of the meanings of that initial set of sentences and of the sentences in those defaults. This result is important because the modal representation allows the use of powerful automatic deduction systems for Modal Logic and because unlike the original Default Logic, it is easily generalized to the case where quantified variables may be shared across the scope of the components of the defaults thus allowing such defaults to produce quantified consequences. Furthermore, this generalization properly treats such quantifiers since both the Barcan Formula and its converse hold.*

***Keywords**: Default Logic, Modal Logic, Nonmonotonic Logic.*

1. Introduction

One of the most well known nonmonotonic logics [Antoniou 1997] which inherently deals with entailment conditions in addition to possibility conditions in its defaults is the so-called Default Logic [Reiter 1980]. The basic idea of Default Logic is that there is a set of axioms Γ and some non-logical default "inference rules" of the form:

$$\frac{\alpha : \beta_1,...,\beta_m}{\chi}$$

which suggest that χ may be inferred from α whenever each $\beta_1,...,\beta_m$ is consistent with everything that is inferable. Such "inference rules" are not recursive and are circular in that the determination as to whether χ is derivable depends on whether β_j is consistent which in turn depends on what was derivable from this and other defaults. Thus, tentatively applying such inference rules by checking the consistency of $\beta_1,...,\beta_m$ with only the current set of inferences produces a χ result which may later have to be retracted. For this reason, valid inferences in a nonmonotonic logic such as Default Logic are essentially carried out not in the original nonmonotonic logic, but rather in some (monotonic) metatheory in which that nonmonotonic logic is defined. [Reiter 1980] explicated this intuition by defining Default Logic in terms of the set theoretic proof theory metalanguage of First Order Logic (i.e. FOL) with the following fixed-point expression: $'\kappa=(\text{dl } '\kappa\ \{'\Gamma_i\}\ '\alpha_i{:}'\beta_{ij}/'\chi_i)$ where dl is: $(\text{dl } '\kappa\ \{'\Gamma_i\}\ '\alpha_i{:}'\beta_{ij}/'\chi_i) =\text{df} \cap\{p: (p\supseteq(\text{fol } p))\wedge(p\supseteq\{'\Gamma_i\})\wedge\forall_i((('\alpha_i\varepsilon p)\wedge\wedge_{j=1,mi}('(\neg\beta_{ij})\not\in '\kappa))\rightarrow('\chi_i\varepsilon p))\}$ where $'\alpha_i$, $'\beta_{ij}$, and $'\chi_i$ are the closed sentences of FOL occurring in the ith default "inference rule" and $\{'\Gamma_i\}$ is a set of closed sentences of FOL. A closed sentence is a sentence without any free variables. fol is a function which produces the set of theorems derivable in FOL from the set of sentences to which it is applied. The quotations appended to the front of these Greek letters indicate references in the metalanguage to the sentences of the FOL object language. Interpreted doxastically this fixed-point equation states:

The set of closed sentences which are believed is equal to
the intersection of all sets of closed sentences which are potentialially believed such that:
the closed sentences derived by the laws of FOL from the potential beliefs are themselves potentially believed,
the closed sentences in $\{'\Gamma i\}$ are potentially believed,
and for each i, if the closed sentence $'\alpha_i$ is potentially believed
and for each j, the closed sentence $'\beta_{ij}$ is believable then the closed sentence $'\chi_i$ is potentially believed.

The purpose of this paper is to show that all this metatheoretic machinery including the formalized syntax of FOL, the proof theory of FOL, the axioms of a strong set theory, and the set theoretic fixed-point equation is not needed and that the essence of Default Logic is representable as a necessary equivalence in a simple (monotonic) Modal Quantificational Logic. Interpreted as a doxastic logic this necessary equivalence states:

That which is believed is logically equivalent to some potential belief such that:
Γ is potentially believed
and for each i, if α_i is potentially believed and for each j, β_{ij} is believable then χ_i is potentially believed.

thereby eliminating all mention of any metatheoretic machinery.

The remainder of this paper proves that this modal representation is equivalent to Default Logic. Section 2 describes a formalized syntax for a FOL object language. Section 3 describes the part of the proof theory of FOL needed herein (i.e. theorems FOL1-FOL9). Section 4 describes the Intensional Semantics of FOL including the meaning operator (i.e. the laws M0-M7) and the relationship of meaning and modality to the proof theory of FOL (i.e. the laws R0, A1, A2 and A3 and the theorems C1, C2, C3, and C4). The modal version of Default Logic, called DL, is defined in section 5 and explicated with theorems MD1-MD7 and SS1-SS2. In section 6, this modal version is shown by theorems DL1 and DL2 to be equivalent to the set theoretic fixed-point equation for Default Logic. Figure 1 outlines the relationship of all these theorems to the final theorems DL2, FOL9, and MD7.

