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PRIMITIVE IDEALS AND SYMPLECTIC LEAVES OF QUANTUM MATRICES

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Abstract. It is proved that there exists a bijection between the primitive ideals of the algebra of regular functions on quantum $m \times n$-matrices and the symplectic leaves of associated Poisson structure.

Motivation. When the noncommutative algebra A is a deformation of a commutative algebra B, one expects to find a close correspondence between the primitive ideals of A and the symplectic leaves of the associated Poisson structure on the variety rm Max (B). It is known, for instance, that there exists a bijection between the primitive ideals of the algebra $\mathbf{C}_q[G]$ of regular functions on the quantum group and the symplectic leaves of G (see [1, 2, 3]). We generalize this result to the case of the quantum $m \times n$-matrixes here.

Content. Let $\mathcal{V}$ be the set of compex $m \times n$- matrixes $(m \leq n)$ nondegenerated in the sense that all minors of maximal order are not zero, $\mathbf{C}[\mathcal{V}]$be the

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algebra of regular functions on $\mathcal{V}$, $\mathbf{C}_q[\mathcal{V}]$ be the quantum analogoue of $\mathbf{C}[\mathcal{V}]$. We decompose $\operatorname{Prim}\mathbf{C}_q[\mathcal{V}]$ (Theorem 2.10, Corollary 2.11) like it is done in [2]. The standard techniques of reduction modulo an ideal and localization give us the algebra $\widetilde{\mathcal{A}}_w$ (see section 3). We prove that $\widetilde{\mathcal{A}}_w$ is an algebra of twisted Lourent polynomials (Proposition 3.1) and prove the theorem about the bijection between the primitive ideals of $\mathbf{C}_q[\mathcal{V}]$ and the symplectic leaves of $\mathcal{V}$ (Theorem 3.9).

Notations. Denote by $\mathcal{M}$ the algebra of regular functions on the linear space of the complex $m \times n$-matrixes. Its quantum analoge is an algebra $\mathcal{M}_q$ (see [4, Def. 1.2]) defined as the associative $\mathbf{C}$-algebra generated by mn elements x_i^j, $1 \le i \le m$, $1 \le j \le n$ with the relations:

$$\begin{aligned}
x_i^j x_k^j &= q^{-1} x_k^j x_i^j && \forall i < k,\\
x_i^j x_i^k &= q^{-1} x_i^k x_i^j && \forall j < k,\\
x_i^j x_l^k &= x_l^k x_i^j && \forall i < l, j > k,\\
\left[x_i^j, x_l^k\right] &= (q^{-1} - q) x_i^k x_l^j && \forall i < l, j < k,
\end{aligned}$$

where q is general, i.e. a nonzero complex number which is not a root of unity. Let C_I^J be the element of the algebra $\mathcal{M}_q$ defined by formulas:

$$C_I^J = \sum_{\sigma \in S_p} (-q)^{l(\sigma)} x_{i_1}^{j_{\sigma(1)}} \dots x_{i_p}^{j_{\sigma(p)}} = \sum_{\sigma \in S_p} (-q)^{-l(\sigma)} x_{i_{\sigma(1)}}^{j_1} \dots x_{i_{\sigma(p)}}^{j_p}.$$

C_I^J is called the quantum minor (or q-minor for short). The algebras $\mathbf{C}[\mathcal{V}]$ and $\mathbf{C}_q[\mathcal{V}]$ are defined as the respective localizations of algebras $\mathcal{M}$ and $\mathcal{M}_q$ with respect to the multiplicative sets generated by all minors (q-minors in the case $\mathbf{C}_q[\mathcal{V}]$) of order m.

It is known (see [4, 1]) that the q-minor admits an expansion with respect to rows (columns), the expansion of the q-minor with respect to another row (column) is equal to zero, the quantum analogoue of the Laplace theorem holds.

For the multiindex $I=\{i_1 \dots, i_p\}$ we define the operation $[I]_s=\{i_1, \dots, i_s\}$, $s \le p$ and denote by $\left[C_I^J\right]_s$ the s-th main q-minor. If $i < i_t$ (or $i > I$) $\forall t$ then we shall write $i < I$ (resp. $i > I$). All multiindices are ordered, $E_p = \{1, \dots, p\}$.

Definitions. Let A be an associative algebra generated by $a_1, \dots, a_k$. The element b of A is called normal, if there exist integer exponents φ_i such that $ba_i = q^{\varphi_i} a_i b$ $\forall i$. The algebra A is called twisted, if all its generators are normal. It is imortant for us that all prime (and, hence, all primitive) ideals of twisted Lourent algebra are generated by their intersections with the center (see [5, 2.3]).

1. Preliminaries. Let $I = \{i_1, \ldots, i_p\}$, $J = \{j_1, \ldots, j_p\}$. Denote $I_t = I \backslash i_t \cup i$.

Lemma 1.1. a) *If* $i < I$, $j \in J$, *then* $C_I^J x_i^j - q x_i^j C_I^J = 0$;

b) *if* $i \in I$, $j \in J$, *then* $C_I^J x_i^j - x_i^j C_I^J = 0$;

c) *if* $i_k < i < i_{k+1}$ *for some* $k < p$, $j \in J$, *then there exist complex numbers* $c_t \in \mathbf{C}^*$ *such that* $C_I^J x_i^j - q x_i^j C_I^J = \sum\limits_{t<k+1} c_t x_{i_t}^j C_{I_t}^J$;

d) *if* $i > I$, $j \in J$, *then* $C_I^J x_i^j - q^{-1} x_i^j C_I^J = 0$;

e) *if* $i < I$, $j > J$, *then* $C_I^J x_i^j - x_i^j C_I^J = 0$;

f) *if* $i \in I$, $j > J$, *then* $C_I^J x_i^j - q^{-1} x_i^j C_I^J = 0$;

g) *if* $i_k < i < i_{k+1}$ *for some* $k < p$, $j > J$, *then there exist complex numbers* $g_t \in \mathbf{C}^*$ *such that* $C_I^J x_i^j - x_i^j C_I^J = \sum\limits_{t<k+1} g_t x_{i_t}^j C_{I_t}^J$;

h) *if* $i > I$, $j > J$, *then there exist complex numbers* $h_t \in \mathbf{C}^*$ *such that* $C_I^J x_i^j - x_i^j C_I^J = \sum\limits_{t=1}^{p} h_t x_{i_t}^j C_{I_t}^J$.

