The Information Content of Canadian Implied Volatility Indexes

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Abstract

For predicting future volatility, empirical studies find mixed results regarding two issues: (1) whether model free implied volatility has more information content than Black-Scholes model-based implied volatility; (2) whether implied volatility outperforms historical volatilities. In this thesis, we address these two issues using the Canadian financial data. First, we examine the information content and forecasting power between VIXC — a model free implied volatility, and MVX — a model-based implied volatility. The GARCH in-sample test indicates that VIXC subsumes all information that is reflected in MVX. The out-of-sample examination indicates that VIXC is superior to MVX for predicting the next 1-, 5-, 10-, and 22-trading days' realized volatility. Second, we investigate the predictive power between VIXC and alternative volatility forecasts derived from historical index prices. We find that for time horizons lesser than 10-trading days, VIXC provides more accurate forecasts. However, for longer time horizons, the historical volatilities, particularly the random walk, provide better forecasts. We conclude that VIXC cannot incorporate all information contained in historical index prices for predicting future volatility.

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1. INTRODUCTION

Accurately forecasting future volatility plays a central role in financial markets for asset pricing in general and particularly important for derivatives, portfolio construction, and risk management. Academics and practitioners have developed a variety of methods to predict future volatility. These methods can be loosely classified into two groups. One uses various econometric models to forecast future volatility from historical asset prices. These models include ARCH/GARCH (autoregressive conditional heteroskedasticity /generalized autoregressive conditional heteroskedasticity), EWMA (exponentially weighted moving average), and long memory ARFIMA (autoregressive fractionally integrated moving average), Riskmetrics, etc.

Another group uses option prices to forecast future volatility. This kind of volatility is called implied volatility because it is implied from option prices. Two types of implied volatility are widely used in practice: model-based and model free implied volatility. The former relies on a specific option pricing formula, such as Black-Scholes (B-S) (1973)/Merton (1973), Hull and White (1987), or Heston (1993), etc. Given a specific formula and all its parameters except for the volatility and equating the observed option price to its model value, one can solve for volatility.

Finance academics have questioned assumptions of some option formulas, e.g., B-S assumes a constant volatility of the price of underlying asset over the life of an option, Hull and White (1987) assumes no risk premium for stochastic variance of assets prices, and the effect of taxes is usually ignored in option models.

The information set of option market is larger than the information set of the underlying asset market, because option traders consider not only the past information but also the probability of relevant events that can happen in the future. From theory aspects, the implied volatility should be more efficient than any of volatility forecasts derived from historical asset prices. Efficient, here, means that all information about future volatility has been incorporated in the implied volatility.

The empirical finding that implied volatility can not subsume all information embedded in historical prices may be due to option model misspecifications. Several studies recommend the model free measure of implied volatility (Britten-Jones and Neuberger, 2000; Jiang and Tian, 2005, among others). The so called model free implied volatility is not conditional on any option pricing formula. It can be computed from a set of option prices directly. In theory, the model free measure is better than model-based one, as the model free approach extracts future volatility information from various options. The model-based measure, however, uses only close to-the-money options.

Most leading organized exchange markets have issued implied volatility indexes, which represent the market consensus estimate of future monthly volatility. In February 1993, the CBOE (Chicago Board Options Exchange) introduced a B-S model-based volatility index calculated from the S&P 100 index (OEX) options with ticker symbol VIX. In September 2003, the CBOE issued a model free volatility index based on the S&P 500 index (SPX) options. The original VIX was replaced by VXO. Throughout the paper, we use VXO and VIX to refer to implied volatility index calculated from OEX and SPX options, respectively. Following the CBOE, the Germany Deutsche Börse introduced VDAX in May 1994; the French Marché des Options Négociables de Paris created two volatility indexes, VX1 (short-term index to capture future 31 calendar day's volatility) and VX6 (long-term index to capture future 185 calendar day's volatility) in October 1997.