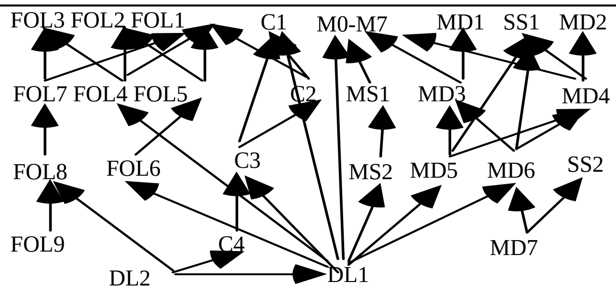

Figure 1: Dependencies among the Theorems

2. Formal Syntax of First Order Logic

We use a First Order Logic (i.e. FOL) defined as the six tuple: ($\rightarrow$, #f, $\forall$, *vars*, *predicates*, *functions*) where $\rightarrow$, #f, and $\forall$ are logical symbols, *vars* is a set of variable symbols, *predicates* is a set of predicate symbols each of which has an implicit arity specifying the number of associated terms, and *functions* is a set of function symbols each of which has an implicit arity specifying the number of associated terms. The sets of logical symbols, variables, predicate symbols, and function symbols are pairwise disjoint. Lower case Roman letters possibly indexed with digits are used as variables. Greek letters possibly indexed with digits are used as syntactic metavariables. γ, γ_1,...γ_n, range over the variables, ξ, ξ_1...ξ_n range over sequences of variables of an appropriate arity, π,π_1...π_n range over the predicate symbols, ϕ, ϕ_1...ϕ_n range over function symbols, δ, δ_1...δ_n, σ range over terms, and α,α_1...α_n, β, β_1...β_n,χ, χ_1...χ_n, Γ_1,...Γ_n,φ range over sentences. The terms are of the forms γ and (ϕ δ1...δn), and the sentences are of the forms ($\alpha\rightarrow\beta$), #f, ($\forall\gamma$ α), and (π δ_1...δ_n). A nullary predicate π or function ϕ is written as a sentence or a term without parentheses. $\varphi\{\pi/\lambda\xi\alpha\}$ represents the replacement of all occurrences of π in φ by $\lambda\xi\alpha$ followed by lambda conversion. The primitive symbols are shown in Figure 2 with their intuitive interpretations.

Symbol	Meaning
$\alpha\rightarrow\beta$	if α then β.
#f	falsity
$\forall\gamma\ \alpha$	for all γ, α.

Figure 2: Primitive Symbols of First Order Logic

The defined symbols are listed in Figure 3 with their definitions and intuitive interpretations.

Symbol	Definition	Meaning		Symbol	Definition	Meaning
$\neg\alpha$	$\alpha\rightarrow$ #f	not α		$\alpha\wedge\beta$	$\neg(\alpha\rightarrow\neg\beta)$	α and β
#t	$\neg$ #f	truth		$\alpha\leftrightarrow\beta$	$(\alpha\rightarrow\beta)\wedge(\beta\rightarrow\alpha)$	α if and only if β
$\alpha\vee\beta$	$(\neg\alpha)\rightarrow\beta$	α or β		$\exists\gamma\ \alpha$	$\neg\forall\gamma\neg\alpha$	for some γ, α

Figure 3: Defined Symbols of First Order Logic

The FOL object language expressions are referred in the metalanguage (which also includes a FOL syntax) by inserting a quote sign in front of the object language entity thereby making a structural descriptive name of that entity. A set of sentences is represented as: {'Γ_i} which is defined as: {'Γ_i: #t} which in turn is defined as: {s: $\exists$i(s='Γ_i)} where i ranges over some range of numbers (which may be finite or non-infinite). With a slight abuse of notation we also write 'κ, 'Γ to refer to such sets.

3. Proof Theory of First Order Logic

First Order Logic (i.e. FOL) is axiomatized with a recursively enumerable set of theorems as the set of axioms is itself recursively enumerable and its inference rules are recursive. The axioms and inference rules of FOL [Mendelson 1964] are those given in Figure 4. They form a standard set of axioms and inference rules for FOL.

MA1: $\alpha \rightarrow (\beta \rightarrow \alpha)$ MR1: from α and $(\alpha \rightarrow \beta)$ infer β
MA2: $(\alpha \rightarrow (\beta \rightarrow \rho)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \rho))$ MR2: from α infer $(\forall\gamma\ \alpha)$
MA3: $((\neg\alpha) \rightarrow (\neg\beta)) \rightarrow (((\neg\alpha) \rightarrow \beta) \rightarrow \alpha)$
MA4: $(\forall\gamma\ \alpha) \rightarrow \beta$ where β is the result of substituting an expression (which is free for the free positions of γ in α) for all the free occurrences of γ in α.
MA5: $(\forall\gamma(\alpha \rightarrow \beta)) \rightarrow (\alpha \rightarrow (\forall\gamma\ \beta))$ where γ does not occur in α.

Figure 4: Inferences Rules and Axioms of FOL

In order to talk about sets of sentences we include in the metatheory set theory symbolism as developed along the lines of [Quine 1969]. This set theory includes the symbols ε, $\not\in$, $\supseteq$, $=$, $\cup$ as is defined therein.

The derivation operation (i.e. fol) of any First Order Logic obeys the Inclusion (i.e. FOL1), Idempotence (i.e. FOL2), and Monotonic (i.e. FOL3) properties:

FOL1: (fol 'κ)$\supseteq$'κ Inclusion

FOL2: (fol 'κ)$\supseteq$(fol(fol 'κ)) Idempotence

FOL3: ('$\kappa\supseteq$'Γ) $\rightarrow$ ((fol 'κ)$\supseteq$ (fol 'Γ)) Monotonicity

From these three properties we prove the following theorems of the proof theory of First Order Logic:

FOL4 ((fol 'κ)$\supseteq$(fol 'Γ))$\leftrightarrow$((fol 'κ)$\supseteq$'Γ) proof: The proof divides into two parts: (1) ((fol 'κ)$\supseteq$(fol 'Γ)) $\rightarrow$ ((fol 'κ)$\supseteq$'Γ). By FOL1 the hypothesis implies the conclusion. (2) ((fol 'κ)$\supseteq$'Γ)$\rightarrow$((fol 'κ)$\supseteq$(fol 'Γ)) By FOL3 the hypothesis implies (fol(fol 'κ))$\supseteq$(fol 'Γ) which by FOL2 implies the conclusion. QED.

