Formalizing Affordances in Situation Theory

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Submitted in partial fulfillment
of the requirements for the degree of

Master of Science

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Abstract

The representation of a perceptual scene by a computer is usually limited to numbers representing dimensions and colours. The theory of affordances attempted to provide a new way of representing an environment, with respect to a particular agent. The view was introduced as part of an entire field of psychology labeled as 'ecological,' which has since branched into computer science through the field of robotics, and formal methods. This thesis will describe the concept of affordances, review several existing formalizations, and take a brief look at applications to robotics. The formalizations put forth in the last 20 years have no agreed upon structure, only that both the agent and the environment must be taken in relation to one another. Situation theory has also been evolving since its inception in 1983 by Barwise & Perry. The theory provided a formal way to represent any arbitrary piece of information in terms of relations. This thesis will take a toy version of situation theory published in CSLI lecture notes no. 22, and add to the given ontologies. This thesis extends the given ontologies to include specialized affordance types, and individual object types. This allows for the definition of semantic objects called environments, which support a situation and a set of affordances, and niches which refer to a set of actions for an individual. Finally, a possible way for an environment to change into a new environment is suggested via the activation of an affordance.

Acknowledgements

First and foremost I would like to thank Professor Michael Winter for the guidance and assistance that he has provided me, particularly once he became my supervisor. His help was invaluable to me. As well, thanks to Professor Ivo Düntsch who was of great help when it came time to select a field in which to work, and in directing me toward a topic which I am capable of excelling in. I would also like to thank my exceedingly generous parents Monica and Carl Lenarcic who supported me financially, and encouraged my continued academic studies. Lastly, and most personally, I would like show my appreciation for my loving partner Megan Armstrong who supported me throughout my studies.

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Chapter 1

Introduction

James J. Gibson spent his life researching vision, in an attempt to explain how humans and other animals perceive their environment. One of the concepts he is most famous for is the creation of affordances. He described what they are, the information in the environment which is available to pick them up, and the process by which we pick them up, namely a direct process. Though the theory is contrary to those who believe our brains act as computers, performing complex operations on the sensory input it receives, there are those who have embraced the idea, and attempted to formalize the notion in some way. Many suggestions have been presented, but each attempt was responded to with a major critique.

In chapter two, this thesis provides a background overview of James Gibson's early career which leads into a review of his first, and last books. The last book he wrote, "*The Ecological Approach to Visual Perception*," was the culmination of his career which expertly described what he believed to be the problem with experimental psychology at the time, and what sort of direction should instead be pursued. This included what he described as opportunities for action afforded to an animal by its surrounding environment. The term *affordances* was applied to these opportunities. Gibson did not however, specify a formal definition, so his successors in the field of ecological psychology attempted to publish formal definitions but have never agreed upon anything except that both animal and environment be taken in relation to each other as Gibson proclaimed. A review of the existing attempts at formalization is given in chapter three.

Gibson's revolutionary ideas were not restricted to psychology however. The term 'ecological' which he branded to many of his theories such as 'ecological optics' and 'ecological information', also found its way into the field of robotics. Simulations can be done to represent Gibson's notions of affordances and how they are perceived, but they do not allow for the so-called unboundedness of the visual world for the agents, or the idea of persistence underlying change. Computer science has provided a basis with which to test some of Gibson's thoughts in a real world environment rather than a simulation. As computing power increases, computations on the go become easier to perform and robots can begin to demonstrate some of the simple affordance 'perceiving and acting upon' techniques in a real world environment. Chapter four will review some work performed in the field of ecological robotics and demonstrate how it has emerged from Gibson's work.

Situation theory, just like a portion of Gibson's writings, was introduced as a new theory of information. Situation theory provided the mathematical foundations to situation semantics which provided a theoretical way of reasoning in real world situations. Essentially it allows for the description of, and possibly the transitions between, situations found in the real world, which can be arbitrarily complex or simplistic depending on what must be represented or conveyed. A review of situation theory is provided to begin chapter five, and subsequent mathematical background is discussed next. The toy version of situation theory we have chosen to use in the modeling of affordances is described. Then the formal description we have used to add the two types of objects to the toy version of situation theory is provided, as well as the definition of two more semantic objects, and one possible method of having situations change into new situations through the enaction of an affordance available in the first.

Further work still needs to be done in modeling how situations change, and more notations could be added to simplify the specification of states, or situations. If a full version of situation theory were used instead of the toy version, examples would be much more complex, the dynamics would involve parameters and anchors, and it would allow for the possibility of defining arbitrary types.

Chapter 2

Gibson's Early Career

James Jerome Gibson was born January 27th, 1904 to a railway surveyor and a country school teacher. He started university in 1921 in the field of philosophy but after a course in experimental psychology in his senior year, he transferred to psychology, which he would complete all of his degrees in. He completed his B.S. in 1925, his M.A in 1926, and in 1928 he published his Ph.D. which refuted a recent thesis by Wulf [61]. Wulf, according to Gibson, concluded that a human's reproduction of perceived images show a change either towards sharpening or leveling-the exaggeration or weakening of characteristics. Gibson did not believe this distinction to be useful, so he suggested that if one was to consider 'nearly equal' sized figures, and a subject were to reproduce them as equal, reproduction could be considered sharpening of the 'nearly equal size' characteristic, but leveling of the characteristic 'unequal size', thus rendering Wulf's publications quite ambiguous. Gibson found in his experiments that subjects' memories spontaneously changed toward simpler or more compact configurations [38]. Within a year of his Ph.D. he published an article entitled, "The Reproduction of Visually Perceived Forms." An extension of his doctoral thesis, it examined how humans store and reproduce figures, with focus on changes made unbeknownst to the perceiver. This was the first of many works by Gibson to examine how the human brain perceives and stores information. In what follows, I will attempt to review most of Gibson's work, however due to time and space constraints, I will focus on articles written before his three books, as well as the books themselves.

2.1 Gibson (1929)

The Reproduction of Visually Perceived Forms [17]

In an attempt to "observe the ways in which the reproduction of visually perceived forms varies from the original stimuli," James Gibson performed studies which tested how subjects perceived and reproduced, certain images [17]. Randomly produced images were used, yet these images contained resemblances and differences which were not intended in their design. They were simply randomly created shapes, not necessarily closed or regular, one category made of straight lines, and the other curved lines. One such example involves four lines arranged in roughly a trapezoid, yet none of the corners were joined. Subjects still assimilated the image with a lampshade, both completing the gaps at the corners subconsciously, and at the same time attributing an everyday item to an abstract image. Another example is a triangle with a rectangle placed so that it overlaps part of the top of the triangle. Several different images were noted by observers. A pyramid with a top on it, an axe, an anvil, and a bell were all listed as possibly depicted items. Upon use in these studies, many such abstract images were given a significance by each subject. Everyone, it seems, performs some perceptual compensations we are unaware of, thus meaning no two people derive the exact same meaning from an image. Gibson might suggest that this is because no two people have exactly the same perceptual history, though he may not have made this claim in the early stages of his career, as his beliefs continued to evolve throughout his academic work. A common tendency of the mind, is to adjust perceived images when they are being stored or recalled. These adjustments are presumably in the interest of easing the work the mind must do, or to allow it to perform the work at all.

Object assimilation and verbal analysis are examples of the mind's tendencies which Gibson defined. They occur when a subject perceives a figure, and encodes that figure to more closely resemble a familiar object, or a verbally given description. Other properties or features the mind seems to alter without our awareness are the closing of gaps in figures, or the straightening of lines in figures. Gibson observed that the mind does not, however, curve already straight lines, and if gaps in an image are widened, as was occasionally noted, instead of closing the gaps, the mind may have perceived each side of the gap as its own figure and simplified each side. In determining frequencies, Gibson found that object assimilation and figure assimilation occur most frequently as the most prominent effect taking place. Secondary effects within the same instance were not surveyed, as they are assumed to be less important having been dominated by the primary effect. It is not hard to believe that humans perceive the world with a bias toward images they have already encountered. It is obviously only possible to know what an object is, if you have come across this object before, either physically, verbally, pictorially, descriptively, or in some other way. Otherwise through relating it to other familiar objects which form the bias.

The final question posed by Gibson in this early article asks, "Is the change in a reproduction of a perceived form caused by the influence of past perceptions on the perception and memory of that form, or is the change cause by the nature of the form itself?" [17]. He replies that no evidence was observed in support of forms which cause a change by nature, and all of his experiments can be explained by the supposition that the experience of the individual brought about habitual modes of perception which condition the changes observed.

2.2 Gibson (1933)

Adaptation, After-effect, and Contrast in the Perception of Curved Lines [18]

Gibson continued to investigate how humans' minds modify perceived figures and in 1933 put forth another publication. This was the second article he wrote for the Journal of Experimental Psychology entitled "Adaptation, after-effect, and contrast in the perception of curved lines" [18]. In it he describes an experiment in which the participant wears prism glasses which shift the entire visual field 15^o to the right. Vertical lines were also perceived as curved as a result of the prisms. The conclusions drawn from the experiment suggest that though human perception does perform subtle constant adaptations we are unaware of, it is also quite adaptive to permanent changes (and reversals of those changes). It is also important to note the experiments performed by Ivo Köhler shortly following Gibson's publication of his first book([24]), which also investigated the eyes remarkable ability to compensate for distortions [44]. Köhler's publications were in German however, and did not receive as much attention, but have since been cited as quite similar to Gibson's by Johansson [41]. After experiencing the world shifted to the right for several days, the participant in Gibson's experiment reported that the shift was becoming more natural each day. After four days, totalling 45 hours, the prism glasses were removed, and the subject reported that vertical lines and edges were now unmistakably curved in the opposite direction. This residual effect lasted to some noticeable extent for two days.

This was not to say that this curious effect is only present in extended periods of adaptation. It was reported that as little as one hour was necessary to detect the curvature after-effect. Gibson conducted an experiment of this nature, to evaluate the amount of the negative after-effect, and to measure the adaptation while subjects were wearing the prism glasses. The results surprised him. He expected that having subjects run their hand along a meter stick, or any straight edge, would coax the adaptation of the perceptual system towards the new, curved, environment, but instead he found that as long as a subject was watching his hand move along the meter stick, the curved visual perceptual system completely dominated the tactile perceptual input by indicating to the brain that the stick was in fact curved. This effect ceased if the subject's eyes were closed or looking away. It would seem, based on these experiments, that perception is highly adaptive and also that it cannot synchronize itself to more than one orientation system. After removing the prism glasses all subjects reported an after-effect which was opposite to the experience they had just encountered. These results all pointed Gibson towards a theory of perception which should be relative to each individual. If one individual experienced an accident in which the perceptual system was altered, it should stand to reason that any complete theory of perception would be able to accommodate any changes.

Other conclusions drawn from the experiments in this paper, include that all negative after effects only occur in places where the perceptual system had begun to adapt to new input, and that the periphery of the field of vision shows a greater adaptation and stronger after effect than does the midpoint on the fovea. Also, this type of after effect differs, in fact, from the after effect of colour, in that the after effect of colour does not carry from one eye to the other. When focusing on colour, after effect is only present in the stimulated eye, whereas the after effect caused by focus on a line looking through a bending prism, is reported even when changing the perceiving eye between looking through the prism, and measuring after effect. It must then, be the brain which is mediating the adaptation and after effect with respect to curvature, or else the perceptual system has internal processes which combine the scene in each eye, and these processes would be the cause of the after effect.

2.3 Gibson & Gibson (1934)

Retension and the Interpolated Task [16]

In 1932, James J. Gibson married Eleanor Jack Gibson, and two years later his first article co-written with her was published. It examined the problem of how new memories can impact old memories, called retroactive inhibition. Action which inhibits a past memory. The authors performed an experiment which was intended to identify to what extent engaging in similar or non-similar tasks hinders the recall of memorized pairs of consonants. Previous work, the authors state, worked with the similarity of material alone, and thus did not take into consideration other factors. Little was known until this publication, about the relative importance of operational and functional similarity on retroaction.

The purpose of the experiment was to isolate and compare the decrements in retention caused by similar material, to the decrements caused by similar operation. The four possible cases, plus one further, were used in carrying out the experiment. The five groups were asked to memorize a list of ten pairs of consonants for two minutes, then each was given a different interpolation task for three minutes, followed by 90 seconds to recall the original consonants. Group 1 was asked to memorize another list of consonants, so operation, and material in this interpolated task were similar. Group 2 was given paired digits to memorize, thus having a similar operation (memorization), but not similar material. Group 3 was asked to cross one pair of consonants out of a long list of different pairs. Here the material was similar, but operation was different. Group 4 was asked to cross out a pair of numbers among a long list of pairs. Obviously both material and operation were different. A fifth group was also used, where these subjects were asked to simply view some dramatic pictures. Here the material and operation are certainly different, but also subjects were no longer given what could be known as 'work' but instead were instructed to simply observe the pictures, and wait.

The results expectedly showed that the worst recall was in the first group which was asked to memorize a second list. Memorizing the second list had evidently caused the participants to forget the first list or recall it incorrectly. The groups which were asked to engage in a task which was similar in either operation or memorization alone did about the same, indicating to the Gibsons that each was about of equal importance. It was also clear that these two features were interdependent, because the loss in retention due to both factors, was more than the summed losses of each of the factors alone. The fourth group, which was asked to perform an activity dissimilar in operation and material did better than the first three groups indicating that an interpolated task which was not similar to the task provided the best recall. The fifth group however, did even better than the fourth, possibly because the interpolated task was even more different. The lack of 'work' in the interpolated task for the group which simply viewed pictures, the authors claimed, was the cause for the difference. The gain in recall from group four to group five was also evidence to suggest that there were other features than the two tested here, which influence memory.

2.4 Gibson (1934)

Retroaction and the Method of Recognition [19]

This short article analyzes one simple question of some importance for the theory of memory at the time. Can retroaction be demonstrated to exist when retention is tested by method of recognition? Gibson specifically discusses two German articles written by Heine [37] and Dahl [8]. The first, executed an experiment where she tested the extent to which retroaction could inhibit memories when the memories are retrieved via recognition, instead of recall. She found that no retroaction was present when recognition was used instead of recall. Dahl found that retention being better after a sleep period than a waking one, also holds when retention is tested via recognition. When it comes to the "sleep-effect," recognition and recall testing return the same results. Dahl's reasoning explains that given this, and Heine's findings that the retroaction effect holds for recall but not recognition, the two effects must be distinguished, and superior retention after a sleep period cannot be explained in terms of retroaction. Gibson however did not agree with this as it would destroy the value of a "Sleep and retention" study carried out by Van Ormer [57], and invalidate its evidence supporting the theory that forgetting is largely due to interpolated activities. As the whole argument depends on the study by Heine, Gibson found it necessary to question the study, and re-open the problem to experimental investigation. Especially since he cited two other experiments (yielding only partially conclusive evidence) which contradicted Heine [37].

Gibson then designed an experiment to test for retroaction under more favourable conditions than the experiment by Heine. Ten nonsense words were printed on cards, shown to participants for five seconds each, under the instruction to look at each word and try to remember as many as possible. An interpolated period of five minutes was given, where the experimental group learned by successive presentations another series of 10 nonsense words. The control group was asked to rest and look through a book with no writing except titles. After the interpolated period, all participants were presented 30 words on cards (five seconds each), and asked to report which they recognized from the original list. Ten words were from the original list, 10 had one letter changed, and 10 had two letters changed. A word with two letters changed were presented first, then the corresponding word with one letter changed, then the original set. A recognition score was calculated, one point for correct recognition, and one half point for making an error, but subsequently correcting the error. Twenty subjects were in each group. The experimental group was inferior in recognition scores by 31%, and also in number of errors by 53%. This lead Gibson to conclude his experiment as a contradiction to Heine [37]. He concluded with a declaration that "although more experimental evidence is needed before the issue can be decided, it is at least clear that Heine's results cannot be accepted as proving that retention tested by recognition will under no conditions show a decrement as a result of interpolation" [19].

2.5 Gibson and Hudson (1935)

Bilateral transfer of the conditioned knee-jerk [32]

Gibson wrote this article to follow up his experiment[33] of three years previous. It showed that if one hand was conditioned to withdraw due to electric shock, the other hand reacted based on the same conditioning, even though it had received no training. The question here, was if a conditioned knee-jerk in one leg forms a similar but latent conditioned response in the other leg. The subjects were each instructed to focus their entire attention upon reacting as quickly as possible to the stimulus of a combined light and buzzer by pressing a grip-key which was held in the hand on the side to be trained. After the stimulus began, 0.28 seconds later a blow was delivered to the patellar tendon. With the exception of those few who had very long reaction times, the hand response occurred slightly before the blow on the tendon. The component events of the voluntary reaction were employed as the conditioned 'stimulus'. The subjects were first trained or conditioned by reacting to the buzzer and light with the hand the grip-key was in, and experiencing the hammer blow to the tendon on the same side as the grip-key. After 5 training trials, a test for conditioning was made where the hammer was caught, and conditioned responses by the legs were noted. If no conditioning, 15 more training trials were done, if the subject demonstrated the conditioning in two successive conditioning tests, the experimenter changed the side of the grip-key, and the hammer. The hammer was caught before delivering the blow, and any movement of the legs was noted.

Sixteen subjects participated and of them, 10 were dismissed after the first sitting due to lack of signs of stable conditioning, though 12 gave at least one conditioned response in 65 trials. Four of the subjects manifested stable conditioned responses from the beginning. Gibson concluded that in subjects where the conditioning was strongly established, and equally stable conditioning of the untrained leg had been set up, and presented itself when the subject was told they would now be working on the other leg, and changed the hand for the grip-key. The light-buzzer-stimulus was also deemed non-essential to the conditioned response as on occasion subjects gave a premature hand reaction to the ready signal, and a kick immediately followed. Trials were redone (five times) with the four subjects who showed stable conditioning, which tested for necessary stimuli. First the hand-reaction was eliminated so that the subject held the grip, but did not react with it, then holding the grip-key was eliminated. A verbal condition was then used (the grip-key reintroduced) where instead of a light-buzzer stimulus the experimenter commanded "Ready, clench!" Then along with the physical clench, the subject verbally reacted by saying aloud, "Ready, clench!" The last iteration saw the subjects clenching the grip-key while saying the words sub-vocally. Of the total 240 tests, 159 were deemed to be positive and interestingly transferred conditioned responses were elicited about as often as the original conditioned responses. Gibson & Hudson stated that, "no essential difference appears between the two types, which [in] fact indicates once more that the transferred responses are essentially similar to the original responses." No single component stimuli was found responsible for the conditioning, though the physical clenching response gave better results than any auditory response. Taking into account Gibson's previous experiment, an explanation of the transfer of conditioning responses "was suggested in terms of a generalized system of avoidance responses, effective for various parts of the body, which as a unit might have become conditioned to the buzzer stimulus." The subject had essentially been conditioned on a particular situation, and mirroring that situation to engage the opposite side of the body did not change the conditioning due to the situation. This early explanation by Gibson and his colleague seem to suggest that the body does in fact operate in terms of situations. It may store the conditioning in a similar manner that we begin to think of food conditioned by the stimulus that we are hungry. If some constraints are satisfied, the conditioned response is engaged, but the constraints do not have to be minimal, and possibly any subset of them may yield the same conditioned response.

2.6 Gibson (1936)

A note on the conditioning of voluntary reactions [20]

James Gibson wrote this discussion as a critique of J.M.Stevens ([50],[51]) experiments. "At first sight," Gibson reports, "[the experiments] seem to demonstrate that a voluntary reaction can be *conditioned*." The purpose of the experiments goes further but the validity of it rests on this conclusion. Subjects were asked to react with the right hand (by hitting a contact with a stylus) to one buzzer sound, and with the left hand (a different contact & stylus) to another buzzer sound. Subjects were allowed to get accustomed to the reaction, and then the buzzers were set to go off either together or in close temporal succession. Without warning or notice, tests for conditioning were made by only one of the two buzzers sounding. Gibson criticized Stephens for lack of 'ready' signal, and an inadequate amount of control over variables. Conditioning was determined to have occurred if a subject reacted with a hand that had not been signaled. Gibson declared the subject may not have been giving conditioned reactions, but only what would be called false reactions in the conventional choice-reaction procedure. A reaction of a hand for which no signal was given, may mean only that when the subject is set to make either of two prepared reactions he may make both or the wrong one. The occurrence of such instances "might have been as frequent before the conditioning procedure had begun as afterwards. The practice trials given before the beginning of experiment," Gibson continued, "were not choice reactions and consequently provide no control" [20]. Gibson was very precise about his experiments and always provided these key ingredients. Subjects were ensured to be prepared to participate, were given notice that the experiment was to begin, and every detail was ensured to be controlled from one experiment to the next. As well, tests were also taken before the experiment and after so as to determine if there was a predisposed type of reaction, or if any conditioning in the experiment was legitimately induced during the experiment, and not prior. A voluntary reaction Gibson defined to be a reaction which depends on a preparatory set aroused by verbal instructions. Stephens responded by admitting that to demonstrate conditioned voluntary reactions, one should employ some control, and he wrote that he would ensure such control in future data so that it will have more bearing on the problems he wishes to investigate.

2.7 Gibson (1937a)

Adaptation with Negative After-Effect [22]

This was the first of three articles published at least in part by Gibson in the same year all dealing with the adaptation of the perceptual system, and the after-effect imposed on it when the change made to perception is reversed. He noted that very little time had been spent studying the functional similarities which cut across the sensory categories of our world. Gibson hoped that determining similarities which cut across sense departments might point to new categories and concepts in sensory psychology, and this was his first paper on the matter. It was dedicated to one such principle which he called *adaptation with negative after-effect* or alternatively *successive contrast.* Of course he admits that there are forms of sensory adaptation which do not produce any negative after-effect, but he also boldly stated that 'it seems certain that there cannot be a negative after-effect without adaptation.' A definition of negative after-effect is as follows:

> If a sensory process which has an opposite is made to persist by a constant application of its appropriate stimulus-conditions, the quality will diminish in the direction of becoming neutral, and therewith the quality evoked by any stimulus for the dimension in question will be shifted temporarily toward the opposite or complementary quality.

This effect can be see in sources such as prolonged blue stimulation, where quality shifts to neutral grey, and any other colour stimulus applied now would produce a hue which is shifted toward yellow.

Gibson examined several notions further: dimension, opposite, neutral, appropriate stimulus and shift. Dimension is divided into intensive dimension, sensory continuums such as pressure, size, or distance which he claims run from an absolute zero to a maximum, and oppositional dimension such as temperature (warm-cold), brightness (white-black), or linear shape (concave-convex) which run from the maximum of one quality to neutral to the maximum of its opposite quality. Though upon further review Gibson might have noticed that distance can be classified oppositionally as near-far (and has no maximum), and temperature does in fact have have a continuum of degrees. He later identifies this concern by deeming the minimum and maximum of temperature as those boundary temperature at which pain is experienced. One important notion is the opposition of two qualities, which mandates that qualities with oppositional dimension are incompatible (a surface cannot be both blue and yellow, a line not concave and convex), but when one attempts to produce both qualities one operation cancels the other and the result tends to be neutral (grey or straight in the examples). A third kind of dimensional change is transitive dimension, when a change is made between two different but not opposite standard qualities such as red to yellow. A thought which comes to mind here is that a transitive dimensional shift may be two simultaneous oppositional shifts. When red changes to yellow, it may be that red is shifting to neutral, and a grey on a different (yellow-blue) scale shifts to yellow. Gibson's thesis stated that, "whenever experimental qualities fall into an opposition series, then adaptation with negative after-effect may be expected to occur" [22].

The author discusses some instances where this adaptation and after-effect emerge. Colour, or 'chromatic adaptation' as Gibson cited from Troland (1930), manifests by shifting the hue evoked by any stimulus toward the complementary of the adapting stimulus. Though adaptation along one dimension of colour will interestingly produce a change in all colour-equations. The shift may be seen as toward the neutral center of a colour-circle thus shifting all points in the circle including non-complementary ones. Brightness is a dimension which adheres to the principle. Illustrated quite simply by anyone by moving from a dark room to a lit one or vice-versa. It does not however, present itself in the form of an after-image, but does nevertheless manifest as an after-effect. Temperature is identified as an opposition series. Gibson states that the neutral quality is definite, in that temperature which the body rests. If a certain temperature feels neutral to the touch, and the skin is attuned to a temperature higher than neutral, the pervious neutral will now feel cold, and any stimuli will feel colder than they ordinarily would. Gibson believed that similar mechanisms are at work on the temperature sensitivity of the skin and the brightness sensitivity of the retina. He also discussed formation of lines in the visual field by delimited objects, concavity of a line, tactile shape of an edge citing his earlier research [18], and linear direction (tilting) of lines. Other examples Gibson examined are tactile-kinaesthetic direction (horizontal/vertical direction), visual movement, tactile movement, and even taste. After exposure to sour, distilled water tastes sweet, and sweet tastes sweeter.

Gibson also explains that some of these effects also occur when one dimension is perceived on a background of another. For instance, when a straight line is seen against a background of curved lines, the straight line appears curved in the opposite direction to its background. This effect is present in tilted lines as well as curved ones. To explain the negative after-effect, Gibson wrote that it is "a by-product or incident of the primary fact of adaptation to a norm." "A state of adaptation," he continued, "might be a state of psychological equilibrium." The process, he supposed is a way of keeping the experimental norm or neutral quality in correspondence with the norm of external conditions. The hypotheses for a comprehensive theory are as follows: the normal quality of any dimension is the most frequent one in past experience; the normal quality is correlated with the most stable physiological condition (the one involving the least output of energy); the normal quality is correlated in the long run with the most stable and thus frequent condition of the external environment.

This article, combined with Gibson's previous writings, demonstrates that Gibson did not hesitate to generalize some properties of a system, to a near-universal nature where the properties are guaranteed to emerge. He took his and other studies on after-effect with regard to qualities such as curvature, and colour, and extended the notion to all oppositional dimensions. This leap can later be observed when he shifted from studies on visual perception in humans to his notion of affordances which can be said to be relevant to any animal's perception using any perceptual sense.

2.8 Gibson and Radner (1937b)

Adaptation, After-effect and Contrast in the Perception of Tilted Lines: I. Quantitative Studies [35]

This study investigates and confirms that the adaptation and after-effect which Gibson proved occurs in the perception of curved lines, also occurs in the perception of lines tilted away from horizontal or vertical. The effect was tested to occur in subjects required to look for several minutes at a 20 cm line-segment drawn on a large cardboard square. The line-segment was tilted varying degrees from horizontal and vertical and the impact of the after-effect was found to be minimal at 45°, and maximum at 10° though it was at least somewhat present at all inclinations. Fixation on the midpoint of the line-segment was reported to have a weaker aftereffect than inspection of the entire line-segment, and it was also reported that if the lengths of the lines used for the inspection and testing were greatly different, the effect was diminished. What would not impact the results is if a black line-segment on a white background was used or if the edge between a black area and white area was used. Also interesting, is that if a tilted rectangular cross was used, the effect was present in both lines of the cross.

Gibson concluded, after repeating the experiments restricting the subjects' vision to looking through a tube, that we as humans possess a 'sense' of visual direction which is adaptable. He then repeated the experiments using the restricted vision, controlling aspects such as duration of inspection, and attempting to quantify the expected amount of after-effect based on the deviation from the neutral. Subjects were also asked to set the line at horizontal, vertical, and 45°, 30°, and 60° from vertical. The errors of each inclination for each subject showed that variability of oblique inclinations was 3 to 6 times greater than the horizontal and vertical thus our acuity for the latter was much greater. Gibson's studies he admitted produced too few data, but he cited three other authors who proved our precision for setting lines to vertical and horizontal is greater than other angles. Testing subjects on 5° tilt from vertical, then horizontal, Gibson controlled duration of fixation by testing with 1, 5, 10, 20, 45, 90, and 120 seconds. After-effects ranged from tenths of a degree after one second, rising rapidly for 5 and 10 seconds, up to 1° for 20 seconds. The after-effect continues to rise up to 120°, but quite slowly topping out at 1.5°. Tests were run for 5 and 10 minutes, but yielded not much higher results. The horizontal axis was deemed to produce a greater after-effect than the vertical, a possible explanation I suggest, might be the impact that the horizon has on the most prominent human activity, locomotion.

The interdependence of the horizontal and vertical was also examined. An after-effect obtained on one reference axis did indeed present itself on the other axis (indirect after-effect), though to a smaller extent. The horizontal and vertical directions do in fact seem to behave as aspects of a single system for visual orientation, or a single *spatial framework*, but the link is not rigid as shown by the diminished impact of the indirect after-effect.

Gibson mentions in his introduction of this article that an essential element in visual perception is the notion of boundary, edge, or line. He could not at this early stage of his career, have known that he would go on to report this elementary principle in his books, nor the impact that it likely had on his eventual theory of affordances.

2.9 Gibson (1937c)

Adaptation, After-effect, and Contrast in the Perception of Tilted Lines. II. Simultaneous Contrast and the Areal Restriction of the After-effect [21]

This third (and last) article in the series assumed the reader is familiar with the previous two which explain adaptation, and the after-effect which follows it, in the frontal visual field. In hopes of proving that, "linear direction is functionally akin to a sensory process like that of color," Gibson sought to demonstrate the occurrence of simultaneous contrast in perception of linear direction, and the process of adaptation is a localized process within the visual field. He had already shown from previous reports that direction and curvature exhibit successive contrast where perception of something impacts what is successively perceived, and was now setting out to prove that like curvature, linear direction also manifests simultaneous contrast.

The apparatus used was described in the first paper, with the new addition of a grating of parallel black lines 5 cm in front of the white disk containing the adjustable black line. Subjects were asked to set the line to vertical before and after the parallel lines were superimposed. Three specific amounts of inclination were tested: 10°, 20°, 45° in addition to the horizontal and vertical control. Results confirmed to Gibson that a contrast-effect was definitely present. The effect was most pronounced at 10° (two degrees), but still present at 20° (half degree). At 45°, however, the average contrast was less than a tenth of a degree. The experiment also revealed that the vertical axis provided a similar but slightly stronger contrast effect than the horizontal axis for each inclination, leading Gibson to provide this as evidence that the two axes function as mutually related spatial standards with a limited degree of interdependence.

The next experiment involved two cases. Subjects looking at a vertical line surrounded by a tilted box or a tilted line surrounded by a vertically aligned box. After adaptation in the first case, the line showed a negative tilt, but the square frame appeared normally oriented. It seemed that the relationship between the line and its frame had been altered. In the second case, the contrasting tilt of the line was reported to increase in amount over the adaptation period, along with a decrease in the apparent tilt of the square. The after-effect in the second case, showed the square frame appearing tilted to observers, but only whilst they maintained fixation on the line. If the eyes left the midpoint of the line, moving to the edges of the square, it appeared normally oriented. Results were similar when the box around the line was replaced by two equal length lines on either side of the line. When fixation was on the middle line, it appeared tilted, if fixation was shifted to an outside line, it appeared tilted, and the others approximately vertical. The conclusion drawn, was that the spread of the negative after-effect in the visual field was quite limited and the effect was localized.