Lemma 1.2. a) *If* $i < I$, $j < J$, *then there exist complex numbers* $a_t \in \mathbf{C}^*$ *such that* $C_I^J x_i^j - x_i^j C_I^J = \sum\limits_{t=1}^{p} a_t x_{i_t}^j C_{I_t}^J$;

b) *if* $i \in I$, $j < J$, *then* $C_I^J x_i^j - q x_i^j C_I^J = 0$;

c) *if* $i_k < i < i_{k+1}$ *for some* $k < p$, $j < J$, *then there exist complex numbers* $c_t \in \mathbf{C}^*$ *such that* $C_I^J x_i^j - x_i^j C_I^J = \sum\limits_{t>k} c_t x_{i_t}^j C_{I_t}^J$;

d) *if* $i > I$, $j < J$, *then* $C_I^J x_i^j - x_i^j C_I^J = 0$;

e) *if* $i < I$, $j \in J$, *then* $C_I^J x_i^j - q x_i^j C_I^J = 0$;

f) *if* $i \in I$, $j \in J$, *then* $C_I^J x_i^j - x_i^j C_I^J = 0$;

g) *if* $i_k < i < i_{k+1}$ *for some* $k < p$, $j \in J$, *then there exist complex numbers* $g_t \in \mathbf{C}^*$ *such that* $C_I^J x_i^j - q^{-1} x_i^j C_I^J = \sum\limits_{t>k} g_t x_{i_t}^j C_{I_t}^J$;

h) *if* $i > I$, $j \in J$, *then* $C_I^J x_i^j - q^{-1} x_i^j C_I^J = 0$.

One may to prove the lemma as in [4] by the expansion of q-minors.

Proposition 1.3. *Let* $\mathcal{P} \in \operatorname{Spec} \mathcal{M}_q$. *If the* q*-minor* $C_I^J \notin \mathcal{P}$ *then* $\left[C_I^J\right]_s \notin \mathcal{P}$ $\forall s$.

P r o o f. Note that the exact expression for the complex coefficient h_t from

1.1.h) is $h_t = (q^{-1} - q)q^{p-t}$. Therefore

$$\begin{aligned}
\left[C_I^J\right]_{p-1} x_{i_p}^{j_p} &= (q^{-1} - q)\sum_{t=1}^{p-1}(-q)^{p-1-t} x_{i_t}^{j_p} C_{[I]_{p-1}\setminus i_t \cup i_p}^{[J]_{p-1}} + x_{i_p}^{j_p}\left[C_I^J\right]_{p-1} \\
&= (1 - q^{-2})\sum_{t=1}^{p}(-q)^{p-t} x_{i_t}^{j_p} C_{[I]_{p-1}\setminus i_t \cup i_p}^{[J]_{p-1}} + q^{-2} x_{i_p}^{j_p}\left[C_I^J\right]_{p-1} \\
&= (1 - q^{-2})C_I^J + q^{-2} x_{i_p}^{j_p}\left[C_I^J\right]_{p-1}.
\end{aligned}$$

Assume that $\left[C_I^J\right]_{p-1} \in \mathcal{P}$. Then $\left[C_I^J\right]_{p-1} x_{i_p}^{j_p} - q^{-2} x_{i_p}^{j_p}\left[C_I^J\right]_{p-1} \in \mathcal{P}$, hence $C_I^J \in \mathcal{P}$ — contradiction. Thus $\left[C_I^J\right]_{p-1} \notin \mathcal{P}$. An induction with respect to p complets the proof. □

Let ${}^{\pm}\mathcal{P}_T^J$ be the left ideals of $\mathcal{M}_q$ generated by the following q-minors:

$${}^{+}\mathcal{P}_T^J = \langle C_I^J | I >_{lex} T\rangle, \qquad {}^{-}\mathcal{P}_T^J = \langle C_I^J | I <_{lex} T\rangle,$$

Proposition 1.4. *The ideals* ${}^{+}\mathcal{P}_T^{E_p}$ *and* ${}^{-}\mathcal{P}_T^{E_n\setminus E_{n-p}}$ *are twoside ones.*

Proof. Let us prove the first statement. It is suffices to show that $C_I^{E_p} x_i^j \in {}^{+}\mathcal{P}_T^{E_p}$ for all generators $C_I^{E_p}$ of ${}^{+}\mathcal{P}_T^{E_p}$ and for all generators x_i^j of $\mathcal{M}_q$. All necessary relations are in sections a),…, h) of Lemma 1.1. For instance, let i, j satisfy c). Then $C_I^{E_p} x_i^j - q x_i^j C_I^{E_p} = \sum_{t<k+1} c_t x_{i_t}^j C_{I_t}^{E_p}$. Note that $t < k+1$, hence $I_t >_{lex} I >_{lex} T$. Therefore any term of the right side lies in ${}^{+}\mathcal{P}_T^{E_p}$, and hence $C_I^{E_p} x_i^j \in {}^{+}\mathcal{P}_T^{E_p}$. All other possibilities admits the same analisis. The statement about ${}^{-}\mathcal{P}_T^{E_n\setminus E_{n-p}}$ is proved similarly using Lemma 1.2. □

2. Decomposition theorem.

Algebra $\mathcal{A}$. Let $\mathbf{N}_k$ be the set of segments of the natural numbers with length k. Denote by $S^0 = \{C_{E_m}^J | J \in \mathbf{N}_m\}$, $\mathcal{S}^0$ the multiplicative subset of $\mathcal{M}_q$ generated by S^0, and let $\mathcal{A}$ be the localization of $\mathcal{M}_q$ with respect to $\mathcal{S}^0$.