FOL5: $\forall$p((p=(fol p))$\rightarrow\alpha$)$\leftrightarrow\forall$p(α{p/(fol p)}) and $\exists$p((p=(fol p))$\wedge\alpha$)$\leftrightarrow\exists$p(α{p/(fol p)})

proof: The universal quantifier version follows from the existential quantifier version by running negation through both sides of the bi-implication. The existential version is proven as follows. There are two cases:

(1) ((p=(fol p))$\wedge\alpha$)$\rightarrow$ $\exists$p(α{p/(fol p)}). The existentially quantified p is replaced by p giving:

((p=(fol p))$\wedge\alpha$)$\rightarrow$(α{p/(fol p)}) The the hypothesis is used to replace p in α by (fol p) giving the conclusion.

(2) (α{p/(fol p)})$\rightarrow$ $\exists$p((p=(fol p))$\wedge\alpha$) Letting p in the conclusion be (fol p) gives:

(α{p/(fol p)})$\rightarrow$ (((fol p)=(fol(fol p)))$\wedge$(α{p/(fol p)})) which holds by FOL1 and FOL2.

FOL6: ($\cap${p: (p$\supseteq$(fol p))$\wedge\varphi$})={s:$\forall$p((φ{p/(fol p)})$\rightarrow$(sε(fol p)))} proof: $\cap${p: (p$\supseteq$(fol p))$\wedge\varphi$} By FOL1 this is equivalent to: $\cap${p: (p=(fol p))$\wedge\varphi$}. Unfolding the definition of intersection gives:{s:$\forall$p((pε{p: (p=(fol p))$\wedge\varphi$})$\rightarrow$(sεp))} which is equivalent to: {s:$\forall$p(((p=(fol p))$\wedge\varphi$)$\rightarrow$(sεp))}. By FOL5 this is equivalent to: {s:$\forall$p((φ{p/(fol p)})$\rightarrow$(sε(fol p)))} QED.

FOL7: If α is a sentence of proof theory then: ($\cap${p: (p$\supseteq$(fol p))$\wedge\alpha$})=(fol($\cap${p: (p$\supseteq$(fol p))$\wedge\alpha$}))

proof: From FOL1 it suffices to prove: (sε(fol($\cap${p: (p$\supseteq$(fol p))$\wedge\alpha$})))$\rightarrow$(sε($\cap${p: (p$\supseteq$(fol p))$\wedge\alpha$})). Unfolding the intersections and simplifying gives: (sε(fol{s: $\forall$p(((p$\supseteq$(fol p))$\wedge\alpha$)$\rightarrow$(sεp))}))$\rightarrow\forall$p(((p$\supseteq$(fol p))$\wedge\alpha$)$\rightarrow$(sεp))which is equivalent to: ((sε(fol{s:(sεp)$\wedge\forall$p(((p$\supseteq$(folp))$\wedge\alpha$)$\rightarrow$(sεp))}))$\wedge$(p$\supseteq$(fol p))$\wedge\alpha$)$\rightarrow$(sεp). Folding intersection then gives: ((sε(fol({s:(sεp)}$\cap${s:$\forall$p(((p$\supseteq$(folp))$\wedge\alpha$)$\rightarrow$(sεp))})))$\wedge$(p$\supseteq$(fol p))$\wedge\alpha$)$\rightarrow$(sεp). Using the second hypothesis to replace p by (fol p) and then dropping the second and third hypotheses gives: (sε(fol(p$\cap${s:$\forall$p(((p$\supseteq$(folp))$\wedge\alpha$) $\rightarrow$(sεp))})))$\rightarrow$(sε(fol p)). Folding $\supseteq$ gives: (fol p)$\supseteq$(fol(p$\cap${s:$\forall$p(((p$\supseteq$(fol p))$\wedge\alpha$) $\rightarrow$(sεp))})). Generalizing, it suffices to prove for all α: (fol p)$\supseteq$(fol(p$\cap\alpha$)). Since p$\supseteq$(p$\cap\alpha$) this follows by FOL3. QED.

FOL8: (dl 'κ 'Γ 'α_i:'β_{ij}/'χ_i)=(fol(dl 'κ 'Γ 'α_i:'β_{ij}/'χ_i)) proof: Unfolding dl gives:$\cap${p: (p$\supseteq$(fol p))$\wedge$(p$\supseteq$'Γ)$\wedge$

$\forall_i((('\alpha_i \varepsilon p) \wedge \wedge_{j=1,mi}('(\neg\beta_{ij}) \not\in '\kappa)) \rightarrow ('\chi_i \varepsilon p))\}$. By FOL7 this is equivalent to: $fol(\cap\{p: (p \supseteq (fol\ p)) \wedge (p \supseteq '\Gamma) \wedge \forall_i((('\alpha_i \varepsilon p) \wedge \wedge_{j=1,mi}('(\neg\beta_{ij}) \not\in '\kappa)) \rightarrow ('\chi_i \varepsilon p))\})$ Folding dl then proves the theorem: $fol(dl\ '\kappa\ '\Gamma\ '\alpha_i{:}'\beta_{ij}/'\chi_i)$ QED.