Another key question in understanding perceptual processes is why the negative after-effect of linear curvature transfers from one eye to the other, but the after-effect of colour does not. The tilt effect did indeed transfer from one eye to the corresponding region of the other. According to one theory at the time, lines and objects in the frontal field of view are seen as upright or tilted based on their relation to the horizontal-vertical framework. The framework provides a reference base, but more importantly for this study is itself determined by the main lines of the visual field. The horizon line is an example of a main line. "These main lines of the field, even if...not gravitationally vertical or horizontal, will become so phenomenally because they determine the framework." Gibson cited experimental support for this theory as coming from a German article by Wertheimer (1912) who asked subjects to look through a tube which tilted the image of the room by 45°. Wertheimer found after a period of adaptation that the tilted image appeared vertical and must have changed subjects' entire spatial framework. Gibson did however, deflect this idea as inapplicable to the phenomenon under study as the adaptation there was complete rather than partial as it is here, and occurred at 45° while in this experiment 45° yielded minimal if any adaptation or after-effect. As well, the tilt effect here was localized, and did not impact the spatial framework as a whole, unlike the study done by Wertheimer. The effect here, Gibson explained, "behaves like a partial and local adaptation process akin in many respects to sensory adaptation." A theory of the phenomenon of tilt adaptation, which would explain the results obtained in this experiment should also explain similar results related to other sensory adaptation, i.e. colour, brightness, linear shape, skin temperature etc. Gibson leaves an attempt at a general theory for another paper.

This article brought to the surface Gibson's clear belief that different senses of the body such as hearing, taste, and touch shared perceptual similarities which might be all explained via some meta-theory. Gibson made this belief more clear some years later in publishing *The Senses Considered as Perceptual Systems* [30]. What can also be examined due even to this early article is the possible existence of some underlying thought or brain processes which are common to all senses in much the same way that inheritance and abstraction are now widely used in programming. Finally, Gibson concluded with the hypothesis that any perceptual experience which falls into an 'opposition series' is subject to adaptation with negative after-effect.

2.10 Gibson & Crooks (1938)

A theoretical field analysis of automobile driving [31]

In 1938 Gibson and Crooks chose to take a practical every day experience and analyze it from a theoretical perspective. This article, they footnoted, is the result of discussions between a psychologist and a practical student of driving. Gibson explains his motivation in writing this paper by reminding the reader that driving an automobile is the most important skill demanded of us in that mistakes could result in the greatest risk of injury or death. This paper was an attempt to write a systematic description of a set of concepts needed to drive. The authors attempted first to base his analysis on present-day psychology–habits, attitudes, and response-sequence–but had little success. The task of driving was instead taken to be predominantly perceptual, and the overt reactions involved in it were relatively simple and easily learned. Their analysis then had to be performed on a perceptual level with suitable concepts "like the 'field' of the driver, 'valences', and the general cross-sectional method employed by Lewin (1936)" [31].

The authors took driving as a type of locomotion guided by vision, which utilizes a tool, unlike walking. The goal state would be presence at the destination. The sub-goals are then the perception and avoidance of all encountered obstacles, and the prevention of any collision which might stop locomotion or produce bodily injury. The visual field is essentially what the driver sees during locomotion. What is special about the field is its selective nature under which pertinent elements stand out, and non-pertinent elements recede into the background. The most pertinent objects are obviously those which could potentially cause a collision, and the least pertinent are distant objects to the far right and left. The authors define the *"field of safe travel"* as an indefinitely bounded field lying inside the edge of the road, which consists at any time of the *"field of possible paths which the car may take unimpeded."* The boundaries of the field are determined by objects with a negative 'valence' in perception while the field of safe travel has a positive 'valence', higher along the (usually safer) mid-line. Gibson and Crooks mean by *valence* the emotional response which is created in imagining oneself closer to, or in contact with, that area or object. Steering then is hypothesized as a "perceptually governed series of reactions by the driver...to keep the car headed into the middle of the field of safe travel."

Collisions are understood to be prevented by shifting the direction of motion, or by ceasing motion altogether. Steering controls the former, so the authors turn to the latter. Acceleration was deemed to be a function of our motivation to arrive at our destination, and its relative urgency. It is only possible to accelerate through a field of safe travel. Deceleration on the other hand, occurs when the field of safe travel contracts, and like steering is primarily an avoidance reaction to obstacles. Another relevant principle introduced is the *minimum stopping zone* which is phenomenally the area within which the driver can stop if necessary. The authors note that it may not be consistent with reality as it depends on the perceiver, but does depend on things like the road surface, and the quality of the brakes, etc. Driving was said to feel 'dangerous' when the minimum stoping zone nears the size of the field of view. The driver's principle awareness of how fast he is going is not measured in miles per hour, but instead related to the minimum stopping zone.

The authors also predict a habitual ratio (for each driver) of depth-of-field to depth-of-zone which tends to be maintained in given traffic conditions. It may be seen as an index of cautiousness, and may decrease if the driver is in a hurry. Accidents may occur if a driver is inattentive to the forward field of view, causing the perceived field of safe travel to become incorrectly bounded. If a sudden change is made to the field of safe travel (e.g. a couch falls off the truck ahead), an entirely new field may open up which did not exist an instant earlier. Obstacles such as the shoulder, a curb, or a ditch may shift from a negative valence to the only positive valence in the field.

The authors list factors which limit the field of safe travel. Natural boundaries consist of fixed obstacles, amount of light, weather such as fog or snow, or the horizon at the top of a hill. Inflexibility to manoeuver at higher speeds prevents acceleration past the point where following a turn in the road would be safe. Moving obstacles radiate negative valence from the point where the driver estimates that the obstacle will be closest to him, i.e. the potential collision point. If the obstacle is moving toward the driver, it will be closer than its current location, if it is moving away, it will be farther. Potential obstacles are areas where the driver is effectively blind. Areas behind which the driver cannot see engage a negative valence due to the uncertainty of what is in that location, and the potential for things in that blind spot to move into the safe field of travel. This recognition and personal valence depends on the individuals experience as an amateur driver may not recognize the danger behind a corner, and a collision may ensue. Legal obstacles and taboos modify possible fields of safe travel by restricting the size by means of a speed limit. Though no danger necessarily occurs here, the driver may encode a negative valence with a certain speed due to the fear of prosecution.

Gibson and Crooks had a prediction for the cause of a major driving statistic. One in twenty people died in the span of one year at the time of writing. Their analysis suggested the principle cause was that the drivers did not know how to drive properly. The discussion of the problem with the public unfortunately began with what not to do, and recognition of abnormal driving, rather than what drivers ought to do, and how to drive normally. Concepts must be taught which refer to operations the driver can understand, and instead of memorizing legal obligations, students of driving should be taught a systematic theory of driving, verifiable by observation and experimentation. The theory in this paper was described as an effort in this direction.

This article seemed to be the first glimmer of a much larger theory still to come. In reviewing the task of driving an automobile, Gibson kept his perceptual theories in mind. Jones [42] suggested that this is the earliest writing which contains clear precursors to the affordance concept. This can be see in the constant references to the safe field of travel, which by affordance terms would make that area traverse-able (by car). As well any obstacles would afford danger, and a free path, affords safety. Though Gibson did not know it yet, this would be another link in his sequence of many very influential papers in the field of perceptual psychology.

2.11 Gibson (1950)

Perception of Visual Surfaces [23]

In 1950 Gibson published his first book. He titled the book, *The Perception of the Visual World*; quite telling of the earlier stages of his career in which his experiments tested the visual perception of subjects. Though he had already predicted that similar processes are involved in some aspect of all of the human senses, he would leave the other four senses for his next book. The present book was written about how we see. Many contributing factors to our accurate perception of reality are indexed and analyzed in this book, with the hope of developing a comprehensive theory of visual perception. Though it may fall short of this ultimate goal and does not bluntly describe how visual perception occurs, it does quite nicely evaluate and explain the impact each effect has on what we might perceive.

Sight is both a vastly important, and quite complex process which most people fail to realize as such. Gibson wondered, "How can vision depend on the pictures in the eyes and yet produce a scene which extends to the horizon?" He examined how the world can be projected from three dimensions, down to two inside the eye, and yet restored again in perception, and emphasized that the problem he is writing about is perception, not sensation. In describing fundamental properties of the visual world such as it being upright, stable, without boundaries, coloured, shadowed illuminated textured etc., he states most importantly is that it is filled with things which have meaning. He predicted that explaining these properties of the visual world would go quite far in explaining the whole panorama of visual experience. If the reader is familiar with the theory of affordances Gibson produced over 20 years after publishing this book, he may identify early components of the theory here in the mention of the visual world being filled with things containing *meaning.*

Early in the book, Gibson put forth initial hypotheses of a "Ground Theory" of space perception with the intension of explaining them in later chapters. The first was that, "The elementary impressions of a visual world are those of surface and edge", which was intuitive in its explanation. The second was that,"There is always some variable in stimulation (however difficult it may be to discover and isolate) which corresponds to a property of the spatial world." It essentially states that any perceived quality including distance, depth, edge, contour has some stimulus though it may not be easily discovered or isolated. The third principle stated that, "The stimulus-variable within the retinal image to which a property of visual space corresponds need be only a correlate of that property, not a copy of it." Some qualities such as solidity and depth do not have any replica in the two-dimensional retinal image, but may have qualities which are correlated to them through a well defined mathematical transformation. The fourth hypothesis (declared as most debatable) predicted that, "The inhomogeneities of the retinal image can be analysed by the methods of number theory and modern geometry into a set of variables analogous to the variables of the physical energy." Simply put, the order or pattern of the retinal image or successive images can be considered as a stimulus. Fifth and last was the hypothesis that, "The problem of how we perceive the visual world can be divided into two problems and considered separately, first, the perception of the substantial or spatial world and, second, the perception of the world of useful and significant things to which we ordinarily attend." The spatial world is a background for the perception of significant things or things with meaning. Gibson explained that the world of these significant things is too complex, so our attention to it is selective. He also admitted that this book is primarily concerned with the literal kind of perception for the spatial world, and an examination of the schematic type, or the 'meaning' would be secondary.

The second chapter focused on past theories of perception and predicted that only through perception are abstract sensations such as colours, sounds, touches etc., combined to form an experience. That is, perception yields meaningful 'things' from sensations alone and is subjective to the observer. Gibson reviewed the empiricist and nativist histories in explaining how theories of perception have evolved. He also discussed the origins of Gestalt theory, which objected to sensations being cues for perception, and instead predicted that the brain or cerebral cortex was responsible for organizing sensations into forms. If everything we are aware of is brought about by stimulation of sense organs, and some things have no counterparts in stimulation, they must then be somehow synthesized. Nativism took the synthesis as innate, Empiricism explained it as learned from past experience, and Gestalt theory suggested it was produced by the whole central nervous system. Gibson explained that the question was not *how* a percept is organized, but on *why* a percept is always organized like the particular entity the eye happens to be pointing at.

The next chapter was designated to something Gibson had been studying particular anomalies of, the visual field. Less familiar than the visual world, it is only perceived directly when a great deal of effort is taken to fixate our eyes, and then pay attention to the whole range of what is visible. It is characteristically clear, sharp, and fully defined at the center, but becomes progressively vaguer away from the fovea. According to Gibson, it is in some respects, like a picture, and it is an introspective, analytic phenomenon, the experience on which the doctrine of visual sensations is based. It is bounded by the limitations of our eyes to perceive that which is not directly focused on, but instead perceived peripherally. It is what would inadvertently be inspected if one was to attempt to examine the visual world, which unlike the visual field, wraps around us 360°, has no boundaries, and is always clear and fully detailed.

James Gibson had studied the visual field in quite a few articles prior to the publication of this book. He explained many of the effects he discovered through his studies, including the location, size, and distance, of after-images in the visual field, and the apparent convergence of parallel lines at the horizon line. A distinction he made is that, in the visual field some areas eclipse other ones, yet in in the visual world one object lies in front of another. It is a matter of semantics to some, but an important distinction to Gibson. The visual field was stated as a "reasonably close correlate of the retinal image" (p 43), and fittingly the next chapter was written about the formation of retinal images.

Understanding visual perception starts with physical objects, light, and the eye. Essentially the laws of optics. Solids and liquids in our environments have surfaces against the gases, and these surfaces reflect any available light, which may invariable reach the cornea of the eye, refract to pass through the pupil, and fall on the retina. Of the infinite rays of light projected to this optical image, the most significant according to Gibson, were those originating from the edges of physical surfaces. He outlined the physical interior to the eye including the rods, cones, and the strong connection of the optic nerves to the occipital lobe, but much about the chemical reactions in the eye and brain was not known, and these facts did not seem to lead to any useful theories of perception. The retinal image, a correlate of external objects, however was worth analysis. In it, borders between colours form lines or contours, and areas of colour have visual textures, their culmination may be called the distribution or pattern of light rays. The pattern which reaches the eye may be defined by either a static arrangement of rays, or by the adjacent order of rays. Essentially, each point may either be perceived as a unit as a member of an arrangement, or as relative to the other points currently, or previously perceived. Tying this back, Gibson declared the visual field's correspondence to the anatomical pattern of excitation. The visual world however, corresponds to successive patterns, overlapping each other as the eye moves, united perhaps by a sort of immediate memory.

The retinal image (or visual field) invariably contains some indication of the orientation of the horizon. The ground or water outside, or perhaps the floor below indoors, provide a reference to the horizontal direction, regardless of the orientation of the perceiver. This is one pattern that we universally perceive, though Gibson explained that the retina responds "to a differential intensity in adjacent order over the retina"([23], p 64). That is, we perceive what we perceive based on the pattern of changes between each successive retinal image. The visual quality of texture, he predicts, is elicited by a cyclical change in the order of light intensities. Physical surfaces such as wood, cloth, or earth have regular structures specific to them which form specific patterns on the retinal image, and thus stimulate a texture specific to that surface.

Gibson continued to examine the connection between the objects in the visual world, and the retinal image by looking at things such as the gradient of texture on the eye when stimulated by a surface parallel to the line of sight. In this case textures run from coarse to fine as distance increases. The gradient is constant however when perceiving images perpendicular to the line of sight. These may be clues to explain exactly how perception works. Gibson wrote that the impression of distance in a picture for example, is immediate (i.e. it has a definable stimulus) caused by the gradation of texture elements, while the interpretation of the picture follows as our minds process what we are perceiving. The concept of gradient, defined as an increase or decrease of something along a given axis of dimension, was quite important to Gibson as it appeared to be "admirably adapted for describing the retinal image, since both gradients and steps of stimulation can be found within it." (p73) Gibson cited an author who found evidence that all living tissue is characterized by physiological gradients of metabolism, excitability, and growth. Therefore it was not unnatural to consider the proposal that light-sensitive cells of the retina, and neural tissue in the brain react to gradients of stimulation.

In a quest to identify stimulus variables for visual depth and distance, Gibson made two assumptions about the typical physical world. Easy to believe, he began that objects tend to be in contact with the ground. The second assumption was that objects tend to be distributed evenly over the ground. Essentially there is an average number of blades of grass in square centimeter, or trees in a square kilometer of forest which nature does not usually stray too far from. Gibson postulated a connection between this idea, and the knowledge that our eyes can only perceive structures of certain sizes. This idea can also be seen in later books where he expresses that the level of focus for an organism will be on the scale of the organism under consideration, i.e. no astronomical or microscopic events for macroscopic organisms. By first isolating the motion, and only considering a motionless observer with fixed eyes, Gibson looked to investigate cues for distance perception, among them the density of natural objects compared to the observer's distance from the objects. Ordinary surfaces are rarely both physically smooth and chemically homogeneous, meaning most (unlike glass) will have crests and troughs and so with sufficient light may be seen. It must also be noted that a transition must be possible from objects to textures in the same way that a forest is perceived to consist of trees from nearby, but will be perceived as a surface from a distance or altitude. Based on an analysis of several basic pictures consisting of distributions of simple figures such as circles or lines, on a plane (parallel to the line of sight) beginning very close to the perceiver and ending with dots on a supposed horizon, Gibson concluded that a gradient of texture is, in isolation, a stimulus for the impression of continuous distance on a surface. Though not usually perceived in isolation, this may still contribute to how we perceive depth. He continued this discussion, adding that the gradient of density (correlated to gradient of texture) is what yields the sense of a continuous third dimension.

Other perceptual instances were examined such as how images containing only contour lines, may bring about the perception of a corner (such as a valley or hill) or alternatively an edge (such as a step up or down). With an edge, the contour lines of the image jump, whereas with a corner they bend. Shading which yields the perception of depth, edges, or curved surfaces provides other subtle conclusions, and the reversal or inversion of images was addressed as well. The direction of the source of light, as well as knowledge of an object's concavity or convexity are additional possible cues in perception. The remainder of this chapter as well as a majority of the next was spent exploring different gradients on the retinal image such as those on illumination, convex or concavities in the surface of objects, and contours separating objects from their backgrounds, in the hope of deriving explanations or rules for cues to perception. While the writer very precisely identifies the way in which we perceive, the writing does not seem to elucidate a mathematical formula for perception, but does confesses that the descriptions needed mathematical analysis. While the stereoscopic effect of binocular vision was commonly believed to be the only important basis for depth perception, Gibson points out the success of one-eyed fliers, and that animals without overlapping binocular fields (e.g. rabbits, rats, etc.) seem to discriminate depth by their behaviour. The theory of retinal gradients Gibson discusses in this book, implies the gradient of disparity between overlapping binocular fields is only one of several cues to visual depth.

With regard to visual perception, even a fixed headed observer provided many cases to discuss. Expanding the situation under review to include motion of the head and body, likewise expanded the discussion on rules governing visual perception. The retinal image, Gibson explained is not like the film in a camera in the sense that motion spoils the picture. Motion for the retinal image enriches the information by registering changes in successive images. Motion from head movements, unlike motion from eye movements, according to Gibson, is indeed a stimulus for the perception of space, and a precise sensory correlate of locomotor behaviour. During a continuous motion towards the horizon, objects on the ground or below the horizon move past the observer, while objects above the horizon remain fixed in place. The speed of the objects seeming to move through the retinal image, corresponds to a gradient of velocity which decreases proportional to in increase in distance from the observer. This gradient of velocity as well as that of direction were then shown as stimulus correlates for perceived space and perceived locomotion. Pursuit and saccadic eye movements were examined each providing its own conclusions pertaining to its impact on visual perception. It was then followed by analysis of the focus of expansion when it lies on a surface (indicating impending collision) as opposed to the horizon (where it indicates the direction of travel).

The investigation of motion and its impact on visual perception was at the time fueled by research on aviation psychology during World War II. Gibson studied perception, pertaining to flying an airplane, for several years (1943-1946) employed by the military, and it was that experience that he admittedly relied upon while writing this portion of the book ([23],p.129). One important idea Gibson explained was that complex discriminations of the direction, altitude, and angle of flight for a pilot are learned, but the stimuli for these discriminations are likely as innate. The retinal image would be the same for notice or expert pilots, but the different would be how they react to variations. Experience, he wrote, allows a pilot to make fine, rather than gross discriminations of not only the pilot's own position and direction, but of other objects in the retinal image. It seemed that learning to attend to, or explore, features of the world was not understood at the time of writing, even if it is at all clearer now. A good correlate would be how beginner drivers may observe only the closest of obstacles, but with time and practice, learn to watch the distant focus of expansion to optimize safety. They do not observe more than a novice, but an expert is attentive to finer details.

The types of retinal motion caused by ordinary motion rather than locomotion through an environment, were distinguished next. A rigid motion is one of translation, rotation, or a combination of the two, whereas a deformation is an expansion, contraction, or skewing of all, or part of, the retinal image. Motion of the whole retinal image for example, is due to fixed head position, and saccadic eye movements. Motion of a delimited part of the image occurs due to movement of an object in the physical environment at a right angle to the line of sight. The surrounding image will appear to move during a pursuit movement of the eyes. Deformation of the total image is caused by movement of the head relative to the ground, with eyes fixed on the horizon. Deformation of a delimited part of the retinal image occurs from stationary head and eyes when an object moves in any direction not at right angle to the line of sight. Gibson wrote that all these alternatives coexist with each other to yield the seamless experience of visual perception. Any of them may be considered alone, or summated with others, and lose none of the stimulus function.

Having considered motion in general, Gibson lightly analyzed stimuli for acceleration which are mediated best by stimulation from within muscles, and stimulation from within the inner ear. The retina is insensitive to acceleration. Gibson tried to convince the reader that the perception of a stable upright visual world depends on the co-variation of the visual sense with the so called body sense. We experience a stable, upright, and unbounded visual world, though neither the retinal image nor the visual field have any of these qualities. When we read, successive images do not overlay each other, as the words would all blend together. Instead, we unknowingly join the successive images into a sort of chain, so that we may read an entire sentence rather than be limited to the words we can fit into our fovea. The rules governing this process can be discussed by dividing the idea that the movement of the eye and the movement of the image are reciprocal, into four categories. If the eyeball is mechanically rotated to the right, the scene should appear to move left because the compensating shift is absent. If the eye actively rotates to the right after a clear negative after image is generated in the center of the field of view, the after image should not appear to move, but will appear to be displaced because the compensatory shift is present with no retinal motion to be canceled. Thirdly, if the eye attempts to rotate to the right but does not due to paralyzed eye muscle, the scene should move to the right because of the compensatory shift to the right. Last, if the eyeball is mechanically rotated to the right after a negative after-image has been produced, the after-image should not appear to move due to any compensatory effect, but the scene will be displaced to the left as in the second case. This discussion for Gibson, clearly implied that the directional stability of the visual world might be a product of activities which are inverse to one another.

Considering the upright nature of the visual world next, Gibson asked why the world does not tilt when the image does. As one might believe gravity was ultimately concluded as the main factor in the upright nature of the world as it is sensed through both the inner ear, and the bilateral musculature. The environment around us (floor, walls, horizon) being reference-axes was examined as Koffka believed it explained the upright nature of the world. Gibson, however dismissed the idea as we can escape these reference-axes in certain specialized conditions, but we have no compensation for gravity.

The unboundedness of the visual world lead Gibson to ask why it is unbounded when the stimuli consist of fragmentary images. Saccadic eye movements, and locomotion are two actions humans use to visually explore the world around us. Saccadic movements are successive eye movements which must each be on some object of attention. Locomotion is a sort of continuous transform of the retinal image. Gibson borrowed an analogy from film, where saccadic movements correspond to sudden changes in the picture, and movement of the head corresponds to a continuous but changing image like when a film pans over a scene. Each alternative was considered to be primary, but Gibson concluded that they must be combined. The words he uses to describe how successive images overlap and how our brain may piece together the images, indicate he does not feel comfortable with the traditional view. Those who have read Gibson's work from late in his career may observe that the problems he encountered here seem to be resolvable under the theory of direct perception he advocated later, as the differences between successive images my be stimuli we perceive innately.

Chapter nine focuses on the constancy of sizes and shapes. Even when our perceptual angle to, or distance from an object changes, the shape and size of the object seem to remain constant. The traditional view that our brains' compensate for variations in size or shape based on our memories is refuted by the simple argument that birds surely do not memorize objects, yet can discriminate between large and small objects even when the larger is so much farther its image is smaller. The question of why sizes and shapes remain constant is proposed as a false question, or a result of the belief that perceptions begin as patches of colour in the visual field. In trying to account for the perception of the material world, we must recognize that objects have significance as well as solidity though the latter must be prior. An inconsistency which arose was in considering a protuberance where both sides are physically white, if the left side is lighted and the right side shadowed why does each side appear to be the same colour. The answer given was that possibly the high-to-low step in brightness yields an impression of depth and therefore can't yield an impression of difference in colour. Gibson finishes this topic of discussion by stating how colour is indeed affected by spatial stimulation. "The innate attribute of extensity which color has been supposed to possess turns out to be not the simplest kind of space by merely indeterminate space."([23], p169)

Due to an experiment Gibson described, which artificially removed objects from their backgrounds, and tested observers to determine if two different shapes represent the same object, he concluded that memory does not play a part in shape constancy. Even new objects to an observer maintain shape constancy. The constancy of shape must then, depend on our ability to perceive in three dimensions. The experiment also concluded however, that constancy is often incomplete in that a judgement of the apparent shape of an object requires a compromise between objective shape, and projected shape onto a picture plane. Gibson differed to an article written by Koffka in 1935. According to it, any perception of the stimulus object involves shape and orientation coupled together. Neither is perceived in isolation thus we suppose that perceived orientation combined with apparent shape yields a constant shape.

Next to be discussed were compression of texture manifested by a slanted surface, where texture along one dimension of the projected image is compressed as slant is increased; constancy of size in perceived objects, where a chart twice the size at twice the distance appears as the same size as the original chart; and then how distance is perceived, regarding factors such as adjacency, height in a frame, and visual angle. These topics lead to the discussion of a quality called scale. Perception of scale is the process by which we understand approximately how large an object is through seeing it, and comparing it to other objects being perceived. It seemed to Gibson that apparent size and apparent distance are linked because the size of an particular object is given by the scale of the background at the point to which it is attached. Through an experiment during the war, Gibson also claimed that dimensions of solid objects were judged more accurately than dimensions between them though both displayed constancy of size. Another wartime experiment was provided as evidence that size of an object does not in fact become smaller in perception until it reaches a vanishing point, but instead can apparently be seen with approximately its true size as long as it can be seen at all. The indeterminacy instead is what changes with distance, as it seems to increase as objects move father away.

Chapter ten picked up on the problem of perception of shape without depth from chapter six. How abstract geometrical figures such as square or circle relate to visual perception was unknown at the time of writing, though Gibson did review Euclid's parallel postulate, and how centuries later geometers concluded that parallel lines meet at a vanishing point implying a location called infinity. This progress lead to the conception of our visual field as expanding from one pole and contracting to another 180 degrees away. The progression that still needed to be made was in ceasing to think about forms as a set of geometrical entities and instead focusing on transitions between them (as had been done for colours). Gibson concluded that all experiments where a subject drew a visual pattern from memory or from a recent observation (he did many such experiments) yield no reason why the subject's response is like the original pattern though they do help explain how a person learns to discriminate similar patterns or conceptualize objects.

Meaning is a very broad topic, but based on his question how early ancestors on the Asian plains come to 'know' a tiger, Gibson examined what meaning is associated with concrete objects. Six kinds of meaning were described, the most important of which was the symbolic meaning carried by anything from flags to words to well known symbols. This type of meaning was said to mediate knowledge and form the basis for reasoning, creative imagination, invention, and discovery. How much behaviour is achieved through learning, and how much arises spontaneously through growth, was examined throughout the rest of this chapter. Four oversimplifications are present under the simple theory, "that the visual world is an unlearned experience, that it is meaningless when seen for the first time, and that what one learns is to see the meanings of things."([23],p200)

The first oversimplification, was the assumption that the parts of the visual world such as color, surface, edge, and interspace, are meaningless. From his experiments asking subjects to memorize randomly generated forms, Gibson claimed that even nonsense forms are only relatively meaningless. According to his wife E. Gibson, nonsense forms like all forms must be differentiable from each other to be memorized, and since memorization requires forming a unique response to the stimuli, there must be at least some meaning present. Tautologically, Gibson wrote that surfaces, edges, and shapes have at least the meaning of surfaces, edges, and shapes. The second oversimplification is the assumption that meaning is detachable from concrete spatial qualities, or that things and events can be separated from their meanings through introspection. This cannot be wholly true due to the observation that never before seen tools seem to look different once their use is understood. The perception has new properties it did not have before which were not aroused through retinal stimulation. The conclusion formed was that meaning is sometimes detachable, and in fact becomes more detachable from spatial qualities the more it approaches high level concepts.

Thirdly the implication that all meaning be learned and there are no unlearned meanings was considered an oversimplification as well. Since everyone develops a slightly different meaning for things because of their differing points of view and experiences, we conclude that everyone learns the meaning of the world independently, and slightly differently. To explain exactly what is meant by 'meaning' we review the example given of a nesting bird. Any object which causes the bird to clean its surface and sit on it has the meaning of an egg. In the first nesting season, birds take objects similar to eggs and treat them as such. After the birds have had a chance to learn finer discriminations, they can tell the difference between egg-shaped objects and eggs. In infants there is a stage between about two and six months where they fixate and smile at any face-like object that moves. After that point they learn to discriminate recognizable faces such as their parents. These facts imply that infants do not begin to learn meaning from nothing. We are not born with a set of innate meanings but not all meaning is acquired either. The last oversimplification of the theory is that when meaning is added to objects, it doesn't modify concrete spatial qualities. Duncker showed in 1939 that a piece of cloth in the shape of a leaf was judged as noticeably greener than the same colour cloth in the shape of a donkey. This effect called memory-colour, is where experiences impact the colour we perceive. The same sort of effect was shown for coins, where poorer children judged coins to be larger, or imagined coins were judged as larger than actual coins. This evidence seems to indicate that *meaning* does indeed impact-at least our perception of-the spatial properties of the visual world.

Toward the end of the book insight was provided into how behaviour is mediated by perception, and what place learning has in seeing. Examining cases where cataract patients who were blind since birth, and then surgically given the gift of sight, it was clear that perception of the visual world was quite simple, but differentiating particular objects in a large scene took some patients many months, and needed to be learned. Essentially Gibson concluded that we learn to differentiate between percepts, rather than learn to perceive them.

In the last chapter Gibson made a few key introspective observations. Things seem to look as though they are 'for' some action. They look as though they are capable of being pushed or grasped, or they appear to resist these actions. The visual world seems to invite behaviour. This idea would prove to be the foundation for his future work. He also noted, based on patients who had lost proprioceptor kinesthetic sense (muscle senses) in the lower portion of their body, that since vision could in these patients compensate for the loss allowing them to walk, there must be two forms of the kinesthetic sense. It is also not hard to see seeds of his future 'direct perception' work when Gibson wrote, "Retinal motion is automatically linked to bodily action from birth onward, so long as the eyes are open and there is light to see." What Bishop Berkeley understood was that walking over and touching something confirms our visual space. What he missed was that the expanding visual field confirms muscular-tactile space. Gibson examines last the stimulus-correlates for the perception of oneself. Among them are any senses we might imagine that make us 'feel' ourselves from our sense of touch, to our inner ear, to the deformation of the whole retinal image. Strangely, even our ability to orient ourselves to everything from the sun, to a cup of tea, emphasize our sense of being.

In conclusion, Gibson isolated some factors pertaining to perception, but armed with the knowledge that he published two more major books, we may correctly infer that his full theory of perception was yet to be completed. The book as a whole is a comprehensive overview of the different mechanisms involved in visual perception at a low level. The rejection of behaviorism in favour of his own new ideas how animals sample information from the ambient visual world, was ground-breaking, though his future writings would prove to be less specific and more abstract. An extension of the first page of his last chapter in this book under the heading 'The Motor Theory of Space Perception'. We can tell only now that many mathematical constructions had yet to be scholarly published when Gibson wrote his books, and he may well have taken to foreign ideas such as rough sets, situation theory, or computer simulations.

2.12 Gibson (1976/1986)

Ecological Approach to Visual Perception [28][29]
Originally published in 1979, the same year James J. Gibson died, *The Ecological Approach to Visual Perception* was the culmination of Gibson's entire life's work on visual perception. He reported in the introduction that this was a sequel to *The Perception of the Visual World* though rather different. Cognitivism, instead of behaviorism was the subject of investigation in this book. His preface nicely explained the shift in his investigation of vision. At first, his goal was to use physics of light, and the retinal image, to master the anatomy of physiology of the eye and the brain. It could then, he supposed, be put together into a theory of perception which would be empirically testable. The more he learned about physics, optics, anatomy, and visual physiology, the more complex the task became. As he explained, optical scientists knew about light as radiation, but not illumination; anatomists knew about the eye as an organ, but not what it can do; physiologists knew about nerve cells in the retina, but not how the visual system works. Though scientists could create holograms, prescribe glasses, and cure disease, vision could not be explained.

