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# DYNAMICAL SYSTEMS IN DESCRIPTION OF NONLINEAR RECURSIVE REGRESSION TRANSFORMERS

Mykola Kirichenko, Volodymyr Donchenko, Denys Serbaev

*Abstract: The task of approximation-forecasting for a function, represented by empirical data was investigated. Certain class of the functions as forecasting tools: so called RFT-transformers, – was proposed. Least Square Method and superposition are the principal composing means for the function generating. Besides, the special classes of beam dynamics with delay were introduced and investigated to get classical results regarding gradients. These results were applied to optimize the RFT-transformers. The effectiveness of the forecast was demonstrated on the empirical data from the Forex market.*

*Keywords: empirical functions, learning samples, beam dynamics with delay, recursive nonlinear regressive transformer, Generalized Inverse, Least Square Method.*

*ACM Classification Keywords: G.1.2 Approximation, G.1.3 Numerical Linear Algebra, G.1.6 Optimization*

## Introduction

Approximation of the function represented by its values is classical direction of researches for mathematics both in the deterministic statement, and in statistical variant (see, [4,5], and also [1-3]). Classical results in this area are full enough represented in the specified works. They show importance of superposition-recurrence as means of generation of a class of approximating functions.

Natural way of use of approximation in applied researches is the forecast of values of researched function. Means of forecasting have special importance in the modern unified systems of automation of management of firm: in so-called Business Intelligence systems and, in particular, in their structural elements as DSS.

The Method of optimal constructing of forecasting means, based on application of classical LSM in combination with superposition-recursion, linear transformations of coordinates and component-wise nonlinear transformations in the context of so called RFT-transformers is offered in present article. The variant of building of RFT-transformers together with algorithm of synthesis of basic element was discussed in [6]. Special classes of systems with delay both for single trajectories and for their beam is introduced and investigated in the article, since optimization is connected to representation of researched objects by control systems with delay.

## Recursive Regression Nonlinear Transformer: RFT-transformer

As it was already marked, the idea of recursive regression nonlinear transformer (RFT-transformer) is offered in [6], in variant, which can be named reverse recursion. Such transformer: with reverse recursion [6], and a forward one, which will be offered and considered below, - are constructing by recursive application of the certain standard element, which will be designated by abbreviation ERRT (Elementary Recursive Regression Transformer). Process of construction of the RFT-transformer consists in connection of the next ERRT to already constructed during execution of the previous steps transformer according to one of three possible types of connection. Types of connection which will be designated as “parinput”, “paroutput” and “seq”, realize natural variants of use of an input signal: parallel or sequential over input, - and parallel over output. An input of the again attached ERRT, in variant of reverse recursion [6], is the input of whole RFT, and in the variant of a forward recursion, which will offered below, the input of whole system is an input of the already constructed RFT.

The basic structural element of the RFT-transformer is ERRT - an element [6], which is determined as mapping from $R^{n-1}$ in $R^m$ of a kind:

$$y = A_+ \Psi_u \left( C \begin{pmatrix} x \\ 1 \end{pmatrix} \right), \tag{1}$$

which approximates the dependence represented by training sample $(x_1^{(0)}, y_1^0), \ldots, (x_M^{(0)}, y_M^{(0)})$, $x_i^{(0)} \in R^{n-1}$, $y_i^{(0)} \in R^m$, $i = \overline{1, M}$,

where:

- C–(n×n) - matrix, which performs affine transformation of the vector x$\in R^{n-1}$ - an input of the system; it is considered to be given at the stage of synthesis of ERRT;
- $\Psi$u – nonlinear mapping from $R^n$ in $R^n$, which consists in component-wise application of scalar functions of scalar argument $u_i \in \Im$, $i = \overline{1, n}$ from the given final set $\Im$ of allowable transformations, including identical transformation: must be selected to minimize residual between input and output on training sample during synthesis of ERRT;
- A+– solution A with minimal trace norm of the matrix equation

  $$A X_{\Psi_u C} = Y, \tag{2}$$

  in which matrix $X_{\Psi_u C}$ formed from vector-columns $\Psi_u (C \begin{pmatrix} x_i^{(0)} \\ 1 \end{pmatrix}) = \Psi_u (z_i^{(0)})$, and Y – from columns $y_i^{(0)}, i = \overline{1, M}$ .