FOL9: $('k=(dl\ '\kappa\ '\Gamma\ '\alpha_i{:}'\beta_{ij}/'\chi_i)) \rightarrow ('\kappa=(fol\ '\kappa))$ proof: From the hypothesis and FOL8 $'k=(fol(dl\ '\kappa\ '\Gamma\ '\alpha_i{:}'\beta_{ij}/'\chi_i))$ is derived. Using the hypothesis to replace $(dl\ '\kappa\ '\Gamma\ '\alpha_i{:}'\beta_{ij}/'\chi_i)$ by $'\kappa$ in this result gives: $('\kappa=(fol\ '\kappa))$ QED.

4. Intensional Semantics of FOL

The meaning (i.e. mg) [Brown 1978, Boyer&Moore 1981] or rather disquotation of a sentence of First Order Logic (i.e. FOL) is defined to satisfy the laws given in Figure 5 below . mg is defined in terms of mgs which maps a FOL object language sentence and an association list into a meaning. Likewise, mgn maps a FOL object language term and an association list into a meaning. An association list is a list of pairs consisting of an object language variable and the meaning to which it is bound.

M0: $(mg\ '\alpha) =df\ (mgs\ '(\forall\gamma 1...\gamma n\ \alpha)'())$ where $'\gamma 1...'\gamma n$ are all the free variables in $'\alpha$

M1: $(mgs\ '(\alpha \rightarrow \beta)a) \leftrightarrow ((mgs\ '\alpha\ a) \rightarrow (mgs\ '\beta\ a))$

M2: $(mgs\ '\#f\ a) \leftrightarrow \#f$

M3: $(mgs\ '(\forall\ \gamma\ \alpha)a) \leftrightarrow \forall x(mgs\ '\alpha(cons(cons\ '\gamma\ x)a))$

M4: $(mgs\ '(\pi\ \delta_1...\delta_n)a) \leftrightarrow (\pi(mgn\ '\delta_1\ a)...(mgn\ '\delta_n\ a))$ for each predicate symbol $'\pi$

M5: $(mgn\ '(\phi\ \ \delta_1...\delta_n)a) = (\phi(mgn\ '\delta_1\ a)...(mgn\ '\delta_n\ a))$ for each function symbol $'\phi$

M6: $(mgn\ '\gamma\ a) = (cdr(assoc\ '\gamma\ a))$

M7: (assoc v L) = (if(eq? v(car(car L))) (car L) (assoc v(cdr L)))

where: cons, car, cdr, eq?, if are axiomatized as they are axiomatized in Scheme.

Figure 5: The Meaning of FOL Sentences

The meaning of a set of sentences is defined in terms of the meanings of the sentences in the set as:

$(ms\ '\kappa) =df\ \forall s((s\varepsilon'\kappa) \rightarrow (mg\ s))$

MS1: $(ms\{'\alpha{:}\ \Gamma\}) \leftrightarrow \forall\xi(\Gamma \rightarrow \alpha)$ where ξ is the sequence of all the free variables in $'\alpha$ and where Γ is any sentence of the intensional semantics. proof: $(ms\{'\alpha{:}\Gamma\})$ Unfolding ms and the set pattern abstraction symbol gives: $\forall s((s\varepsilon\{s{:}\ \exists\xi((s='\alpha) \wedge \Gamma)\}) \rightarrow (mg\ s))$ where ξ is a sequence of the free variables in 'a. This is equivalent to: $\forall s((\exists\xi((s='\alpha) \wedge \Gamma))) \rightarrow (mg\ s))$ which is logically equivalent to: $\forall s \forall\xi\ (((s='\kappa) \wedge \Gamma) \rightarrow (mg\ s))$ which is equivalent to: $\forall\xi(\Gamma \rightarrow (mg\ '\alpha))$ Unfolding mg using M0-M7 then gives: $\forall\xi(\Gamma \rightarrow \alpha)$ QED

The meaning of the union of two sets of FOL sentences is the conjunction of their meanings (i.e. MS3) and the meaning of a set is the meaning of all the sentences in the set (i.e. MS2):

MS2: $(ms\{'\Gamma_i\}) \leftrightarrow \forall i \forall\xi_i \Gamma_i$ proof: $(ms\{'\Gamma_i\})$ Unfolding the set notation gives: $(ms\{'\Gamma_i{:}\ \#t\})$

By MS1 this is equivalent to: $\forall_i \forall\xi_i(\#t \rightarrow \Gamma_i)$ which is equivalent to: $\forall i \forall\xi_i \Gamma_i$ QED.

MS3: $(ms('\kappa \cup '\Gamma)) \leftrightarrow ((ms\ '\kappa) \wedge (ms\ '\Gamma))$ proof: Unfolding ms and union in: $(ms('\kappa \cup '\Gamma))$ gives: $\forall s((s\varepsilon\{s{:}\ (s\varepsilon'\kappa) \vee (s\varepsilon'\Gamma)\}) \rightarrow (mg\ s))$ or rather: $\forall s(((s\varepsilon'\kappa) \vee (s\varepsilon'\Gamma)) \rightarrow (mg\ s))$ which is logically equivalent to: $(\forall\alpha((s\varepsilon'\kappa) \rightarrow (mg\ s))) \wedge (\forall s((s\varepsilon'\Gamma) \rightarrow (mg\ s)))$. Folding ms twice then gives:$((ms\ '\kappa) \wedge (ms\ '\Gamma))$ QED.