These facts are not at a level appropriate for the study of perception. The basis for vision in this work was the ambient optic array rather than the retinal image, as it was in his work in 1950. It examined the possibility that neither behaviorism nor mentalism provide a sufficient explanation. Gibson wrote, "Why must we seek explanation in *either* Body or Mind? It is a false dichotomy."[29] in his preface and he kept that principle throughout the book. Another fundamental principle he began with was to "suggest that natural vision depends on the eyes in the head on a body supported by the ground, the brain being the only central organ of a complete visual system." He went on to provide terms for types of vision from *aperture vision*, looking at a piece of a picture like a telescope, to *ambient vision*, what we are able to detect through eye and head movements in our environment, to *ambulatory vision*, achieved from walking and moving about. These types of vision are what we need in life to detect our environments, act in them, and survive. We need to see all the way around objects using different points of observation. He uses this as a basis to partition the book. The first part is about the environment, what there is to be perceived must be clear to discuss perceiving it; the second is about the information for perception, ecological optics as he called it examines the information in light which stimulates our receptors; and the third is about the activity of perception, "not the processing of sensory inputs...but the extracting of invariants from the stimulus flux." A fourth part examines pictures and special kinds of awareness, but will not be discussed here. Gibson wrote that contents of this book formed a proposal of a radically new way of thinking about perception, and this review should reflect that.

Gibson introduced his ecological approach by explaining the role the environment plays. Animal and environment for an inseparable pair where either term implies the other. It is obvious in one direction, but the notion that an environment does not exist without an animal to perceive it might be controversial, but Gibson only meant 'environment' in the strictest sense of the word, and did not mean that the physical world would cease to exist without its being perceived. He restricted the word environment to refer to the terrestrial scale of a living being, thus ruling out the physical scale of atoms and galaxies, and time scale of nanoseconds and eons. The environment is nested in that objects are within other objects such as cells in leaves in trees in canyons in mountains. Basically the environment will be examined at the level of ecology, or a level appropriate to what the organism in question (typically humans) may perceive. The environment also contains higher order modes of perception from things such as speech, writing, microscopes, or pictures, but this review will be brief on these accounts as the focus is how Gibson came to his theory of affordances.

Before discussing the propagation of information, he first ensures this propagation is possible. For this, we need a medium, and air fills that role nicely, as light can openly reverberate until it reaches a sort of steady state. Water is a substance in our medium of air, though if we consider aquatic species, water becomes a medium. Mediums are thus taken with reference to an organism as we shall see becomes a commonality.

Physical reality is less considered here than what is considered ecological reality as the former does not consist of any meanings while the latter does. If we perceived entities of physics or mathematics, meanings would have to be imposed, but Gibsons suggested that we perceive entities of environmental science and thus meanings can be *discovered.* Objects tend to have meanings related to what we can do with them. Likewise the environment allows us to move within it, take cover from it in shelters, drink it in the case of water, or use it in any number of refined ways in the case of fire or other tools. Tools, Gibson claimed, extended the body, and perceptual psychology has since accepted this proposal[45]. The mind it seems can learn to accommodate new tools and incorporate them into our mental model of our body as an extension. Since a tool can shift from being a part of the environment to being a part of the animal, we notice that the boundary between animal and environment can shift, suggesting that the absolute duality of objective and subjective is false. Considering the affordances of things allows us to escape the philosophical dichotomy.

The information available for visual perception is carried by ambient light. This type of light is a result of illumination as opposed to radiant light which causes illumination. Ambient light carries information, while radiant light carries energy. Radiant light originates from atoms and terminates at atoms, while ambient light depends on an environment of surfaces. Ecological optics is interested in ambient light, or as Gibson wrote, "light that has gone astray." The field of ecological optics was founded by Gibson in an article with that title in *Vision Research* in 1961[26]. It borrows from physical, geometrical, and physiological optics, but goes beyond all of them. A distinction between luminous and illuminated, that which emits light, and that which is lit by light, is made. An interesting discussion involved Gibson giving two contradictory assertions, but each can still be considered true in their own right. This is one in a pattern of many instances where Gibson found new ways to examine problems that were previously thought to have no solution, or an obvious solution. Either we never see light, or we only see light (or neither) but both cannot be true. Yet if we consider that we cannot detect the light itself, only what it is that the light illuminates, we may accept that we never see light. And contradicting this is the idea that we need light to illuminate something or we cannot see anything, thus giving weight to the idea that we only see light as it carries the information. This debate was not in fact resolved, but is interesting nonetheless. The information available for perception is said to be carried to an organism only through a flowing array of (structured) stimuli, and not through a single stimulus. As well, the retinal image registers pattern and change from and in ambient light, rather than analyzing a sort of still picture.

Gibson discussed the ambient optic array, but first found it necessary to explain his definition of this. The ambient portion being explained already, and optic being quite clear, Gibson noted that an array is implicitly somehow arranged, and contains structure. Components of the array can be described in terms of visual solid angles, which Gibson reported had been clear since Euclid. The components change when the point of observation changes, unlike the physical objects themselves which persist. Natural perspective according to Gibson, had failed to consider motion of the point of perspective, and this shortcoming was quite a large one. The optic array changes due to locomotion, but maintains non-changing things as a result of the rigid world. Perspective structure changes through displacement of the observation point. Invariant structure is also obtained through displacement of the observation point, but occurs indirectly by observing the underlying consistencies. The invariance in the structure of the ambient optic array does not exist except in relation to the variants. The sources of invariants are the layout and reflectances of the surfaces in the environment. The sources of variant optical structure are moving points of observation and moving sources of illumination. Gibson next examined events, and the information specifying them.

Ecological events as we are concerned with them were given by motions at the level of surfaces and substances in a medium. Gibson broke events down into three categories, changes in the layout of surfaces caused by forces, changes in colour or texture caused by changes in composition, and changes in existence caused by a change in the state of a substance. The first type, mechanical events, covers transforms, what Gibson called rearrangement of the furniture of the earth, such as a rolling ball, a walking animal, or the breaking of a glass. The second type covers chemical events, where the composition of a surface of a substance is altered, such as rust, erosion, or even animals changing their colour to indicate their readiness to mate. The third type destruction or creation of surfaces, occurs due to changes in the state of matter, such as evaporation, destruction, or biological growth. Gibson stated that these events are perceived, but time is not. Time consists of events filling it, but does not exist a priori. With respect to affordances, some events have them, while other specify a change in them. The actual specification of an event, though quite general, is defined as a disturbance in the invariant structure of the optic array. It is these disturbances which carry the information about events in the environment.

Self-perception, Gibson wrote, is unique to each individual. It is easy to believe that each person gets different information about his/her body. Not only visual information is available, because we hear our actions, feel our skin touching objects or the ground, and smell odors around or near us. Visual Kinesthesis is the term Gibson gave to the pickup of information due to any of the three types of movement; head movement relative to the body, limb movement relative to the body, or locomotion of the body relative to the environment. What is more interesting is Gibson's claim that information specifying the self and the environment are inseparable, or that "the supposedly separate realms of subjective and objective are actually only poles of attention." (p116)

The theory of affordances examined how to go from an environment of surfaces to affordances. It will be useful to rewrite one of the most famous of Gibson's quotes.

> The *affordances* of the environment are what it *offers* the animal, what it *provides* or *furnishes*, either for good or ill. The verb *to afford* is found in the dictionary, but the noun *affordance*, is not. I have made it up. I mean by it something that refers to both the environment and the animal in a way no existing term does. It implies the complementarity of the animal and environment.

Affordances, he went on to write, must be taken relative to an animal, unlike physical properties. If a surface has physical qualities of flat, horizontal, extended, and rigid that can be discriminated, it will look support-able or stand-on-able. If it does look this way, the affordances are visually perceived. A note of clarification which Gibson was particular about, was that to perceive an affordance is not to classify an object. We perceive affordances, not qualities and we can enact affordances of objects without classifying them. Gibson suggested that a set of affordances constitutes a niche. A niche refers to how an animal lives rather than where it lives. A niche implies a kind of animal, and an animal implies a kind of niche, in a sort of complementarity of the two terms. This complementarity is even more striking with respect to affordances and individuals. "...an affordance is neither an objective property nor a subjective property; or it is both if you like. An affordance cuts across the dichotomy of subjective-objective and helps us to understand its inadequacy. It is equally a fact of the environment and a fact of behaviour. It is both physical and psychical, yet neither. An affordance points both ways, to the environment and to the observer." (p.129)

Affordances can be simple such as the earth affording support, or much more complex. For example, the behaviour of other people affords our behaviour. Places have affordances such as hiding, finding food, or being dangerous. It can be seen that affordances are not only positive. Fire affords warming, but also burning. Affordances, Gibson wrote, are invariant combinations of variables and are always there to be perceived. They are specified by stimulus information, and are directly perceived. Perceiving an affordance is a process of perceiving a value-rich ecological object. Affordances may also be misperceived. A transparent glass door may appear to afford traversability through it, but attempting to engage this affordance will result in collision. We thus learn to see the affordances of our environment through experience, and learn to adapt to the misinformation of affordances.

The third part of Gibson's book was devoted to visual perception, or more specifically, experimental evidence to support direct perception. Gibson argued against the idea that the perception of the environment was based on retinal pictures, neural pictures, or mental pictures. "Direct perception," he wrote, "is the activity of getting information from the ambient array of light."(p.147) The discussion was broken into the perception of three things: surface layout, changing surface layout, movements of oneself. The perception of surface layout was an update to his ground theory from his 1950 book. He wrote that he would rename ground theory, the theory of the layout of surfaces asserting that perception of surface layout is direct. He did not believe that perception began from two dimensional form perception and the idea that the third dimension was at all 'lost' was taken as ill conceived as it was never really in the environment. Old experiments had never allowed for the perception of invariants over time. Gibson argued that the eye picks up a sequential transform, not forms.

Gibson used experiments to provide evidence of his theory of direct perception. Some such as the visual cliff, or the optical tunnel emphasized the way animals pick up information (and affordances) about their environment directly, but did not empirically prove that the perception is direct. One interesting example was where subjects were asked to judge the size of a wooden stake nearest to a base line of stakes with different heights. Though it had previously been supposed that since objects cease to be visible at some distance, that was done by way of becoming smaller, Gibson's experiment shown conclusively that the size did not decrease. The judgements became more variable, but the mean size was still accurate. Gibson took this to be because of invariant ratios for example the ratio of the size of the stick to the number of texture elements it occluded, or the proportion of the amount of stake above the horizon to the amount below. An invariant law of equal texture for equal amounts of terrain was given, showing for Gibson that size and distance were perceived directly, as well as the only invariant of ecological optics, the horizon. The horizon is a frame of reference against which all other optical motion is judged.

An argument against direct perception is one where a motionless observer peers through a window and what is seen could be any of several forms. The simple counterargument Gibson provided was that this argument fails to consider that observers in general may move about and thus identify which of the several forms is being perceived. What specifies an object for Gibson, are invariants which are themselves 'formless.'

Invariants in motion were used as evidence that the perception of a changing layout is direct. The perception of environmental events seemed to depend on a disturbance in the structure of the ambient optic array. Gibson cited experiments by Gunnar Johansson in which light circles are moved on a screen in a periodic fashion[39]. It seemed that if circles separated by any amount, shared the same periodic movement, observers judged them to be part of a rigid structure moving as a whole, yet if the movement ceased, the connection was lost. Gestalt theory indicated that we must organize the environment in our heads or we would perceive separate individual circles of light. Gibson argued instead that the circles were already organized by their motion, and we perceived the association between similarly moving circles directly. The first experiments to suggest direct perception for Gibson were ones in which both humans and animals, (one at a time) were placed close to a large screen, and a dot was enlarged from a point, to the size of the screen, prompting flinching actions by observers as they reacted instinctively to the threat of collision. Results of an experiment on the perception of slant, implied a certain change of form could yield a constant form with a change in slant, which Gibson pressed into the idea that objects are specified by invariants under transformation.

Gibson relayed what was for him an important experiment performed by Johansson in 1964 where circles of light were again moved about[40]. The circles first were organized to project the appearance of a square, and when it was moved about, or resized maintaining width to height ratios the transforms were rigid. But when he attempted to demonstrate elastic transforms by varying the width and height out of phase from each other, instead of a changing form, observers noted a sort of three dimensional transform where the form was rotating about three different axes. This stunned scientists, and helped Gibson to develop his theory of direct perception. Not only the environment is directly perceived, but also the movements of our own bodies relative to the environment, and relative to our head. Gibson suggested that vision is kinesthetic in that it registers movements of our body in cooperation with our other senses.

Occlusion was a term introduced by Gibson and co-authors in 1969, describing the process by which one object is hidden behind another[34]. The place where the object being hidden, disappears is called an occluding edge. Occluding edges and surfaces are a result of perceiving the world under a natural perspective. A more accurate depiction of our perception of our environment results from imagining many perspective points along a path of observation. Though Gibson writes that it sounds strange to say that one could perceive an environment at no fixed point of observation, it must be true if we admit that one can perceive surroundings during locomotion.

The theory of reversible occlusion was presented as a better explanation to how animals find their way, than the theory of response chains or the theory of cognitive maps. Essentially to find their way, animals need to determine which area to point their focus of expansion at, or which occluding edge hides the target. Experimental support came from an experiment where successive screens had progressive accretion or deletion of texture or structure from one side. Observers unanimously perceived a 'going behind' motion from the deletion of structure, and a 'coming from behind' motion from the accretion of texture. Only time it seemed, can yield certain impressions.

Looking around by way of turning ones head was described as having the visual field sweep across the array. As we moved the visual field by moving our eyes, some things go out of sight, but we are still aware of them. They persist. What we pick up in perception according to Gibson, is the invariant (persistent) structure of the visual world. This is given as possible because what is now hidden to the visual field, what we perceived moments ago, is continuous, or connected to the unhidden environment we perceive now. Gibson also took it upon himself to explain the functions of the visual system such as blinking, in terms of ecological optics. The function of the retina, for example, was to register invariants of structure rather than points of an image, the saccadic eye movements are exploring what information is available for pickup, and the adjustment of the pupil is done to optimize the pickup of information.

The last subject reviewed here is on the theory of information pickup and its consequences. Gibson wished to convey to other psychologists the fallacy in working with only laboratory experiments which universally fail to account for the perceiver's awareness of being in the world. The new theory of information pickup sought to convey several new ideas. First, a new notion of perception, where perceiving is a continuous psychosomatic act not performed by the mind or the body, but by a living observer. Second a new assumption about what there is to be perceived, namely surfaces and occluding edges together with the affordances of the environment, rather than colours, points, and forms. Third, a new concept of information for perception, referring to the specification of the observer's environment. Information for perception cannot be defined or measured in terms of bits, and it is not conserved like matter or energy. Fourth, a new assumption of perceptual systems with overlapping functions, explained that a perceptual system includes all the senses with actions that make use of them. Essentially a perceptual system does not involve solely the eyeball, but the eyes-in-the-head-on-the-body-resting-on-the-ground as a whole system. The fifth and last idea was how optical information pickup entails the concurrent registering of both persistence and change in the flow of structured stimulation. Everything in the world, Gibson said, both persists and changes to varying degrees. It is the job of the perceiver to separate change from nonchange by extracting invariants of structure from the flux of stimulation, while still perceiving the flux. The new theory, contrary to stimulus-sequence theory, which assumes the only way to apprehend persistence is by comparison and subsequent judgement, states that the perceptual system is directly able to perceive the persistence by recognizing the transforms (or lack thereof) which have taken place.

The last section in Gibson's book was devoted to depiction, detailing the perception of pictures, movies, and optical illusions, but we leave it to the reader to investigate as our primary focus was on affordances, and our direct perception of them.

Chapter 3

Background of Affordance Formalizations

After studying the human perceptual systems for so many years, Gibson put forth a new idea: Affordances. Affordances he explains, refer to the part of the environment which in relation to an agent (or animal), specify a possibility for any agent (or animal) of that type to act. The notion is best explained by this famous excerpt from his final work *The Ecological Approach to Visual Perception*:

> The affordances of the environment are what it offers the animal, what it provides or furnishes, either for good or ill. The verb *to afford* is found in the dictionary, but the noun *affordance* is not. I have made it up. I mean by it something that refers to both the environment and the animal in a way that no existing term does. It implies the complementarity of the animal and the environment [29].

The quotation clearly indicates that an affordance cannot be specified without reference to a possible agent which can possibly initiate an action. If the animal and environment complement each other to specify affordances, we can assume both must be referenced in any formal definition. The idea of an affordance itself was officially published in 1977[27].

3.1 Turvey (1992)

Affordances and Prospective Control [56]

In 1992, Michael Turvey considered Gibson's affordance concept in the context of an animal's Prospective Control. PC, as Turvey writes it, is a way of viewing the control of any non-deterministic choice. An animal, is constantly selecting from a seemingly infinite list of possible actions. No animal can continue to exist without taking some form of action; with inaction, or waiting, considered types of action. Put another way, time persists whether we make a decisive choice or no choice at all. The method of selection from this list of possible actions, is what scholars in many fields have been studying for a very long time. Turvey explains that "conducting an act requires that one perceive whether the act as a whole is possible, what subacts are possible with respect to the surface layout, and the possible consequences of current subacts if current (kinetic, kinematic) conditions persist" ([56], p.174). This seems to be somewhat straightforward, because obviously to act, one must perceive that act being possible, but the actual process of deriving the possibility for action must be articulated, and defined with no ambiguity, so that one day it might be possible to simulate an animal in its entirety. As Turvey mentions in his abstract, "...research in the ecological approach to prospective control is ultimately the search for objective laws," and the same I'm sure, can be said of research into the formalism of affordances [56].

His aim was to formulate the ontological basis for scientific research into the affordance concept, yet he dwelled on metaphysical discussions in his article. He decided to examine the reasoning that affordances are real properties of the environment relative to the animal. He presented the ontological position of the ecological approach as follows:

> 3.1 There are only propertied things; neither properties nor individual things are real independently of each other.

This lead to several metaphysical points in support of affordances being an ontological rather than epistemological category. The metaphysical points did unfortunately lead him to his belief that affordances are real parts of the *environment* to be perceived. That premise caused him to present the following formal theory of affordances, which while certainly useful as a first attempt to be improved upon, would receive criticism for years to come.

- Let W_{pq} (a system) $= j(X_p, Z_q)$ be composed of Z (person) and X (stairs). Let p be a property of X and q be a property of Z. p is said to be an affordance of X, and q is the effectivity of Z (i.e. the complement of p) if and only if there exists some property r such that

1. $W_{pq} = j(X_p, Z_q)$ possesses r
2. $W_{pq} = j(X_p, Z_q)$ possesses neither p nor q
3. Neither Z nor X possesses r

The first thing to notice with this definition is that the affordance, as denoted by p, is a property of the stairs Z. Gibson himself, states in several ways that affordances are not a part of an animal *or* an environment, but rather a method of referring to the complementarity, duality, or polarity of the two. Turvey did adhere to the complementarity issue, however in using q as the complement of the affordance p, he explicitly placed affordances in the realm of the environment. If as Turvey claimed, the affordance rests in the environment there would have to be a complementary entity. The word effectivity is used to refer to this entity in the same way as Shaw et al. (1982) when they introduced the term. Another observation easily made is the lack of specification of the function j. Two sections before introducing the definition of affordance, Turvey comments on the actualizing of a disposition, or the engagement of an affordance.

> 7.3 What exhibits an actual or manifest property is the unit formed by Z and its complement X. Thing Z with disposition q joins thing X with disposition p to form thing $W_{pq} = j(X_p, Z_q)$ with manifest property r (j is the joining or juxtaposition function)

Though there are missing links to Turvey's formalism, it was a bold attempt to bring some formality to the area of affordances, and it triggered many responses which have in turn, contributed to work in the idea.

3.2 Greeno (1994)

Gibson's Affordances [36]

James G. Greeno wrote a review of Gibson's Affordances, and the theory's progression, but he did not contribute to the formalism itself. In the article he surveys Gibson's work in the field of ecological psychology, beginning in 1954 with "The visual perception of objective motion and subjective movement', and concluding with Gibson's final and most well known book. Greeno did not argue with, but rather supported the idea that the affordance concept refers to the conditions in the *environment* which allow an animal to both perceive an action as possible, and consequently carry out that action. He carries forward the notion that the opposite or 'dual' in some sense of an affordance is an 'effectivity', 'ability', or 'aptitude' depending on whose notation should be accepted. One paragraph worth noting refers to an article by Neisser [46] in which two kinds of perceptual processes are argued for. *Direct perception* as proposed by Gibson, which provides information for dynamic interaction with the environment, and *recognition* which provides information for identifying and classifying objects and events. The idea was not investigated, but the thought that Gibson's theory of affordances might only apply to occasions when an agent is learning a *new* action or movement, might have merit.

Each of the empirical studies performed in the development of affordances asks participants to perceive some new opportunity for action. These results could be useful in articulating how exactly an agent acquires a new affordance, but the downside to this possibility is that recognition would undoubtedly happen more quickly than affordance acquisition, increasing the difficulty in measuring it experimentally, as well as leaving us with no experimental data pertaining to recognition. Greeno did not see anything unreasonable with using mental symbols in half of Neisser's proposal. He suggested that mental symbols are products of the perceptual process of recognition. As well, he claimed that during recognition new mental symbols are created. A theoretical understanding of these symbols could provide humans with a basis for understanding conceptual entities, such as numbers or the even the affordance concept.

A large portion of the article is dedicated to a discussion of Situation Theory and its application to affordances, with focus on the constraints. The constraints employed in situation theory may form rules which govern the pickup or activation of an affordance. If for instance an animal were attuned to the prototypical example constraint 'where there is smoke, there is fire,' it would be able to sense that the area which contains smoke affords danger. Greeno mentions such constraints as $\ll$action by agent$\gg\Rightarrow\ll$good effects in situation$\gg$ in a situation involving a skilled practitioner. These are indeed constraints which an agent might be aware of, but the above example seems much too general to be of any value.

If the above constraint were to be useful to an agent, it would also have to be attached to information specifying the practitioner and the circumstances under which the constraint will hold. For instance, there is no guarantee that the practitioner may take some action that unknowingly to him, does not bring about a 'good effect'. Greeno acknowledges that these constraints are conditional, but mentions that they will hold whenever participants in the situation or conversation are attuned to the same set of constraints. This does not take into consideration a situation where all involved have the same expectations, yet some new, different result emerges. Overall, Greeno put value into Gibson's work and his suggestion to investigate situation theory will be investigated in a later chapter, and may prove fruitful.

3.3 Sanders (1997)

An Ontology of Affordances [48]

In this commentary, Sanders follows up some of his previous articles, in which he compares Gibson's metaphysical views with Merleau-Ponty and Berkeley. Here, he argues essentially that affordances are not properties of things in the same way humans might attribute the concept 'coarse' to sandpaper, but instead that they exist as ontological primitives. An affordance is a real thing which is composed of parts of some cognitive agent and parts of its environment which allow the agent to act. According to Sanders, an affordance is real but also subsumes its real, composite parts, in the same way that a human brain exists, but does not prevent us from referring to the frontal lobe, the thalamus, etc., as individual parts which also exist.

Sanders discusses affordances as relative to both the subjective and objective poles of attention in the same sort of way velocity is relative to the frame of reference. The many perspectives of motion, he continued were reconciled by the special theory of relativity, which defines the relativistic conception of space-time in such a way that each perspective of each individual can be mathematically modeled. In terms of affordances, the frame of reference would be the ecological scale of the animal in question, or the scale under which it may take action. Only under very particular circumstances could one consider the atomic or galactic scale as ecological. It must first be possible to manipulate some aspect of the atomic or galactic scale, and that opportunity for action must be presently achievable for such scales to be considered. For the majority of situations, it is only the perceptual sphere of the organism-the sum total of everything the organism may perceive with its senses-that must be referenced.

With this in mind, Sanders also takes issue with the notion of effectivities, on the basis that Gibson created the term affordance to refer to the subjective and objective poles, and the idea of an effectivity was created to represent solely the subjective side of an objective affordance. Just because Gibson states that an affordances is an opportunity for action *in the environment,* some of Gibson's followers took this to mean that some counterbalance must reside in the animal to complement the affordance. What Gibson meant however, is that though the opportunity for action rests in the environment, the affordance itself references this opportunity and the animal together completing the affordance relation. The animal it can be said, must exist to be able to perform the action, and so its reference is a necessary part of the affordance concept. Sanders concluded by referencing relativity theory which he used to illustrate that there is no truth about what is in motion, and why different views from different perspectives can be unified. This can be done in much the same way using an ecological approach to ontology, with its reliance upon affordances. With a change in perspective, what may be regarded as an affordance or event may change as well. Sanders claimed that only "by way of the ecological approach that one can understand just why both materialism and idealism seem right from certain delimited perspectives, and ... why neither was ever capable of refutation."

3.4 Stoffregen (2000)

Affordances and Events [52]

Shortly after focusing on perception of affordances for other people, Stoffregen wrote a target article for Ecological Psychology, entitled *Affordances and Events.* In it, he compared literature on event perception, to literature on affordance perception. He also uses quotations from Gibson's book to explore the similarities and differences between affordances and events. Stoffregen relays Gibson's belief that both affordances and events are objects of perception, and then goes on to differentiate between the two. In much of the literature on event perception, Stoffregen explains, events are taken as objective, in the sense that an actor is not referred to. When event perception is empirically studied, no attention is given to entire animal–environment system, but instead the event and its successive perception is studied in isolation from other variables. Event perception also takes the assumption that properties of the animal are perceived separately from properties of the environment. This would mean that to derive an affordance, an animal would have to perform some internal operations to compare the sets of properties, and this internal representation is an obvious violation of Gibson's direct perception based ecological approach. The definition of an event which follows is consistent with the brief and at times ambiguous descriptions written by Gibson in his book, *The Ecological Approach to Visual Perception.* "Events are static (i.e., stationary) and dynamic (i.e., moving) properties of objects and surfaces defined (i.e., measured) independent of the perceiver"[52].

Affordances, in contrast to events, must be defined as a relation between an animal and its environment, that have consequences for behaviour. Stoffregen also produced Turvey's definition of affordances and claimed that it was equivalent for his purposes. Upon closer inspection though, Turvey's definition that affordances are "properties of the environment of an animal that have consequences for the animal's behavior" [56], misrepresented what is meant by an affordance. This excerpt identifies the specification of an affordance residing in the properties of an environment of an animal, which Stoffregen examined in his section on the definition of affordances. At the time of writing, only Turvey and Sanders had authored anything regarding a formal definition of affordances. Stoffregen mentioned his preference to Turvey's attempt, but took issue with the way Turvey represented the affordance as a property or properties of the environment. Instead Stoffregen suggested that p and q be properties of the environment and animal respectively, and the higher order relation r, "constitute[s] the constraint on or opportunity for behavior, that is, the affordance"[52]. This revision seemed to more closely resemble the true spirit of the affordance concept as proposed by Gibson. In support of this revision, Stoffregen cited a study by Mark (1987) who examined each participants ability to judge maximum sitting height with 10 cm blocks on their feet. After 12 trials, the judgement of maximum sitting height improved so that the participants could accurately determine these heights. In contrast, the participants were asked to estimate block height, and the overestimation was consistent at about 30% over the course of all trials. These results indicated that when the higher order relation between animal and environment is modified, the animal may compensate for that discrepancy even without a direct knowledge of its own properties or dimensions. The height of the block on the subject's feet was learned in relation to how it modified the subject's affordances, or capabilities for action, but not as an objective property in isolation.

One proposal Stoffregen delivered which I disagree with was that if infants begin with a sensitivity to events, and acquire a sensitivity to affordances with development of action skills, this would force a cognitivist conclusion. In other words, we would have to perform mental processing on our visual stimuli of events to derive opportunities for action. A truly Gibsonian viewpoint, as rejected by the author, would be that as infants, our perceptual systems are advanced only enough to detect events, and as our perceptual systems become attuned to the environment around us and our bodies become able to take action, we begin perceiving affordances. This is not to say that we must necessarily then cognitively process events to obtain affordances. Stoffregen states that this view would suggest that events are more easily detected by novices, but I propose that it may be the case that the research Stoffregen cites, may have a different definition of event than ones associated with the debated term in affordance literature. In addition to this work, Stoffregen also published a 'Theory and Research' section to this paper [53].

3.5 Chemero (2000)

What Events Are [3]

In the same issue of *Ecological Psychology* Anthony Chemero argues against some of Stoffregen's conclusions. He claims that animals do perceive events, but a new conceptualization of them is needed. Instead of an event referring to any change in an animal's physical surroundings, Chemero proposed that, "events are changes in the layout of affordances of the animal–environment system." Essentially, when an affordance comes into existence, disappears, or is modified, this would be classified as an event. Chemero maintained that though Stoffregen's article presented some aspects of Gibson's theory which are often forgotten, and it attempted to improve on the existing affordance formalism, one conclusion in particular would have widespread consequences for ecological psychology and the ecological method in general.

> More generally, I have suggested that events may not be perceived; it may be that only affordances are perceived. If so, this would raise serious questions about the utility of research in event perception in the context of the ecological approach.

Here, Chemero quotes Stoffregen's article and points out that if the quote is accurate, much of the work in the entire journal of Ecological Psychology would be useless. There is a way, he continues, to accept the valuable points Stoffregen reenforces, yet reject the hypothesis that humans and animals do not perceive events. If events are redefined as an instance in which the ambient optical array specifies that an affordance has become possible to actualize, or no longer possible, we can conclude that humans do perceive events, but the question shifts to whether any are perceived directly or all are perceived indirectly.

Any animal can perceive when actions move from possible to impossible or vice versa. Chemero suggests that instead of the traditional experiments which have tested affordances in the past, new experiments should emerge. Instead of providing subjects with many situations, and asking them to judge if an action is performable in each situation, experimenters should allow subjects to continuously perceive a situation which is changing, and respond with the point at which the action becomes possible or impossible. This would be an effective method of studying affordances and events, as this critical point (with Chemero's new definition), can be regarded as an event. This critical point, in fact, is explored in [5].

3.6 Gibson, E.J. (2000)

Where is the Information for Affordances [15]

In a third paper in the same issue of Ecological Psychology, Eleanor J. Gibson responded to Stoffregen's target article. She reported her belief that to perceive an affordance (which humans do regularly) perception of events must be possible, perception of the relations between events must be possible, and it must be possible for the perceiver to acquire the information for all of this. The information which specifies an affordance, she writes, is in the events both external to, and within the perceiver [15]. It can be clearly observed then, that Gibson's widow believed events to exist both completely within a perceptual agent, as well as external to the agent. The repercussions to taking events in this manner would be that neither Stoffregen's view that events are not perceived, nor Chemero's view that events are solely changes in the layout of affordances, could be taken as accurate. Yet some portion of both ideas is incorporated, even if the beliefs put forth were not dependant on either of them. Stoffregen's belief that events are not perceived is bluntly taken as wrong, but his idea that events can be viewed as independent of an agent is somewhat accepted. With regard to events which are external to some perceiver, we could classify them as independent, however if the event triggered the appearance or disappearance of an affordance, it must necessarily be related to a perceiver. If events inside and outside the perceiver are taken equally, we presumably would categorize them together, though they may have subtle differences. Internal events would refer to bodily changes within an animal, may or may not alter the layout of affordances either. Chemero's description of events is somewhat incorporated, and in some sense overloaded. Eleanor Gibson does seem to confirm Chemero's description of events as changes in the layout of affordances, but she also implies that not every event is necessarily a change in an affordance.