Lemma 2.1. *Let* $\mathcal{P} \in \operatorname{Spec}\mathcal{M}_q$. *If* $\mathcal{P}\bigcap S^0 = \emptyset$ *then* $\mathcal{P}\bigcap S = \emptyset$, *where* S *is the set of all* q*-minors of order* m.

Proof. We shall prove the equivalent statement: if there exists a q-minor $C_{E_m}^J \in S\bigcap\mathcal{P}$ then there exists a q-minor $C_{E_m}^{J'} \in S^0\bigcap\mathcal{P}$.

a) If $J \in \mathbf{N}_m$ then the statement holds. If $J \notin \mathbf{N}_m$ then let us denote $T = \{j_m - m + 1, \ldots, j_m\} \in \mathbf{N}_m$. By expansion of q-minor, one may show that

$$C_{Em}^J\left[C_{E_m}^T\right]_{m-1} - q^{-\langle J,T\rangle+2}\left[C_{E_m}^T\right]_{m-1} C_{E_m}^J = q^{-\langle J,T\rangle}(1-q^2)C_{E_m}\left[C_{E_m}^J\right]_{m-1},$$

where $\langle J, T\rangle = \sharp\{j \in J | j < T\}$. Hence $C^T_{E_m}\left[C^J_{E_m}\right]_{m-1} \in \mathcal{P}$. If $C^T_{E_m} \in \mathcal{P}$ then the lemma is proved.

b) If $C^T_{E_m} \notin \mathcal{P}$ then $\left[C^J_{E_m}\right]_{m-1} \in \mathcal{P}$. We apply a) to $\left[C^J_{E_m}\right]_{m-1}$. The order of the q-minor is lower. Hence after m steps we get what is required. □

Proposition 2.2. $\operatorname{Spec} \mathbf{C}_q[\mathcal{V}] \cong \operatorname{Spec} \mathcal{A}$.

Proof. Note that $\mathbf{C}_q[\mathcal{V}]$ and $\mathcal{A}$ are different localizations of the same algebra $\mathcal{M}_q$. Therefore

$$\begin{aligned} \operatorname{Spec} \mathbf{C}_q[\mathcal{V}] &= \{\mathcal{P} \in \operatorname{Spec} \mathcal{M}_q | \mathcal{P} \bigcap S = \emptyset\}, \\ \operatorname{Spec} \mathcal{A} &= \{\mathcal{P} \in \operatorname{Spec} \mathcal{M}_q | \mathcal{P} \bigcap S^0 = \emptyset\}. \end{aligned}$$

Now the proposition follows from Lemma 2.1. □

Decomposition of $\operatorname{Spec}\mathcal{A}$**.** Let $w = (w_+, w_-) \in S_m \times S_m$. Denote

$$P^+_{w_+} = \bigcup_{i=1}^{m-1} \{C^{E_i}_K | K >_{lex} w_+^{-1}(E_i)\}, \quad S^+_{w_+} = \bigcup_{i=1}^{m-1} \{C^{E_i}_{w_+^{-1}(E_i)}\},$$

$$P^-_{w_-} = \bigcup_{i=1}^{m-1} \{C^{E_n \backslash E_{n-i}}_K | K <_{lex} w_-^{-1}(E_m \backslash E_{m-i})\}, S^-_{w_-} = \bigcup_{i=1}^{m-1} \{C^{E_n \backslash E_{n-i}}_{w_-^{-1}(E_m \backslash E_{m-i})}\},$$

$$P_w = P^+_{w_+} \bigcup P^-_{w_-}, \quad S_w = S^+_{w_+} \bigcup S^-_{w_-},$$

and let $\mathcal{P}^\pm_{w_\pm}$, $\mathcal{P}_w$ be the left ideals of $\mathcal{M}_q$ generated by $P^\pm_{w_\pm}$ and P_w respectivelly; $\mathcal{S}^\pm_{w_\pm}$, $\mathcal{S}_w$ be the multiplicative subsets of $\mathcal{M}_q$ generated by $S^\pm_{w_\pm}$, S_w. Note that the ideals $\mathcal{P}^\pm_{w_\pm}$, $\mathcal{P}_w$ are in fact twoside. For instance $\mathcal{P}^+_{w_+} = \sum_{i=1}^{m-1} {}^+\mathcal{P}^{E_i}_{w_+^{-1}(E_i)}$, and any term of the right side is a twoside ideal by 1.4.

Proposition 2.3. *Let* $\mathcal{P} \in \operatorname{Spec}\mathcal{A}$*. Then there exists a unique* $w_+ \in S_m$ *such that* $\mathcal{P} \supseteq \mathcal{P}^+_{w_+}$, $\mathcal{P} \cap S^+_{w_+} = \emptyset$.

First we shall prove some lemmas.

Lemma 2.4. *Let* $\mathcal{P} \in \operatorname{Spec}\mathcal{A}$, $p \leq m-1$*. Then there exists a unique multiindex* T *with length* p *such that* $C^{E_p}_T \notin \mathcal{P}$, $C^{E_p}_I \in \mathcal{P}\ \forall\ I >_{lex} T$.