The meaning operation may be used to develop an Intensional Semantics for a FOL object language by axiomatizing the modal concept of necessity so that it satisfies the theorem:

C1: $('\alpha\varepsilon(fol\ '\kappa)) \leftrightarrow ([]\ ((ms\ '\kappa) \rightarrow (mg\ '\alpha)))$

for every sentence $'\alpha$ and every set of sentences $'\kappa$ of that FOL object language. The necessity symbol is represented by a box: []. C1 states that a sentence of FOL is a FOL-theorem (i.e. fol) of a set of sentences of FOL if and only if the meaning of that set of sentences necessarily implies the meaning of that sentence. One modal logic which satisfies C1 is the Z Modal Quantificational Logic described in [Brown 1987; Brown 1989] whose theorems are recursively enumerable and which extends the weaker possibility axioms used in [Lewis 1936; Bressan 1972; Hendry & Pokriefka 1985]. Z includes all the laws of S5 modal Logic [Hughes & Cresswell 1968] whose laws are given in Figure 6. κ and Γ represent arbitrary sentences of the intentional semantics.

R0: from α infer $([]\ \kappa)$	A2: $([](\kappa \rightarrow \Gamma)) \rightarrow (([]\kappa) \rightarrow ([]\Gamma))$
A1: $([]\kappa) \rightarrow \kappa$	A3: $([]\kappa) \vee ([]\neg[]\kappa)$

Figure 6: The Laws of S5 Modal Logic

These S5 modal laws and the laws of FOL given in Figure 4 constitute an S5 Modal Quantificational Logic similar to [Carnap 1946; Carnap 1956], and a FOL version [Parks 1976] of [Bressan 1972] in which the Barcan formula: $(\forall\gamma([]\kappa))\rightarrow([]\forall\gamma\kappa)$ and its converse hold. The R0 inference rule implies that anything derivable in the metatheory is necessary. Thus, in any logic with R0, contingent facts would never be asserted as additional axioms of the metatheory. The defined Modal symbols used herein are listed in Figure 7.

Symbol	Definition	Meaning		Symbol	Definition	Meaning
$<>\kappa$	$\neg\,[]\,\neg\kappa$	α is logically possible		$[\kappa]\,\Gamma$	$[]\,(\kappa\rightarrow\Gamma)$	β entails α
$\kappa\equiv\Gamma$	$[]\,(\kappa\leftrightarrow\Gamma)$	α is logically equivalent to β		$<\kappa>\,\Gamma$	$<>\,(\kappa\wedge\Gamma)$	α and β is logically possible

Figure 7: Defined Symbols of Modal Logic

From the laws of the Intensional Semantics we prove that the meaning of the set of FOL consequences of a set of sentences is the meaning of that set of sentences (C2), the FOL consequences of a set of sentences contain the FOL consequences of another set if and only if the meaning of the first set entails the meaning of the second set (C3), and the sets of FOL consequences of two sets of sentences are equal if and only if the meanings of the two sets are logically equivalent (C4):

C2: $(\text{ms}(\text{fol}\ '\kappa))\equiv(\text{ms}\ '\kappa)$ proof: The proof divides into two cases: (1) $[(\text{ms}\ '\kappa)](\text{ms}(\text{fol}\ '\kappa))$. Unfolding the second ms gives: $[(\text{ms}\ '\kappa)]\forall s((s\varepsilon(\text{fol}\ '\kappa))\rightarrow(\text{mg}\ s))$. By the soundness part of C1 this is equivalent to: $[(\text{ms}\ '\kappa)]\forall s(([(\text{ms}\ '\kappa)](\text{mg}\ s))\rightarrow(\text{mg}\ s))$. By the S5 laws this is e: $\forall s(([(\text{ms}\ '\kappa)](\text{mg}\ s))\rightarrow[(\text{ms}\ '\kappa)](\text{mg}\ s))$ which is a tautology.

(2) $[(\text{ms}(\text{fol}\ '\kappa))](\text{ms}\ '\kappa)$ Unfolding ms twice gives: $[\forall s((s\varepsilon(\text{fol}\ '\kappa))\rightarrow(\text{mg}\ s))]\forall s((s\varepsilon'\kappa)\rightarrow(\text{mg}\ s))$

which is: $[\forall s((s\varepsilon(\text{fol}\ '\kappa))\rightarrow(\text{mg}\ s))]((s\varepsilon'\kappa)\rightarrow(\text{mg}\ s))$ Backchaining on the hypothesis and then dropping it gives: $(s\varepsilon'\kappa)\rightarrow(s\varepsilon(\text{fol}\ '\kappa))$. Folding $\supseteq$ gives an instance of FOL1. QED.

C3: $(\text{fol}\ '\kappa)\supseteq(\text{fol}\ '\Gamma)\leftrightarrow([(\text{ms}\ '\kappa)](\text{ms}\ '\Gamma))$

proof: Unfolding $\supseteq$ gives: $\forall s((s\varepsilon(\text{fol}\ '\Gamma))\rightarrow(s\varepsilon(\text{fol}\ '\kappa)))$.By C1 twice this is:$\forall s(([(\text{ms}\ '\Gamma)](\text{mg}\ s))\rightarrow([(\text{ms}\ '\kappa)](\text{mg}\ s)))$

By the laws of S5 modal logic this is equivalent to: $([(\text{ms}\ '\kappa)]\forall s(([(\text{ms}\ '\Gamma)](\text{mg}\ s))\rightarrow(\text{mg}\ s)))$. By C1 this is: $[(\text{ms}\ '\kappa)]\forall s((s\varepsilon(\text{fol}\ '\Gamma))\rightarrow(\text{mg}\ s))$. Folding ms then gives: $[(\text{ms}\ '\kappa)](\text{ms}(\text{fol}\ '\Gamma))$. By C2 this is : $[(\text{ms}\ '\kappa)](\text{ms}\ '\Gamma)$. QED.