The following quotation illustrates the authors view of events: "The information for an affordance is to be found in events that include the relevant environmental features, the activity of the organism, and the consequences that ensue as well as the relations among these" [15]. If the information specified for an affordance lies in the events, it would seem to imply that events first occur, possibly several of them, and either the culmination of them, or possibly the final or most recent event (as judged temporally), may specify an affordance if an animal is capable of perceiving and acting upon it. She does not however take one step farther. We may accept the structure discussed by Gibson here, but she leaves the task of identifying how animals actually pickup the information for events, and how this information is specified, to future ecological psychologists.

3.7 Wells (2002)

Gibson's Affordances and Turing's Theory of Computation [60]

In 2002, A.J. Wells examined some of the previous models of affordances, and then outlined an approach using Turing's theory of computation. "Affordances," according to Gibson, as Wells relates it, "are ecological; they are relational; they are facts of the environment and behavior; sets of them constitute niches; they are meanings; they are invariant combinations of variables; and they are perceived directly." All of these components make it quite challenging to integrate them all into a suitable theory of affordances, or we may discover one day that Gibson left us with exactly what we needed to specify affordances accurately. Wells also investigates two of Gibson's ideas which can possibly be regarded as contradictory. Gibson clearly believed that affordances are relational; that they refer to, or as Gibson put it 'point to', both the environment and the observer. Gibson also claimed that affordances are facts of the environment, and not dependant on the observer. The second claim seems to indicate that the affordance does not in fact 'point to' the animal. Wells did not see any contradiction to these claims, and described Gibson's view of affordances as *asymmetric interdependence.* Asymmetric because of the heightened importance placed on the environment, and interdependent because the terms are complemented by each other to create an affordance. Wells further examined the seven features of affordances he recites at the beginning of his article, then discussed coalitions as models for ecosystems from Turvey et al.(1981).

A discussion of the Turing machine which will write the infinite sequence of the natural number, explains exactly how the Turing machine works and then reviews of Turvey and Greeno's input toward the formalization of affordances were given, but I will not reiterate them again here. Wells points out to the reader that "every Turing machine has components modeling the agent and components modeling the external environment." This is quite a keen observation when discussing J.J. Gibson's theories. A Turing machine has a read/write head which, according to the ecological approach to perception, could be an animal or 'agent' which directly perceives its environment and behaves according to its perception and intention or 'state'. The tape would be the agents environment, and a state is something common to both Turing machines and the ecological approach though the latter might prefer the words intention or mood to the word state. Wells defines an affordance A, and effectivity E as follows:

$$A = (q, a) \tag{3.7.1}$$

$$E = (b, p, k). \tag{3.7.2}$$

The terms a and b, are environment referential, p and q, are animal referential terms, and k refers to both as a representation of the movement of an animal relative to its environment. A represents the perception side of the affordance, where an animal in state q perceives a, and E represents the action side where behaviour b causes the animal to change to state p and move in direction k. Because A represents the perception, and E the action it is fitting based on Gibson's belief that action and perception are inseparable, that instructions for a Turing machine are combinations of configurations and action. Wells' Turing machine instruction for affordances has the form

$$(A, E) = ((q, a), (b, p, k)) \tag{3.7.3}$$

which refers to a situation where an animal perceives an affordance A, and performs effectivity E. Thus it is obvious that Wells subscribes to the notion of effectivities in opposition to affordances. As well, "a set of instructions constitutes the 'machine table' for a Turing machine," so since an instruction represents an affordance and its associated effectivity, a set of them, "in Gibson's (1979/1986) terms, a machine table specifies a niche" [60]. Wells continues to explain, q is a member of finite set Q which enumerates the functional states of an animal (not necessarily state of mind), a is a member of finite set S of types of entity in the environment. He is also quite clear to reiterate just as Turing's states were, Q and S are finite, and the same goes for sets pertaining to effectivities. Formally, an effectivity E contains $p \in Q$, $b \in S$, and k has been modified from Turing's original work to include a set M of possible movements which might include such elements as 'sit,' 'stand,' or 'grasp.' The movements possible in original Turing Machines were only left, right, or no movement, but this was due to the restricted nature of the environment as a one dimensional array of tape. An animal in its environment has uncountably many possible actions, but the set M contains possible movements, and is necessarily finite.

Wells then runs through the Turing machine which will write the infinite list of natural numbers. In it he elucidates the process a Turing machine takes and its connection to affordances. Both affordances and Turing's machine require no internal processing, independent of the environment. Affordances are professed to be this way by Gibson, and a Turing machine only reads the tape, and reacts based on what is read, and the state it is in. As Wells states, "information is stored by acting on the environment, not by modifying the functional states of the machine," which is quite consistent with Gibson's writings, though this may need to be revised since animals may be capable of both changing and creating new functional states throughout their lifetime. The parallels between affordance and Turing machines are quite striking, as seen in the agent who acts based on a combination of its state and what it perceives about the environment. This could be a reference to either theory.

One functionality which Turing machines are not capable of is the creation or modification of the agent's own states. Secondary tapes may be permissable, to store states in the environment, but an animal I would argue after leaning a task does not store the entire process of say 'breaking a nut open with a rock' in the environment, but merely a trigger to such actions (i.e. a rock and nearby unopened nut). When a child learns to walk, this is a state that has never been encountered before, unless one counts the perception of another human walking, in which case we take this as the instance where the state is introduced. This state is added to the possible actions the child has already encountered. A Turing machine however is not given the capability to change its current states, so any behaviours which emerge from a Turing machine are a result of states initialized before the machine is turned on. One could argue that an animal may have all possible functional states 'built in' and they are simply 'turned on' when an experience causes them to be. This could also be argued against however, that it would mean all actions any human takes are in some way innate but inactive, and waiting to be activated but surely the ability to weld or fire a bow and arrow are not innate. Gibson would claim that even these skills are picked up via some tenant of direct perception.

Wells contrasted further the theories of affordances and Turing machines, by reminding the reader that Gibson saw the environment as prior to animals as the source of perceptual information, and then stating that the tape is the source of perceptual information for a Turing machine. "If the environment were changed, by specifying a two-dimensional tape, for example, the structure of internal states would also have to change"[60]. Thus, the states and the structure of the states, that we might talk about in a theory of affordances would not even resemble states, and structure of the states in a one-dimensional tape Turing machine.

A configuration consisting of a state and input, is said to be a fact of the environment and a fact of behaviour. The times and places a configuration was or will be written on the tape are facts of the environment, and the state coupled to the input are tied to an effectivity, or a behaviour. This is another way affordances seem to integrate smoothly with Turing machines, with more proof given in that sets of configurations constitute niches. The three components of the niche as given by Gibson, that it specifies the way of life of the animal, it contains a coherence across all affordances, and it suggests that different animals that share aspects of their life have affordances in common, are all seen in a configuration table. The table specifies how rather than where an animal functions referring to its way of life. All configurations in a table have the same animal component and some configurations may certainly be identical in different animals. Wells argues further however, that "a niche,...,is a habitat plus a set of affordances" referring to the tape or environment of an agent as the habitat. He suggests it 'insufficient' to call the set of configurations of a Turing machine a niche, because it neglects the Turing machine's dependance on the structure of the tape at initialization.

Wells overviewed ‘the status of internal states in abstract machine theory,’ with the conclusion that contra Gibson, “the notion of an internal state does not have to be understood in terms of memory traces, even though the history of an organism’s interaction with an environment has an impact on its structure.” The learning of an affordance, he claims, adds to the set of internal states such that a new behaviour is added to the learner’s possible behaviours, but the animal need not have a memory of the learning. An interesting proposition, since Turing machines do not have the capability to acquire new states, though if a Turing machine were created which was allowed to add internal states post-initialization, it could presumably be run on traditional computational hardware, and thus necessarily be possible to program as a TM with two internal states. To conclude his article, Wells noted the parallels between a theory of affordances, and the theory of computation, and confirmed his support for effectivities because of the way they integrate into Turing machines.

3.8 Steedman, M. (2002)

Formalizing affordance [49]

Mark Steedman at the University of Edinburgh, Scotland, saw that the notion of affordance lacked a formal expression, and so he proposed one possible formalism using Linear Dynamic Event Calculus (LDEC). First, the existential, and universal quantifiers are defined as follows.

$$n \geq 0 \;\Rightarrow\; [\alpha](y = F(n)) \tag{3.8.1}$$

$$n \geq 0 \;\Rightarrow\; \langle\alpha\rangle(y = F(n)) \tag{3.8.2}$$

The intended meaning of the first implication is that if initially n is greater or equal than 0, then after any possible execution of the program α the value of y will be equal to $F(n)$, where F is some function on integers. Notice that α can be a nondeterministic program offering multiple possible runs depending on internal choice. Furthermore, in the first equation nothing is being said about termination of the program α; it might not terminate at all. The second statement requires that given the same initial situation there is one possible run of α that terminates with $y = F(n)$.

The third equation, shown below, is a composition of programs such as α, named *sequence*, and is “an operation related to functional composition over events, viewed as functions from situations to situations” [49]. The main purpose of the composition is to join behaviour to elicit larger goals. The example given by Steedman generates a plan for getting outside, by joining *push door* and *go through door*, to get *push'*; *go-through'* [49].

(3.8.3) $$[\alpha][\beta]P \Rightarrow [\alpha;\beta]P$$

Next the author defines a linear logical implication and demonstrates its possible use in the simplified example of a door which may be open or shut.

(3.8.4) $$\text{shut}(x) \multimap [\text{push}(y,x)]\text{open}(x)$$

(3.8.5) $$\text{open}(x) \multimap [\text{push}(y,x)]\text{shut}(x)$$

(3.8.6) $$\text{in}(y) \multimap [\text{go-through}(y,x)]\text{out}(x)$$

(3.8.7) $$\text{out}(y) \multimap [\text{go-through}(y,x)]\text{in}(x)$$

Linear implication uses up the proposition on the left, so that it may no longer take part in any proofs, and adds the proposition on the right. Steedman then explains how the rules are interpreted, and further discusses the door example. The calculus so far defined is not new as it resembles certain versions of modal logic. If initially a situation can be modeled by: in(you)∧door(d)∧shut(d), then applying the above rules allows for the derivation of the following situations:

(3.8.8) $$[\text{push}(you,d)]\text{open}(d)$$

(3.8.9) $$[\text{push}(you,d)]\text{door}(d)$$

(3.8.10) $$[\text{push}(you,d)]\text{in}(you)$$

but not for the situation

(3.8.11) $$[\text{push}(you,d)]\text{shut}(d)$$

Preconditions, allowing the conception of these such situations and the planning of them, are discussed next. Two examples follow:

(3.8.12) $$\text{door}(x) \wedge open(x) \Rightarrow \text{possible}(\text{go-through}(y,x))$$

(3.8.13) $$\text{door}(x) \Rightarrow \text{possible}(\text{push}(y))$$

These examples are indeed as intuitive as they look. Another needed definition is the transitive property of the possibility relation.

(3.8.14) $$\text{possible}(\alpha) \wedge [\alpha]\text{possible}(\beta) \Rightarrow \text{possible}(\alpha;\beta)$$

As Steedman reports: "This says that in any situation in which it is possible to α, and in which actually doing α gets you to a situation where it is possible to do β, is a situation in which it is possible to do α *then* β.[49]" This 'event sequence composition' permits the stringing together of actions to create higher level behaviour. If an agent is in a world where in(you)$\wedge$door(d)$\wedge$shut(d) is given, (3.8.12) is regarded as a goal which would have the agent search for a suitable series of actions to yield the *out(you)* result. One such proof for the existence of such a plan, is the composition of *push* and *go-through* actions seen here.

$$\alpha = \text{push}(you, d); \text{go-through}(you, d) \tag{3.8.15}$$

Steedman suggests *backward-chaining* from the goal, on the consequents of rules, using transitivity, as a way to produce this proof. The result of performing the action-sequence α, changes two of the three given facts in the factual conjunction known about the world. It yields $out(you) \wedge door(d) \wedge open(d)$. The calculus is further developed in another paper. The author now turns to formalizing affordances using LDEC. Using a discussion of an experiment done by Köhler(1925), Steedman suggests that it be uncontroversial "...that affordances like egress are indexed in such animals by object-concepts like *door*, rather than by end-states like being *out*, and that planning proceeds *reactively* by forward chaining from what is the case rather than backward chaining from the goal. [49]" The following are derived from equations (3.8.4) through (3.8.7) with $\leadsto$ read as 'yields'.

$$\text{push}(y, x) \leadsto \left\{ \begin{smallmatrix} shut(x) \multimap open(x) \\ open(x) \multimap shut(x) \end{smallmatrix} \right\} \tag{3.8.16}$$

$$\text{go-through}(y, x) \leadsto \left\{ \begin{smallmatrix} in(y) \multimap out(y) \\ out(y) \multimap in(y) \end{smallmatrix} \right\} \tag{3.8.17}$$

The affordances of a door, or doors in general, are given through a simple equation Affordances(door) $= \left\{ \begin{smallmatrix} \text{push} \\ \text{go-through} \end{smallmatrix} \right\}$ representing a set of functions. Steedman then defines the affordance-based door-schema *door′* as a function mapping doors into (second-order) functions from their affordances (i.e. push, go through, etc.) to their results.

$$door' = \lambda x_{door} \cdot \lambda p_{\text{Affordances}(door)} \cdot px \tag{3.8.18}$$

$$door' = \lambda x_{door} \cdot \mathbf{T}x \tag{3.8.19}$$

The second equation is a result of a primitive combinator **T** or *type raising* which is defined in general by $\mathbf{T}x \equiv \lambda p \cdot px$, as the operation which turns "an object of a given type into a function over those functions that apply to objects of that type." One drawback to the method used by Steedman is that 'Affordances(x)' should produce a list of affordances that an agent has ever encountered with x, but according to Gibson's views, the available affordances are not stored in a list within the agent, but rather in the optic array, and processing to access each affordance should not be needed as it is in Steedman's formalism. Steedman does recognize that restricting the affordances to those already stumbled across will not allow for new affordances to be built. He admits that the affordance-based door-schema $door'$ would be much more useful generalized over a wider range of affordances, allowing for an agent to view a door as something far removed from normal use such as a raft. But human capability for this generalization is quite limited by perceptual processes and in the modes of interaction with objects as Gibson insisted. Combinatory Categorical Grammars were also examined but are quite complex and do not seem to conform to Gibson's notions.

3.9 Jones (2003)

What is an Affordance [42]

In what was essentially a literature review, Jones analyzed the evolution of Gibson's affordance concept. Beginning from Gibson's 1938 article on automobile driving, Jones claims that Gibson's account of the ratio between a minimum stopping zone, and the field of safe travel, is quite similar to the concept of an affordance ratio published by Warren[59]. after Gibson had already passed away. Jones also noted how Gibson's optical center of expansion, first written about in 1947, also evolved into a more complete theory of optic flow. It seems that many of Gibson's original works were of a much greater value than he or anyone else might have believed at the time of writing. Though many of his ideas were used to develop new theories and ideas, the original work also provides the academic community with a view of the development process, and the ability to apply some of his early ideas to new or modern problems. Some ideas, such as the postulation that meaning is not detachable from objects seen first in 1950, can still be used in considering new formalisms of the affordance concept. One disadvantage to Gibson's work, as observed by Jones, is that Gibson never fully "explicated what he meant by perceiving things with reference to an animal"[42]. Jones believed, as do I, that this lack of specification is what has led to debates between the location of the affordance, and the existence of some animal side duality to an affordance, among others. With reference to the lack of specifications early on, Jones asks about Gibson: "Was he continuing to formulate his thoughts and did not want to commit himself to something about which he was unclear? Or could he have been motivated by other factors?" To which he responds that Gibsons regard to the concept of affordances was still evolving, and if Gibson had been able to publish more, his thinking would have changed further. In particular, I would submit that Gibson's understanding of the concept was changing to accommodate newly published works he had taken an interest in. With this suggestion, it seems clear that had James Gibson been able to learn about modern relational algebra, or possibly the topics in situation theory (see later), he may well have suggested that an affordance can be modeled as a particular relation which stands between the animal and environment. He may also have saved us many years work in trying to correctly formalize the concept. Jones concluded the article citing the symposium he organized at the 2002 North American meeting of the International Society for Ecological Psychology, and referring readers to four articles in the special issue, which are devoted to affordances. A discussion of these articles follows.

3.10 Chemero, Klein, and Cordeiro (2003)

Events as Changes in the Layout of Affordances [5]

Three years after Chemero proposed this new definition of events, he published a commentary co-authored by two colleagues. In it, they report their findings from an experiment involving affordance and event perception. In [3], Chemero described an experiment in which an affordance is slowly introduced or removed, and the critical point at which the perceiver detects the affordance appearing or disappearing would be an event according to his re-conceived notion of events. Now, the authors describe their experiment in which participants were asked to report the instant their gap-crossing affordance vanished or was introduced. The participants' leg length, and stride length were also measured, and this itself led to a result. Based on correlations between measurements, perceived stepping ability seems more relevant than actual step length in detecting the maximum sized crossable gap. This would demonstrate that though an animal's perception of its actual ability is more important than its proportions. The authors take the results to support "the claim that events can be profitably understood as changes in the layout of affordances" [5]. The second result demonstrated by this experiment, is in support of using the term *event* to refer to these appearances or disappearances of affordances. Stoffregen[53] claimed that Chemero did not offer an argument for why "event" should refer to these changes, but here the authors argue that "conceiving of events as changes in the layout of affordances gives ecological psychologists the ability to do empirical work on event perception with a good conscience." Lastly, this leads the authors to refute another of Stoffregen's claims, stating that if affordances are evolutionarily valuable to perceive, the appearance or disappearance of this affordance would also be evolutionarily valuable to perceive. Anyone who is familiar with the ecological approach would agree that affordances are quite valuable to perceive, because this is the basis that Gibson created them on. An opportunity for action is certainly valuable, or no actions could ever be taken and actions are obviously necessary for continued survival. The appearance of one of these opportunities would be valuable for the same reason, as it signifies the presence of this opportunity, and the disappearance of an opportunity would keep the animal safe or alive. Indeed, in some ways we could also consider the disappearance of an affordance to be an affordance itself which could be said to afford safety.

3.11 Stoffregen (2003a)

Affordances are Enough: Reply to Chemero et al. [54]

Having read the article by Chemero, Klein, and Cordeiro before it was published, Stoffregen felt the need to restate his previous position. It had seemed to him that literature on affordances, and literature on events, "had evolved and were continuing to evolve more or less independently," and thus his comparison between them yielded them as different, distinct entities [52]. In the present paper, Stoffregen continues to reject the application of the term *event* solely to changes in the layout of the animal–environment system, but he does put forth an interesting discussion. It seems that both affordances and events have affordances. Stoffregen considered a volcanic eruption, in which the end points of the eruption have particular affordances, and the layout of the affordances might be changed by the eruption and consequential lava flow. The eruption here, has been shown to constitute an event (in Chemero's sense of the word), and yet it definitely contains dangerous affordances. Stoffregen also discusses how walking, can be regarded as an event (in Chemero's sense), when it changes the layout of affordances, but also as an affordance itself, allowing for exploration. The results of these examples seem to contradict Stoffregen's previous point about affordances and events being distinct. Walking in this case, is regarded as both an affordance, and an event. He concluded the paper by reviewing that affordances have a nested structure, and using this as an argument that:

> changes in the layout of affordances have no special status in the ontology of the ecological approach to perception and action and therefore do not constitute a distinct category of perceivables. For this reason, changes in the layout of affordances do not need a special name (such as *event*) [54].

Stoffregen and Chemero agreed up to this point in time, that events not pertaining to an animal, have no motivation for study under the ecological approach. They agreed that affordances, and changes in the layout of them are perceived. But the term 'event' continued to be a matter of dispute.

3.12 Chemero (2003)

An Outline of a Theory of Affordances [4]

In this outline of the theory of affordances, Anthony Chemero attempts to clarify the exact definition of an affordance. Rather than a property of the animal–environment system, he declared affordances to be *relational.* That is, the affordance itself is a two way reference pointer to both the animal and the environment, though which parts of each is still an open problem. Chemero brings up the radical empiricism of James, and uses the empiricist notion of perception to illustrate the foundation of affordances he is presenting. A radical empiricist would claim that since perception is an act which includes the thing perceived, if two animals perceive the same object, their minds each include the object, and thus the two minds overlap. The name given to this is the *two minds problem.* This would hold true for the perception of an affordance resting solely in the environment. Chemero accounted for this problem using the empiricist notion that everything we perceive is real, including relations between things. If affordances are indeed relations between an animal and an environment, they are real, may be perceived by each animal, and may overlap without any minds themselves overlapping.

The basic logical structure of the affordance is given by Chemero as follows:

(3.12.1) Affords-ϕ(Environment, Organism) where ϕ is a behaviour

and is refined by discussing what it is about the environment and organism that is referred to. On the organism side of the relation, Chemero acknowledged the many experimenters who identified ratios between body scale and the environment. He retaliates however by implying that the experimenters have not been studying affordances as they believed. Based on research by Cesari, Formenti and Olivato (2003), body scale is, for Chemero, a stand in for the ability of the animal, and none of the experimenters measured ability, but only rather dimensions. The elderly for example, cannot necessarily climb stairs that satisfy the 0.88 ecological invariant found by Warren[59] because of the constrained 'ability' to lift the leg. Chemero also takes issue with the word properties in terms of the environment. The environment contains 'features', not properties, so the new structure is as follows:

(3.12.2) Affords-ϕ(feature, ability) where ϕ is a behaviour

Chemero then defines the ability side of the affordance, to be functional properties, rather than dispositions in Turvey's sense. If these abilities were dispositions, they would necessarily become actualized in prime circumstances, which is to say that any time an animal might possibly climb a step, it does. This is obviously incorrect, so in regarding abilities as functional properties Chemero allowed for the organism to choose its actions, and since functional properties are given to depend on evolutionary history, affordances as well are tied to evolution. Chemero claims though, that his definition of affordances "does not assume that affordances are resources that exert selection pressure." He also claims that if they did, affordance could not imply niches or sets of affordances as Gibson defined them, though I cannot find any reason why not.

Chemero places the animal in a 'perceives' relation to the affordance as follows: Perceives [animal, affords-ϕ (feature, ability)]. The feature and ability relata however, are left out when defining what the animal itself will perceive. Humans, and perhaps only humans, he writes, have the capability to perceive things about their own abilities and features of their environment without perceiving the affordance alone. Most animals have a simplified 'perceives' relation: Perceives [animal, affords-ϕ].

Three topics remain to be discussed, niches, events, and the existence of affordances in the absence of animals. Niches are according to Gibson, the set of affordances for a particular animal. Chemero's formal definition follows: If S is the set of all possible situations, each ability a_i specifies a subset of S, s_i, where the ability can be enacted. If an animal has abilities $a_i, \ldots, a_n$, that organism's niche will be the union of each subset of S for each ability. Based on prior papers ([3],[5]), Chemero takes an event to be a change in the layout of affordances. Most events occur as changes in the environment which modify the niche for an animal by adding or removing an affordance, however breaking a leg will also eliminate the 'climbable' affordance from a niche as well. Lastly, Chemero states that "affordances do not disappear when there is no local animal to perceive and take advantage of them," but also that "affordances do depend on the existence of some animal that could perceive them"[4]. Though it is tempting to disagree, considering that an affordance is an opportunity for action, and there is no opportunity if there is no animal, a simply reference to direct perception forces an agreement with Chemero. According to direct perception, no processing is performed on the information we acquire from the world. Gibson was clear that affordances are directly perceived, and thus must exist prior to being perceived so that they may be picked up by an animal.

3.13 Stoffregen (2003b)

Affordances as Properties of the Animal–Environment System [55]

In this article Stoffregen admits his lack of focus on the formal definition of affordances, because he had been seeking to develop and clarify the concept of *affordance* by contrasting it with the concept *event*. He critiqued Turvey's formal definition of affordance mainly taking issue with attributing an affordance to be a property of some environment for multiple reasons. One reason was that as a property of the environment, it must have some complementary side in the realm of the animal. Along the same lines, an affordance should be a property of the animal–environment system as a whole, the way Gibson described it as equally a part of the animal as part of the environment. Stoffregen takes issue as well with the notion of an affordance as a disposition. Though an affordance is something potential, able to be actualized as a disposition is, an affordance is not necessarily actualized in suitable conditions like a disposition. An animal, or agent, may possibly take a certain perceived action, but that does not force it to do so. Stoffregen cited Turvey's recognition of the problem, and explained that the juxtaposition function j was defined to filter affordance p and effectivity q from an array $m \times n$ of dispositions, but Turvey neglects to elaborate on any method of filtering or what precisely j does, so this renders his definition incomplete for any real case [55]. One bandage solution Turvey might respond with here, might be that the 'suitable' circumstances he listed as a characteristic of dispositional properties and thus affordances, might in fact be those conditions under which the action is possible, including conditions where the action leads to a goal compliant with the intentionality of the animal, and possibly even the agent initializing the action. His failure to specify the juxtaposition function's capability or methods of filtering cannot be reconciled as easily.

Another issue Stoffregen takes with Turvey's definition, is that if affordances and effectivities are part of the environment and animal respectively, knowledge of the complementarity of the two must be obtained indirectly by conjoining the directly (but separately) obtained information about affordances and effectivities. Requiring internal processing to derive an affordance is not compliant with the theory of direct perception, and affordances as defined by Gibson were directly perceived so this juxtaposition function it seems, will not fit with a theory of affordances. If Turvey's previous definition of affordance was labeled simply a property of the environment, the effectivity was labeled a property of the animal, and the term *affordance* was re-assigned to relate these two properties, this objection might fade away. Stoffregen investigates affordances as properties of the animal–environment system and reworks Turvey's definition to accommodate just this.

Stoffregen's new definition of affordance begins in the same way as Turvey's by referring to $W_{pq} = (X_p, Z_q)$ as, by example, a person-climbing stairs system, but then p is simply a property of X rather than an affordance of X, and q merely a property of Z rather than an effectivity. Instead of a juxtaposition function to join p and q, p/q defines a higher order property h (of the animal–environment system). Then h is an affordance of the system W_{pq} if and only if W_{pq} possesses h, and Z nor X alone possesses h. Now, Stoffregen states "affordances are properties of the animal–environment system, and they exist only at the level of the animal–environment system" [55]. Also of significance, the new definition does not refer to or include behaviour (i.e. the actualization of affordances) because affordances are only given as opportunities for action, not for an implemented action, though I would regard this as a semantical distinction. Stoffregen also stated that h is a fact rather than a potential which I believe to be a matter of semantics as well. The emergent property h, should be regarded as a fact of potentiality since Stoffregen even wrote that "If h is within a certain range of values, then stair climbing is possible; that is it may happen in the future." Clearly this means that h (within a set of values) has the potential to be actualized, and if h falls outside those values, it is not an affordances, and thus nothing discussed here applies.

It is worth noting that Stoffregen does not constrain affordances to refer to physical properties of an animal. "Any property of an animal can bear a relation to some property of the environment that gives rise to an affordance, including biomechanical properties such as leg length, and other types of properties such as strength and flexibility, skills, beliefs, and emotional states"[55]. All of these are important, though the largest concern is finding a way to formally integrate non-numerical properties of animals such as emotional states into the affordance definition. Behaviour is defined as well in a similar but definitely different manner than affordances. It "happens at the conjunction of complementary affordances and intentions or goals." Let $W_{pq} = m(h, i)$(e.g. an animal–environment system) be composed of different affordances h, and intentions i, were both are properties of the animal–environment system. A given behaviour b (e.g. climbing stairs) will occur if and only if (and when) an affordance and its complementary intention co-occur at the same point in the space–time continuum, where m is a psychological choice function [55]. He later stipulates that he cannot at present define m.

Stoffregen next states that affordances may be specified and detected prospectively, but also makes the claim that the existing set of affordances in a given animal–environment system determines from the unlimited set of intentions, which are satisfiable at any given place and time. This claim that affordances select intentions seems inaccurate because an animal is capable of an intention which is not possible based on its naivety and also capable of intentions which reach far beyond the referenced place and time, possibly infinitely. An example might be an intention to one day in the future climb a certain set of stairs.

The implications which his proposed definition might have on the ecological approach to perception and action are examined as well. According to the definition, affordances might be perceived without prior perception of required properties of the animal or environment. Accordingly, if properties of the animal–environment system may be perceived directly, there may be no motivation for perception of properties of the animal or environment separately. Stoffregen argues that under the ecological approach to perception and action affordances are perceivable, but nothing else is. He argues that there is no reason based on the ecological approach to perception and action, for perception of anything which does not have a consequence for action. Events are thus not perceivable according to Stoffregen. He also regards effectivities in a similar manner as Warren[59], that they are lower level constituents of the higher order relations that constitute affordances. Effectivities in his definition are properties of the animal as they are in Turvey's, but to Stoffregen they are subordinate to affordances.

Stoffregen revised Turvey's formal definition of affordances, and explored the ramifications for the field of ecological psychology, but he concluded his article with an important issue for the ecological approach. His conclusion reminds the reader that the detection and specification of what precisely and formally constitutes an affordance within the animal–environment system is still an open problem.

3.14 Kirlik (2004)

On Stoffregen's Definition of Affordances [43]

The following year, Alex Kirlik produced a commentary, "On Stoffregen's Definition of Affordances," directed at revealing a crucial flaw in Stoffregen's definition. Kirlik could not find any objection to Stoffregen's[55] definition by the presentation of an affordance which the definition cannot handle. He instead, examined the other direction of attack by claiming that the lack of constraints on properties p and q in the definition allows any relation of properties within an animal–environment system to be an affordance. Trouble will arise when we attempt to let the informal definition, "opportunity for action" guide us to select properties which will necessarily give way to an affordance.

According to the definition, h should be an "opportunity for action" but no restrictions are placed on the selection of p and q except that they be properties of an animal and environment. Kirlik thus, suggested an example where p is the property "was shipped to the United States from a British cathedral in 1894" and q is the property "had diphtheria as a baby." What-ever the emergent property $p/q = h$ turned out to be (e.g. "a person who had diphtheria as a baby is now climbing stairs that were shipped to the United States from a British cathedral in 1894") Kirlik did not believe it to be "*constitutive*" of the existence of an affordance. The properties p and q here might give way to the precise example relation, but the opportunity for action 'climbability' certainly does not depend on when stairs were shipped, or having had diphtheria. A stronger example Kirlik used, assigned property p (e.g., of Cleveland Browns Stadium) to be " has a capacity of 70,000 fans for football" and assigned q to be the property of a person who "is not sitting in San Diego." Because Stoffregen's definition did not even require p and q to be in spatiotemporal contact, this can be considered a legitimate system, and the higher level property $p/q = h$ of the system can be considered an affordance. Kirlik voiced his suspicion that Turvey[56] created the 'juxtaposition' operator to deal with the problems above. Turvey did not however, actually deal with the problems, but instead, push them into the realm of the juxtaposition function which he did not define.