In Canada, the Montréal Exchange disseminates the B-S model-based MVX (Montréal volatility index) since December 2, 2002. The construction of the MVX follows the methodology for computing the CBOE VXO index. The iShare S&P/TSX 60 index fund options are used for computing MVX. As of October 15, 2010, the Montréal Exchange uses a new volatility index, VIXC (volatility index in Canada) to replace MVX. The VIXC, a model free implied volatility, is computed by the same methodology as the one of the CBOE VIX and is based on the S&P/TSX 60 index options. The S&P/TSX 60 is a list of the 60 largest companies on the Toronto Stock Exchange as measured by market capitalization. The VIXC estimates the next 30-calendar (22-trading) day's volatility in Canadian stock market. The main objective for such a change is to introduce options and futures on this implied volatility index.

A large number of studies examine the forecast accuracy of implied volatility for future volatility in diverse option markets. Mixed results have been found whether implied volatility outperforms forecasts based on historical asset prices, such as

ARCH/GARCH volatility, Riskmetrics volatility, etc. In addition, it is inconclusive as to whether B-S model-based implied volatility outperforms model free implied volatility in terms of forecasting future volatility. For instance, Jiang and Tian (2005) find that the model free volatility produces more accurate forecast than the B-S implied volatility. Andersen, Frederiksen, and Staal (2007), however, document that the CBOE VIX is less accurate for forecasting future volatility than the B-S implied volatility.

This thesis has three objectives. First, we aim to examine the forecast ability of the VIXC to forecast future volatility in the Canadian stock market. Second, we will examine whether the VIXC outperforms prediction methods based on historical asset prices, which include random walk, GJR-GARCH, and Riskmetrics. Third, we will compare forecasting ability between the recently introduced VIXC and MVX.

This study has several potential contributions. First, to the best our knowledge, no study examines the forecast ability of the implied volatility for the Canadian stock market. The predictive ability of the VIXC for future realized volatility will provide useful implications for participants in the Canadian stock market.

Second, given conflicting results in previous studies about the ability to forecast future volatility by using different methods, this thesis contributes to this stream of literature by providing new evidence from the Canadian stock market.

Third, no study examines whether model-based implied volatility outperforms model free implied volatility in a relatively less liquid option market. The Canadian stock market provides an excellent stage for such an examination. Because both MVX and VIXC are disseminated by an official exchange, the Montréal Exchange, the measurement errors produce the least effect for statistical inference. This thesis provides empirical evidence for such a comparison.

2. LITERATURE REVIEW

We define time series forecasts of volatility as a volatilities that are computed from historical returns of underlying asset with various econometric models, such as ARCH/GARCH (Hamiliton, 1994), Riskmetrics (1996), lagged standard deviation (STD), stochastic volatility (SV), random walk, long memory ARFIMA (Hamiliton, 1994), etc.

A large body of literature examines the relationship between implied volatility and future volatility outcome. The general conclusion is that implied volatility is biased and is superior to time series forecasts of volatility (Poon and Granger, 2003, 2005). However it is not clear that whether implied volatility is efficient (Poon and Granger, 2003).

2.1 Implied volatility derived from stock index options

2.1.1 The S&P 100 index options

Previous empirical literature reports mixed results on the predictive power between implied volatility and time series forecasts of volatility. A few studies document that implied volatility is a biased and inefficient estimator for future volatility. Moreover, the time series forecasts of volatility are more informative than implied volatilities. Day and Lewis (1992) examine information content of volatility implied from the S&P 100 index call options from 1983 to 1989. Their time series volatilities were constructed from EGARCH/GARCH model and lagged STD. The time series forecasts of volatility have incremental information for future volatility beyond that contained in implied volatility, and vice versa. Their conclusions, however, suffered from a maturity mismatching problem (Christensen and Prabhala, 1998). They examine the weekly forecasting ability of implied volatility derived from options with a longer life (up to 36 trading days). Canina and Figlewski (1993) examine the same index from 1983 to 1987. They argue that implied volatility does not contain incremental information for future volatility with the presence of time series volatility. The lagged STD is their proxy for time series volatility and their results show that the lagged STD has higher correlation to future volatility. In contrast, the implied volatility is weakly correlated to future volatility. Therefore, the lagged STD is superior to the implied volatility for forecasting purposes. Their findings, however, suffer from telescoping effects (Christensen, Hansen, and Prabhala, 2002).