Proof. Let us assume that such T does not exist and expand the q-minor $C^{E_m}_{E_m}$ with respect to the first p columns. Then by the Laplace theorem $C^{E_m}_{E_m} \equiv 0 (\operatorname{mod} \mathcal{P})$. But $C^{E_m}_{E_m}$ is invertable in $\mathcal{A}$ — contradiction. Hence T exists. Now let us write the set $\{J | C^{E_p}_J \notin \mathcal{P}\}$ with respect to the lexicographic order and denote by T the leading term. The lemma is proved. □

Lemma 2.5. *Let $\mathcal{P}$, p, T be from 2.4, $i > T$, $j \in E_p$. Then $x_i^j \equiv 0 (\mathrm{mod}\, \mathcal{P})$.*

P r o o f. Let $T_k = T \backslash t_k \cup i$. Then $T_k >_{lex} T$ and $C_{T_k}^{E_p} \equiv 0 (\mathrm{mod}\, \mathcal{P})$. Let us expand $C_{T_k}^{E_p}$ with respect to the j-th column:

$$C_{T_k}^{E_p} = \sum_{l=1}^{k-1} (-q)^{j-l} x_{t_l}^j C_{T_k \backslash t_l}^{E_p \backslash j} + \sum_{l=k}^{p-1} (-q)^{j-l} x_{t_{l+1}}^j C_{T_k \backslash t_{l+1}}^{E_p \backslash j} + (-q)^{j-p} x_i^j C_{T_k \backslash i}^{E_p \backslash j}$$

If $x_i^j \not\equiv 0 (\mathrm{mod}\, \mathcal{P})$ then

$$\begin{aligned} C_{T_k \backslash i}^{E_p \backslash j} &\equiv -(-q)^{p-j} (x_i^j)^{-1} \left\{ \sum_{l=1}^{k-1} (-q)^{j-l} x_{t_l}^j C_{T_k \backslash t_l}^{E_p \backslash j} + \sum_{l=k}^{p-1} (-q)^{j-l} x_{t_{l+1}}^j C_{T_k \backslash t_{l+1}}^{E_p \backslash j} \right\} \\ &\equiv -(-q)^{p-j} (x_i^j)^{-1} \sum_{l \neq k} s(k,l) (-q)^{j-l} x_{t_l}^j C_{T_k \backslash t_l}^{E_p \backslash j} \, (\mathrm{mod}\, \mathcal{P}); \end{aligned}$$

where $s(k,l) = \begin{cases} 1 & \text{for } k < l, \\ -q^{-1} & \text{for } k > l. \end{cases}$ Let us expand $C_T^{E_p}$ with respect to the j-th column using the fact that $T \backslash t_k = T_k \backslash i$:

$$\begin{aligned} C_T^{E_p} &= \sum_{k=1}^{p} (-q)^{j-k} x_{t_k}^j C_{T \backslash t_k}^{E_p \backslash j} = \sum_{k=1}^{p} (-q)^{j-k} x_{t_k}^j C_{T_k \backslash i}^{E_p \backslash j} \\ &\equiv \sum_{k=1}^{p} (-q)^{j-k} x_{t_k}^j \left\{ -(-q)^{p-j} (x_i^j)^{-1} \sum_{l \neq k} s(k,l) (-q)^{j-l} x_{t_l}^j C_{T_k \backslash t_l}^{E_p \backslash j} \right\} \\ &\equiv (-q)^{p-j+1} (x_i^j)^{-1} \sum_{k<l} \Big\{ (-q)^{j-k} x_{t_k}^j s(k,l) (-q)^{j-l} x_{t_l}^j C_{T_k \backslash t_l}^{E_p \backslash j} \\ &\qquad\qquad + (-q)^{j-l} x_{t_l}^j s(l,k) (-q)^{j-k} x_{t_k}^j C_{T_l \backslash t_k}^{E_p \backslash j} \Big\} \\ &\equiv (-q)^{p-j+1} (x_i^j)^{-1} \sum_{k<l} \Big\{ (-q)^{2j-k-l} x_{t_k}^j x_{t_l}^j C_{T_k \backslash t_l}^{E_p \backslash j} \\ &\qquad\qquad - q^{-1} (-q)^{2j-k-l} x_{t_l}^j x_{t_k}^j C_{T_l \backslash t_k}^{E_p \backslash j} \Big\} \\ &\equiv (-q)^{p-j+1} (x_i^j)^{-1} \sum_{k<l} (-q)^{2j-k-l} x_{t_k}^j x_{t_l}^j (C_{T_k \backslash t_l}^{E_p \backslash j} - C_{T_l \backslash t_k}^{E_p \backslash j}) (\mathrm{mod}\, \mathcal{P}). \end{aligned}$$

As above $T_k \backslash t_l = T_l \backslash t_k$. Therefore $C_T^{E_p} \equiv 0 (\mathrm{mod}\, \mathcal{P})$ — contradiction. Hence $x_i^j \equiv 0 (\mathrm{mod}\, \mathcal{P})$. □

Lemma 2.6. *Let $\mathcal{P}$, p, T be from 2.4. Then for all i, j*

$$C_T^{E_p} x_i^j \equiv q^{\delta_{E_p,j}-\delta_{T,i}} x_i^j C_T^{E_p} (\operatorname{mod} \mathcal{P}) \quad \textit{where } \delta_{J,k} = \begin{cases} 0 & \textit{for } k \notin J, \\ 1 & \textit{for } k \in J. \end{cases}$$

P r o o f. If i, j are as in sections a), b), c), e), f), g) or h) of Lemma 1.1, then we obtain the statement as in 1.4. If $i > T$, $j \in E_p$, then we use that fact that by the Lemma 2.5 $x_i^j \equiv 0 (\operatorname{mod} \mathcal{P})$. □

Lemma 2.7. *Let $\mathcal{P}$, p, T be from 2.4. Then for all K, L*

$$C_T^{E_p} C_K^L \equiv q^{\sharp\{E_p \cap L\} - \sharp\{T \cap K\}} C_K^L C_T^{E_p} (\operatorname{mod} \mathcal{P})$$

Proof. Let the order of the q-minor C_K^L is equal to r. Then by Lemma 2.6

$$\begin{aligned} C_T^{E_p} C_K^L &\equiv C_T^{E_p} \textstyle\sum_{\sigma \in S_r} (-q)^{l(\sigma)} x_{k_1}^{l_{\sigma(1)}} \dots x_{k_r}^{l_{\sigma(r)}} \\ &\equiv \textstyle\sum_{\sigma \in S_r} q^{A_\sigma} (-q)^{l(\sigma)} x_{k_1}^{l_{\sigma(1)}} \dots x_{k_r}^{l_{\sigma(r)}} C_T^{E_p} (\operatorname{mod} \mathcal{P}). \end{aligned}$$