C4: $((\text{fol}\ '\kappa)=(\text{fol}\ '\Gamma))\leftrightarrow((\text{ms}\ '\kappa)\equiv(\text{ms}\ '\Gamma))$ proof: This is equivalent to $(((\text{fol}\ '\kappa)\supseteq(\text{fol}\ '\Gamma))\wedge((\text{fol}\ '\Gamma)\supseteq(\text{fol}\ '\kappa)))\leftrightarrow([(\text{ms}\ '\kappa)](\text{ms}\ '\Gamma))\wedge([(\text{ms}\ '\Gamma)](\text{ms}\ '\kappa))$ which follows by using C3 twice.

5. Default Logic Represented in Modal Logic

The fixed-point equation for Default Logic may be expressed as a necessary equivalence in an S5 Modal Quantificational Logic supplemented with propositional quantifiers [Fine 1970; Bressan 1972] which obey the normal laws of Second Order Logic (i.e. laws analogous to MR2, MA4, and MA5 given in Figure 4 where γ is now a propositional variable), as follows: $\kappa\equiv(\text{DL}\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)$

where DL is defined as: $(\text{DL}\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)=_{df}\exists p(p\wedge([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow[p]\chi_i))$

where the propositional variable p does not occur in Γ, α_i, β_{ij}, and χ_i. When the context is obvious $\Gamma\ \alpha_i{:}\beta_{ij}/\chi_i$ is omitted and just $(\text{DL}\ \kappa)$ is written. The idiom $\exists p(p\wedge([]\varphi))$ may be intuitively read as a nominal as the (possibly infinite) disjunction of all propositions such that $[]\varphi$. When $[]\varphi$ holds for only a finite number of propositions: $\varphi_1,\ldots,\varphi_n$ then $\exists p(p\wedge([]\varphi))$is equivalent to: $\varphi_1\vee\ldots\vee\varphi_n$, but there is in no requirement that φ holds for only a finite or even only a denumerable number of propositions.

The first two theorems state that DL entails Γ and any conclusion χ_i of a default whose entailment condition holds in DL and whose possible conditions are possible with κ.

MD1: $[(\text{DL}\ \kappa\ \ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\Gamma$

proof: Unfolding DL gives: $[\exists p(p\wedge([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([p]\chi_i)))]\Gamma$. Since p is not free in Γ, pulling $\exists p$ out of the hypothesis of the entailment gives:

$\forall p((([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow([p]\chi_i)))\rightarrow([p]\Gamma))$ which is a tautology. QED.

MD2: $(([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij})) \rightarrow ([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\chi_i)$

proof: Unfolding both occurrences of DL gives:

$(([\exists p(p\wedge([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow([p]\chi_i)))]\alpha_i)\wedge(\wedge_{j=1,mi}(<\kappa>\beta_{ij})))$

$\rightarrow([\exists p(p\wedge([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([p]\chi_i)))]\chi_i)$

Since p is not free in α_i and χ_i, pulling $\exists$p out of the hypotheses of the outer two entailments gives:

$((\forall p((([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([p]\chi_i)))\rightarrow([p]\alpha_i)))\wedge(\wedge_{j=1,mi}(<\kappa>\beta_{ij})))$

$\rightarrow\forall p((([p]\Gamma)\wedge\wedge_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([p]\chi_i)))\rightarrow([p]\chi_i))$

Instantiating the p in the hypothesis to the p in the conclusion gives:

$((((([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow([p]\chi_i)))\rightarrow([p]\alpha_i))\wedge(\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\wedge([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([p]\chi_i))) \rightarrow([p]\chi_i)$

which simplifies to just: $(([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij})\wedge([p]\Gamma)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([p]\chi_i))) \rightarrow([p]\chi_i)$

Forward chaining using the first and second hypotheses on the fourth proves the theorem. QED.

The concept (i.e. ss) of the combined meaning of all the sentences of the FOL object language whose meanings are entailed by a proposition is defined as follows: $(ss\ \kappa) =df\ \forall s(([\kappa](mg\ s))\rightarrow(mg\ s))$. SS1 shows that a proposition entails the combined meaning of the FOL object language sentences that it entails. SS2 shows that if a proposition is necessarily equivalent to the combined meaning of the FOL object language sentences that it entails, then there exists a set of FOL object language sentences whose meaning is necessarily equivalent it:

SS1: $[\kappa](ss\ \kappa)$

proof: By R0 it suffices to prove: $\kappa\rightarrow(ss\ \kappa)$. Unfolding ss gives: $\kappa\rightarrow\forall s(([\kappa](mg\ s))\rightarrow(mg\ s))$

which is equivalent to:$\forall s(([\kappa](mg\ s))\rightarrow(\kappa\rightarrow(mg\ s)))$ which is an instance of A1. QED.

SS2: $(\kappa\equiv(ss\ \kappa))\rightarrow \exists s(\kappa\equiv(ms\ s))$

proof: Letting s be {s: $([\kappa](mg\ s))$ gives $(\kappa\equiv(ss\ \kappa))\rightarrow (\kappa\equiv(ms\{s: ([\kappa](mg\ s))))$. Unfolding ms and lambda conversion gives: $(\kappa\equiv(ss\ \kappa))\leftrightarrow (\kappa\equiv\forall s(([\kappa](mg\ s))\rightarrow(mg\ s)))$. Folding ss gives a tautology. QED.

The theorems MD3 and MD4 are analogous to MD1 and MD2 except that DL is replaced by the combined meanings of the sentences entailed by DL.