In effect, ERRT represents empirical regression for linear regression y on $\Psi$u(C$\begin{pmatrix} x \\ 1 \end{pmatrix}$), constructed with method of the least squares, with previous affine transformation of system of coordinates for vector regressor *x* and following nonlinear transformation of each received coordinate separately.

*Remark 1.* Further we shall assume that functions of component-wise transformations from $\Im$ would have a necessary degree of smoothness where it is necessary.

Task of synthesis of ERRT by an optimal selection of nonlinear transformations of coordinates on the given training sample was introduced and solved in already quoted above work [6]. The solution of a task of synthesis is based on methods of the analysis and synthesis of the pseudoinverse matrices, developed in [7-9]. Particularly, reversion of Grevil's formula [10] was received in these works that recurrently allows recalculating pseudoinverse matrices at replacement of a line or a column of the appropriate matrix.

## Generalized Recursive Regression Transformers with Forward Recursion

Recursion in construction of the RFT-transformer in variant of forward recursion offered below will be considered in the generalized variant in which several ERRTs is used in recurrent connection to already available RFT-structure. Total quantity of recurrent references we shall designate through N, and quantity of ERRTs used on a step m – through $k_m$ , $m = \overline{1, N}$ . The common number of ERRTs, used for construction of whole transformer will be designated through T: $T = \sum_{m=1}^{N} k_m$ .

Variants of the generalized forward recursion which depend from type of connection of attached ERRT: parinput, paroutput and seq, - are represented on figures 1-3. Where $\hat{y}$ designates an output of already available RFT-structure or an output of the same structure after connection of the next ERRT from the total number $k_m$ of such elements, attached on a step m of the recursion: $m = \overline{1, N}$ . Each figure is accompanied by the system of equations which determine transformation of a signal on the next step of recursion.

- Type parinput:

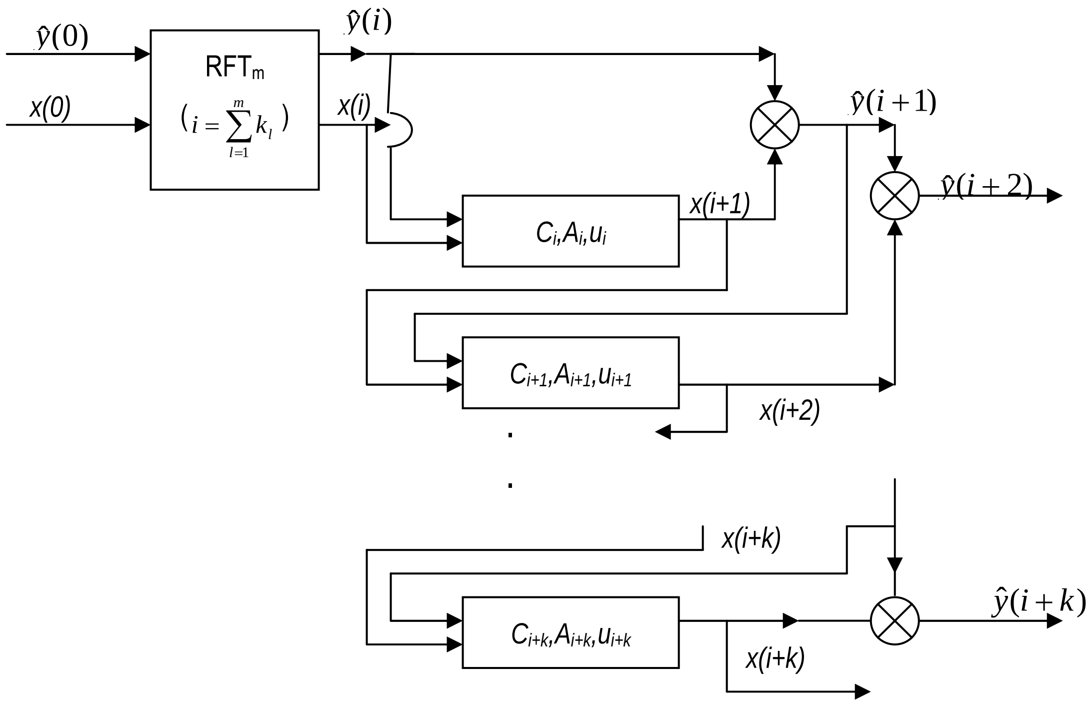


Figure 1. Scheme of connection of parinput type – forward recursion.