We shall show that A_σ does not depend on σ. Indeed, by Lemma 2.6, $C_T^{E_p} x_{k_j}^{l_{\sigma(j)}} \equiv q^{\delta_{E_p, l_{\sigma(j)}} - \delta_{T,k_j}} x_{k_j}^{l_{\sigma(j)}} C_T^{E_p} (\operatorname{mod} \mathcal{P})$. Therefore

$$\begin{aligned} A_\sigma &= \sum_{j=1}^{r} (\delta_{E_p, l_{\sigma(j)}} - \delta_{T,k_j}) = \sum_{j=1}^{r} \delta_{E_p, l_{\sigma(j)}} - \sum_{j=1}^{r} \delta_{T,k_j} \\ &= \sum_{j=1}^{r} \delta_{E_p, l_j} - \sum_{p=1}^{r} \delta_{T,k_j} = \sharp\{E_p \cap L\} - \sharp\{T \cap K\}. \end{aligned}$$

□

Lemma 2.8. *Using the Lemma 2.4 let us construct from the given ideal $\mathcal{P}$ the collection of multiindices T_1, T_2,$\dots T_{m-1}$, T_m where p is the length of T_p. Then $T_1 \subset T_2 \subset \dots T_{m-1} \subset T_m$.*

P r o o f. Let $p < s$. By Lemma 2.7,

$$C_{T_p}^{E_p} C_{T_s}^{E_s} \equiv q^A C_{T_s}^{E_s} C_{T_p}^{E_p} (\operatorname{mod} \mathcal{P}), \quad C_{T_s}^{E_s} C_{T_p}^{E_p} \equiv q^B C_{T_p}^{E_p} C_{T_s}^{E_s} (\operatorname{mod} \mathcal{P})$$

where $A = \sharp\{E_p \cap E_s\} - \sharp\{T_p \cap T_s\}$, $B = \sharp\{E_s \cap E_p\} - \sharp\{T_s \cap T_p\}$. It is clear that $A = -B$. Therefore $\sharp\{T_s \cap T_p\} = \sharp\{E_s \cap E_p\} = p$. Hence $\sharp\{T_s \cap T_p\} = \sharp T_p$, $T_p \subset T_s$. □

P r o o f o f P r o p o s i t i o n 2.3. By Lemma 2.4 for any prime ideal $\mathcal{P}$ from $\operatorname{Spec}\mathcal{A}$ there exists an unique collection of multiindices $T_1 \subset T_2 \subset \ldots T_{m-1} \subset T_m$. Denote

$$w_+^{-1}(1) = T_1, w_+^{-1}(2) = T_2 \backslash T_1, \ldots, w_+^{-1}(m) = T_n \backslash T_{m-1}.$$

The proposition is proved. □

The following proposition may be proved as above.

Proposition 2.9. *Let $\mathcal{P} \in \operatorname{Spec}\mathcal{A}$. Then there exist unique $w_- \in S_m$ such that $\mathcal{P} \supseteq \mathcal{P}^-_{w_-}$, $\mathcal{P} \cap S^-_{w_-} = \emptyset$.*

Using Propositions 2.3 and 2.9 one proves the following theorem.

Theorem 2.10. *Denote* $\operatorname{Spec}_w\mathcal{A} = \{\mathcal{P} \in \operatorname{Spec}\mathcal{A} | \mathcal{P} \supseteq \mathcal{P}_w, \mathcal{P} \bigcap S_w = \emptyset\}$. *Then*

$$\operatorname{Spec}\mathcal{A} = \bigsqcup_{w \in S_m \times S_m} \operatorname{Spec}_w\mathcal{A},$$

where $\bigsqcup$ denotes the disjoint union.

Corollary 2.11. *Denote* $\operatorname{Prim}_w\mathcal{A} = \{\mathcal{P} \in \operatorname{Prim}\mathcal{A} | \mathcal{P} \supseteq \mathcal{P}_w, \mathcal{P} \bigcap S_w = \emptyset\}$. *Then*

$$\operatorname{Prim}\mathcal{A} = \bigsqcup_{w \in S_m \times S_m} \operatorname{Prim}_w\mathcal{A},$$

where $\bigsqcup$ denotes the disjoint union.

3. Bijection theorem.

Algebras $\mathcal{A}_w$ and $\widetilde{\mathcal{A}}_w$. Let $\mathcal{S}_w$ be the multiplicative subset of $\mathcal{M}_q$ generated by $S_w \cup S^0$. Denote $\mathcal{A}_w = (\mathcal{M}_q/\mathcal{P}_w)_{\mathcal{S}_w}$. The analysis of $\operatorname{Spec}\mathcal{A}$ reduces to the analysis of $\operatorname{Spec}\mathcal{A}_w$. Denote by $\widetilde{S}^{\pm}_{w\pm}$, $\widetilde{S}^0$, $\widetilde{S}_w$, the subset of $S^{\pm}_{w\pm}$, S^0, S_w respectivelly, consisting of all main q-minors. Let $\widetilde{\mathcal{S}}_w$ be the multiplicative subset of $\mathcal{M}_q$, generated by $\widetilde{S}_w$. Let $\widetilde{\mathcal{A}}_w = (\mathcal{M}_q/\mathcal{P}_w)_{\widetilde{\mathcal{S}}_w}$. By Proposition 1.3, $\operatorname{Spec}\mathcal{A}_w = \operatorname{Spec}\widetilde{\mathcal{A}}_w$.

Proposition 3.1. *The algebra $\widetilde{\mathcal{A}}_w$ is an algebra of twisted Lourent polinomials generated by the q-minors from $\widetilde{S}_w$.*

First let us prove some lemmas.