MD3: $[ss(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i)]\forall i\Gamma_i$

proof: By R0 it suffices to prove $(ss(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i))\rightarrow\forall i\Gamma_i$ which is equivalent to:

$(ss(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i))\rightarrow\Gamma_i$. Unfolding ss gives: $(\forall s(([(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i)](mg\ s))\rightarrow(mg\ s)))\rightarrow\Gamma_i$ which by the meaning laws M0-M7 is equivalent to: $(\forall s(([(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i)](mg\ s))\rightarrow(mg\ s)))\rightarrow(mg\ '\Gamma_i)$. Backchaining on $(mg\ '\Gamma_i)$ with s in the hypothesis assigned to be $'\Gamma_i$ in the conclusion shows that it suffices to prove:

$([(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i)](mg\ '\Gamma_i))$ which by the meaning laws: M0-M7 is equivalent to: $([(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i)]\Gamma_i)$ which by the laws of S5 is equivalent to: $([(DL\ \kappa\ \forall i\Gamma_i\ \alpha_i{:}\beta_{ij}/\chi_i)]\forall i\Gamma_i)$ which is an instance of MD1. QED.

MD4: $(([ss(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge(\wedge_{j=1,mi}<\kappa>\beta_{ij}))\rightarrow ([ss(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\chi_i)$

proof: Unfolding the last ss gives:

$(([ss(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([\forall s(([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)](mg\ s))\rightarrow(mg\ s))]\chi_i)$

Instantiating s in the hypothesis to $'\chi_i$ and then dropping the hypothesis gives:

$(([ss(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([(([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)](mg\ '\chi_i))\rightarrow(mg\ '\chi_i))]\chi_i)$. Using the meaning laws M0-M7 gives: $(([ss(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([(([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\chi_i)\rightarrow\chi_i)]\chi_i)$.Backchaining on χ_i, it suffices to prove: $(([ss(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\chi_i)$

By SS1 and the first hypothesis it suffices to prove:

$(([(DL\kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow([(DL\ \kappa\ \Gamma\ \alpha_i{:}\beta_{ij}/\chi_i)]\chi_i)$ which is an instance of MD2. QED.

Finally MD5, MD6, and MD7 show that talking about the meanings of sets of FOL sentences in the modal representation of Default Logic is equivalent to talking about propositions in general.

MD5: $(\exists p((ms\ p)\wedge([(ms\ p)](\forall i\Gamma_i))\wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow[(ms\ p)]\chi_i)))\equiv(DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)$

proof: The proof divides into two entailments:

(1) $[\exists p((ms\ p)\wedge([(ms\ p)]\forall i\Gamma_i)\wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}(<\kappa>\beta_{ij}))\rightarrow[(ms\ p)]\chi_i))]\ (DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)$

DL is unfolded giving: $[((ms\ p)\wedge([(ms\ p)](\forall i\Gamma_i))\wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow[(ms\ p)]\chi_i))]$

$\exists p(p\wedge([p]\forall i\Gamma_i)\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow[p]\chi_i))$

Instantiating the quantified p in the conclusion to be (ms p) produces a tautology.

(2) $[(DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)]\ \exists p((ms\ p)\wedge([(ms\ p)]\forall i\Gamma_i)\wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow[(ms\ p)]\chi_i))$

p is assigned to be the set: $\{s: [(DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)](mg\ s)\}$.

Since p only occurs in (ms p) and since $(ms\{s: ([(DL\ \kappa)](mg\ s))\})$ is equivalent to $(ss(DL\ \kappa))$ we get:

$[(DL\ \kappa)]((ss(DL\ \kappa))\wedge([(ss(DL\ \kappa))]\forall i\Gamma_i)\wedge\forall_i((([(ss(DL\ \kappa))]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow([(ss(DL\ \kappa))]\chi_i)))$

which holds by theorems SS1, MD3, and MD4. QED.

MD6: $(ss(DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i))\equiv(DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)$

proof: In view of SS1, it suffices to prove: $([(ss(DL\ \kappa))](DL\ \kappa))$. Unfolding the second occurrence of DL gives: $[(ss(DL\ \kappa))]\exists p(p\wedge([p](\forall i\Gamma_i))\wedge\forall_i((([p]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow[p]\chi_i))$. Letting p be $(ss(DL\ \kappa))$ then gives:

$[(ss(DL\ \kappa))]((ss(DL\ \kappa))\wedge([(ss(DL\ \kappa))](\forall i\Gamma_i))\wedge\forall_i((([(ss(DL\ \kappa))]\alpha_i)\wedge\wedge_{j=1,mi}<\kappa>\beta_{ij})\rightarrow([(ss(DL\ \kappa))]\chi_i)))$

which holds by theorems MD3 and MD4. QED.

MD7: $(\kappa\equiv(DL\ \kappa(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i))\rightarrow\exists s(\kappa\equiv(ms\ s))$

proof: From the hypothesis and MD6 $\kappa\equiv(ss(DL\ \kappa))$ is derived. Using the hypothesis to replace $(DL\ \kappa)$ by κ in this result gives: $\kappa\equiv(ss(DL\ \kappa))$, By SS2 this implies the conclusion. QED.

6. Conclusion: The Relationship between Default Logic and the Modal Logic

The relationship between the proof theoretic definition of Default Logic [Reiter 1980] and the modal representation is proven in two steps. First theorem DL1 shows that the meaning of the set dl is the proposition DL and then theorem DL2 shows that a set of FOL sentences which contains its FOL theorems is a fixed-point of the fixed-point equation of Default Logic with an initial set of axioms and defaults if and only if the meaning (or rather disquotation) of that set of sentences is logically equivalent to DL of the meanings of that initial set of sentences and those defaults.