I now propose some obvious restrictions based on the examples Kirlik illustrated. The juxtaposition function in Turvey's definition, or the higher order property h in Stoffregen's, must maintain spatiotemporal contact between the component properties. If the "things" never come into proximity to each other, the referred to animal can't act in the referred to environment. The properties also, must be simple enough to be directly acquired, so a capacity of 70,000 or being shipped in 1894 do not qualify as proper component properties in an affordance. The authors must remember that affordances, as Gibson defined them, are directly picked up via the optical array.

3.15 Şahin et al. (2007)

To Afford or Not to Afford: A New Formalization of Affordances Toward Affordance-Based Robot Control [47]

From the Middle East Technical University in Ankara, Turkey, Erol Şahin, Maya Çakmak, Mehret R Doğar, Emre Uğur, and Göktürk Üčoluk, co-wrote an article which examined the concept of affordances, as well as looked at how the concept of affordances could change the way we design the control of primarily autonomous robots. First the authors reviewed the developments in the concept starting from Gibson, and with that, provided citations emphasizing Gibson's firm belief that an affordance is not simply a property of the environment, but rather that it "...points both ways, to the environment and to the observer" [29]. One criticism provided was that of experiments which focused mainly on the perception aspect of affordances. Other cognitive processes, such as learning, high level reasoning, and inference mechanisms, they report, were not investigated or discussed, so they provide a cursory attempt at bridging the gap and an attempt to relate affordances to high-level perception. Taking an argument from E.J. Gibson, the authors seem to agree that learning is not " 'enriching the input,' but discovering the critical perceptual information in that input." A process termed *differentiation.* In past discussion regarding the affordance concept, only the pickup of basic information such as 'sit-able' is examined, while the pickup of higher level information such as *my* chair is not considered. The expression of possession, or other higher level attributes, is not a capability of the basic affordances.

With a look at neurophysiology, the authors discuss some researchers who had tried to elicit which neurons fire during which activities or actions performed by subjects. The findings that the same neurons fire while performing a particular action as when we observe someone performing that action, provided results which support the idea that an affordance can be acquired by observing it being performed. The authors continued to draw parallels between Gibson's way of thinking and other areas of research. The field of reactive or "behaviour-based robotics was motivated by criticism of the then dominant robotic architectures, which favored modeling and inference" [47]. This is in line with Gibson's creation of the affordance concept in criticism of the then dominant theory of perception which also involved modeling and inference. One proponent of the behaviour-based approach, Rodney Brooks, claimed that "the world is its own best model," and used this notion in designing several robots. They can be said to simulate affordances, in the sense that they do not represent the process going on internally, but rather react based on perceptual input alone.

This strand of robotics also supports my belief that for a robot to operate autonomously it must be allowed, in real time, to learn how to control its actuators, how to navigate, and what its purpose is. These capabilities, in my opinion, cannot be programmed or built into the robot, but rather the architectures (representing pickup and control of affordances) must be available to allow the robot to construct this control for, and of, itself. There is something to be said here, about the human life cycle in which, the first several years are much less productive than the middle or later years. The authors also mention two important aspects of robot control using affordances. A robot must be able to both learn the consequences of certain behaviour and also learn to pickup the invariant properties of the environment which afford certain behaviour. The consequences can be learned through repetition, and gaining an understanding of why results occur from given behaviour. The pickup of invariant properties, opposingly, is much more difficult to distinguish. Indeed, it is only through a very deep understanding of Gibson's life's work that we may begin to formalize the ecological theory of perception, into a usable model for a so called behaviour-based autonomous robot. Şahin and his co-authors also reviewed the formalisms published by Turvey, Stoffregen, and Chemero, as well as Steedman. Each of the first three authors follow one thread of ecological psychology literature while Steedman's formalization skips the issues of invariance, and instead "focuses on developing a representation where object schemas are defined in relation to the events and actions they are involved in" [47]. None of the reviewed formalisms was sufficient to develop an affordance-based robot control architecture, so the authors begin to develop their own account of how the affordance concept can be exploited.

The reason given for previous confusion regarding the placement of affordance, is the existence of not one, but three perspectives of affordances. Explicitly differentiating between the agent, environmental, and observer perspectives of affordances, is the first step Şahin et al. take. The agent perspective includes its possibilities for interaction with the environment. This is the most essential perspective to the development of autonomous robot control so it is examined further below. The environmental "...perspective attaches affordances over the environment as extended properties that can be perceivable by the agents" [47]. In some regard, the authors suggest that the environment almost literally, screams out the affordances it possesses, though this would make the acquisition of affordances much easier to a robot as it would only have to ask or sample the environment to determine what actions it affords. It may be somewhat difficult in designing this system, as each object in the environment must know what it affords any agent, prior to even coming into contact with them. The environmental perspective, in my opinion, while valuable, should be regarded as implicitly built up through contact with agents. It could be argued that a ball, does not 'say' anything until there is some perceiving agent it may communicate its affordances to, and even then, only affordances already learned by the agent are truly available. The observer perspective, however, I believe to be much more valuable, just as the authors acknowledge the importance Gibson placed on this perspective. Gibson stated that though a child begins by perceiving her own affordances, “she must learn to perceive the affordances of things for other observers as well” [28]. The observer perspective is regarded as any situation where an agent may perceive a second agent (possibly of the same species) in its environment. This is possibly one way in which affordances are acquired. If the second agent is observed during the execution of an affordance by the first, one might think that the first agent would acquire the possibility to attempt this affordance, even if its execution is not yet possible.

The authors then attempted to extend the basic affordance relation (environment, agent) by further embedding the affordance in larger relations. Instead of the environment and agent, each must be replaced by the relevant relata. To ensure clear use of terms, *entity* is used to denote environmental relata as perceived by the agent, *behaviour* is used to denote “...the agent’s embodiment that generates the perception-action loop that can realize the affordance,” [47] and *effect* is used to represent the outcome of the interaction between the agent and the environment. The formalism progressed in the paper first to include this effect, taking the form (effect, (entity, behaviour)), though the authors claim that the relation at this point resides in the interacting agent, because “all three components are assumed to be sensed through proprioception of the agent” [47]. Şahin at al. defend the explicit use of *effect*, by observing that even in Gibson’s writings, if a lift-ability affordance is considered, the expected effect *lifted* is implied. The addition also eases the complexity of generating classes of affordances, if the classes are based on the effects they obtain. Equivalence is discussed next with respect to a hypothetical humanoid robot trying to discover affordances in its operating environment. Given two affordance relation instances, one including a black can as the entity, and one including a blue can, the two instances can be combined to yield the relation instance (lifted, ($\left\{ {blue{-}can \atop black{-}can} \right\}$, lift-with-right-hand)). This relation can be compacted using perceptual invariants to determine that any can, regardless of colour, can be a part of this relation instance. Thus allowing for the robot to extend instances, to classes of instances based on the join of two instances and the extraction of their invariant parts. The entity equivalence class would somehow provide information as to what parts are invariant, and what parts may change. In the authors example, knowledge of (lifted, ($\langle$*-can $\rangle$, lift-with-right-hand)) for a robot, refers to the possibility that the robot may lift any coloured can with its right hand, and that can will become lifted. If, in the example, the robot’s right hand is engaged (possibly already lifted something), it might use the rule that two affordances with the same effect, on the same entities, contain equivalent behaviours. That is, lift-with-left-hand might be substituted because it achieves the same effect, and is thus an equivalent behaviour. The same sort of equivalence is also defined over affordances and effects. If two entity-behaviour pairs produce the same effect, they are equivalent. Similarly, if an agent wishes to join multiple relation instances it must be able to discern if the effects generated are equivalent, so equivalence classes of four different perspectives must be considered. At this point the authors redefine their formalism of affordances as follows:

$$(\langle \text{effect} \rangle, \langle (\text{entity,behaviour}) \rangle) \tag{3.15.1}$$

The crucial note at this point is that this relation is between equivalence classes. The authors continue to complicate the definition by adding the capability of modeling the affordances from an observer perspective. They add an agent equivalence class, in direct relation to the entity-behaviour tuple though when dealing with only one agent, it is omitted. A version of the formalism is also given according to a situation where an animal engages in an affordance relation with only another animal, and not the environment, so the 'entity' part is dropped.

In a discussion of the implications of the formalism, the authors admit that a crucial part of the formalism which governs how relation instances can be generated, and how they may be merged, has yet to be studied. They also admit that two control systems would be necessary, the second system would complement the first by learning exceptions to general relations acquired by the first. Planning is another issue closely related to affordances. The authors propose that some notion of planning would be implicit in the affordance relation as the effect of each affordance should be known. If then, an agent were to desire a certain effect, the corresponding affordance may be accessed. Planning is also implicit in actualizing many affordances, seen during the initial parts of the agent's action. To actualize the affordance 'grasp-able' an agent may have to first traverse to the item to be grasped, and this traversal can be seen as implicit in the grasping affordance. If traversal to the object were impossible, the grasping affordance would not be possible or even necessarily perceived.

Traversal is a more basic affordance, and is examined further by the authors using a trained virtual robot, as well as a real robot. The robot contained seven pre-coded actions ranging from moving sharply left to sharply right, and a range scanner granting ability to judge distance. The robot was then able to generate relation instances with entity being the raw perceptual vector, behaviour being in the range 1-7 of movement behaviour, and effect being a 0 or 1 indicating failure or success. An exploration phase allowed the robot time to populate a list of relation instances, then it was trained during which time it learned entity equivalence classes, and finally it underwent testing. The authors revealed that entity equivalence classes were in fact discovered by the trained classifiers after the second half of of the training phase. The robot also was able to learn through which sized apertures traversability was afforded. The authors here, take a moment to suggest that though traversability is clearly related to the robots width, this does not mean that this information is explicitly used by the agent. The robot possesses no knowledge of its size so it cannot possibly use that information explicitly, yet it could come up with this knowledge by measuring the smallest aperture through which it can traverse. This idea is in opposition to those who believe there is a ratio or higher order value which is directly responsible for determining if an affordance is able to be executed. These values are undoubtedly satisfied each time an affordance is performed, but the agents do not themselves have any explicit awareness of the ratios, so obviously they are not the crucial determinant in deciding possible affordances. Lastly, Şahin et al. report that their on-going work will allow for continuous range of movements, instead of a discrete set, and the effects will not be grouped as simply a success or failure. Though they did circumvent Gibson's idea that affordances are properties of the animal–environment system and are not internalized, the idea that the affordance can be taken from this system and projected into the agent, environment, or observer, is quite an interesting one. Examining methods behind this projection may prove quite useful.

3.16 Chemero & Turvey (2007)

Gibsonian Affordances for Roboticists [6]

In 2007, after the debate regarding the formalities of affordances had relaxed for a few years and at the same time as Sahin et al. [47] published, Chemero and Turvey collaborated to write "Gibsonian Affordances for Roboticists." The principal objective was to use hypersets to compare Gibsonian([4],[56]), and representationalist ([47],[58]) understandings of the notion 'affordance'. The most important observation made of the hyperset graphs presented, was that the graphs representing Gibsonian understandings of affordances, proposed by Turvey [56] and Chemero [4], contained loops thus creating non-wellfounded hypersets. The representationalist definition, however, as proposed by Vera and Simon [58] and Sahin et al. [47], do not contain loops, and are therefore well-founded. "This means that the Gibsonian, but not the representationalist, definitions understand animal-environment systems as complex systems" [6]. The Gibsonian definition can also be said to take the animal and environment as a unified system, where each is unintelligible separately, unlike the representationalist definition. The authors take two consequences from these conclusions. First they make dynamical systems theory the most natural explanation of action. If the animal-environment system is unified, the authors state it all by necessitates explaining behaviour using non-linearly coupled dynamical equations. Secondly, the conclusions grant credit to the claim that perception is direct, because as one all encompassing system, there is no need for the animal to represent the environment. The representationalist view, assumes an animal must gather data about an entity external to it, combine that mentally represented data with mentally represented actions, select an action, and finally execute that action externally. The authors claim that taking the animal and environment as separate, forces the two main tenets of Gibsonian ecological psychology to be incompatible. "In spite of Sahin et al.'s claims to the contrary, if affordances are mental representations, perception of them cannot be direct" [6].

Another noteworthy difference the authors point out between the Gibsonian and representationalist perspectives, is that the Gibsonians do not subscribe to the notion that cognition is computation. Only representationalists are said to be consistent with computationalism. Many mainstream cognitive scientists reject Gibson's views, solely on the basis that he and his followers reject the computationalist stance. They then attempt to develop a notion of affordances which is computation-friendly, but Gibson and those that carry on his beliefs, purposely reject computationalism along with representationalism.

Chemero and Turvey also describe two consequences which cannot be derived from Gibsonian views on affordances. The first is that the complexity of the animal-environment system does not take it out of the realm of computational modeling and simulation. And the second is that the incompatibility of Gibsonian affordances and computationalism does not make Gibsonian affordances irrelevant to roboticists. Complex systems are in fact computable, though it may seem unreasonable to expect a computer to be capable of dealing with objects defined in terms of one another. As the authors state, "there is no necessary connection between circularity and non-computability" and "hyperset membership has been, shown to be Turing-computable in polynomial time" [6]. Google (though it applies to search engines in general) is cited as an application where searches must index a web of pages which are undoubtedly circularly linked. There are many instances of links on one page which lead to another, and links on that page leading back, yet Google is still able to navigate circularly linked web pages without getting caught in (infinite) loops. This illustrates that defining affordances circularly does not imply their inability to be computed or simulated. It may be claimed that to be useful to roboticists who use micro-processors, affordances must be taken as representations. As a Gibsonian roboticist, one would have to commit to constructing robot-environment systems, rather than simply just robots. That each member of the system must be built for one other, requires a new way of thinking, but does not preclude it from being done. To support this claim the authors provide citations ([12],[13]) to several papers (which I review below) which begin a path toward the creation of a robot-environment system based on the tenets of Gibson's ecological psychology.

Chapter 4

Ecological Robotics

4.1 Duchon & Warren (1994)

Robot Navigation from a Gibsonian Viewpoint [12]

Though James J. Gibson's work was dedicated to understanding human and animal cognition, there is nothing Gibson has written which cannot be applied to robots as well. Andrew Duchon and William Warren took robot navigation, and programmed Gibson's ideas regarding optic flow into it. Citing Rodney Brooks [2] as the first to design a behaviour-based robot, the authors reject the "sense-model-plan-act" notion of robots as Brooks did, and review ecological psychology principles that might help build a robot. The optic array as Gibson[25] referred to it, refers to all rays which converge to the point of observation. As the point of observation moves, the optic flow will also shift. As an animal navigates its environment, the optical array consists of objects moving toward the animal from the focus of expansion, and objects moving away from the focus of expansion toward the exterior regions of the optic field. These objects moving toward the outer regions of the optic field will-if navigation continues toward the focus of expansion-seem to move from in front, to beside, to behind, the animal. Objects which do not move away from the focus of expansion when navigation is (at constant speed) directed towards that focus, are said to have a 'time to contact' defined by the equation $\tau_g(t) = r(t)/v(t)$ with r representing distance to the retina, and v representing the velocity. A Law of Ecological Optics is defined by the authors as

$$\text{flow} = f(\text{Force}) \qquad (4.1.1)$$

which means that the flow is a function of the forces acting on the observer. The inverse of this law is called the Law of Specification, whereby the force acting on an observer is specified by the optic flow, and is defined as follows:

$$\text{Force} = g(\text{flow}). \tag{4.1.2}$$

The law of specification however, does not account for displacement of the environment or external forces so a Law of Control is defined by equating the difference in intended force with a function on the difference in optic flow. To remain in a constant position in changing environment, an agent provide the forces necessary to cancel out the environmental changes. A fly in a rotating drum, must provide the (unequal) forces necessary if it wishes to hover above a constant spot inside the drum. According to research by Warren(1988) on this subject, we know that a fly can produce these forces needed to hover above a moving spot without any training, so the ability to move to satisfy optical flow equations seems to be innate.

Since these control laws are satisfied during an animal's natural navigation, it seemed of value to Duchon and Warren to make use of them. All that is needed for a robot to register the optic array and optic flow is a CCD camera and a computer. So the authors constructed such a robot, and programmed two possible navigation strategies. This was the first time that the fields of ecological psychology and mobile robotics crossed over into each other. The first control strategy, called the *Balance Strategy*, "acts to equate the rate of optic in the left and right halves of the visual field, as observed in some insects" [12]. It is defined with regard to ΔR as the amount of relative rotation from the current heading in degrees, $\dot{\mathrm{w}}$ the magnitude of optic flow on that side of the visual field as, and a constant k.

$$\Delta R = k(|\dot{\mathrm{w}}_L| - |\dot{\mathrm{w}}_R|) \tag{4.1.3}$$

One situation the authors note where this strategy would be useful is moving down a hallway. If an agent moved closer to one side of the hallway, that side's flow would increase, and it would move to equate flow to both sides, by moving away from the closer wall. This balance strategy, however, would not allow for an agent to navigate through a cluttered environment. If an agent with this strategy were placed in such a situation, it would likely have difficulty trying to balance the many objects moving around it.

The alternate strategy employed by Duchon and Warren, is called the *Avoid-Closest Strategy.* Here, the agent turns away from the area of the optic array with the lowest time to contact. The Avoid-Closest control law is defined as follows:

$$\Delta R = k\left(\frac{1}{\tau_{\min}} \times \frac{1}{\mathrm{pos}(\tau_{\min})}\right) \tag{4.1.4}$$

where $\tau_{\min}$ represents the lowest time to contact in the visual field, and pos() specifies the visual angle relative to the current heading. The minimum time to contact is found from the average time to contacts given for each of the columns in the optic array. The amount of rotation is best described by this quotation: "The shorter the time to contact, and the closer the column is to the heading direction, the more the agent will turn away." A complementary strategy called *Seeking-Farthest* was described, but thought to have unfortunate consequences when the visually farthest point is adjacent in the scene to the visually closest point. In addition to the two control strategies, other actions were programmed into the robot. It was found that if the average illumination fell below a certain threshold, the robot could not detect its environment and perform accordingly. So to fix this little bug, the authors added an emergency reflex of stopping, turning 90^o and then resuming the main control strategy, whenever the optic array became too dark. The other practical difficulty was in regard to the balance strategy. When directed straight toward a uniform wall, both sides of the optic array are balanced, so the agent had no action to correct. This would unfortunately lead the robot to collide with the wall, so one additional emergency procedure was included. If the time to contact was below a threshold of, for example 4 frames, the robot stopped, and turned 90^o before resuming the main strategy. The last practicality issue that was run into was attributed to the robot having no sense of its own size. The camera's visual field was 60^o wide yet the base was 12 inches wide, so at times, the robot anticipated safe passage, but only the camera, and not the base itself were able to pass through. This issue was not compensated for.

The robot's specifications included three computers: one for sampling the NTSC standard video information down to a 128x32 pixel array, one to run Camus'(1994) optic flow algorithm allowing information to be extracted from image sequences in real time, and the last to run the control strategy, and physically control the base. The results obtained from this experiment were quite promising, but also demonstrated that other movements, or strategies should be employed so that certain situations do not require emergency procedures. Both strategies, it is reported, were effective but susceptible to dark regions where they register no flow. In the balance strategy, if one side of the array contains no flow, no possible behaviour can make the other side equal. In the avoid closest strategy, it becomes the relatively closest point to be avoided and this will always be on one side. The effectiveness of the experiment is given in figures illustrating the robot's paths. Several times a human hand suddenly appeared in the robots optic array, and the robot each time avoided the hand. The authors conclude with their intention to provide additions to the control laws to allow a robot to perform actions such as pursuit, approach, docking, and escape, using optic flow alone. The proposed thought was to include constants in the balancing equation to allow a robot to follow walls, or to adapt to new situations in which it might not be able to achieve optic flow balance.

Within the article Duchon and Warren mention the Seek-Farthest strategy, but also disregard it. While it may be true that adjacent near and far points is a cause for concern to a robot, if some sort of intention is integrated into the robot, it could balance the optic flow sufficiently as not to collide with anything, but while doing so, also seek out the farthest point. If the robot had 'modes' as some way of representing intention, each behaviour could be associated with a mode. In escape or exploration mode, seeking the farthest point might be highly prioritized, while docking mode would entail a slow steady approach to an object. Also the dark regions in the field of view could be programmed as obstacles, or as unknown regions. This 'meaning' can be dealt with in its own way. As the robot is given the capability of recognizing what certain areas 'afford', it can act accordingly. Unknown regions for instance could be explored, or avoided. Attaching a light to the robot or using a less cluttered environment, might also allow the experimenters to focus on the issue of optic flow based control strategies instead of hard coding emergency behaviours which distract from the main task of the experiment which was to test the Gibsonian approach to visually guided navigation.

4.2 Duchon, Warren, and Pack-Kaelbling (1995)

Ecological Robotics: Controlling Behavior with Optic Flow [13]

The next year, 1995, Andrew Duchon and William Warren teamed up with Leslie Pack-Kaelbling to produce a paper which more closely examined the intersection between the fields of behaviour-based robotics and ecological psychology, specifically control laws based on optic flow. First they review what optic flow is, and explain a few equations. If an observer is moving toward its focus of expansion (FOE) which has distance r from the observer, at a constant velocity v, time-to-contact or more strictly, time-to-passage, is "specified by $\tau_g = r/v$ where τ_g is the 'optic variable' *tau*-global"[13]. Immediacy is defined as the inverse, $\eta = 1/\tau_g$.

Laws of Control are equations which remain satisfied throughout any movement which relies on optic flow. It is said that animals utilize the information in the optic array by acting to achieve a desired type of optic flow. The changes in the internal forces of the animal such as forces caused by wings, legs, etc. is equal to a function of the change in the optic flow.

$$\Delta F_{\text{internal}} = g(\Delta\text{flow}) \tag{4.2.1}$$

This equation is inspired by research on flies which the authors report utilize a balance strategy particularly when placed in a rotating drum where they will produce the differential thrust needed to track one side of the drum as it rotates. This however, is one of many equations that the fly will follow. The control law being satisfied at any one time will depend on the goal, intention, or 'mode' of the fly. Only one set of affordances will be utilized or perhaps even perceived by the fly at one time, and that set is dependant on the fly's 'action mode'. The authors example is that a foraging fly will seek only affordances of nourishment and landing, and anything else would afford avoidance. A fully nourished fly in contrast, will seek somewhere that affords safe landing, while a flower may even afford avoidance.

The authors admit that their previous work [12] may not be sufficient for survival of an agent. The Avoid-Closest strategy while advantageous as a component, by itself will only cause an agent to wander around, not hitting things. This hardly allows for goal states unless there is a task to perform while not hitting things. One task particularly related to survival of an agent is the game of tag. Either as a predator attacking its prey such as a lion attacking a herd, or some prey escaping the predator, such as a zebra which avoids being taken down, the game of tag is one which is closely related to animal behaviour and evolution. The game of tag then, was used in the design of new optical flow based control strategies for a robot. This robot also contained four 'action modes' which acted somewhat like goals or intentions. The 'watching' mode was used as a ground state during which the robot was sensitive to, while waiting for, certain optical flow patterns to be detected. If the target is detected, the robot centers it in its field of view, and will track it until the trial is over. If the target approaches the robot (given by $\eta > m_e$), it will flee until it escapes or is caught. If the target moves away from the robot (given by $\eta < -m_e$) the agent will chase it. The chasing behaviour consists of 'shadowing' where it attempts to match speed with the prey, and then, when optical flow is balanced or constant, 'docking' where a controlled approach is made until distance falls below a threshold indicating the prey being caught. If the target appears, and subsequently recedes out of the field of view, it is considered an escape. After any of the modes is completed (i.e. robot is caught, escapes, catches the target, or fails to) the process stops, and begins again.

Fixating on the target was a task that was active whenever the target was detected. Obtaining a clear view and centering the target was the first task of fixation upon detection. Equations are given to allow the robot to rotate its left and right sides based on the optical position of the target relative to the center of the field of view. A more complex agent, the authors state, could take into account the optical velocity of the target allowing it to predict where the target will be after its own rotation is made.

The authors quote James Gibson's work which describes how the expansion of a textured contour in the optic array may trigger a flight or attempted escape response of a prey animal, and also how the contraction indicates something moving away and possibly safety from that thing. They use the expansion as it is, but suggest also that contraction might trigger pursuit by a predator. These two actions form the basis of the robot upon which this paper is based. In each of these higher level actions than fixation, only the sum of the change in force needed to be considered because the rotational acceleration (the difference in left/right forces) is taken care of by the control laws for fixation. Escape is done by retreating backwards from the incoming target until either the agent is captured by the target coming within a certain distance, or the agent escapes by getting a certain distance away from the target. A chase however, consists of two parts, each of which may be functional components to other larger behaviour than chasing. The first action is shadowing, which is as the name suggests, keeping a constant distance behind the prey target. This may occur even when an animal has no intension of fighting or chasing, but is simply following its mother, or watching its territorial boundaries. Once shadowing is successful, indicated by matching the speed of the target (possibly zero velocity) the agent begins a docking sequence. A soft contact is the objective during docking rather than a hard contact representing an attack. The threshold between hard and soft contact is said to lie with the derivative of time to contact at $\dot{\tau} = -0.5$. Values less than -0.5 will result in hard collisions while values greater than -0.5 will yield soft collisions. According to cited references the authors state that human subjects use the strategy implemented to control braking during an approach to a stop sign in a closed-loop display.

The experiments done in this paper were divided into two groups. Those in which the robot performed segmentation during the perception process, and those in which it did not. Segmentation is the process of simplifying an image by partitioning it into sets of pixels. In this experiment segmentation was done by summing the optical flow vectors in each column of the array, and subtracting the sum of previous column, to get dist(j) for column j. A self moving target should have the highest dist() values because its motion on the two boundary sides should be in completely opposite directions, so the two columns with the highest values were taken as the boundaries to the target.

The equipment used was the same as the robot from their previous experiment, with three computers each running their own task, and feeding the results to the next step in the sequence. The first, subsampled the field of view to a 128 x 32 pixel array, the second performed Camus'(1994) real-time algorithm for optic flow, while the third performed the control laws and sent actions to the base. The algorithm by Camus is not reviewed except to say that the real-time is accomplished by a time-space tradeoff forgoing a search over all pixels, for a search in a 1 pixel radius, over several frames. The target was either a 4x12, or 3x5 inch piece of cardboard coloured randomly, or the first author's legs. The target was introduced to the robot's field of view, as either approaching or retreating, with speeds limited to those appropriate given the robot's capabilities. The trials were then divided up based on the distance the target started at, whether the chase or escape was successful, whether the choice to chase or escape was correct, and of course those done using segmentation and those without.

The results of the experiment seem to indicate that segmentation made no large difference in performance, but a robot guided by optical flow alone is quite a promising prospect. The authors state that "it is difficult to compare the performance of the control laws with and without segmentation," because the robot's behaviour is actually a reaction to the experimenter's behaviour which was not constant throughout the trials. The authors may want to keep in mind with future experiments that a comparison is only viable if the input is the same for each experiment, or each type of experiment. The chases were claimed to be one viable indication of performance, depending on what type of contact was made with the target. While segmentation did not change the percentage of hits and misses (for the real, hard, far, chases on day 2), with segmentation, more successful chases resulted in physical contact with the target, while without segmentation more were successful docks using τ. Segmentation seems to be useful in hunting, but tracking a mate, might be more successful using a slower 'docking' type approach. The authors express that performance without segmentation may be a result of a simple environment, which may be advantageous where the prey composes of a majority of the field of view. Segmentation would be an asset however, if prey will be passing behind objects, so that "information about the accretion and deletion of texture" can be made use of [13].

To conclude their paper, the authors relate that knowing which control law to use, may be the largest concern when trying to navigate based on optic flow. A few concerns I have are that the authors program the robot to center the target in its field of view whenever it is detected, but in a true chase, the predator is usually concerned with where its prey will be when it gets to it rather than where it currently is. Aiming in the direction which the prey is headed would be a more accurate representation of the tag or chase scenario (a formula for this is given but not implemented because of complexity). Alternately, when the robot acts as the prey, it is able to continuously detect the predator which also does not accurately represent a life or death hunt as the prey is not usually running backward. One final note to make is that the authors do manage to integrate at least two control laws at a time, since fixation was done continuously, even while other behaviours were active. This is the way an animal might interact with its environment, when it for instance, stalks its prey, while also avoiding collisions with obstacles. Learning to nest control laws will definitely be of large concern in ecological robotics.

4.3 Duchon, Warren, and Pack-Kaelbling (1998)

Ecological Robotics [11]

After three papers, and a poster, on topics concerning robotic control using optic flow, Andrew Duchon and William Warren co-operated again with Leslie Pack-Kaelbling to write a paper which leaves no ambiguity based on the title 'Ecological Robotics'. After a review of Gibson's principles of optic flow and direct perception, the authors discuss some optic flow control laws and one source of inspiration which came from studies of how flies control their movement. A summary of the principles behind ecological robotics illustrated the authors' views within the ecological approach to perception and action. Agent and environment are treated as a system, and the agent's behaviour emerges from the dynamics of this system. The agent's task is to map available information to the available control parameters to achieve a desired state of the system. The environment provides enough information to allow for adaptive behaviour, and the environment need not be represented in the agent by a central model. Task-specific memory and learning are allowed however. This experiment, it is then explained, will involve implementing optic flow control laws as taken from [12], as well as actions programmed in [13], and exploring their culmination, and limits, both in reality as well as a simulation.

The authors cite Gibson[30] as having referred to mechanisms such as their balance strategy as "the principle of symmetrical stimulation in orientation." The balance strategy dictates that when an agent is translating, closer objects move faster across the retina, than farther objects, and take up more of the field of view as well. Accordingly, the optical control law has the agent turn away from the side of greater flow while taking into account the other side of the field of view to ensure it does not collide with anything on either side.

There were two robots used in the experiment, the smaller of which, now named Louie, was used in previous experiments, ([12],[13]), had a 30.5 cm base, a camera 75 cm off the ground, and a 60^o field of view. The larger robot, named Ramona, had a 61 cm base, a camera 120 cm off the ground, and a 120^o field of view. The runs were performed in an atrium containing tables, chairs, and a few trees, with mostly textureless surfaces. Ramona, because of its height, had its camera tilted down 45^o . Due to a capability of faster speeds it also had a speed function programmed which reduced speed in a cluttered environment (when more flow was detected), and allowed for maximum speed in an open setting.

The robots were implemented in such a way that rotational velocity which was obviously controlled, was subtracted from translational velocity. Only optic flow due to translation was used as input to the control laws. Compared to the other two cited attempts to control a robot with optic flow, this experiment used normalized sums of the optic flow for the entire left and right halves of the scene, while the other two experiments used maximum values of optic flow within the left and right halves instead. Two justifications are that a normalized sum is less susceptible to noise, and it incorporates information from more distant objects as well.