In parinput type of connection already constructed structure approximates an output of training sample by its input, and set of ERRTs - resulting residual of such approximation depending from an input of training sample. Transformation of the information at this type of connection is described by the following system:

$$
\begin{aligned}
&x(i+j) = A_{i+j-1}\Psi_{u_{i+j-1}}(C_{i+j-1}x(i)),\\
&\hat{y}(i+j) = \hat{y}(i+j-1) + A_{i+j-1}\Psi_{u_{i+j-1}}(C_{i+j-1}\cdot x(i)).\\
&i = \sum_{l=1}^{m} k_l,\ j = \overline{1, k_{m+1}}
\end{aligned}
\tag{3}
$$

- Type paroutput:

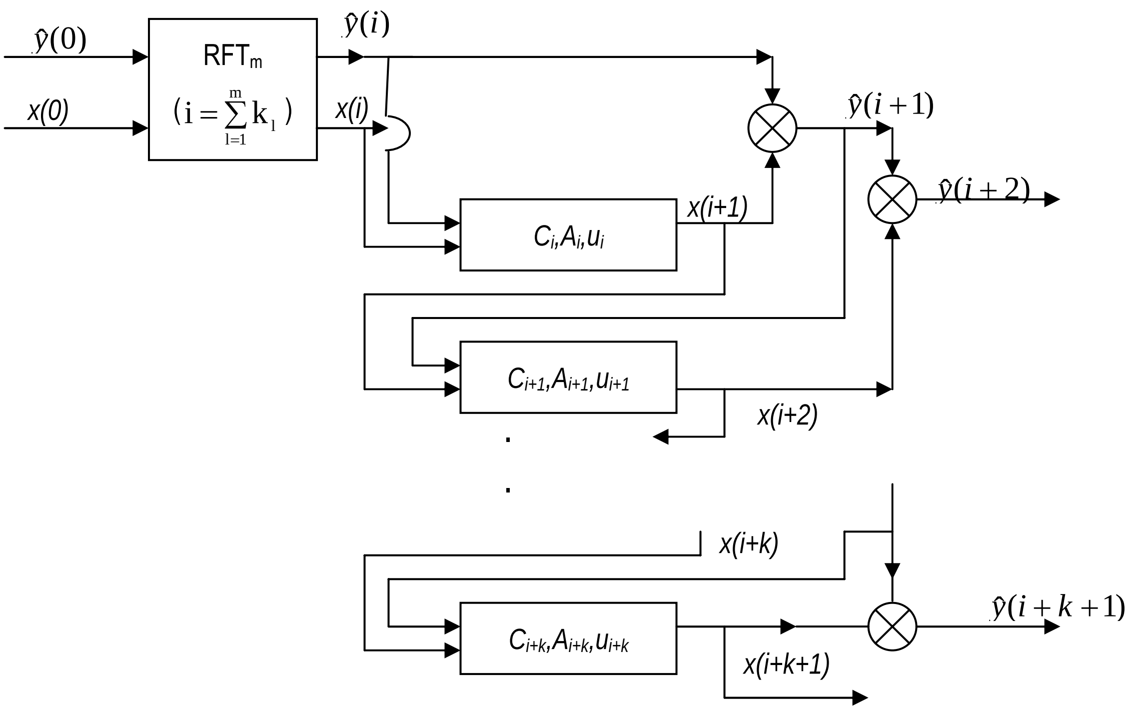


Figure 2. Scheme of connection of paroutput type – forward recursion.

The system describing transformation inside the transformer and communication between an input and an output, at this type of connection looks like:

$$
\begin{aligned}
&x(i+j) = A_{i+j-1}\Psi_{u_{i+j-1}}(C_{i+j-1}x(i+j-1)),\\
&\hat{y}(i+j) = \hat{y}(i+j-1) + A_{i+j-1}\Psi_{u_{i+j-1}}(C_{i+j-1}\cdot x(i+j-1)).\\
&i = \sum_{l=1}^{m} k_l,\ j = \overline{1, k_{m+1}}
\end{aligned}
\tag{4}
$$

- Type seq:

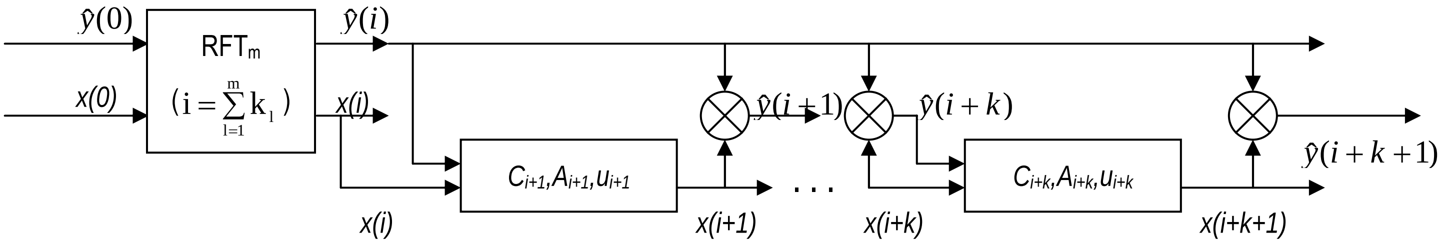


Figure 3. Scheme of connection of seq type – forward recursion.

The equations describing transformation of the information on the next step of recursion look as follows:

$$x(i+j) = A_{i+j-1}\Psi_{u_{i+j-1}}(C_{i+j-1}x(i+j-1)),$$
$$\hat{y}(i+j) = \hat{y}(i) + A_{i+j-1}\Psi_{u_{i+j-1}}(C_{i+j-1}\cdot x(i+j-1)).$$
$$i=\sum_{l=1}^{m} k_l,\ j=\overline{1,k_{m+1}} \qquad (5)$$

In this scheme of connection RFT$_{m-1}$ approximates an output part of training sample by input part, and set of ERRTs approximates residual which depends from an output of the next ERRTs.

Entry conditions for all types of connections are described by equations:

$x(0) = x$ – an input of whole RFT-transformer, (6)

$\hat{y}(0)=0,\ \hat{y}(1)=x(1)$ for all types of connections. (7)

According to (6) in a training mode the inputs of RFT-transformer are $x_{i1}^{(0)}$ : $x_i^{(0)} \in R^{n-1}$ , $y_i^{(0)} \in R^m$ , $i=\overline{1,M}$ , and outputs are – $y_i^{(0)} \in R^m$ , $i=\overline{1,M}$ .

Connections of recursive construction of the RFT-transformer are determined so, that standard functional of the least squares method is minimized during its construction, i.e.

$$\sum_{i=1}^{M} \| y_i^{(0)} - RFT(x_i^{(0)}) \|^2 . \qquad (8)$$

Equations (3) - (7) for N steps of recursion with common number T of used ERRTs, $T=\sum_{m=1}^{N} k_m$ , $k_m$ – quantity of ERRTs, used on a step with number $m=\overline{1,N}$ , and also efficiency functional (8) - represent mathematical model of RFT-transformer.

## Discrete control systems with delay

Equations (3)-(8) represent the system of the recurrent equations being certain generalization of a classical control system with discrete time (for details see [12-16]) and first of all in referring to delay.

### 2.1. The simple and combined control systems with delay

*Definition.* Simple, accordingly - combined, - nonlinear control system with delay on a time interval $\overline{0,N}$ is a control system whose trajectories are defined by system of recurrent equations (9), accordingly - (10), entry conditions (11) and efficiency functional (12) and represented below:

$$x(j+1) = f(x(j-s(j)), u(j), j) \quad , \tag{9}$$

$$x(j+1) = f(x(j), x(j-s(j)), u(j), j) \tag{10}$$

$$j = \overline{0, N-1}, \qquad x(0) = x^{(0)} , \tag{11}$$

$$I(U) = \Phi(x(\mathrm{N})) , \tag{12}$$

where function s(j): s(0)=0, $s(j) \in \{2,..., j\}$ , $j = \overline{0, N-1}$– is known.

Evidently, systems with delay for a beam of trajectories are determined. Entry conditions of trajectories of a beam we shall designate $(x(0))_i = x_i^{(0)}$ , $i = \overline{0, M}$ , M– quantity of trajectories of a beam. Trajectories of a beam for both types of systems with delay we shall designate by the appropriate indexation: $x_i(j)$ , $j = \overline{0, N-1}$ , $i = \overline{0, M}$ .

Let's define efficiency functional for a beam of dynamics which we shall consider dependent only from final states of trajectories of a beam, with equation:

$$I_p(U) = \sum_{i=1}^{M} \Phi^{(i)}(x_i(N)) . \tag{13}$$

*Remark 2.* Evidently, efficiency functional, as well as for classical control systems, may be defined on all trajectory or trajectories. However, systems with delay at which efficiency functional depends only from final states of a trajectory will be considered in context of RFT-transformers.