Lemma 3.2. *Let $l(y)$ denotes the number of inversions of the permutation y.*

a) *For any permutation $w_+ \in S_m$ there exist $u_{j_0} = (j_0, j_0+1) \in S_m$ and $v_+ \in S_m$ such that*

$$w_+ = v_+ u_{j_0}, \quad l(w_+) = l(v_+) + 1, \quad v_+^{-1}(E_{j_0}) < v_+^{-1}(j_0+1).$$

b) *For any permutation* $w_- \in S_m$ *there exist* $u_{i_0} = (i_0, i_0+1) \in S_m$ *and* $v_- \in S_m$ *such that*

$$w_- = v_- u_{i_0}, \quad l(w_-) = l(v_-) + 1, \quad v_-^{-1}(E_m \backslash E_{m-i_0}) > v_-^{-1}(i_0).$$

This lemma is a particular case of known properties about Wejl groups.

Lemma 3.3. *Let* w_+, v_+, j_0 *be from* 3.2. *Then* $\widetilde{S}^+_{w_+} = \widetilde{S}^+_{v_+} \cup C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}$.

P r o o f. First let us show that
a) $[w_+(E_j)]_i = [v_+(E_j)]_i \ \forall j \neq j_0, i \leq j$;
b) $[w_+(E_{j_0})]_i = [v_+(E_{j_0-1})]_i \ \forall i < j_0$;
c) $[w_+(E_{j_0+1})]_i = [v_+(E_{j_0+1})]_i \ \forall i \leq j_0$.
Note that $w_+^{-1}(j) = v_+^{-1}(j) \ \forall j \neq j_0, j_0+1$. In addition $w_+^{-1}(j_0) = v_+^{-1}(j_0+1)$; $w_+^{-1}(j_0+1) = v_+^{-1}(j_0)$. Therefore $w_+^{-1}(E_j) = v_+^{-1}(E_j) \ \forall j \neq j_0$. This proves a). Let us prove b):

$$w_+^{-1}(E_{j_0}) = w_+^{-1}(E_{j_0-1}) \cup w_+^{-1}(j_0) = v_+^{-1}(E_{j_0-1}) \cup v_+^{-1}(E_{j_0+1}).$$

In addition $v_+^{-1}(j_0+1) > v_+^{-1}(E_{j_0-1})$. Hence if $i < j_0$ then

$$\left[w_+^{-1}(E_{j_0})\right]_i = \left[v_+^{-1}(E_{j_0-1}) \cup v_+^{-1}(E_{j_0+1})\right]_i = \left[v_+^{-1}(E_{j_0-1})\right]_i.$$

Let us prove c): $w_+^{-1}(E_{j_0+1}) = w_+^{-1}(E_{j_0-1}) \cup w_+^{-1}(j_0) \cup w_+^{-1}(j_0+1)$. In addition $w_+^{-1}(j_0) > w_+^{-1}(j_0+1)$ and $w_+^{-1}(j_0) > w_+^{-1}(j_0-1)$. Therefore

$$\left[w_+^{-1}(E_{j_0+1})\right]_{j_0} = w_+^{-1}(E_{j_0-1}) \cup w_+^{-1}(j_0+1) = v_+^{-1}(E_{j_0-1}) \cup v_+^{-1}(j_0) = v_+^{-1}(E_{j_0}).$$

If $i \leq j_0$ then $\left[w_+^{-1}(E_{j_0+1})\right]_i = \left[v_+^{-1}(E_{j_0})\right]_i$ and c) is proved. Now recall that

$$\left[C^{E_j}_{w_+^{-1}(E_j)}\right]_i = C^{E_i}_{\left[w_+^{-1}(E_j)\right]_i}, \quad \left[C^{E_j}_{v_+^{-1}(E_j)}\right]_i = C^{E_i}_{\left[v_+^{-1}(E_j)\right]_i}.$$

Therefore a), b) imply that $\widetilde{S}^+_{v_+} \subset \widetilde{S}^+_{w_+} \backslash C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}$, and on other hand a), c) imply that $\widetilde{S}^+_{v_+} \supset \widetilde{S}^+_{w_+} \backslash C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}$. The equality follows. $\square$

Lemma 3.4. *Let* w_+, v_+, j_0 *be from* 3.2. *Then* $\mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})} \rangle \subset \mathcal{P}^+_{v_+}$.

P r o o f. Note that $w_+^{-1}(E_{j_0}) >_{lex} v_+^{-1}(E_{j_0})$. Indeed

$$\begin{aligned} w_+^{-1}(E_{j_0}) &= w_+^{-1}(E_{j_0-1}) \cup w_+^{-1}(j_0) >_{lex} w_+^{-1}(E_{j_0-1}) \cup w_+^{-1}(j_0+1) \\ &= v_+^{-1}(E_{j_0-1}) \cup v_+^{-1}(j_0) = v_+^{-1}(E_{j_0}). \end{aligned}$$

In addition $w_+^{-1}(E_j) = v_+^{-1}(E_j)\ \forall j \neq j_0$. Therefore (see Section 1 for the definition of ideals ${}^+\mathcal{P}_T^J$) ${}^+\mathcal{P}^{E_j}_{w_+^{-1}(E_j)} = {}^+\mathcal{P}^{E_j}_{v_+^{-1}(E_j)}\ \forall j \neq j_0$, $\langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \subset {}^+\mathcal{P}^{E_{j_0}}_{v_+^{-1}(E_{j_0})}$ and ${}^+\mathcal{P}^{E_{j_0}}_{w_+^{-1}(E_{j_0})} \subset {}^+\mathcal{P}^{E_{j_0}}_{v_+^{-1}(E_{j_0})}$. Hence

$$\begin{aligned} &\mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle = \sum_{j=1}^{m-1} {}^+\mathcal{P}^{E_j}_{w_+^{-1}(E_j)} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \\ &= \sum_{j\neq j_0}^{m-1} {}^+\mathcal{P}^{E_j}_{w_+^{-1}(E_j)} + {}^+\mathcal{P}^{E_{j_0}}_{w_+^{-1}(E_{j_0})} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \subset \sum_{j\neq j_0}^{m-1} {}^+\mathcal{P}^{E_j}_{w_+^{-1}(E_j)} + {}^+\mathcal{P}^{E_{j_0}}_{v_+^{-1}(E_{j_0})} \\ &= \sum_{j\neq j_0}^{m-1} {}^+\mathcal{P}^{E_j}_{v_+^{-1}(E_j)} + {}^+\mathcal{P}^{E_{j_0}}_{v_+^{-1}(E_{j_0})} = \mathcal{P}^+_{v_+}. \end{aligned}$$