The above conclusions may suffer measurement errors as well. They all use closing prices of call options to construct implied volatilities. As the option and stock market do not close at the same time, the non-simultaneous prices between index and option markets cause negative first order serial correlation in the implied volatility (Harvey and Whaley, 1991; Hentschel, 2003). Furthermore, implied volatility derived from call options is significantly different from the one from put options (Harvey and Whaley, 1992). Therefore, the use of only call options may introduce more measurement errors into implied volatilities.

Fleming, Ostdiek, and Whaley (1995) examine the CBOE VXO from 1986 to 1992. They find that the CBOE VXO is a biased but efficient forecast for future volatility. Fleming (1998) comes to the same conclusion by examining OEX call and put options separately from 1985 to 1992. The implied volatility outperforms several types of time series volatility for predicting future volatility.

Christensen and Prabhala (1998) examine the OEX options with a longer period from 1983 to 1995. They criticize the sampling procedure used by Canian and Figlewski (1993). They use non-overlapping monthly samples for their regression tests. They conclude that implied volatility is biased but contains incremental information for future volatility beyond that is revealed by lagged STD. Their results indicate that implied volatility is more accurate in predicting future volatility after 1987. In addition, after adjusting measurement errors with instrumental variables, they found that implied volatility is an unbiased and efficient estimator. The lagged STD does not have any incremental information.

Christensen, Hansen and Prabhala (2002) find that OEX option market is efficient. Their testing period spans from 1993 to 1997. In contrast to Christensen and Prabhala (1998), Christensen, Hansen, and Prabhala (2002) find that implied volatility is biased. Doran and Ronn (2005) use the same sampling procedure and the same test methodology used by Christensen and Prabhala (1998). With a longer examination interval from 1986 to 2004, they indicate that the unbiasedness finding of Christensen and Prabhala (1998) depends on time periods under examination. Instrument variables used by Christensen and Prabhala (1998) cannot help to reduce the bias associated with implied volatility.

Corrado and Miller (2005) document that the CBOE VXO is a biased but efficient estimator of future volatility. Their examination period spans from 1988 to 2003. The CBOE VXO incorporate all future volatility information contained in the lagged STD.

Blair, Poon, and Taylor (2001) examine the forecasting ability of the CBOE VXO for time horizons: 1-, 5-, 10-, and 20-day, from 1987 to 1995. They use high frequency 5-minute intra daily returns to compute daily realized volatility. Their in-sample results show that time-series volatilities derived from 5-minute OEX returns have no incremental information that is reflected in VXO. Out-of-sample tests showed that the VXO provides more accurate forecasts than alternative time-series volatilities. The forecast from the combination of VXO and time-series volatilities can not improve forecast accuracy significantly. The CBOE VXO almost includes all the information contained in time-series volatilities.

Contrary to the finding that implied volatility is superior to time series forecasts of volatility, Koopman, Jungbacker, and Hol (2005) find that the CBOE VXO contains less information than daily realized volatility for predicting the next day's volatility in the period from 1997 to 2003. Their long memory ARFIMA model and unobserved ARMA components model produce the best and second best forecast, respectively. Furthermore, the R-squared values from regressing subsequent realized volatility on GARCH+RV and GARCH +IM forecasts, 0.605 and 0.419, respectively, clearly indicate the superiority of realized volatility to implied volatility.