Phase trajectories of simple systems with delay are defined by a set of functions f(z, u, j), $j = \overline{0, N-1}$ with one argument z, responsible for a phase variable, and for combined one - a set f(z,v,u,j), $j = \overline{0, N-1}$ with two variables: z, v, which respond for phase variables. Gradients on a phase variables will be designated accordingly $grad_z f$ , $grad_v f$ , in this last case.

The problem of optimization for both types of such systems with delay is being solved, as well as in a classical case, with construction of the conjugate systems and functions of Hamilton. The assumptions of the smoothness providing correct construction of conjugate systems and functions of Hamilton, and also their use for gradients calculations on controls completely coincide with classical and further will be considered automatically executed.

### Optimization in simple control systems with delay

*Definition.* The conjugate system of a simple control system with delay (9), (11), (12) we shall name the following recurrent equation concerning p(k), $k = \overline{N,0}$ :

$$p(k) = \sum_{j \in \{j: j-s(j)=k, j>k\}} grad_{x(k)} \{ p^T(j+1) f(x(k), u(j), j) \} \quad , \qquad k = \overline{N-1, 0} \tag{14}$$

with the initial condition

$$p(N) = -grad_{x(N)}\Phi(x(N)) \tag{15}$$

Accordingly, in the case of a beam of trajectories the conjugate systems are defined for each trajectory by equations:

$$p^{(i)}(N) = -grad_{x_i(N)}\Phi^{(i)}(x_i(N)), \tag{16}$$

$$p^{(i)}(k) = \sum_{j\in\{j:j-s(j)=k,\, j>k\}} grad_{x_i(k)}\{p^{(i)^T}(j+1)f(x_i(k),u(j),j)\}$$

$$k = \overline{N-1,0},\, i = \overline{1,M}. \tag{17}$$

Function of Hamilton for simple system with delay is defined by a classical equation:

$$H(p(k+1),x(k-s(k)),u(k),k) = p^T(k+1)f(x(k-s(k)),u(k),k),\, k = \overline{N-1,0}. \tag{18}$$

For a beam of trajectories of a simple control system with delay the set of functions of Hamilton $H^{(i)}$, $i = \overline{1,M}$ for each of trajectories $x_i(k)$, $k = \overline{N-1,0}$.

*Theorem 1.* The gradient on control from the efficiency functional which depends only from final states of trajectories of a beam, for a simple control system with delay is defined by equations:

$$grad_{u(k)}I(U) = -\sum_{i=1}^{M} grad_{u(k)}\left\{p^{(i)^T}(k+1)f(x^{(i)}(k-s(k)),u(k),k)\right\} = -grad_{u(k)}\sum_{i=1}^{M} H_v^{(i)}(x_i(k),u(k),k),$$

$$k = \overline{0,N-1} \tag{19}$$

*The evidence.* The evidence will be carried out precisely the same as in a classical case: first - for one trajectory, and then by use of additivity of efficiency functional on trajectories of system.

### Optimization in the combined control systems with delay

*Definition.* The conjugate system for the combined control system with delay is the system determined by recurrent equations:

$$p(k) = grad_z\{p^T(j+1)f(x(k),x(k-s(k)),u(j),j) \; +$$

$$+ \sum_{j\in\{j:j-s(j)=k,\geq kj\}} grad_v\{p^T(j+1)f(x(k),x(j),u(j),j)\},$$

$$k = \overline{N-1,0},\, i = \overline{1,M} \tag{20}$$

with the initial condition

$$p(N) = -grad_{x(N)}\Phi(x(N)). \tag{21}$$

Accordingly, function of Hamilton H(p(k+1),x(k),x(k–s(k),u(k),k)) of the combined system is defined by a equation:

$$H(p(k+1),x(k),x(k-s(k),u(k),k)) = p^T(k+1)f(x(k),x(k-s(k)),u(k),k). \tag{22}$$

As before, the upper index (i), $i = \overline{1,M}$: $p^{(i)}(k)$, $H^{(i)}$, $k = \overline{N-1,0}$, will define objects for trajectories of a beam.