Simulations were also performed by placing a simulated robot in a simulated environment designed to demonstrate behaviours such as "avoiding a small object, going through a small aperture, negotiating a corner, going down a hall, and interacting with other agents" [11]. The agent was simulated as a point with a field of view able to vary from 5 to 360 degrees, but a triangle was drawn around it for other agents to perceive. When a single agent was placed in a fixed environment and given a fixed speed, there was an emergent behaviour observed. The agent first wandered around avoiding the walls, but settled into a circular path around one of the few walls. A very small aperture could only be traversed once gaussian noise had been introduced to the system. It was also determined that a small field of view such as 60 degrees gave rise to much worse behaviour than an 120 or 150 degree field of view, but there is some upper bound, because 200 degrees caused poor performance. I would expect an optimal value here to be around 180 degrees so that the agent will not avoid things it has passed, but will take into account as much of the scene in *front* of it as possible.

Another simulated environment had an agent navigate a square passage, with walls blocking half of the hall on both sides at varying distances apart. The "best" path was either the safest path, taken by keeping a larger distance from walls (184.58 units), or the shortest path (147.93 units). Duchon et al. determined that their 'balance strategy' lead to a balance of the safest and shortest paths. The average length was 164.85 which was approximately half way between the safest and shortest paths. With small amounts of noise however ($\sigma = 0.01$), the path became shorter on average over ten runs to 157.73 units, but increasing the noise to $\sigma = 0.02$ lengthened the path to 175.42 units on average, though with this level of noise the agent only completed the runs 50 percent of the time.

The game of TAG investigated in [13] was reviewed explaining the control laws behind each of the different states. Fixating on the target involves keeping the target centered in the field of view and the target's subsequent approach or withdrawal causes the agent to enter either an escape or a chase state. The escape involved backing away from the target causing outward optical flow, until a certain threshold distance was achieved, and chasing involved shadowing, then docking. The shadow component was to match speeds with the prey, and the docking was intended as a slow soft contact as opposed to a hard attack.

The results obtained from the experiment, were varied. More than half of the approaches led to false chases, slightly higher without segmentation than with it. Across the nine successful escapes in 20 trials, five were reasonable, three short, and one long. The approach seems feasible, but is still in need of some adjustments to be more robust, and reliable. When the robot was chasing the target, 70 percent of the 20 trials resulted in successful chases, equal number with and without segmentation. The difference came in the type of contact. With segmentation most hits were actual contact, but without segmentation a majority were successful docks using the time to contact. Segmentation seems to decrease the odds of the robot incorrectly chasing an approaching target, however among successful chases, it increases the odds of a 'hard' contact, rather than a dock with the target. I would submit that segmentation is only an asset to a robot which must 'bring down' its target, rather than track it, or even evade it. The optical flow approach itself was shown to be more useful in chasing, or tracking a target (i.e. perceiving it visually), than in evading one (i.e. specialized type of avoidance behaviour).

The authors admit that "the quality of the segmentation algorithm was poor," and "the robot would often follow 'ghosts'." This is obviously one area which needs to be improved, possibly simply by using newer, faster computers to run the real time algorithms. The algorithm does allow recovery from this temporary poor behaviour however, because it will usually perceive the target to have jumped to a new location in the next time step, and adjust its heading accordingly.

Learning a few things from the runs and simulations on Louie, the authors modified a few control laws when experimenting with their larger robot Ramona. In this robot the shadowing then docking, was reduced to a single control law where the robot increased its speed until it catches up with the target, then slows down gradually. Because of this change only a dock or failure can result from a chase. The robot was equipped with a possible 180^o field of view when watching, but this was narrowed to 90^o upon moving (analogous to attention).

An interesting observation was that when the target wandered at an angle to the watcher's heading, one side of the target was closer, causing more outward flow on that side, which in turn caused the robot to underestimate time to contact, and slow down. The authors suggest that evidence for the utility of such a behaviour can be seen in prey animals who have developed zig-zagging as an escape tactic to enable them to both see their predator, and to fool the predator into believing it is doing better than it is. They also provide reference to a study that suggested that humans have separate 'what' and 'how' pathways in the brain, claiming that humans just like the robots tested, do not need to model what an object is to maneuver around it. These robots accordingly model only the 'how' pathway and still function usefully. The authors finally suggest neural networks as a means of satisfying the many soft constraints of affordances to choose a single output or action.

Chapter 5

Situation Theory

Situation Theory is concerned with mathematically describing the meaning of a situation, or the information conveyed. The concept of information is not definitely defined in that it is quite difficult to discuss quantities of information or to generalize types of information. Classical logic is concerned with only one type of information: truth. Only the truth of a statement, or validity of an argument can be identified, unless the configuration of the environment in which the truth takes place can somehow be represented.

Situation Theory uses nine basic types to lay a foundation towards a comprehensive theory of information. We will describe each of them below using definitions from [9] and [10] though only seven will be used in my thesis. The three types, *time*, *space*, and *individual*, are simple types which are used to specify the temporal and spatial locations, and the participatory agents. They can be self-referential such as 'now' or 'here', or they may be precise such as 'Brock University J block' or '12:02 pm'. Individuals are usually represented by some reference which conveys the intended knowledge. An individual might be given by name, title, or reference and may be as specific or general as one wishes such as 'Professor Michael Winter', 'Dean of Science', 'my father', or simply 'Peter'.

- TIM: the type of a temporal location
- LOC : the type of a spatial location
- IND : the type of an individual

The *relation* type must reference the number of parameters suitable to use it. If we consider the use of a relation such as 'giving' if it was given as a 3-place relation referencing the giver, the receiver, and what is being given, it would be invalid to apply giving to Paul, Adam, Walrus, and gift as it contains too many parameters and does not make sense. By the same reasoning we cannot have a 3-place relation 'giving' without it being applied to 3 objects as it would not convey any information at all. Some relations have more than one possible arity. The name alone ('giving') does not specify the relation, but a name coupled with an arity does, i.e.(giving,3). The relation might possibly have two places or three, and the items which fill the roles may have different restrictions in different scenarios. For example, it might be 2-ary, one reference to the giver, and one to the receiver, or it may have one for the giver, and one for the item being given. Minimally, a relation must have a name, a number of components equal to the arity (and of suitable type), and a polarity to identify its truth value. The type of a *polarity* is the most simple type, consisting of values of either zero or one. A polarity of zero indicates the relation is not true, while one indicates the relation and its components hold true. The inclusion of the polarity could seem possibly redundant, though the functionality given from being able to represent false items of information may outweigh any performance benefit from reducing the size of the definition.

- REL^n: the type of an n-place relation
- POL: the type of a polarity

One of the more important types in situation theory is that of infons. Situation Theory, as an attempted beginning toward a way to represent any and all information, attempts to do so in terms of basic building blocks it calls *infons*. Inspired by physics-based particles such as photons, and neutrons, the infon is described as the most fundamental or *basic* of the representational components. Infons are important as they emphasise the study of information as an empirical science, though this does not imply the physical existence of an infon. Each infon contains several values, each is one of the types listed below, or else of a higher order defined type constrained to the situation. Each infon is an object of form: $\ll r, a_1, \ldots, a_n, \varepsilon \gg$ where r is an n-place relation, $a_1, \ldots, a_n$ are objects appropriate for the respective argument places of r, and ε is a polarity of value 0 or 1.

Any *situation* will involve at least implicitly a time, a place, and an individual. These may not be definite, and they may even make the statement that nobody was or will be in a situation, that the time was in the past, or that the place is on earth. The situation type is also used to specify generalities among different situations. For example s could be defined as any situation where a person is running.

- INF: the type of an infon
- SIT: the type of a situation

As an example of infons consider the following two simple infons:

$$\ll \text{'less than'}, 2, \pi, 1 \gg$$
$$\ll \text{'sum of'}, 7, 3, 5, 0 \gg$$

We can see that 2 and π stand in the 'less than' relation, and 7 does not fall in the 'sum of' relation with 3 and 5. Note that there is nothing preventing an infon from representing an incorrect piece of information. The object $\ll$ 'sum of', $7, 3, 4, 0 \gg$ is indeed an infon, but it is not modeled by our world since in our reality $3 + 4 = 7$. To be modeled in our reality ε would have to be 1 instead of 0. When we write 'modeled' we mean that if s is some situation and σ is some infon, $s \models \sigma$ denotes the fact that σ is true in s, s *supports* σ, or s *models* σ. Any situation will have at least 1 but arbitrarily many infons which it supports, showing that the information the infon conveys is true in that situation.

For any abstract situation s, (supporting a set of infons) it will be considered *coherent* if and only if for no $r, a_1, ..., a_n$ are both of the following valid:

- $s \models \ll r, a_1, ..., a_n, 1 \gg$
- $s \models \ll r, a_1, ..., a_n, 0 \gg$

Thus all situations in our reality are coherent as something cannot both be true and untrue at the same time. To allow for many states to be supported by one situation we allow for a situation to model an entire set of infons. If I is a set of infons, and s is a situation, we say that $s \models I$ if and only if $s \models \sigma$ for all $\sigma \in I$. Another name for an infon we will use later is a 'state'.

As we've mentioned any situation will involve (at least implicity), a time, a place, an individual, and some relation between them. If any of these components is unknown, a *parameter* type can take the place of an unknown value. Parameters, usually denoted with dots above them, are introduced for making reference to arbitrary objects of a given type. For each basic type T (other than PAR), there exists an infinite collection $\dot{T}_1, \dot{T}_2, ...$ of basic parameters used to denote arbitrary objects of type T. For the purposes of explaining Situation Theory we have included a description of parameters, but for simplification they will not be used once we work with a model.

The basic types also include a *type* type, so the theory allows for arbitrary levels of focus or abstraction depending on the situation and agents referred to, and the current intentionality of the agent. It also allows for the following to hold true: "For any object x in the theory, there is at least one type such that x is of that type"([10], p51). This simply states that every object is of some type. For example, every person on the earth is the type of a person which might be defined like this:

$$[IND_1 | w \models \ll person, IND_1, LOC_i, TIM_i, 1 \gg].$$

This represents all individuals subject to a restriction that the world w, models some infon, in which the individual is a person associated with some location and time.

- PAR: the type of a parameter
- TYP: the type of a type

Situation Theory also has methods to assign values to parameters which we will briefly discuss here, but will not factor into my thesis. To give the parameters values we use *anchors.* An anchor for a set A of basic parameters, is a function defined on A which assigns each parameter T_n in A an object of type T. If f is an anchor for A and T_n is a parameter in A then this is a fact: $w \models \ll \text{of-type}, f(T_n), T, 1 \gg$.

If $\dot{p}$ is to be a parameter for a person, it should be restricted so that whenever f anchors $\dot{p}$ to an object a in situation s, then in s, a is a person (Thus parameters restrict anchors). Roughly, a parameter $\dot{p}$ results from tagging a parameter $IND_i \in PAR$ with 'condition' C of being a person writing $\dot{p} = IND_i \upharpoonright C$. The parameter can be anchored like so: $f(\dot{p}) = f(IND_i) \upharpoonright C$. For a description of C the reader may refer to above where the type of all persons is given.

If σ is a parametric infon, and f is an anchor for some/all free parameters in σ: $\sigma[f]$ will represent the (parameter-free) infon that results from replacing each v in the domain of f that occurs free in σ by its value $f(v)$.

If I is a set of parametric infons, and f is an anchor for some/all free parameters in infons in I, we define $I[f] = \{\sigma[f] | \sigma \in I\}$.

These are more developed notions than we will be able to accommodate in this thesis, but we will now introduce some mathematical terminology, and then work with a toy version of Situation Theory which does not make use of types, parameters, or anchors, and infons are called states.

5.1 Mathematical Preliminaries

In this chapter, we will introduce and discuss a toy version of Situation Theory. Most of the following was derived from Lecture Notes in Computer Science: Situation Theory and its Applications, Volume 1, By Robin Cooper. The focus was on the first chapter "Replacement Systems and the Axiomatization of Situation Theory," written by Peter Aczel [7]. After its introduction, we will extend the model using the notion of *affordance* in order to provide a new formal description of affordance relations within Situation Theory.

5.1.1 Sets, Classes, and Categories

Before diving into signatures, algebras, form systems, and ontologies, we will provide a cursory overview of some of the concepts needed.

We first distinguish between sets and classes taking the definition of sets from [14]. In set theory, only two concepts are considered primitive in that they are not defined in terms of other concepts. Essentially a set is a collection of objects or members to be regarded as a single entity. The other primitive concept is membership. Any single object is an element or member of a set if that object is in the set. We denote that an object x is a member of set X by writing $x \in X$. One important notion on sets will be the concept of *subset*. A set A is a subset of set B if every member of A is also a member of B. To identify this we write $A \subseteq B$.

Another operation on sets we will use is the union operator. If we are given any number of sets, and wish to extract the union of them, we construct a set which includes all elements from each set, but in the event of an element being a member of two or more sets, we simply include it once. Algebraically, given sets $\{A_1, \ldots\}$ we define the union of them as follows:

$$\bigcup_{i \in I} A_i = \{x | x \in A_i \text{ for some } i \in I\}.$$

A similar operator may be used called the disjoint union symbolized by $\uplus$. When the normal union is taken over many sets which contain an identical element, that element is only found in the union once. When the disjoint union is taken over many sets with an identical element, each element carries the information of which set it is from.

$$\biguplus_{i \in I} A_i = \{(i, a) | a \in A_i, i \in I\}$$

To refer to a collection of all sets, we do not call this collection a set, but instead a class. A class is a collection of objects like a set is. Every set may also be called a class, but a class which may be considered a set is called a small class. These classes which are considered sets as well are defined by the property that we can eventually list all members of it. A proper class however, is one which is too big to be called a set. A particular class which will be utilized in the following sections is the class of all sets. This class (and only this class) will be represented by the variable V.

We find it necessary to introduce ZFC$^-$ formally. Zermelo-Fraenkel (ZF) set theory with 'C' the axiom of choice, will be what we will work with here. In the ZF axiomatization, eight axioms are given. We refer the reader to the literature to review them.

It will be necessary to assume the Anti-Foundation Axiom (AFA) to accomplish some of our goals, so we provide a definition. The axiom was published by Peter Aczel in 1988[1]. It essentially allows sets to contain themselves as members, so that every collection of objects (even a collection which contains itself as a member) may be considered a set. The smallest non-well-founded set is the Quine Atom which may be represented like so: $x = \{x\}$. Thus the set contains only one object, namely itself.

A notion on sets and classes which we will need is the power set. The powerset of a set X, written $powX$ is the set of all subsets of X, including the empty set, and the set X itself. $powX = \{x | x \subseteq X\}$. The power-class of a class also uses the notation $powA$ for class A and is defined by: $powA = \{x \in V | x \subseteq A\}$. [1]

A relation is a collection of ordered pairs described as a subset of the product of the two classes which it relates. R is a relation if $R \subseteq V \times V$. If R is a relation we may write xRy to indicate $(x, y) \in R$.

A function F is a relation that is total and univalent. A total relation is one where for all x there is some y such that $(x, y) \in F$. That is, every x from the source class is related to some element y in the target class. A univalent relation is one where for all x, y, z if $(x, y) \in F$ and $(x, z) \in F$ then $y = z$. Essentially, any value may only be related to one single value.

Functions of relations which we will need are the domain and range. The domain function, written *dom*, may be applied to a relation, and will return the set of all possible input values. Formally, $domR = \{x|xRy$ for some $y\}$. The range, written *ran*, when applied to a relation will return the set of possible output values otherwise called the image of the function. $ranR = \{y|xRy$ for some $x\}$. The range is a subset of the codomain which refers to the entire set the function is mapping values to, even if some values are not mapped to. The membership relation $\in$ is the class $\{(x, y)|x \in y\}$, and for each class A we have that $\in_A = \in \cap (A \times A)$.

If A is a class, and I is a set, A^I will represent the class of all functions from I to A. The set I however is not-necessarily-finite. We will see in the next section how we will regard A^I if I is finite.

Originally a branch of mathematics, category theory has branched into theoretical computer science, abstracting the notions of sets, and functions. A category C is defined by the following properties:

- a collection Ob_C of objects, denoted by $a, b, \ldots$
- a collection Mor_C of morphisms (arrows) denoted by $f, g, \ldots$
- two operations dom, cod assigning to each arrow f two objects respectively called domain (source) and codomain (target) of f
- an operation *id* assigning to each object b a morphism id_b (identity of b) such that $dom(id_b) = cod(id_b) = b$
- an operation $\circ$ (composition) assigning to each pair f, g of arrows with dom(f)=cod(g) an arrow $f \circ g$ such that $dom(f \circ g) = dom(g), cod(f \circ g) = cod(f)$
- identity and composition must also satisfy the following two conditions:

1. identity: for any arrows f, g such that $cod(f) = b = dom(g)$ it must hold that $id_b \circ f = f$ and $g \circ id_b = g$
2. associativity: for any arrows f, g such that $dom(f) = cod(g)$ and $dom(g) = cod(h)$ it must hold that $(f \circ g) \circ h = f \circ (g \circ h)$

A morphism f with source a and target b will be denoted $f : a \rightarrow b$. When dealing with two classes, say A and B, a function or morphism from one to the other $f : A \rightarrow B$ is also a relation $f \subseteq A \times B$ such that for each $a \in A$ there is some unique $b \in B$ (also written $f(a) \in B$) such that afb. If we are given two objects a and b, the collection of all morphisms from a to b will be denoted $C[a, b]$.

A category is called small if both the class of objects, and the class of morphisms, are actually sets and not proper classes. A category with either a proper class of objects, or a proper class of morphisms will be called not-small, or large. If we call a category superlarge, not only is the category not small in that it is not a set, but it is much larger than a set in that it may be unbounded.

In category theory, a functor is a mapping (or morphism) between categories. If we have categories A and B, a functor $F : A \rightarrow B$ will associate each object $x \in A$ with an object $F(x) \in B$. As well, it will associate with each morphism $f : x \rightarrow y \in A$ a morphism $F(f) : F(x) \rightarrow F(y) \in B$ so that the following two conditions hold: $F(id_X) = id_{F(X)}$ for every object $x \in A$ and $F(g \circ f) = F(g) \circ F(f)$ for all morphisms $f : x \rightarrow y$ and $g : y \rightarrow z$. Functors will preserve identity morphisms, and composition of morphisms.

As well we will mention initial objects and so should provide a definition of them. If we are given a category C, an object 0 is initial if and only if for any $b \in Ob_C$ there is a unique $f \in C[0, b]$. Essentially there must be a unique function with the initial object as a source, which terminates at every other object in the category.

Initial Algebras are also of interest. Taking the definition from [1], an Initial Algebra is an initial object in the category of F-algebras. It is defined relative to a fixed functor $F : C \rightarrow C$ where C is a category.

1. $\mathcal{A} = (A, \alpha)$ is an algebra if $\alpha : FA \rightarrow A$ in C. The algebra will be called *full* if $\alpha : FA \leftrightarrow A$ is a bijection.
2. Given $\mathcal{A} = (A, \alpha)$ and $\mathcal{B} = (B, \beta)$, π is a homomorphism from $\mathcal{A}$ to $\mathcal{B}$ written $\pi : (A, \alpha) \rightarrow (B, \beta)$, if $\pi : A \rightarrow B$ such that the diagram

commutes.

$$\begin{tikzcd}
A \arrow[r, "\pi"] & B \\
FA \arrow[u, "\alpha"] \arrow[r, "F\pi"'] & FB \arrow[u, "\beta"']
\end{tikzcd}$$

Obviously, $id : A \to A$ is a homomorphism from (A, α) to (A, α) for every algebra (A, α). Moreover, if f is a homomorphism from (A, α) to (B, β) and g is a homomorphism from (B, β) to (C, γ), then $g \circ f$ is a homomorphism from (A, α) to (C, γ). i.e. F-algebras with homomorphisms form a category.

3. $\mathcal{A} = (A, \alpha)$ is an initial algebra if and only if for every algebra $\mathcal{B} = (B, \beta)$ there is a unique homomorphism $\mathcal{A} \to \mathcal{B}$.

5.1.2 Signatures and their Algebras

A signature is a class of symbols usually labeled as Ω. Each of the symbols ω in the class has as an arity the set $v\omega$. A signature is *finitary* if for every symbol ω, $v\omega$ is a finite set, and *standard finitary* if for every ω, $v\omega = \{1, \ldots, n\}$ with $n \in \mathbb{N}$. In the case of a standard finitary signature, ω is called an *n-place function symbol.*

If we take Ω as a signature, an Ω-algebra $\mathcal{A} = (A, \omega^{\mathcal{A}})_{\omega \in \Omega}$ consists of a class A and a map $\omega^{\mathcal{A}} : A^I \to A$ for each symbol ω of arity I. If we are dealing with a standard finitary signature, $I = \{1, \ldots, n\}$ for $n \in \mathbb{N}$, and then A^I can be read as an n-fold cartesian product like so: $A^n = \overbrace{A \times \cdots \times A}^{n}$. The map $w^{\mathcal{A}}$ can now be identified with an n-place operation on A. In addition, if $n = 0$, then A^0 is a singleton set. These operators with arity 0 can be identified with elements of A.

An **Ω-algebra** homomorphism is defined to be a map between two Ω-algebras.

5.1.3 Class Operations and F-Algebras

Associated with each signature Ω is an operation $\Omega[]$ that associates with each class A a new class $\Omega[A] = \{(\omega, f) | \omega \in \Omega \ \& \ f \in A^{v\omega}\}$.

Note the class T of terms is the smallest class such that $\Omega[T] \subseteq T$ and in fact T is a fixed point of $\Omega[]$; i.e., $\Omega[T] = T$. $\Omega[]$ is an example of a set continuous class operation and it is this fact about $\Omega[]$ that implies that the class T exists. A class operation F is *set-continuous* if it is *monotone*, i.e., if $A \subseteq B$ then $FA \subseteq FB$ and *set-based*; i.e., if $a \in FA$ then $a \in FA_0$ for some $A_0 \in powA$. The proof for the following can be found in Aczel (1988).

Theorem 11 *If F is a set continuous operation then there is a smallest class A such that $FA \subseteq A$ and a largest class B such that $B \subseteq FB$. Both are fixed points of F.*

Consider the category of classes and functions between them. For any class operation F, a functor on the category of classes, call (A, α) an *F-algebra* if A is a class and $\alpha : FA \to A$. We call an F-algebra (A, α) full if $\alpha : FA \cong A$ is a bijection. Also, every fixed point A of F determines the full F-algebra (A, id_A). If Ω is a signature and $\mathcal{A} = (A, \omega^{\mathcal{A}})_{\omega \in \Omega}$ is an Ω-algebra then it determines the $\Omega[]$-algebra (A, α) where $\alpha(\omega, f) = \omega^{\mathcal{A}} f$ for $(\omega, f) \in \Omega[A]$.

By reading the above equation from right to left as a definition, given the map α, of each operation $\omega^{\mathcal{A}}$, it is clear that every $\Omega[]$-algebra (A, α) arises in the above way from a unique Ω-algebra $\mathcal{A}$. Moreover $\mathcal{A}$ is full if and only if (A, α) is full. This one-one correspondence between Ω-algebras and $\Omega[]$-algebras suggests a generalization of the theory of signatures and algebras to suitable F and F-algebras. To get a smooth generalization we need to make some assumptions about F.

5.1.4 Standard Functors and Full Algebras

If Ω is a signature then the signature operation $\Omega[]$ can be made into a functor on the (superlarge) category of classes by defining, for each map $h : A \to B$ the map $\Omega[h] : \Omega[A] \to \Omega[B]$ where, for $(\omega, f) \in \Omega[A]$,

$$\Omega[h](\omega, f) = (\omega, h \circ f).$$

It is easily checked that this gives us a functor. It is in fact a standard functor. A functor F on the category of classes is *standard* if it is set continuous as a class operation and *preserves inclusion maps*, i.e., if $A \subseteq B$ and $i_{A,B} : A \hookrightarrow B$ is the inclusion map, with $i_{A,B} a = a$ for all $a \in A$, then $F i_{A,B}$ is the inclusion map $i_{FA,FB} : FA \hookrightarrow FB$. The latter property map also be expressed by saying that if $f : A \to B$ then Ff depends only on the class $\{(a, fa) | a \in A\}$ and not on the codomain B of the map $f : A \to B$. It implies that if $f : A \to B$ and $A' \subseteq A$ then $F(f|A') = (Ff)|(FA')$, i.e., that

F preserves restriction.

We have defined the notion of an F-algebra for any class operation F. In order to have a suitable notion of homomorphism between F-algebras we need to assume that F is a functor.

If (A, α) and (B, β) are F-algebras, where F is a functor on the category of classes, then a map $\pi : A \to B$ is a *homomorphism* from (A, α) to (B, β), written $\pi : (A, \alpha) \to (B, \beta)$ if the diagram below commutes.

$$\begin{tikzcd}
A \arrow[r, "\pi"] & B \\
FA \arrow[u, "\alpha"] \arrow[r, "F\pi"'] & FB \arrow[u, "\beta"']
\end{tikzcd}$$

i.e., $\pi(\alpha a) = \beta(F\pi a)$ for all $a \in FA$. Note that if (A, α) and (B, β) are the $\Omega[]$-algebras associated with the Ω-algebras $\mathcal{A}$ and $\mathcal{B}$ then $\pi : \mathcal{A} \to \mathcal{B}$ is an Ω-algebra homomorphism if and only if $\pi : (A, \alpha) \to (B, \beta)$ is an $\Omega[]$-algebra homomorphism.

With this general notion of homomorphism for F-algebras we get the category of F-algebras and hence the notion of an initial F-algebra, which is necessarily unique up to isomorphism if it exists.

Theorem 12 (The Initial Algebra Theorem) *For any Functor F on the category of classes any initial F-algebra is full. If F is standard then an initial F-algebra exists. Moreover the least fixed point A of F determines the initial F-algebra (A, id_A).*

We have excluded the Final Coalgebra Theorem and a discussion of Strongly Standard functors, as they are not needed for the work that is done here, and their descriptions can be found in [7].

5.1.5 Form Systems

A *form system* is defined to be comprised of three parts: A class A of objects, a mapping $C : A \to powX$ called the 'component' map, which assigns to each element in A a subset of objects (referred to as Ca) from the class X of *components*, and a 'dot' operation used to combine a form and a map, to obtain a new form. The system however, is also taken 'over' a class X. By restricting X we can obtain replacement systems or ontologies as we will see in the next two sections. If we take X to be the same class as A, the form system is called a *replacement system* but if X is defined as the class

V of all sets, we call the form system an *ontology*. The class X in any case, contains all items possible to fill component roles of objects. If a mapping operation $\sigma : Ca \rightarrow X$ will map each component $t \in Ca$ of object a to a new component $t_{\text{new}} \in X$, then the dot operation combines the map with the object to get $\sigma \cdot a$ which is a new form. This new form is obtained from a by replacing each of its components using σ.

To illustrate a form system, we use the example of the absolute function. If we take A to be the class of all integers, X would refer to the set of natural numbers, and the mapping C would be the absolute function itself. An integer taken from A has a single component, namely its absolute value. We define one possible σ_1 to map any component number to that number plus one. Therefore, σ_1 is defined for all natural numbers and can always be applied to the components of an object. If we take an example object -2 in our class A of integers, and perform the component map we yield its single component $C(-2) = 2$. The 'dot' operation in our example yields a different effect for values of $a < 0$ and $a \geq 0$. σ_1 joined to a positive a will map its component to its component's successor thus increasing the integer object by one. If a was negative however, increasing the value of its component, would decrease a. In our example $\sigma \cdot (-2) = (-3)$ and $\sigma \cdot 2 = 3$.

To illustrate further we provide another example. If we consider as objects lists of elements, or any stack, or queue data structure, we can imagine that the components of a list are the elements within that list. We may then assign any arbitrary mapping of elements to new elements through the sigma operation. As a concrete instance, consider the list of numbers: L = [1,2,2000]. The component map C(L) would yield the three numbers in the list. The dot operation on elements in the list may be defined in any way we wish. For instance we might give that: $\sigma \cdot 1 = 27$, $\sigma \cdot 2 = 603$, and $\sigma \cdot 2000 = 11$. The object $\sigma \cdot L$ will be constructed by replacing each element in the list with the newly mapped element like so: [27,603,11]. This is all that is required to define a form system.

Two further rules for form systems follow. The form $\sigma \cdot a$ is equal to a if performing σ on a component of a yields that same component. The form which results from σ' applying the dot operation to $\sigma \cdot a$ is to be referred to as $\sigma' \cdot (\sigma \cdot a)$. This form is equal to $(\sigma' \circ \sigma) \cdot a$ if σ' can map each of the components of $\sigma \cdot a$ to an element in X.

Formally, $(A, C, \cdot)$ is a form system over class X if A is a class, $C : A \rightarrow powX$ and for each $a \in A$, if $\sigma : Ca \rightarrow X$ then $\sigma \cdot a \in A$ such that:

$$\begin{array}{lrcl}
(5.1.1) & C(\sigma \cdot a) & = & \{\sigma x | x \in Ca\} \\
(5.1.2) & \sigma \cdot a & = & a \text{ if } \sigma x = x \text{ for } x \in Ca \\
(5.1.3) & \sigma' \cdot (\sigma \cdot a) & = & (\sigma' \circ \sigma) \cdot a \text{ if } \sigma' : C(\sigma \cdot a) \to X
\end{array}$$

A replacement system and ontology as we shall see below, are formally defined simply by giving the restriction of X where a replacement system assigns X to be the same class as A, and an ontology restricts X to the universal class V. In a replacement system, the σ function accepts parameters just as it does in a form system, but instead yields forms from the class A, while σ in an ontology yields forms from the universal class V.

To see how to get a full algebra (A, α) for ontology $\mathcal{U}$ review section 5.1.7 on ontologies, specifically the syntactic examples. In addition, that paragraph will explain how using the forms of the ontology $\mathcal{U}$ we can determine a replacement system $\mathcal{U}(A, \alpha)$. To examine the Representation Theorem which would show how each replacement system has the form $\mathcal{U}(A, \alpha)$ for some full algebra (A, α) for some ontology $\mathcal{U}$, please review [7] for the details.

5.1.6 Replacement Systems

One of the two basic ideas in the model of Situation Theory which we are concerned with here, is replacement systems. A replacement system is comprised of objects, where each object has components which are themselves objects in the system. Those components may be replaced by other components to yield a new object. For example, consider as objects, terms built from integers, and the binary operation of addition and multiplication with the usual precedences. A typical object a might be represented by $2 \times 3 + 4$. The components of a term are its subterms, or in this example, break down the equation by the least significant operation. The components of a thus are $\{2 \times 3, 4\}$. Replacement can be modeled by a function σ mapping terms to terms. For example if $\sigma(2 \times 3) = 1$ and $\sigma(4) = 5$ then the object $\sigma \cdot a$ is obtained from a by replacing the components using σ, and results in the object $1 + 5$.

Set Theoretic Examples

If V is the class of all pure sets, i.e. V is the largest class A so that $A = powA$, then $(V, C, \cdot)$ is a replacement system, where for $a \in V$, $Ca = a$, and if

$\sigma : a \to A$ then $\sigma \cdot a = \{\sigma x | x \in a\}$

In fact any fixed point A of pow would give a replacement system in this way. More generally any fixed point A of any *subpowerclass* operation $P[]$ would give a replacement system. To define these operations let us call a class P of sets *image closed* if for any function f, $domf \in P \Rightarrow ranf \in P$.

An image closed class P determines the subpowerclass operation $P[]$, where for any class X, $P[X] = \{u \in P | u \subseteq X\}$.