Consistent with Koopman, Jungbacker, and Hol (2005), Corrado and Truong (2007) find that the time series volatilities constructed from the Parkinson (1980) approach appears to dominate the CBOE VXO. Their test period spans from 1990 to 2006. They augment the standard GJR-GARCH (Glosten, Jagannathan, and Runkle, 1993) model by including two exogenous variables: the daily implied volatility, denoted by GJR-GARCH+IM, and Parkinson (1980) volatility, denoted by GRJ_GARCH+Rng. Their results show that the GJR-GARCH+Rng specification produces less forecasting errors than does the GJR+IM specification across horizons: 1-, 10-, and 20-day forecasts.

In summary, most recent studies find that volatility implied from the S&P 100 index options is superior to time series volatility and is efficient for forecasting purpose. However, most studies only compare implied volatility to a few kinds of time series volatility, such as ARCH/GARCH volatility and lagged STD. It is obvious that different forecast approaches have unequal ability to capture future volatility information. We argue that the efficiency conclusion should be based on comprehensive comparisons between implied volatility and a wide range of time series volatilities, e.g., comparing implied volatility with long memory ARFIMA forecast, GARCH forecast, EWMA forecast, etc. In next section, we review the findings with respect to the S&P 500 index options.

2.1.2 The S&P 500 index options

Andersen and Bollerslev (1998) introduce the "realised/integrated" measure of volatility, denoted by RV, which uses the summation of high frequency intraday squared assets returns to proxy the unobserved true volatility. Henceforth, a number of studies use RV as comparison benchmark and show that forecast accuracy improves significantly (e.g., Poteshman, 2000; Martens and Zein, 2004). The R-squared value from regression tests is significantly improved when researchers use RV as the proxy of the true latent volatility. The conclusion regarding unbiasedness, however, is still uncertain.

Poteshman (2000) examines the forecast ability of SPX options from 1988 to 1997. He uses non-overlapping samples and constructs implied volatility series from Black-Scholes (1973) model. He finds that implied volatility is a biased but efficient estimator for future monthly volatility. The extent of biasedness decreases in a sequential manner when he constructs the latent volatility with daily index close prices, daily future close prices, and 5-minutes future prices, respectively. In addition, he uses Heston (1993) model to derive implied volatilities in the period from 1993 to 1997. His multivariate regression tests, which include implied and lagged monthly STD as explanatory variables, show that implied volatility is an unbiased and efficient predictor for future monthly volatility. His conclusion that variance risk should be priced is consistent to that of Chernov (2001), Bakshi and Kapadia, (2003), among others.

Shu and Zhang (2003) use Christensen and Prabhala (1998) sampling skills and use 5-minute index returns to construct realized volatility in the sample period from 1995 to 1999. Unlike Poteshman (2000), they argue that although implied volatility derived from either Black-Scholes (1973) or Heston (1993) model is an efficient estimator of future volatility, the Wald test indicates that both kinds of implied volatility are biased. In addition, the Black-Scholes volatility is better than Heston (1993) volatility for forecasting purpose. These two studies come to different conclusions regarding whether variance risk should be priced. One reason may due to the proxy of comparison benchmark. Poteshman (2000) constructs realized volatility from 5-minute future returns; Shu and Zhang (2003) use 5-minute index returns. The R-squared values from regressing RV on lagged RV suggest that RV built from future returns is more accurate that RV built from index returns. Another reason may due to the different methodology used to estimate the parameters of the Heston (1993) model. The estimated parameters of Heston (1993) model in these two studies are significantly different.