□

Lemma 3.5. *Let* w_+, v_+, j_0 *be from* 3.2. *Then*

$$x^j_{v_+^{-1}(j_0+1)} \in \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \quad \forall j \leq j_0, \qquad x^{j_0+1}_{v_+^{-1}(j_0+1)} \notin \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle.$$

P r o o f. Note that $v_+^{-1}(j_0+1) > v_+^{-1}(E_{j_0}) \ni v_+^{-1}(1) = w_+^{-1}(1)$. Therefore $x^1_{v_+^{-1}(j_0+1)} \in {}^+\mathcal{P}^{E_1}_{w_+^{-1}(E_1)} \subset \mathcal{P}^+_{w_+} \subset \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. Let us expend the q-minor $C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}$ with respect to $w_+^{-1}(j_0)$-th row and apply induction:

$$C^{E_{j_0}}_{w_+^{-1}(E_{j_0})} = \sum_{j=1}^{j_0} c_j x^j_{w_+^{-1}(j_0)} C^{E_{j_0}\setminus j}_{w_+^{-1}(E_{j_0-1})}, \quad c_j \in \mathbf{C}^*.$$

The first $j_0 - 1$ terms belong to $\mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. Hence $x^{j_0}_{w_+^{-1}(j_0)} C^{E_{j_0-1}}_{w_+^{-1}(E_{j_0-1})} \in \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. By definition $C^{E_{j_0-1}}_{w_+^{-1}(E_{j_0-1})} \notin \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. Exchanging $w_+^{-1}(j_0)$ by $v_+^{-1}(j_0+1)$ we see first statement is proved.

Let us prove the second statement. Note that $\mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \subset \mathcal{P}^+_{v_+}$. Hence by the above $x^j_{v_+^{-1}(j_0+1)} \in \mathcal{P}^+_{v_+}\ \forall j \leq j_0$. Let us expand the q-minor

$C^{E_{j_0+1}}_{v_+^{-1}(E_{j_0+1})} \notin \mathcal{P}^+_{v_+}$ with respect to $v_+^{-1}(j_0+1)$-th row.

$$C^{E_{j_0+1}}_{v_+^{-1}(E_{j_0+1})} = \sum_{j=1}^{j_0+1} c_j x^j_{v_+^{-1}(j_0+1)} C^{E_{j_0+1}\setminus j}_{v_+^{-1}(E_{j_0})}, \quad c_j \in \mathbf{C}^*.$$

The first j_0 terms lie in $\mathcal{P}^+_{v_+}$. Therefore $x^j_{v_+^{-1}(j_0+1)} C^{E_{j_0+1}\setminus j}_{v_+^{-1}(E_{j_0})} \notin \mathcal{P}^+_{v_+}$. This implies that $x^{j_0+1}_{v_+^{-1}(j_0+1)} \notin \mathcal{P}^+_{v_+}$. Now we obtain what is required from the inclusion $\mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \subset \mathcal{P}^+_{v_+}$. □

Lemma 3.6. *Let w_+, v_+, j_0 be from* 3.2. *Then* $\mathcal{P}^+_{w_+} = \mathcal{P}^+_{v_+} \setminus \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$.

Proof. Let us prove the inclusion $\mathcal{P}^+_{v_+} \subset \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. Note that

$${}^+\mathcal{P}^{E_j}_{v_+^{-1}(E_j)} = {}^+\mathcal{P}^{E_j}_{w_+^{-1}(E_j)} \subset \mathcal{P}^+_{w_+} \subset \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \quad \forall j \neq j_0.$$

Hence it suffices to show that $C^{E_{j_0}}_I \in \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle \quad \forall C^{E_{j_0}}_I \in {}^+\mathcal{P}^{E_{j_0}}_{v_+^{-1}(E_{j_0})}$. If $v_+^{-1}(j_0+1) \in I$ let us expand the q-minor $C^{E_{j_0}}_I$ with respect to $v_+^{-1}(j_0+1)$-th row. We get what is required by applying of Lemma 3.5. If $v_+^{-1}(j_0+1) \notin I$ let us denote $I' = I \cup v_+^{-1}(j_0+1)$. Note that

$$I' = I \cup v_+^{-1}(j_0+1) >_{lex} v_+^{-1}(E_{j_0}) \cup v_+^{-1}(j_0+1) = v_+^{-1}(E_{j_0+1}) = w_+^{-1}(E_{j_0+1}).$$

Therefore $C^{E_{j_0+1}}_{I'} \in \mathcal{P}^+_{w_+} \subset \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$ We expand $C^{E_{j_0+1}}_{I'}$ with respect to $v_+^{-1}(j_0+1)$-th row:

$$C^{E_{j_0+1}}_{I'} = \sum_{j=1}^{j_0+1} c_j x^j_{v_+^{-1}(j_0+1)} C^{E_{j_0+1}\setminus j}_{I'\setminus v_+^{-1}(j_0+1)}, \quad c_j \in \mathbf{C}^*.$$

The first j_0 terms lie in $\mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$ by Lemma 3.5. Therefore $x^{j_0+1}_{v_+^{-1}(j_0+1)} C^{E_{j_0+1}\setminus j_0+1}_{I'} \in \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. By Lemma 3.5 $x^{j_0+1}_{v_+^{-1}(j_0+1)} \notin \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$, therefore $C^{E_{j_0+1}\setminus j_0+1}_{I'} \in \mathcal{P}^+_{w_+} + \langle C^{E_{j_0}}_{w_+^{-1}(E_{j_0})}\rangle$. The inclusion is proved. According to Lemma 3.4 this implies the equality. □

The following lemmas may be proved by same method.