It is not necessary for A to be a fixed point of $P[]$. It suffices to have a bijection $\alpha : P[A] \cong A$.

This determines a replacement system $\mathcal{U}_P(A, \alpha) = (A, C, \cdot)$, where for each $a \in A$, if $a = \alpha u$ then $Ca = u$ and if $\sigma : u \to A$ then $\sigma \cdot a = \alpha(\{\sigma x | x \in u\})$.

This last most general family of set theoretical examples determines all the replacement systems $(A, C, \cdot)$ where $C : A \to powA$ is injective. Note that for any replacement system $(A, C, \cdot)$ there is an image closed class P such that $P[A] = \{Ca | a \in A\}$. In fact we can let P be the class of all sets $ranf$ for f a function with $domf = Ca$ for some $a \in A$.

It is easy to give a complete survey of the image closed classes P. First, note that a class P of sets is image closed if and only if $P \cup \{\emptyset\}$ is image closed. So we need only consider those classes P with $\emptyset \in P$, the remainder being obtained by taking $\emptyset$ out.

Image Closed Class: A class P of sets, with $\emptyset \in P$, is image closed if and only if either $P = powV$ or else $P = \{a \in powV | card(a) < \kappa\}$ for some cardinal number $\kappa > 0$. In other words, a class of sets is image closed if either it is the set of all subsets of the universal set, or if it is all elements with an upper bound on cardinality κ.

Syntactic Examples

The variable 'free terms' of a signature determine a replacement system. We will generalize this example so as to apply to any full algebra of a signature. We will use a general notion of a (single-sorted) signature where there can be a proper class of symbols and any set can be used as the arity of a symbol, i.e., as the set indexing the argument positions of the symbol. So a signature Ω consists of a class of symbols, each symbol $\omega \in \Omega$ having some set $v\omega$ as its arity. The result of 'applying' a symbol ω of arity $I = v\omega$ to a family of arguments, with a_i filling the argument role having index $i \in I$, will be written $\omega(\cdots a_i \cdots)_{i \in I}$.

We can assume that this is really the ordered pair (ω, f) where f is the function with domain I such that $fi = a_i$ for $i \in I$. In the case when $I = \{1, \dots, n\}$ then it is natural to write $\omega(a_1, \dots, a_n)$. The class of variable free terms of the signature is the least fixed point of the operation $\Omega[]$, where for any class X, $\Omega[X] = \{\omega(\cdots x_i \cdots)_{i \in I} | \omega \in \Omega \ \& \ I = v\omega \ \& \ x_i \in X$ for $i \in I\}$.

If A is the class of variable free terms or more generally if A is any fixed point of $\Omega[]$, then $(A, C, \cdot)$ is a replacement system, where for each $a = \omega(\cdots a_i \cdots)_{i \in I} \in A$, $Ca = \{a_i | i \in I\}$, and if $\sigma : Ca \to A$ then $\sigma \cdot a = \omega(\cdots \sigma a_i \cdots)_{i \in I}$.

More generally from any (A, α) such that $\alpha : \Omega[A] \cong A$ we get a replacement system $\mathcal{U}_\Omega(A, \alpha) = (A, C, \cdot)$ where for each $a \in A$, if $a = \alpha (\omega(\cdots a_i \cdots)_{i \in I})$ then $Ca = \{a_i | i \in I\}$ and if $\sigma : Ca \to A$ then $\sigma \cdot a = \alpha(\omega(\cdots \sigma a_i \cdots)_{i \in I})$.

5.1.7 Ontologies

The other basic idea in this model of Situation Theory is that of ontologies. An ontology, like a replacement system, contains objects. Contrary to a replacement system however, the components of an object are not necessarily objects themselves, but instead may be any arbitrary objects. We notice the two kinds of examples of replacement systems display a common pattern. In each case there is a class operation F, and given a pair (A, α) with $\alpha : FA \cong A$, we can obtain a replacement system $(A, C, \cdot)$. Below, a common generalization is given. The key observation is that in each of our examples the class operation F is what we call an ontology operation $\mathcal{U}[]$. If $\mathcal{U} = (U, C_\mathcal{U}, \cdot_\mathcal{U})$ is an ontology then its ontology operation defined as follows. For each class X let $\mathcal{U}[X] = \{u \in U | Cu \subseteq X\}$.

Set Theoretical Examples: If P is an image closed class then $\mathcal{U}_P = (P[V], C, \cdot)$ is an ontology where $Cu = u$ for all $u \in P[V]$, and if $\sigma : u \to V$ then $\sigma \cdot u = \{\sigma x | x \in u\}$. Now $\mathcal{U}_P[]$ is the subpowerclass operation $P[]$.

Note that $V = (V, C_V, \cdot_V)$ where $Cu = u$ and $\sigma \cdot u = \{\sigma x | x \in u\}$, can also be taken as an ontology when it is applied to itself like so: $V[V] = \{u \in V | u \subseteq V\}$ which is equal to V itself.

Syntactic Examples: If Ω is a signature then $\mathcal{U}_\Omega = (\Omega[V], C, \cdot)$ is a signature ontology where for $u = \omega(\cdots v_i \cdots)_{i \in I} \in \Omega[V]$, $Cu = \{v_i | i \in I\}$ and if $\sigma : Cu \to V$ then $\sigma \cdot u = \omega(\cdots \sigma v_i \cdots)_{i \in I}$.

Now $\mathcal{U}_\Omega[]$ is the signature operation $\Omega[]$.

Let $\mathcal{U} = (U, C_{\mathcal{U}}, \cdot_{\mathcal{U}})$ be an ontology. Then we define (A, α) to be the *$\mathcal{U}$-algebra* or simply an algebra (for U) if A is a class and $\alpha : \mathcal{U}[A] \to A$. It is full if α is a bijection $\alpha : \mathcal{U}[A] \cong A$. Each full algebra (A, α) determines a replacement system $\mathcal{U}(A, \alpha) = (A, C, \cdot)$ where for $a \in A$, if $a = \alpha u$ then $Ca = C_{\mathcal{U}} u$ and if $\sigma : Ca \to A$ then $\sigma \cdot a = \alpha(\sigma \cdot_{\mathcal{U}} u)$.

Call replacement systems $\mathcal{U}(A, \alpha)$ obtained in this way *$\mathcal{U}$-replacement systems.*

Discrete Ontology

A discrete ontology is one in which no element has components. Formally, if we have an ontology $(At, C, \cdot)$ where $Ca = \emptyset$ for each atom $a \in At$, we call the ontology discrete. Any class of objects can be defined as a discrete ontology by regarding the objects in the class as objects in the ontology, and defining all component maps to be empty.

Generalized Signature Ontology

Here we must generalize the definition of signature ontology given in [7] so that we may extend it. The generalized version of signature ontology has components which are restricted by the ontology $\mathcal{U}$. If we choose the set theoretic ontology induced by V as $\mathcal{U}$, we obtain the signature ontology as defined above. Here, define a generalized signature ontology as $\mathcal{U}_{\Omega,\mathcal{U}} = (\Omega[U], C, \cdot)$ where Ω is a signature, $\mathcal{U} = (U, C_{\mathcal{U}}, \cdot_{\mathcal{U}})$ and U is a subclass of V (which as we have seen can be taken as an ontology itself). The component map is defined to accept an element from $\Omega[U]$ and return a set of elements from U. The components of this ontology are given by $Cu = f(v\omega)$ where $u = (\omega, f)$ with $f \in U^{v\omega}$. To create new elements $\sigma \cdot u = (\omega, g)$ where $g(i) = \sigma(f(i))$ for $i \in v\omega$. If we are given a standard signature (i.e. $I = v\omega$ where $I = \{1, \ldots, n\}$ and $n \in \mathbb{N}$) the following definitions work as well. The components can be seen like so $Cu = \{\cdots x_i \cdots\}$ if $u = \omega(\cdots x_i \cdots)$. For new elements, the σ operation works by performing σ on each component of the form like so: $\sigma \cdot u = \omega(\cdots \sigma x_i \cdots)$.

The components themselves are members of the class U, thus any set of components satisfies: $\{\cdots x_i \cdots\} \in U \subseteq powV$, as long as $U \subseteq_{\text{class}} V$.

Product Ontology

We define a product ontology as follows: assume we have two ontologies $\mathcal{U}_1 = (U_1, C_1, \cdot_1)$ and $\mathcal{U}_2 = (U_2, C_2, \cdot_2)$. Their product is then defined by the following three definitions:

$$\begin{align} \mathcal{U}_1 \times \mathcal{U}_2 &= (U_1 \times U_2, C, \cdot) \tag{5.1.4}\\ \text{where } C(u_1, u_2) &= C_1 u_1 \cup C_2 u_2 \tag{5.1.5}\\ \text{and } \sigma \cdot (u_1, u_2) &= (\sigma|_{U_1} \cdot_1 u_1, \sigma|_{U_2} \cdot_2 u_2) \tag{5.1.6}\end{align}$$

where $\sigma|_{U_1}$ represents the restriction of σ to U_1 (but if $x \notin U_1$ then $\sigma|_{u_1}$ is not defined), or more formally:

$$\sigma|_{u_1}(x) = \sigma(x) \Leftrightarrow x \in U_1 \tag{5.1.7}$$

The product ontology has universe of objects represented by the cross product of U_1 and U_2, a specialized component map, and a dot operation used to create new forms. The specialized component map takes a pair (u_1, u_2) with the first term u_1 from the universe U_1 and the second term u_2 from the universe U_2. It will then retrieve the components of the first term using the component map C_1 from $\mathcal{U}_1$, the components of the second term using the component map C_2 from $\mathcal{U}_2$, and it will then return the union of these components.

The sigma operation works in a similar way in that it works over one ontology, then the other, then merges the results. If we wish to obtain a new object pair, σ is performed on an object pair (u_1, u_2) and the resultant object will contain a new pair of two new terms. The first term would be the result of performing σ on u_1 in ontology $\mathcal{U}_1$, but the operation must be robust enough to work on any object u not necessarily in universe U_1. We thus restrict σ to the universe of objects U_1, and define this restriction to mean that $\sigma|_{U_1}$ is only possible on objects from U_1. The operation $\sigma|_{U_1}$ performed on an object in U_2 would simply be invalid. The operation $\sigma|_{U_2}$ is defined in the same way.

Sum Ontology

It will also be necessary to have defined a sum ontology so that we may create one ontology from many. We begin with the ontologies we wish to sum, each referred to as $\mathcal{U}_k = (U_k, C_k, \cdot_k)$ for some k. The ontology we wish to compute will look like this: $\mathcal{U} = \Sigma_{k=1\ldots n}(\mathcal{U}_k)$ where n is the number of ontologies to be summed.

$$\begin{aligned}
&(5.1.8) & \mathcal{U} &= (\biguplus_{i=1\ldots n} \{\mathcal{U}_i\}, C, \cdot) \\
&(5.1.9) & \text{where } C(u) &= C_k u \text{ if } u \in U_k \\
&(5.1.10) & \text{and } \sigma \cdot u &= \sigma \cdot_k u \text{ if } u \in U_k
\end{aligned}$$

The sum ontology has a universe of objects which is simply a disjoint union of the universes of the subontologies. The component map for the new ontology is defined based on the component maps for the subontologies. The component map from the ontology the object came from is used, and the dot operation also takes its definition from the subontology the object came from.

5.2 Toy Version of Situation Theory

We utilize the toy version of situation theory taken from the lecture notes cited above. In its current format, the theory contains four types of objects: Atoms, Sets, States, and Situations. These will be described below, and then an ontology for each is defined.

- Atoms by definition have no components. An atom *may* be an n-place($n \geq 0$) atomic relation (from the set REL).

  $At = REL \uplus IND \uplus (TIM \uplus LOC \uplus) \ldots$

- Sets are objects of the form $\{t_1, t_2, \ldots\}$ where $t_1, t_2, \ldots$ are elements of the set A of objects. The collection of elements is always small, and for every small collection of objects there is a set with those objects as elements. The notation for sets will be traditional, where $x \in X$ will mean that the object x is in the set X.

- States are objects of the form $\ll r, a_1, \ldots, a_n, \varepsilon \gg$ with r as an n-place atomic relation of the state, $a_1, \ldots, a_n$ as objects called arguments, and $\varepsilon \in \{0, 1\}$.

- Situations are objects which have states 'holding' in them denoted by $s \models w$ for state w holding in situation s. For every set of states, there exists a situation with exactly these states holding in it. If all the states which hold in situation s_1 also hold in s_2 we denote this by $s_1 \sqsubseteq s_2$. We also say that s_1 is a subsituation of s_2, and s_2 is a supersituation of s_1.

$$s_1 \sqsubseteq s_2 \iff \text{if } s_1 \models w \text{ then } s_2 \models w \text{ for all } w$$

At this point we wish to define a set containing as elements each of the four types. Define $\mathcal{T}$ to be the set of types such that $\mathcal{T} = \{atom, set, state, sit\}$. We will now define an ontology for each type.

For the toy version of Situation Theory we are using here, an ontology $\mathcal{U}$ will be outlined by the definition of a subontology for each type. Assume we are given a class At of atom names and pairwise disjoint subclasses (empty intersection) $R_n \subseteq At$ of n-place atomic relation names for $n \in N$. As described in [7], we define four ontologies with the intention of unifying them via summation to create a single ontology. In a formal manner, we define $\mathcal{U}_t$ for $t \in \mathcal{T}$ having already defined $\mathcal{T}$ as the set of the four types.

- $\mathcal{U}_{\text{atom}}$ is the discrete ontology $(At, C, \cdot)$ where $Ca = \emptyset$ for each atom $a \in At$. Also notice that this ontology can be regarded as a sum of ontologies over the subcategories of atom including the relations, individuals, times, places, etc.

- $\mathcal{U}_{\text{set}}$ is the set theoretic ontology $\mathcal{U}_P$ where P is the image closed class $powV$ (image closed $\equiv$ contains all limit points).

- $\mathcal{U}_{\text{state}}$ is the signature ontology $\mathcal{U}_\Omega$ where Ω is the signature having the class of symbols $\bigcup_{n\geq 0}(R_n \times \{0,1\})$ with each symbol in $R_n \times \{0,1\}$ having arity $\{1,\ldots,n\}$.

- Finally $\mathcal{U}_{\text{sit}} = pow\mathcal{U}_{\text{state}}$, where for any Ontology $\mathcal{U} = (U, C_{\mathcal{U}}, \cdot_{\mathcal{U}})$ the ontology $pow\mathcal{U} = (powU, C, \cdot)$ with $\forall u \in powU : Cu = \bigcup\{C_{\mathcal{U}}u' | u' \in u\}$ and if $\sigma : Cu \to V$ then $\sigma \cdot u = \{(\sigma|Cu) \cdot_{\mathcal{U}} u' | u' \in u\}$

Now we can define a sum ontology $\mathcal{U} = \Sigma_{t\in\mathcal{T}}\mathcal{U}_t$ i.e. $\mathcal{U}$ is the ontology $(U, C_{\mathcal{U}}, \cdot_{\mathcal{U}})$ where $U = \{(t,u) | t \in \mathcal{T} \;\&\; u \in U_t\}$ and if $(t,u) \in U$ then $C_{\mathcal{U}}(t,u) = C_{\mathcal{U}_t}u$ and if $\sigma : C_{\mathcal{U}}(t,u) \to V$ then $\sigma \cdot_{\mathcal{U}} (t,u) = \sigma \cdot_{\mathcal{U}_t} u$.

If $\mathcal{A} = (A, \alpha)$ is any full algebra for $\mathcal{U}$ then we get a $\mathcal{U}$-replacement system which can be viewed as a universe of objects for the toy situation theory. This will need to be redone if new types are needed, so that their ontologies may be included in the sum ontology. The class is partitioned into four classes A_t, one for each type t, where $A_t = \{\alpha(t,u)|u \in \mathcal{U}_t[A]\}$. Such a class A is called a model of the situation theory.

To see how the full algebra will be needed we look at the atoms. The syntax of an atom is somewhat imprecise. People may be syntactically referred to with any name, but in this example we will look at the name 'Peter'. Syntactically 'Peter' is an object with a name, but in the model the object must have some meaning. Within the model, our atom 'Peter' can be given a superscript full algebra (like so: $\text{Peter}^{\mathcal{A}}$) to indicate that we have one such full algebra and we are referring to a semantical person, rather than a syntactic name. Now, using the homomorphism of the algebra we can make reference to the semantical 'Peter' object. We form the following definition which would hold true if we substituted any atom for Peter: $\text{Peter}^{\mathcal{A}} = \alpha(\text{atom}, \text{'Peter'})$. Formally:

- The atoms (given $\mathcal{A}$) have the form $at^{\mathcal{A}} = \alpha(\text{atom}, at)$ for $at \in At$. They make up the class A_{atom} which itself consists of a summation of relations, and other atomic objects such as times and places for instance.

- The sets have the form $\alpha(\text{set}, a)$ where $a \in powA$. The symbol $\in$ will be interpreted by the membership relation $\in^{\mathcal{A}} \subseteq A \times A_{\text{set}}$ which is defined by $b \in^{\mathcal{A}} \alpha(set, a) \Leftrightarrow b \in a \in powA$ for all $b \in A$. The sets form the class $A_{\text{set}} = \{\alpha(\text{set}, a)|a \in powA\}$. The interpretation (by the algebra) of sets is as follows: $\{t_1, t_2, \ldots\}^{\mathcal{A}} = \alpha(\text{set}, \{t_1^{\mathcal{A}}, t_2^{\mathcal{A}}, \ldots\})$.

- States are objects which have the form $\alpha(\text{state}, (r, \varepsilon)(t_1, \ldots, t_n))$ for $t_1, \ldots, t_n \in A, r \in R_n$, and $\varepsilon \in \{0, 1\}$. The interpretation for r, the n-place atomic relations of $\mathcal{A}$ will follow the form of atoms. We define $\ll r, t_1, \ldots, t_n, \varepsilon \gg^{\mathcal{A}} = \alpha(\text{state}, (r, \varepsilon)(t_1^{\mathcal{A}}, \ldots, t_n^{\mathcal{A}}))$ for $t_1, \ldots, t_n \in A, r \in R_n$, and $\varepsilon \in \{0, 1\}$. The states are members of the class A_{state}.

- Situations of $\mathcal{A}$ have the form $\alpha(sit, s)$ where $s \in pow(\mathcal{U}_{\text{state}}[A])$ (i.e. s is a set of states). Expanding situation s to its state components, the following is defined: $s^{\mathcal{A}} = \alpha(sit, \{w^{\mathcal{A}}|s \models w\})$. Together all the possible situations form the class $A_{sit} = \{\alpha(sit, s)|s \in powA_{\text{state}}\}$.

The $\models$ symbol will refer to the holding relation $\models^{\mathcal{A}} \subseteq A_{sit} \times A_{state}$ for state w in a situation s. The relation $\models^{\mathcal{A}}$ on $\mathcal{A}$ is semantically defined by the following:

$$(5.2.1) \qquad \alpha(sit, s) \models^{\mathcal{A}} w \Longleftrightarrow w \in s \in pow(\mathcal{U}_{\text{state}}[A]).$$

Since situations will have states holding in them, we will have a necessity for a way to determine if one situation is contained within another. If some situation s_2 will support states $\{a_1, a_2, a_3, a_4\}$, and another situation s_1 supports states $\{a_2, a_3\}$, we can obviously see that the states holding in s_1 are a subset of the states holding in s_2. This makes s_1 a subsituation of s_2, and s_2 a supersituation of s_1. The component map on situations unfortunately returns the objects from within the states which hold in it. Without the ε value we cannot use the component maps to indicate if one situation is contained by another because the situation supporting more states, may have $\varepsilon = 0$ for some states. We compute the interpretation of $s_1 \sqsubseteq^{\mathcal{A}} s_2$ given a full algebra $\mathcal{A} = (A, \alpha)$ formally. The relation is given by: $\sqsubseteq^{\mathcal{A}} \subseteq A_{\text{sit}} \times A_{\text{sit}}$.

$$\begin{aligned}
&(\forall s_1, s_2 \in pow U_{\text{state}}[A]) : \\
&\qquad \alpha(sit, s_1) \sqsubseteq^{\mathcal{A}} \alpha(sit, s_2) \\
\Leftrightarrow &\qquad \alpha(sit, s_1) \models^{\mathcal{A}} w \Rightarrow \alpha(sit, s_2) \models^{\mathcal{A}} w \text{ for all } w \in A_{\text{state}} (\text{by def of } \sqsubseteq) \\
\Leftrightarrow &\qquad w \in s_1 \Rightarrow w \in s_2 \text{ for all } w \in A_{\text{state}} (\text{by def of } \models^{\mathcal{A}}) \\
\Leftrightarrow &\qquad s_1 \subseteq s_2
\end{aligned}$$

So for $s_1, s_2 \in pow A_{\text{state}}$ we have $\alpha(sit, s_1) \sqsubseteq^{\mathcal{A}} \alpha(sit, s_2)$ if and only if $s_1 \subseteq s_2$.

To make reference to a situation existing in our reality we define a situation s as actual if and only if there is a model A, i.e., a full algebra of U, and for every $r \in R_n$ a relation $r^{\mathcal{A}} \subseteq A^n$ so that

1. If $s \models \ll r, t_1, ..., t_n, 1 \gg$, then $(t_1^{\mathcal{A}}, \ldots, t_n^{\mathcal{A}}) \in r^{\mathcal{A}}$
2. If $s \models \ll r, t_1, ..., t_n, 0 \gg$, then $(t_1^{\mathcal{A}}, \ldots, t_n^{\mathcal{A}}) \notin r^{\mathcal{A}}$

5.2.1 A Preliminary Example

Note that any object or item which is not given an object type (i.e. hair, room, etc.) is implicitly an Atom (i.e. it has type At).

Object Types	Real Objects in example
Atoms	Peter $\in$ IND $\subseteq$ Atom in $\in$ REL (2-ary) has $\in$ REL (2-ary) broken $\in$ REL (1-ary) chair $\in$ Atom (0-ary) leg $\in$ Atom (0-ary)
States	$w_1 = \ll$ in, Peter, room, 1 $\gg$ $w_2 = \ll$ in, chair, room, 1 $\gg$ $w_3 = \ll$ has, Peter, $w_4, 1 \gg$ $w_4 = \ll$ broken, leg, 0 $\gg$ $w_5 = \ll$ has, room, stairs $\gg$ $w_6 = \ll$ supports, chair, Peter,1 $\gg$ $w_7 = \ll$ has, Peter, $w_8, 1 \gg$ $w_8 = \ll$ is, hair, brown,1 $\gg$
Situations	$s_1 \models w_1$ $s_2 \models \{w_1, w_2, w_3, w_6, w_7\}$ $s_3 \models \{w_1, w_2, w_6\}$ $s_4 \models w_1, s_4 \models w_2$
Derived Facts:	$s_1 \sqsubseteq s_4 \sqsubseteq s_3 \sqsubseteq s_2$

In this example, the first atom is the person Peter. The other three atoms are simple relations and members of the class REL, a subset of the atoms. The first two, 'in' and 'has', are 2-ary relations so they take 2 atomic objects each. The last atom, 'broken' is 1-ary and thus only accepts one object. There are eight states above. The first w_1 relates the information that 'Peter' is in a room. The information surrounding the room is not identified, but for the intended information to be conveyed, where the room is, or what kind of room it is, may not be necessary. The second state w_2 represents the information that a chair is also in the room. The state w_3 conveys that Peter possesses the information conveyed by the state w_4. The state w_4 claims that the possessor of it, does not have a broken leg, because $\varepsilon = 0$ in this case. The next state w_5 declares that the room has stairs. If supported by a situation w_6, would lead us to conclude that a chair in that situation supports Peter. The second last state w_7 would mean that Peter possesses the last state w_8 meaning that Peter's hair is indeed brown.

Next we are given four situations. It is also important to recognize here, that there implicitly exists a situation for every combination of the above states. Focusing on our named situations, we see that s_1 supports only w_1, or more specifically, in the situation s_1, Peter is in the room. Any situation which supports w_1 can now be viewed as a super-situation of s_1. In particular in our example, if some other situation $s_x \models w_1$ then we can say that $s_1 \sqsubseteq s_x$. Our second situation s_2 supports more states. In s_2 Peter and the chair are in the room, Peter does not have a broken leg, the chair supports Peter, and Peter has brown hair. Of course, nothing is said about Peter's hair in s_1, but this means it could be anything. It should be noted that s_2 supports all of the states holding in each of the other three situations. In the situation s_3, Peter and the chair are in the room, and the chair supports Peter, but we do not know for sure if Peter can sit down as nothing is said about the state of his legs, nor his capabilities in general. The final situation s_4 is only slightly larger than s_1. In addition to Peter being in the room alone, here he has access to a chair as well.

Observe that sets are not used here. They may be utilized in certain relations which necessarily reference a set. For instance the relation 'ancestors of' might be a 2-ary relation between an individual, and a set, where the set contains all the ancestors of the individual.

Semantics of the Example

What is shown above is simply the syntax for describing the situations. The representation however, is still missing. For the representation, we make use of a full algebra $\mathcal{A} = (A, \alpha)$. Observe the representation for the first situation s_1. Given the full algebra we represent the semantic object referencing s_1 with a superscript algebra like so: $s_1^{\mathcal{A}}$. The situation will break down as follows:

$$\begin{aligned} s_1^{\mathcal{A}} &= \alpha(sit, \{w_1^{\mathcal{A}}\}) \\ &= \alpha(sit, \{\alpha(state, (in, 1)(i_1^{\mathcal{A}}, room^{\mathcal{A}}))\}) \\ &= \alpha(sit, \{\alpha(state, (in, 1)(\alpha(atom, i_1), \alpha(atom, room)))\}) \end{aligned}$$

5.3 Addition of Affordances as Objects

This toy version of situation theory lacks an explicit method of describing Gibson's concept of affordances. It may be possible to describe affordances as specific instances of existent objects, but to allow certain dynamics to emerge, we will define affordances explicitly. It may also be a concern that organizing a set of affordances from which an organism must select, deters from Gibson's theory that affordances are perceived with no mediating images in the brain. If we assume however, that the participatory agents have no knowledge of the system, but simply live and act by it, it can preserve the theory of direct perception. Agents do not actively choose from a list of affordances, but the list of affordances is existent nonetheless, just as there are so many opportunities for action on an everyday basis, that we inadvertently select one being ignorant of the list of possibilities as a whole. While the affordances are in fact in the environment, it should be clear that the individuals are also a part of the environment, and do not have knowledge of, or access to, anything they do not perceive directly.

In addition to the four types already given, we define a new type, namely affordances. Each affordance, references exactly three objects: an affordance name Φ which also describes the affordance (e.g. 'sit-able' or 'supports'), s a situation, and i an individual. The affordance action will be from the set of affordance actions AFF, which is part of the atoms (AFF $\subset$ Atoms). The syntax for an affordance will be similar to states. The name or action of the affordance will be first, the situation the affordance resides in next, and finally the individual who might perform the affordance.

- Affordances are objects of the form $\phi = \ll \Phi, s, i \gg$, where ϕ represents the identification, Φ the action or name of the affordance from the set AFF, $s \in A_{\text{sit}}$ represents the minimal situation necessary for the affordance, and $i \in A_{\text{IND}}$ represents some individual capable of performing the affordance. The set of full affordances will be referred to as aff.

When we talk about affordances, we want to include opportunities when they may be performed. Instead of allowing affordances (in addition to states) to hold in situations, we define 'environments' to make reference to a sort of coupling of a situation with a set of affordances. This will not however, be made an explicit object type.

- Environments will have the syntax e and will be defined by their properties using the relation $\vdash$. The set of all environments will be referred to as Env. If we are given that $e \vdash w$ for some state w it will be the case that w is true in e. As a shorthand, we may also refer to an entire set of states holding in an environment like so: $e \vdash \{w_1, \ldots, w_n\}$. This will indicate that each state in the set is true in e. From this, we can use the fact that for any set of states, there exists a situation modeling exactly those states, to refer to an environment modeling a situation directly. If we are given a situation s, $e \vdash s$, will stand for s being modeled by the environment, or more precisely, each of the states modeled by s hold in e. If we are given an affordance ϕ and the fact that $e \vdash \phi$, then we may conclude that ϕ is contained in the environment e. Similarly, we may refer to an entire set such as $e \vdash a$ where a is the set of affordances $a = \{\phi_1, \ldots, \phi_n\}$.

From the above we can see that the following equations hold true:

$$\begin{aligned} e \vdash s \quad &\Leftrightarrow \quad s \models w \Rightarrow e \vdash w \text{ for all states } w \\ e \vdash \{\phi_1, \ldots, \phi_n\} \quad &\Leftrightarrow \quad e \vdash \phi_1 \wedge \ldots \wedge e \vdash \phi_n \end{aligned}$$

We need to ensure that affordances in the environment will be performable within the situation described by the environment. To do this we will define 'proper' environments, and for practical purposes, restrict our attention to those environments only.

- An environment e will be called proper if and only if $e \vdash \ll \Phi, s, i \gg \Rightarrow e \vdash s$ for all affordances $\ll \Phi, s, i \gg$. The set of proper environments will be termed pEnv.

Next, we define an ontology for affordances which will become a member of the set $\mathcal{T}$ of separate types, so that it may be incorporated into the ontology $\mathcal{U}$. An ontology for environments will not be defined as they will not be an object type, nor will they be permitted to stand as components of other objects.

- $\mathcal{U}_{\text{aff}}$ is the generalized signature ontology $\mathcal{U}_{\text{AFF}, \mathcal{U}_{\text{sit}} \times \mathcal{U}_{\text{IND}}}$ where AFF is the signature having the class of symbols $\Phi_1, \Phi_2, \cdots =$AFF $\subseteq$ Atoms with AFF as the affordance names.

The argument $\mathcal{U}_{\text{sit}}$ represents the ontology of situations, and $\mathcal{U}_{\text{IND}}$ represents the ontology on the class of individuals (currently a subset of atoms). These arguments in the generalized signature ontology ensure that the components of an affordance are always pairs of one situation, and one individual, together with an action from the set AFF. Note that the situation, and individual are both the minimal descriptions required for the affordance to be 'do-able'. For instance, if we consider a monkey in the jungle, we could list every possible observable state for that monkey (including the weather, the colour of the monkey's fur, or the monkey's gender, for example), but a situation which only describes that a branch is 'grasp-able' is what will be used to define the affordance. This means that for any affordance $\ll \Phi, s, i \gg$, either s or any supersituation of s will allow for the affordance to be performed.

Remaking the ontology and replacement system

Now our previous definition of an ontology $\mathcal{U}$ made of four sub-ontologies can be extended to include a fifth sub-ontology, affordances. If $\mathcal{U} = \Sigma_{t \in \mathcal{T}} \mathcal{U}_t$ where $\mathcal{T} = \{\text{atom, set, state, sit, aff}\}$ we will get similar definitions as above for the universe of objects U for $\mathcal{U}$, the component map, and σ, however objects will now belong to one of five classes including affordances.

If $\mathcal{A} = (A, \alpha)$ is a full algebra for our new $\mathcal{U}$, we then get a new $\mathcal{U}$-replacement system which can (again) be viewed as a universe of objects for our toy situation theory. The class is now partitioned into five classes (A_t), one for each type $t \in \mathcal{T}$.