Several studies also find that implied volatility is a biased but efficient estimator for future volatility. For instance, Corrado and Miller (2005) examine CBOE VIX from 1990 to 2003. They use STD that is constructed from daily index returns as the latent volatility. The lagged STD and GJR-GARCH volatility are alternative approaches for forecasts. Szakmary, Ors, Kim, and Davidson III (2003) use Bridge implied volatility derived from Black (1976) model to examine forecast efficiency of SPX options in period from 1983 to 2001. They use STD to stand for the latent volatility. The lagged STD and GARCH volatility are two alternatives of time series forecasts of volatility. Noh and Kim (2006) use Black-Scholes (1973) model to derived implied volatility in the sample period from1994 to 1999. They examine forecasting ability between B-S implied volatility and a set of lagged realized volatility constructed from 10-, 30-, 60-, 120-minute, and daily index returns. Besides finding that implied volatility is an efficient estimator, they also find larger R-squared value resulting from multivariate regression when higher frequency returns are used to compute realized volatility. This finding is consistent to that of Andersen and Bollerslev (1998), Poteshman (2000), Shu and Zhang (2003).

Jiang and Tian (2005) examine prediction accuracy of three measures: model free, Black-Scholes, and lagged daily realized volatility from 1988 to 1994. They find that model free implied volatility outperforms both the Black-Scholes and lagged daily realized volatility and is an efficient estimator. Regressing realized volatility against model free, Black-Scholes, and lagged daily realized volatility shows that the mode free volatility contains all information embedded in the other two volatilities. In addition, the

Black-Scholes volatility is better than the lagged daily realized volatility for forecasting future volatility, but is inefficient.

Busch, Christensen, and Nielsen (2006) find implied volatility is unbiased and is efficient in the period from 1990 to 2002. They use monthly non-overlapping options and compute implied volatility from the modified Black (1976) formula. The 5-minute returns of SPX futures are used to construct comparison benchmark. The lagged realized volatility was viewed as a time series forecast of volatility.

The studies mentioned above in this section all show that implied volatility derived from SPX options is superior to time series volatilities and is an efficient predictor for future volatility. These studies draw their conclusions on comparison between implied volatility and a few time series forecasts of volatilities. The scenario may change if one compares implied volatility with a wide range of time series volatilities, e.g., long memory ARFIMA volatility, ARMA volatility, etc. We list some contrary findings next.

Martens and Zein (2004) contend that the implied volatility derived from SPX options is an inefficient predictor for future volatility. Their test spans 1994 to 2000. The latent volatility is constructed from the summation of 5-minute intraday and 30-minute intra-night returns. Their results show that implied volatility contains all information embedded in GARCH volatility. However, with the presence of long memory ARFIMA volatility as an extra explanatory variable, they find that ARFIMA volatility does contain incremental information. The best forecasting accuracy can be obtained by combining the implied and ARFIMA volatility.

Becker, Clements, and White (2006) argue that the CBOE VIX can not incorporate all the volatility information that is embedded in several time series forecasts of volatility. Thus, VIX is an inefficient estimator. Their data spans from 1990 to 2003. Their time series forecasts of volatility include: GARCH, GJR-GARCH, SV, ARMA, ARFIMA, and EWMA. The subsequently realized volatility is built from 30-minute SPX index returns.

Becker, Clements, and White (2007) examine the CBOE VIX information content from 1990 to 2003. They use a set of time-series forecast of volatilities, including GARCH, SV, ARFIMA, MIDAS, and lagged RV. They first decompose the VIX into two components: one has information captured by all time-series volatilities; the other one has the residual information. These two components are orthogonally constructed by regressing VIX onto the space spanned by all time-series forecasts of volatilities. They then examined whether the second component has a relation with the future realized volatility. They showed that VIX does not have any incremental information that is captured by these time-series volatilities.

Corrado and Truong (2007) enlarge standard GJR-GARCH model by including Parkinson(1980) volatility and the daily CBOE VIX. By setting restriction on the coefficients of these two exogenous variables, the model fitness can be examined by the likelihood ratio. With the presence of CBOE VIX, the significance of other coefficients of the GJR-GARCH model indicates that the CBOE VIX does not contain all the information embedded in historical prices. Further, the “P” value (Blair, Poon, and Taylor, 2001), RMSE (root mean square error), and MAE (mean absolute error) from out-of-sample forecasts all indicate that the Parkinson (1980) volatility weakly outperforms the CBOE VIX.