DL1: $(ms(dl(fol\ '\kappa)\{'\Gamma_i\}\ '\alpha_i{:}'\beta_{ij}/'\chi_i))\equiv(DL(ms\ '\kappa)(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)$

proof: $(ms(dl(fol\ '\kappa)\{'\Gamma_i\}\ '\alpha_i{:}'\beta_{ij}/'\chi_i))$ Unfolding the definition of dl gives:

$ms(\cap\{p: (p\supseteq(fol\ p))\wedge(p\supseteq\{'\Gamma_i\})\wedge\forall_i((('\alpha_i\varepsilon p)\wedge\wedge_{j=1,mi}('(\neg\beta_{ij})\not\in(fol\ '\kappa)))\rightarrow('\chi_i\varepsilon p))\})$. By FOL6 this is:

$ms\{s:\forall p((((fol\ p)\supseteq\{'\Gamma_i\})\wedge\forall_i((('\alpha_i\varepsilon(fol\ p))\wedge\wedge_{j=1,mi}('(\neg\beta_{ij})\not\in(fol\ '\kappa)))\ \rightarrow('\chi_i\varepsilon(fol\ p))))\ \rightarrow(s\varepsilon(fol\ p)))\}$

Using C1 four times, C3, and FOL4 this is equivalent to: $ms\{s:\forall p((([(ms\ p)](ms\{'\Gamma_i\}))\wedge\forall_i((([(ms\ p)](mg\ '\alpha_i))\wedge\wedge_{j=1,mi}\neg([(ms\ '\kappa)](mg\ '\neg\beta_{ij})))\ \rightarrow([(ms\ p)](mg\ '\chi_i))))\rightarrow([(ms\ p)](mg\ s)))\}$

By the meaning laws M0-M7 this is equivalent to:

$ms\{s:\forall p((([(ms\ p)](ms\{'\Gamma_i\}))\ \wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}\neg([(ms\ '\kappa)]\neg\beta_{ij}))\rightarrow([(ms\ p)]\chi_i)))\quad\rightarrow([(ms\ p)](mg\ s)))\}$

By MS2 this is equivalent to:

$ms\{s:\forall p((([(ms\ p)](\forall i\Gamma_i))\ \wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}\neg([(ms\ '\kappa)]\neg\beta_{ij}))\rightarrow([(ms\ p)]\chi_i)))\quad\rightarrow([(ms\ p)](mg\ s)))\}$

Folding <> gives: $ms\{s:\forall p((([(ms\ p)](\forall i\Gamma_i))\wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}(<(ms\ '\kappa)>\beta_{ij}))\ \rightarrow([(ms\ p)]\chi_i)))\rightarrow([(ms\ p)](mg\ s)))\}$

By S5 Modal Quantificational Logic this is equivalent to:

$ms\{s: ([\exists p((ms\ p)\wedge([(ms\ p)](\forall i\Gamma_i))\wedge\forall_i((([(ms\ p)]\alpha_i)\wedge\wedge_{j=1,mi}(<(ms\ '\kappa)>\beta_{ij}))\rightarrow([(ms\ p)]\chi_i)))]\ (mg\ s))\}$

By MD5 this is equivalent to: $ms\{s: ([(DL(ms\ '\kappa)(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)](mg\ s))\}$

Unfolding ms and lambda conversion gives: $\forall s(([(DL(ms\ '\kappa)(\forall i\Gamma_i)\alpha_i{:}\beta_{ij}/\chi_i)](mg\ s))\rightarrow(mg\ s))$

Folding ss gives: ss(DL(ms 'κ)(∀iΓ$_i$)α$_i$:β$_{ij}$/χ$_i$). By MD6 is equivalent to:(DL(ms 'κ)(∀iΓ$_i$)α$_i$:β$_{ij}$/χ$_i$) QED.

DL2: ((fol 'κ)=(dl(fol 'κ){'Γ$_i$} 'α$_i$:'β$_{ij}$/'χ$_i$))↔((ms 'κ)≡(DL(ms 'κ)(∀iΓ$_i$)α$_i$:β$_{ij}$/χ$_i$))

proof: By FOL8 (fol 'κ)=(dl(fol 'κ){'Γ$_i$} 'α$_i$:'β$_{ij}$/'χ$_i$) is equivalent to: (fol 'κ)=(fol(dl(fol 'κ){'Γ$_i$} 'α$_i$:'β$_{ij}$/'χ$_i$)).

By C4 this is equivalent to: (ms 'κ)≡(ms(dl(fol 'κ){'Γ$_i$} 'α$_i$:'β$_{ij}$/'χ$_i$)).

By DL1 this is equivalent to: (ms 'κ)≡(DL(ms 'κ)(∀iΓ$_i$)α$_i$:β$_{ij}$/χ$_i$) QED.

Theorem DL2 shows that the set of theorems: (fol 'κ) of a set 'κ is a fixed-point of a fixed-point equation of Default Logic if and only if the meaning (ms 'κ) of 'κ is a solution to the necessary equivalence. Furthermore, by FOL9 there are no other fixed-points (such as a set not containing all its theorems) and by MD7 there are no other solutions (such as a proposition not representable as a sentence in the FOL object language). Therefore, the Modal representation of Default Logic (i.e. DL), faithfully represents the set theoretic description of Default Logic (i.e. dl).Finally, we note that (∀iΓ$_i$) and (ms 'κ) may be generalized to be arbitrary propositions Γ and κ giving the more general modal representation: κ≡(DL κ Γ α$_i$:β$_{ij}$/χ$_i$).

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