To represent the set of possible objects, we now have partitions so that $A = \bigcup \{A_{\text{atom}}, A_{\text{set}},\ A_{\text{state}}, A_{\text{sit}}, A_{\text{aff}}\}$. The set A_{state} for example would be the set of states for the full algebra $\mathcal{A}$.

Semantics of Affordances and Environments

Now that an ontology has been defined for affordances, we assume we have a full algebra which we will call $\mathcal{A} = (A, \alpha)$, to give the semantics. The affordance names $\Phi_1, \Phi_2, \dots \in AFF \subseteq At$ of $\mathcal{A}$ will have the form $\Phi^{\mathcal{A}} = \alpha(\text{atom}, \Phi)$ much like other atoms.

- The affordances (given $\mathcal{A}$) have the form $\ll \Phi, s, i \gg^{\mathcal{A}} = \alpha(\text{aff}, \Phi(s^{\mathcal{A}}, i^{\mathcal{A}}))$ for $\Phi \in A_{\text{AFF}} \subset A_{\text{atom}}, s \in A_{\text{sit}}, i \in A_{\text{IND}}$. All of the affordances together make up the class $A_{\text{aff}} = \{\alpha(\text{aff}, a) | a \in \mathcal{U}_{\text{AFF}}[A]\}$.
- Environments will be members of the class $A_{Env} = A_{\text{sit}} \times pow(A_{\text{aff}})$. An environment will be semantically decribed as follows:

$$e^{\mathcal{A}} = (\alpha(sit, \{w^{\mathcal{A}} | e \vdash w\}), \{\phi^{\mathcal{A}} | e \vdash \phi\}).$$

The 'contained in' relation will have several interpretations since states, sets of states, or situations may stand in relation to an environment. There will need to be a fourth interpretation for affordances contained in environments as well. First, we examine the interpretation with only one state. The interpretation for our 'contained in' relation on environments and states will look like so: $\vdash^{\mathcal{A}} \subseteq A_{\text{Env}} \times A_{\text{state}}$. Alternatively, we may refer to a set of states, indicating each element of the set holds in the environment. This interpretation will be defined similar to above, but with the powerset of A_{state} instead. Formally: $\vdash^{\mathcal{A}} \subseteq A_{\text{Env}} \times pow(A_{\text{state}})$. Third, if we wish to determine if a situation s is contained in an environment (i.e. $e \vdash s$), we would need to discover if for any state w, if $s \models w$ then $e \vdash w$. The interpretation for our 'contained in' relation $\vdash$ on environments and situations is given by $\vdash^{\mathcal{A}} \subseteq A_{\text{Env}} \times A_{\text{sit}}$.

$$(\alpha(sit, s), as) \vdash^{\mathcal{A}} w \Leftrightarrow w \in s \in pow(\mathcal{U}_{\text{state}}[A])$$

For the derived notion $e \vdash s$ we obtain

$$(\alpha(sit, s_1), as) \vdash^{\mathcal{A}} \alpha(sit, s_2) \Leftrightarrow s_2 \subseteq s_1$$

To show that an affordance ϕ is contained in an environment e, we write $e \vdash^{\mathcal{A}} \phi$ but the superscript algebra will be dropped when there is no confusion. Similar to the above equation, $\vdash$ for affordances is interpreted as follows: $\vdash^{\mathcal{A}} \subseteq A_{\text{Env}} \times A_{\text{aff}}$. Lastly, we also extend this definition to allow for an entire set of affordances to be contained in an environment: $\vdash^{\mathcal{A}} \subseteq A_{\text{Env}} \times pow(A_{\text{aff}})$.

$$(\alpha(sit, s), as) \vdash^{\mathcal{A}} \phi \Leftrightarrow \phi \in as$$

Furthermore, we must examine the property of an environment being called proper. Environment e is proper if for every affordance in the environment, the situation component of the affordance must be 'contained in' ($\vdash$) the environment. The class of all proper environments will be termed A_{pEnv} and by the equations below we can see that $A_{\text{pEnv}} \subseteq A_{\text{Env}}$.

The environment e where $e \vdash^{\mathcal{A}} s_1$ and $e \vdash^{\mathcal{A}} a$ with $a = \{\phi_1, \ldots\}$ as a set of affordances, is 'proper' if

$$e = (\alpha(sit, s_1), as) \in pEnv \Leftrightarrow s_2 \subseteq s_1 \text{ for all } \alpha(\text{aff}, \Phi(s_2, i)) \in as$$

5.3.1 A Full Example

Note that any object or item which is not given an object type (i.e. hair, room, etc.) is implicitly of the type Atom.

Object Types	Real Objects in example
Atoms	Peter $\in$ IND $\subseteq$ Atom in, has $\in$ REL (2-ary) broken $\in$ REL (1-ary) chair, leg $\in$ Atom (0-ary)
States	$w_1 = \ll$ in, Peter, room, 1 $\gg$ $w_2 = \ll$ in, chair, room, 1 $\gg$ $w_3 = \ll$ has, Peter, $w_4, 1 \gg$ $w_4 = \ll$ broken, leg, 0 $\gg$ $w_5 = \ll$ has, room, stairs, 1 $\gg$ $w_6 = \ll$ supports, chair, Peter, 1 $\gg$ $w_7 = \ll$ has, Peter, $w_8, 1 \gg$ $w_8 = \ll$ is, hair, brown,1 $\gg$
Situations	$s_1 \models w_1$ $s_2 \models \{w_1, w_2, w_3, w_6, w_7\}$ $s_3 \models \{w_1, w_2, w_6\}$ $s_4 \models \{w_1, w_2\}$
Affordances	$\phi_1 = \ll$ sit, s_3, Peter $\gg$ $\phi_2 = \ll$ see, s_4, Peter $\gg$
Environments	$e \vdash s_2$ $e \vdash \{\phi_1, \phi_2, \ldots\}$
Known Facts:	$s_1 \sqsubseteq s_4 \sqsubseteq s_3 \sqsubseteq s_2$

Here we would like to simply take the time to describe what a particular affordance being present really means. If an individual Peter possesses the affordance $\ll$ sit, Peter, $s_3 \gg$, it means that Peter may 'sit' in the situation s_3 or any supersituation thereof.

Now that the affordances are introduced, possibilities for action are more readily accessible. What we cannot yet do however, is claim that an individual possesses an affordance without the use of a state containing a relation such as 'has'. To make identifying actions an individual may take easier, we must revisit the notion of an individual.

5.4 The Niche and the Individual

With affordances and environments defined, actions and when they may be performed are much easier to determine. What we are missing is a method of attributing an affordance, (or set of possible affordances) to an individual. We do not want to rely on the relation 'has' (as we began to in the example) to indicate that an individual has an opportunity for action, as it would require a very large number of states. Any organism, according to Gibson will be a member of some affordance relations, which correspond to actions it may take. Unfortunately in our toy example of Situation Theory, individuals had been placed as a subset of the atoms, which by definition have no components. We will require each individual to make reference to a niche which is the only concept of Gibson's that has yet to be implemented here. We therefore refine the definition of individuals by creating their own object type, so that they may contain components, and we define an ontology for niches as well, for use in the new ontology for individuals. A niche will contain affordance actions only, rather than full affordances. According to Gibson, a niche is a group of individuals capable of performing the same actions. This could be modeled in the full Situation Theory by types of individuals placed in the environment. This approach requires anchors, parameters, etc. Since we are only working with the toy example of Situation Theory in this thesis we have chosen a different but equivalent way to model a niche. The type above can be defined externally as all the individuals with the same niche, i.e., for a given set n we choose $\{\ll x, n_1, n_1' \gg | \; n_1 = n\}$. Niches in our example can be seen as the type of all individuals that can perform the same actions though types are not actually used. We believe a niche represented as a set of affordance actions, adheres to what Gibson described[29].

- A niche represents a set of affordance names. The first set $n = \{\Phi_1, \ldots\}$ $\Phi_i \in AFF$ in an individual corresponds to affordance actions which the individual is capable of performing according to his/her physical ability. This set is not dependent on any situation or even current properties of the animal. Even though an eagle may not currently be able to fly because of a broken wing, it has the physical proportions, and requirements to do so once it has healed, so the affordance action 'fly' would be in the eagle's niche n. This set will not change for any given individual. We wish to emphasize that niches consist of affordance actions or names, but not entire affordances. Membership of affordance actions in niches, or in the niche n of an individual, will be defined by the $\in$ relation.

- The second niche n' of the individual references the affordances currently available for enaction. We will be able to see that n' will necessarily always be a subset of n. It is fairly intuitive that actions you can currently perform are a subset of all actions you could ever perform in the future. If an eagle breaks its wing, the action 'fly' will be removed from n', and when the eagle regains its ability to fly, it will be added back.

- Each individual i will reference three components: a name x, a niche set n, and a second niche n', the set of the affordances currently performable. They will appear in the general form, $i = \ll x, n, n' \gg$. Once defined we will be able to write $\Phi \in i$ to mean that the individual i possesses the affordance action Φ in its niche n, or $\Phi \in' i$ to mean that i has Φ in its niche n' meaning it is currently able to perform the affordance.

We have now redefined individuals to be their own new object type rather than a subset of atoms. The set of individuals will now be referred to as Ind, differing from the previously defined IND by two lowercase letters. That is, IND will contain individual names, whereas Ind will contain full individuals with component niches and names. An individual, according to this new object type will be seen like so: $i = \ll x, n, n' \gg$ where $x \in \text{IND}$ is a name or reference for the individual, n is the set of all the affordances the individual is capable of performing upon ever being in a situation allowing it, and having the will to do so, and n' is the set of actions which the individual could presently enact due to spatial and temporal restrictions, as well as capabilities.

Once individuals have been given components, particularly the two niches, certain affordances might be invalid. While the language allows for an affordance $\ll \text{fly}, s, i_1 \gg$ with the individual defined as $i_1 = \ll$ 'Peter',{sit, stand, hear, see, walk, $\ldots\}, \{$hear, see, walk, $\ldots\} \gg$ to exist, we need to ensure only certain affordances are valid. The example just given should indeed be considered to be invalid since i_1 does not refer to an individual which is capable of flight. To be performable, the individual the affordance relates to, must possess the action of the affordance in their niche. Syntactically an affordance is now expanded to accommodate for individuals with components. An affordance will now look like this when the individual is expanded: $\phi = \ll \Phi, s, \ll x, n, n' \gg\gg$ where $\Phi \subseteq$AFF refers to the action of the affordance. In a practical application, only affordances which are performable in some possible situation need be considered so we restrict our attention to these affordances. They will satisfy the following equation and will be termed *proper affordances* with the set of them being called pAff.

$$\ll \Phi, s, i \gg \text{ is proper } \Leftrightarrow \Phi \in' i$$

Formally we will need an ontology for the individual but not explicitly for niches as only individuals will be an object type. Niches will not be permitted to stand in arbitrary argument roles. An ontology for niches will still be required though, as it will be used in the specification of the individual, due to the restriction of components.

- For niches, we give the ontology $\mathcal{U}_{\text{niche}} = pow(\mathcal{U}_{\text{AFF}})$ where we borrow the definition of $pow\mathcal{U}$ for $\mathcal{U}$ from the ontology of situations. $\mathcal{U}_{\text{AFF}}$ is the discrete ontology $(A_{\text{AFF}}, C, \cdot)$ where $Ca = \emptyset$ for each affordance $\Phi \in A_{\text{AFF}} \subseteq A_{\text{Atom}}$.
- $\mathcal{U}_{\text{Ind}}$ is the generalized signature ontology $\mathcal{U}_{\text{IND}, \mathcal{U}_{\text{niche}} \times \mathcal{U}_{\text{niche}}}$ where IND represents the signature with class of symbols as names of individuals and $IND \subseteq$ Atom. Individuals will not have arbitrary object components, but simply two niche components and a name component.

After defining the ontology for individuals we need to add the new type to the set of types $\mathcal{T}$. Individuals will be the last type we add, so the final definition of the set of object types is as follows:

$$\mathcal{T} = \{\text{atom, set, state, sit, aff, Ind}\}.$$

By the same process as before, we define $\mathcal{U}$ to be the sum ontology over the ontologies for each of the types in $\mathcal{T}$ which now includes individuals. If $\mathcal{A} = (A, \alpha)$ is any full algebra for $\mathcal{U}$ we get a $\mathcal{U}$-replacement system which we view as our universe of objects. The class A (and the domain of α) is partitioned into six classes for the six types. The semantics of the niche and the individual which follow must, as usual, be based on a full algebra $\mathcal{A}$.

- An individual will have the form $\ll x, \{\Phi_1, \ldots\}, \{\Phi'_1, \ldots\} \gg^{\mathcal{A}} = \alpha(\text{Ind}, x(\{\Phi_1^{\mathcal{A}}, \ldots\}, \{\Phi'_1 A, \ldots\}))$. The class of individuals each with names and niches will be called A_{Ind}.

As well, we interpret the relation $\in$ between affordance actions, and individuals like so: $\in^{\mathcal{A}} \subseteq A_{\text{AFF}} \times A_{\text{Ind}}$. Formally: $\Phi \in^{\mathcal{A}} i \Leftrightarrow \Phi \in^{\mathcal{A}} n \Leftrightarrow \Phi \in n$ for

$i = \ll x, n, n' \gg$ where $\in$ without a reference algebra is the set membership relation. In addition, to indicate that an affordance action is in the niche n' of an individual i, we provide an interpretation for $\in'$: $\in'^{\mathcal{A}} \subseteq A_{\text{AFF}} \times A_{\text{Ind}}$. Formally, $\Phi \in'^{\mathcal{A}} i \Leftrightarrow \Phi \in'^{\mathcal{A}} n' \Leftrightarrow \Phi \in n'$ for $i = \ll x, n, n' \gg$.

It will also be necessary to judge if an affordance action is in a niche. Once again we must be given a full algebra $\mathcal{A} = (A, \alpha)$. The membership relation for affordance actions in niches is given in the same way as membership in a set and is defined as follows: $\Phi^{\mathcal{A}} \in^{\mathcal{A}} \alpha(Ni, n) \Leftrightarrow \Phi \in n \in pow(A_{\text{AFF}})$.

Proper Affordances

The semantic meaning of proper affordances will be interpreted as follows:

- An affordance $\alpha(\text{aff}, \Phi(s, i))$ is proper if and only if $\Phi \in'^{\mathcal{A}} i$.

Essentially, for an affordance to be proper, the animal which possesses the affordance must also have the corresponding affordance action in their niche n'. To restrict the scope of possible affordances in an environment, we say that to be supported by a proper environment an affordance must be proper as well.

$$pEnv \vdash \phi \Rightarrow \phi \text{ is proper}$$

Since it is obvious that any proper affordance is also simply an affordance, we acknowledge that the class of proper affordances, denoted by pAff, will be a subset of the class of affordances. That is: $A_{\text{pAff}} \subseteq A_{\text{aff}}$.

5.4.1 Redefinition of our Ontology

Now that we have concluded our additions to the theory, we can now redefine our replacement system $\mathcal{U} = \Sigma_{t \in \mathcal{T}} \mathcal{U}_t$. With the addition of the 'individual' type, the set $\mathcal{T}$ is now defined like so: $\mathcal{T} = \{$ atom, set, sit, state, aff, Ind $\}$. The class of objects A is now partitioned into our six types, and isomorphism α will have a different definition for each of the four original types and each of the two newly identified types.

5.4.2 Updated Example

Object Types	Real Objects in example
Niches	$n_1 = \{\text{see, hear, sit, grasp}, \ldots\}$ $n'_1 = \{\text{see, hear, sit}, \ldots\}$
Individuals	$i_1 = \ll \text{Peter}, n_1, n'_1 \gg \in A_{\text{Ind}}$

5.5 Dynamics of the System

What we have described so far, has made it possible to describe situations, their components, and particularly affordances. What is missing is the ability to model how a situation changes to another, how one environment changes to the next, and how states are formed or modified.

First, to make the dynamics simpler we remind the reader of the idea of an affordance being in an environment. Some affordance ϕ will be in some environment e if it is given that $e \vdash \phi$. Equivalently for a set of affordances, $e \vdash \phi \Leftrightarrow \phi \in a \wedge e \vdash a$ where $a = \{\phi, \ldots\}$.

Now we will introduce some dynamics by the introduction of the $\rightarrow$ or 'yields' relation on environments. It will enable one environment to yield a new environment through the enaction of an affordance. If in some environment, an individual has some affordance (in its niche n'), and if that individual chooses to enact the affordance, it will cause the environment to shift to a new one. The yields relation is based on a function f called the enacting function. Formally, we define

$$(e_1 \rightarrow e_2) \Longleftrightarrow (\exists \phi)(e_1 \vdash \phi \wedge f(e_1, \phi) = e_2).$$

The function f takes one proper environment, and a proper affordance, and returns a new proper environment where the affordance has just been enacted. This $f : \text{pEnv} \times \text{pAff} \rightarrow \text{pEnv}$ is actually a partial function so that $f(e, \phi)$ is defined if and only if $e \vdash \phi$, i.e., the affordance ϕ is enactable in the given environment.

5.5.1 Non-deterministic yields relation

If Peter is standing in front of a blue chair and a red chair, he may choose to sit in either, or neither, thus making the yields relation non-deterministic. An algebraic example follows:

Object Types	Real Objects in example
Atoms	near $\in$ REL (2-ary) sitting on $\in$ REL (2-ary) supports $\in$ REL (2-ary) 'Peter' $\in$ IND $\subset$ Atom blue chair, red chair $\in$ Atom
Ind	$i_1 = \ll$ 'Peter'$, n_1, n'_1 \gg \in Ind$
Niches	$n_1 = \{see, hear, sit, grasp, \dots\}$ $n'_1 = \{see, hear, sit, \dots\}$ $n'_2 = \{see, hear, stand, \dots\}$
States	$w_1 = \ll$ near, i_1, blue chair, 1 $\gg$ $w_2 = \ll$ near, i_1, red chair, 1 $\gg$ $w_3 = \ll$ supports,blue chair,i_1,1 $\gg$ $w_4 = \ll$ supports,red chair,i_1,1 $\gg$ $w_5 = \ll$ sitting-on,i_1,blue chair,1 $\gg$ $w_6 = \ll$ sitting-on,i_1,red chair,1 $\gg$
Situations	$s_1 \models \{w_1, w_3\}$ $s_2 \models \{w_2, w_4\}$ $s_3 \models \{w_1, w_2, w_3, w_4\}$ $s_4 \models w_5$ $s_5 \models w_6$
Affordances	$\phi_1 = \ll$ sit$, i_1, s_1 \gg$ $\phi_2 = \ll$ sit$, i_1, s_2 \gg$ $\phi_3 = \ll$ stand$, i_1, s_4 \gg$ $\phi_4 = \ll$ stand$, i_1, s_5 \gg$
Environments	$e_1 \vdash s_3$ $e_1 \vdash \{\phi_1, \phi_2, \dots\}$ $e_2 \vdash s_4$ $e_2 \vdash \{\phi_3, \dots\}$ $e_3 \vdash s_5$ $e_3 \vdash \{\phi_4, \dots\}$
Enacting Functions	$f(e_1, \ll$ sit$, i_1, s_1 \gg) = e_2$ $f(e_1, \ll$ sit$, i_1, s_2 \gg) = e_3$
Known Facts:	$s_1 \sqsubseteq s_3$ and $s_2 \sqsubseteq s_3$

$\text{sit} \in \text{AFF} \;\&\; \text{sit} \in n_1$

Our prototypical person Peter finds himself situated near two differently coloured chairs, either of which is capable of supporting him. He may sit on the blue chair, thus engaging the yields relation $e_1 \rightarrow e_2$, or he may sit on the blue chair, engaging the yields relation $e_1 \rightarrow e_3$. The fact that Peter is given these alternatives illustrates that the yields relation itself is non-deterministic. Also note that $e_1 \vdash s_3$, $s_1 \sqsubseteq s_3$, and $s_2 \sqsubseteq s_3$ ensuring that the affordances are proper and enactable.

5.5.2 Yields not always confluent

One question that may be asked of the yields relation is whether or not it is confluent. Confluence is a property of rewriting systems which dictates that if given two choices, taking either of them still allows you to arrive in some situation which was possible to arrive at via the other choice. If the yields relation were to be confluent, any action we take would not rule out any other action from been taken. Intuitively we know that at least some choices we make rule out other choices from ever being possible, but an example is provided after the formal definition, to illustrate how yields is not confluent.

First take $\rightarrow^*$ to be the reflexive, transitive, closure on the yields ($\rightarrow$) relation. A relation $\rightarrow$ is called confluent if for all x, y_1, y_2 the properties $x \rightarrow^* y_1$ and $x \rightarrow^* y_2$ imply that there is a z with $y_1 \rightarrow^* z$ and $y_2 \rightarrow^* z$. This definition can be visualized by the following diagram:

$$\begin{aligned} \text{If } (\forall x, y_1, y_2)[(x \rightarrow^* y_1) \quad &\wedge \quad (x \rightarrow^* y_2)] \text{ then} \\ (\exists z)[(y_1 \rightarrow^* z) \quad &\wedge \quad (y_2 \rightarrow^* z)] \end{aligned}$$

$$\begin{tikzcd} & x \arrow[dl, "*"'] \arrow[dr, "*"] & \\ y_1 \arrow[dr, "*"'] & & y_2 \arrow[dl, "*"] \\ & \exists z & \end{tikzcd}$$

A confluent relation will be deterministic in all terminating cases, a property that we do not expect from the possible futures. Through an example we will demonstrate that the relation is in fact not always confluent. If Peter holds a match, and is sitting in front of two firecrackers, he has several options. He could light the match, or not, but upon lighting it he might watch it burn out, or light one firecracker, or the other. If he lights the match, and chooses to ignite the red firecracker, it will burn away, just as the blue firecracker would. When he makes the decision to ignite one of the firecrackers, he is entering an environment where it will be impossible to ignite the other firecracker. If he lights the red one, he has no other matches to light the blue one, and vice versa. Admittedly, he may also watch the match slowly burn away, but he still has no manner of igniting either firecracker. This example scenario illustrates how the yields relation is not always confluent.

Object Types	Real Objects in example
Atoms	near, grasping, ignite $\in$ REL (2-ary) burnt $\in$ REL (1-ary) red firecracker $\in$ Atom blue firecracker $\in$ Atom match $\in$ Atom 'Peter' $\in$ IND $\subset$ Atom
Ind	$i_1 = \ll \text{'Peter'}, n_1, n'_1 \gg \in \mathit{Ind}$
Niches	$n_1 = \{\text{see, sit, grasp, strike, ignite} \ldots\}$ $n'_1 = \{\text{see, grasp, strike, ignite,} \ldots\}$
States	$w_1 = \ll \text{near}, i_1, \text{red firecracker}, 1 \gg$ $w_2 = \ll \text{near}, i_1, \text{blue firecracker}, 1 \gg$ $w_3 = \ll \text{grasping}, i_1, \text{match}, 1 \gg$ $w_4 = \ll \text{lit, match}, 1 \gg$ $w_5 = \ll \text{burnt, match}, 1 \gg$ $w_6 = \ll \text{burnt, red firecracker}, 1 \gg$ $w_7 = \ll \text{burnt, blue firecracker}, 1 \gg$
Situations	$s_1 \models \{w_1, w_2, w_3\}$ $s_2 \models \{w_1, w_2, w_3, w_4\}$ $s_3 \models \{w_1, w_3, w_4\}$ $s_4 \models \{w_2, w_3, w_4\}$ $s_5 \models \{w_5, w_6\}$ $s_6 \models \{w_5, w_7\}$

	$s_7 \models w_3$
Affordances	$\phi_1 = \ll \text{strike}, s_1, i_1 \gg$ $\phi_2 = \ll \text{ignite}, s_2, i_1 \gg$ $\phi_3 = \ll \text{see}, s_3, i_1 \gg$ $\phi_4 = \ll \text{see}, s_4, i_1 \gg$
Environments	$e_1 \vdash s_1$ $e_1 \vdash \{\phi_1, \ldots\}$ $e_2 \vdash s_2$ $e_2 \vdash \{\phi_2, \ldots\}$ $e_5 \vdash s_5$ $e_5 \vdash \{\phi_3, \ldots\}$ $e_6 \vdash s_6$ $e_6 \vdash \{\phi_4, \ldots\}$
Enaction Functions	$f(e_1, \ll\text{strike}, s_7, i_1 \gg) = e_2$ $f(e_2, \ll\text{ignite}, s_3, i_1 \gg) = e_5$ $f(e_2, \ll\text{ignite}, s_4, i_1 \gg) = e_6$
Known Facts:	$s_7 \sqsubseteq s_1 \sqsubseteq s_2$ $e_2 \vdash s_3$ and $e_2 \vdash s_4$

Here we identify a situation we begin with. Peter is presently holding a match, and near to two different coloured firecrackers. In this example something such as their distance apart, is such that Peter cannot light both firecrackers with the single match he possesses. This can be derived from the fact that our functions detailing the shifting of environments, show that after lighting one firework, the match is burnt, thus Peter cannot light another. We assume all environments and affordances are proper. Finding himself in e_1, we suppose Peter chooses to enact his 'strike' affordance, shifting environments to e_2. We highlight that $e_1 \rightarrow e_2$. He then finds himself in e_2 where he may now ignite one or the other firecracker. If he chooses to light the red one, he will be in environment e_5, if he chooses to light the blue one, he will be in environment e_6. Algebraically: $f(e_2, \ll\text{strike}, s_3, i_1 \gg) = e_5$ and $f(e_2, \ll\text{strike}, s_4, i_1 \gg) = e_6$. Once he has made his choice, he will find that he has no matches which are unburnt and thus he has no way of igniting the other firecracker. In short, environment e_2 may yield environment e_5 or environment e_6, but not both. Neither e_5 nor e_6 can yield any environment (call it e_{imp}) in which both e_5 and e_6 had occurred.

$$\begin{tikzcd}
& e_2 \arrow[dl] \arrow[dr] & \\
e_5 \arrow[dr, dotted] & & e_6 \arrow[dl, dotted] \\
& e_{imp} &
\end{tikzcd}$$

The regular arrows indicate the ability for e_2 to yield e_5 and e_6, but the dotted arrows indicate the fact that e_y is an unobtainable environment.

5.5.3 Examining how n' changes

In the previous examples, the niche n' never changed as the individuals never lost any of their affordance actions. Though it is not a relatively frequent occurrence, sometimes an animal will either enact an affordance incorrectly, or enact an affordance whose result was not expected. To illustrate this we extend the previous example. Imagine that when Peter finds himself in e_2, he cannot decide which firework to light, or else does not wish to light either. In any case, he enacts a 'waiting' affordance which allows the lit match to burn his finger. The partial function may be applied as it is shown in the table, thus leading Peter into environment e_7 where most notably, he has a burnt finger. The burning of his finger causes his niche of currently available actions n'_1 to change to n'_2 where strike, and ignite are no longer performable due to his burnt finger. Once his burn heals to a point where he may strike, or ignite, these affordance actions are replaced into his niche of currently available actions.

Object Types	Objects in Example
Atoms	burnt (1-ary) has (2-ary)
Niches	$n'_1 = \{\text{see, grasp, strike, ignite}, \dots\}$ $n'_2 = \{\text{see, grasp}, \dots\}$
States	$w_8 = \ll \text{has, Peter}, w_9, 1 \gg$ $w_9 = \ll \text{burnt, finger}, 1 \gg$
Situations	$s_7 \models \{w_3, w_5, w_8\}$

Affordances	$\phi_5 = \ll wait, s_2, i_1 \gg$
Environments	$e_2 \vdash \{\phi_2, \phi_5, \ldots\}$ $e_7 \vdash s_7$
Enaction Functions	$f(e_2, \ll \text{wait}, s_2, i_1 \gg) = e_7$

Chapter 6

Conclusion and Future Work

The formalization of affordances provided conforms to the writings of James Gibson and thus the notion which he envisioned. By the addition of an affordance type, individuals which are no longer atomic, may check their niches to decide if certain actions are possible. If the individual is capable of performing an action, perceives that the environment surrounding it allows for that affordance to take place, and the individual wishes to perform the action, he or she may then do so. According to the formalization presented, the change of environment and its supported situation is governed by the enacting function, and the yields relation.

The work done in this thesis involves only a toy version of situation theory, and thus may be extended to include all notions of situation theory. Here types, parameters, and anchors were not taken advantage of because of their complexity. Utilizing the 'type' type, higher order types could be devised, and representing the type of all people, or the type of any set of objects or individuals who can be uniquely identified is possible. Niches in this alternate model would be defined differently. In addition, the dynamics provided here do not come close to describing all methods of one environment yielding a new one. The dynamics could be modified so that instead of an external function f, one could have a possible future relation entailing situations or environments which are, and which are not, possible futures from a current situation.

6.1 States which affect Yields Relation

Since we are attempting to model the real world, we must admit that the number of relations, atoms, sets, states, etc., is infinite. New operations $\in REL$ may be available in the future which do not exist now. If the partial function f determines possible source and target environments, an example of a relation which might impact f, is the 'possible future situation' relation. This relation nested in a state given $\epsilon = 1$, would describe possible situations which are attainable from the current situation, while a value of $\epsilon = 0$ would indicate the situations which are unattainable. If a state such as $\ll$ possible future situation, $s_{imp}, \{\Phi_1, \ldots, \Phi_n\}, 0 \gg$ holds in a situation s_1 it must not be possible for $s_1 \rightarrow^* s_{imp}$. That is, s_1 cannot yield the impossible situation s_{imp} in any number of repeated applications of the yields relation. It must also be the case that this state remains valid in future situations which result from this one. It is worthwhile to note that the inclusion of the list of possible affordances in this impossible situation allows the extraction of an environment from the state.

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Appendix A

Variable Names

Table A.1: Naming Conventions

Variable	**What it represents**
at/At	$At = REL \uplus IND \uplus \ldots$
$a, b, \ldots$	arbitrary objects in A
$t, t_1, \ldots$	arbitrary objects in A
$w, w_1, \ldots$	states/infons
$s, s_1, \ldots$	situations
$e, e_1, \ldots$	sets
$n, n', n_1, n'_1 \ldots$	niches
$\phi, \phi_1, \ldots$	complete affordances
$\Phi, \Phi_1, \ldots$	affordance action names
$r, r_1, \ldots$	n-place relations
A_x	class A restricted to objects of type x
$\mathcal{A} = (A, \alpha)$	full algebra = (class, homomorphism)
ε	polarity (0 or 1)

Table A.2: Relations

Relation	**What relation is on**
$\in$	'in' relation to indicate an object is a member of a set
$s \models w$	'holds' relation used for state w holding in situation s
$e \vdash w$	'contained in' relation for state w in environment e
$e \vdash \phi$	'contained in' relation for affordance ϕ in environment e
$e \vdash s$	'contained in' relation for situation s in environment e
$\Phi \in n$	'in' relation for affordance action Φ in niche n
$\Phi \in i$	'in' relation for affordance action Φ in niche n of individual i
$\Phi \in' i$	'in' relation for affordance action Φ in niche n' of individual i

Table A.3: Class Names

Class	**Contents of Class**
V	class of all sets
atom	atoms
sit	situations
set	sets
state	states
Ind	individuals
REL	relations
AFF	affordance names
aff	full affordances
Env	Environments
pAff	proper affordances
pEnv	proper environments