*Theorem 2.* Gradients on the appropriate controls from the efficiency functional which depends only from final values of trajectories of the combined control system with delay, are defined by gradients from the appropriate functions of Hamilton:

$$grad_{u(k)} I_p(U) = -\mathrm{grad}_{u(k)} \mathrm{H}(\mathrm{p}(\mathrm{k}+1), \mathrm{x}(\mathrm{k}), \mathrm{x}(\mathrm{k}-\mathrm{s}(\mathrm{k})), \mathrm{u}(\mathrm{k})), \mathrm{k})) . \tag{23}$$

And, hence, for a beam of dynamics the appropriate gradient is defined by equation:

$$grad_{u(k)} I_p(U) = -\sum_{i=1}^{M} \mathrm{grad}_{u(k)} \mathrm{H}^{(i)}(\mathrm{p}^{(i)}(\mathrm{k}+1), \mathrm{x}_i(\mathrm{k}), \mathrm{x}_i(\mathrm{k}-\mathrm{s}(\mathrm{k}), \mathrm{u}(\mathrm{k}), \mathrm{k})) . \tag{24}$$

*The evidence*. The evidence will be carried out in a standard way for use of the conjugate systems and functions of Hamilton.

## RFT-transformers and control systems with delays

As it was already marked RFT-transformer may be represented by a control system with delay. More precisely, the following theorem is fair.

*Theorem 3.* Regression RFTN-transformer with the direct N-times recursive reference to $k_m, m = \overline{1,N}$ ERRT elements on each of the steps of the recursion is represented by the nonlinear combined beam of dynamics with delay on an interval $\overline{0,T}$ , $T = \sum_{l=1}^{N} k_l$ :

with a phase variable

$$z(t) = \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix},\ z_i(t) \in R^m, i = 1,2\ , t = \overline{0,T}\ ; \tag{25}$$

- by the system of the recurrent equations which determines trajectories of a beam:

$$z^{(i)}(t+1) = f(z^{(i)}(t), z^{(i)}(t-k(t)), C_t, t) = \begin{pmatrix} f_1(z^{(i)}(t), z^{(i)}(t-k(t)), C_t, t) \\ f_2(z^{(i)}(t), z^{(i)}(t-k(t)), C_t, t) \end{pmatrix}, \tag{26}$$

  with $f_1, f_2 \in R^m,\ t = \overline{1,T-1}$ , $i = \overline{1,M}$ , dependent on topology of the RFT-transformer and determined by equations (29)-(31 );

- initial conditions:

$$z^{(i)}(0) = \begin{pmatrix} x_i^{(0)} \\ 0 \end{pmatrix}, i = \overline{1,M}\ , \tag{27}$$

  where $z^{(i)}(0)$ , $i = \overline{1,M}$ initial conditions of trajectories of a beam, and $x_i^{(0)}$ , $i = \overline{1,M}$ – elements of an input of training sample;

- efficiency functional I(C), $C = (C_1, \ldots, C_T)$ , which depends on matrices $C_1, \ldots, C_T$ as on controls:

$$I(C) = \sum_{k=1}^{M} \| y_k^{(0)} - z_2(T) \|^2 , \tag{28}$$

where $y_k^{(0)}$ , $k = \overline{1,M}$ components of an output of training sample.

The proof can be found in [8].

The proved statement enables using of methods of optimization of the theory of control for optimization of already constructed RFTN-transformer on residual size depending on matrices $C_0, C_1, ..., C_{T-1}$. This statement also is a subject of the following theorem.

*Theorem 4.* By presence of continuous derivatives up to the second order inclusive of functions of family $\Im$ RFT - transformer may be optimized by gradient methods with gradients of the efficiency functional on matrices C, as on parameters.

*The proof.* According to the theorem 3 RFT-transformer may be represented by the combined control system with delay, and according to the theorem 2 gradients on the appropriate controls - parameters of the RFT-transformer are defined by equation (24). Importance of the given theorem lies in that it gives exhaustive interpretation of Back Propagàtion algorithm, outlining at the same time borders of the specified method

RFT-transformers in various variants: linear, nonlinear, and also with gradient optimization - were used for the forecast of values on days in such high volatility market described by modeling uncertainty, as the currency market. Such use demands the decision of some questions, concerned formation of training sample, choice of an interval of forecasting and some other which remain behind frameworks of discussion. We shall note only, that the standard set of characteristics was predicted: close rate, the maximal and minimal rates for the day. It is necessary to note, that the prediction well catches turns of a exchange rate and together with other technical parameters forecasts on the basis of suitable RFT - transformer may be effectively used in the appropriate market

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