Lemma 3.7. *Let w_-, v_-, i_0 be from* 3.2. *Then*

$$\widetilde{S}^-_{w_-} = \widetilde{S}^-_{v_-} \cup C^{E_n\setminus E_{n-i_0}}_{w_-^{-1}(E_n\setminus E_{n-i_0})}.$$

Lemma 3.8. *Let w_-, v_-, i_0 be from* 3.2. *Then*

$$\mathcal{P}^-_{w_-} = \mathcal{P}^-_{v_-} \setminus \langle C^{E_n\setminus E_{n-i_0}}_{w_-^{-1}(E_n\setminus E_{n-i_0})}\rangle.$$

Proof of Proposition 3.1. Let $e = (e, e) \in S_m \times S_m$ be the pair of the unit perturbations. As in [4, Theorem 1.24] one can show that $\widetilde{\mathcal{A}}_e$ is twisted Lourent algebra generated by q-minor from $\widetilde{S}_e$. Applying Lemma 3.2 $l(w_+)$ times (resp., $l(w_-)$ times), we get the unit perturbation from w_+ (resp., from w_-). In the chane of algebras $\widetilde{\mathcal{A}}_e, \dots, \widetilde{\mathcal{A}}_w$ any algebra is constructed from the previous one by addition of the single element (see Lemmas 3.3, 3.6 and 3.7, 3.8). Lemma 2.7 implies that the added element is normal for all steps. The proposition is proved. □

Bijection theorem. Let us introduce the Poisson bracket in $\mathbf{C}[\mathcal{V}]$ (as in [6, § 6] for instance) by the formula:

$$\forall f, g \in \mathbf{C}[\mathcal{V}]: \quad \{f, g\} = \lim_{q\to 1} \frac{f * g - g * f}{q - 1},$$

where $*$ denotes the multiplication in $\mathbf{C}_q[\mathcal{V}]$. Thus the algebra $\mathbf{C}[\mathcal{V}]$ comes the Poisson algebra associated with $\mathbf{C}_q[\mathcal{V}]$.

Denote by R_w, Q^0, Q_w, $\widetilde{Q}_w$ the sets of minors in $\mathcal{M}$, analogous to P_w, S^0, S_w, $\widetilde{S}_w$ ones in $\mathcal{M}_q$. Denote by $\mathcal{U}$ the subset of the linear space of complex $m \times n$ -matrices defined by the conditions: $D^J_I \neq 0$ $\forall D^J_I \in Q^0$. Let $\mathcal{U}_w$ be the algebraic submanifold of $\mathcal{U}$ defined by the conditions: $D^J_I = 0$ $\forall D^J_I \in R_w$, $D^J_I \neq 0\, \forall D^J_I \in Q_w$; and $\widetilde{\mathcal{U}}_w$ be the subset of $\mathcal{U}_w$ defined by the conditions: $D^J_I \neq 0$ $\forall D^J_I \in \widetilde{Q}_w$. The limit $q \to 1$ shows that

$$\operatorname{Symp}\mathcal{V} = \operatorname{Symp}\mathcal{U}, \quad \operatorname{Symp}\mathcal{U} = \bigsqcup_{w\in S_m\times S_m} \operatorname{Symp}\mathcal{U}_w, \quad \operatorname{Symp}\mathcal{U}_w = \operatorname{Symp}\widetilde{\mathcal{U}}_w.$$

Thus the set of the symplectic leaves of $\mathcal{V}$ decomposes into union of classes, the classes are parametrized by the pairs of permutations $w \in S_m \times S_m$, and $\operatorname{Symp}_w\mathcal{V} = \operatorname{Symp}\widetilde{\mathcal{U}}_w$.

Theorem 3.9. *There exists a bijection β between* $\operatorname{Prim}\mathbf{C}_q[\mathcal{V}]$ *and* $\operatorname{Symp}\mathcal{V}$ *such that*

1. $\beta(\mathrm{Prim}_w \mathbf{C}_q[\mathcal{V}]) = \mathrm{Symp}_w \mathcal{V}$,
2. $\dim \beta(\mathcal{P}) = \mathrm{GKdim}\, \mathbf{C}_q[\mathcal{V}]/\mathcal{P} \quad \forall \mathcal{P} \in \mathrm{Prim}\, \mathbf{C}_q[\mathcal{V}]$,

where GKdim *denotes the Gelfand-Kirillov dimention.*

Proof. The algebra $\widetilde{\mathcal{A}}_w$ is twisted by 3.1. Hence (see [5, 2.3]), all primitive ideals of $\widetilde{\mathcal{A}}_w$ are generated by the maximal ideals of its center. On the other hand, the symplectic leaves of $\widetilde{\mathcal{U}}_w$ are defined by the collection of Kazimir functions (see [6, § 2]) plaing the role of central elements in Poisson algebra $\mathbf{C}[\widetilde{\mathcal{U}}_w]$. If the pair of generators C_I^J, C_K^L of the algebra $\mathcal{A}_w$ satisfies the relation

$$C_I^J C_K^L = q^{\varphi_{IK}^{JL}} C_K^L C_I^J,$$

then the limit $q \to 1$ shows that

$$\{D_I^J, D_K^L\} = \varphi_{IK}^{JL} D_I^J D_K^L$$

for the respective generators D_I^J, D_K^L of $\mathbf{C}[\mathcal{U}_w]$. Therefore both the generators of Center $\widetilde{\mathcal{A}}_w$ and the generators of Cazimir $\mathbf{C}[\widetilde{\mathcal{U}}_w]$ are defined by integer solutions of the same system $\Phi\bar{t} = 0$. The bijection is proved. The statement about the dimetion statement follows from the fact that

$$\mathrm{GKdim}\,\mathrm{Center}\, \mathcal{A}_w = \dim\ker\Phi = \mathrm{GKdim}\,\mathrm{Cazimir}\, \mathbf{C}_q[\mathcal{U}_w].$$

The theorem is proved. □

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