With the similar methodology of Corrado and Truong (2007), Nishina, Maghrebi, and Holmes (2006), Maghrebi, Kim, and Nishina (2007) find that the CBOE VIX does not fully incorporate future volatility information embedded in SPX historical prices.

Andersen, Frederiksen, and Staal (2007) investigate the forecast quality of the CBOE VIX and B-S implied volatility in the SPX option market in the period from 1990 to 2002. The time series forecast of volatility is constructed from long memory ARFIMA model. They use non-overlapping monthly samples and construct realized volatility with 5-minute SPX index returns. Among these three volatilities, only ARFIMA forecast is unbiased. Although all these volatilities have significant information for future volatility, the ARFIMA volatility outperforms both B-S implied volatility and the CBOE VIX. Furthermore, the B-S implied volatility outperforms the CBOE VIX. However, neither the ARFIMA volatility nor the B-S volatility contains all the information for future volatility. The combination of implied and ARFIMA volatility produces the best forecast of future volatility.

In summary, although the conclusion with respect to forecasting efficiency appears to be controversial, we argue that the implied volatility from SPX option prices probably is not an efficient estimator of future volatility. Neither of the studies that come to the efficiency conclusion compare long memory ARFIMA forecast with implied volatility.

These studies normally use lagged STD and/or GARCH kinds of volatility to stand for the competing alternatives to implied volatility. These time series volatilities cannot capture all the information about future volatility from historical asset prices. When high frequency intra day returns are used to construct long memory ARFIMA volatility forecast, implied volatility can not incorporate all information embedded in ARFIMA volatility. The most accurate forecast may be obtained by the combination of implied and time series forecasts of volatility, as suggested by Andersen, Frederiksen, and Staal (2007).

2.1.3 The NASDAQ 100 index options

Simon (2003) documents that implied volatility is biased but outperforms two alternatives of implied volatility: GJR-GARCH and EWMA forecasts of volatility. He uses monthly STD as the latent volatility. His test spans from 1995 to 2002. When regressing the level of volatility on VXN (Volatility index of NASDAQ) and its competitors, either GJR-GARCH or EWMA volatility, the VXN incorporates all information contained in the time series volatilities. When replacing the level of implied volatility by its first difference, VXN can not fully incorporate information embedded in EWMA volatility. The GJR-GARCH volatility still do not have incremental information.

In contrast to the biasedness finding, Corrado and Miller (2005) indicate that the VXN is roughly unbiased. They also find VXN outperforms alternative forecasts derived from historical index prices. The latent volatility is built from STD as well. Their test spans from 1995 to 2003. Corrado and Truong (2007) examine VXN forecasting efficiency from 1997 to 2006. They also document that VXN dominates the time series forecasts of Parkinson (1980) volatility.

In summary, these three studies find that the CBOE VXN outperforms its alternatives. The CBOE VXN has significant information with regard to future volatility. In next section, we review the findings for non-US stock index options.

2.1.4 The non-US stock index options

In contrast to the US index options, non-US index options are commonly traded with relatively smaller volume. We use the ratio of total notational value of stock index options to total market capitalization in a country to infer its option market liquidity. At the end of 2009, the ratio is 0.5597 for the second largest stock market in the world, Japan Tokyo Stock Exchange (TSE); the 4th largest stock market, NYSE Euronext (Europe) is of 0.5597; the 7th largest market, Hong Kong Exchanges is of 0.2710; the 8th largest stock market, Canada TSX Group is of 0.00109. Quite different from these low ratios, in the US, the ratio is about 1.3449 (see statistical reports of World Federal of Exchanges, http://www.world-exchanges.org). Figlewski (1997) suggests that the more liquid the option market is, the more efficient the option prices. The illiquidity feature of non-US option markets may cause their option prices to be inefficient and introduces more error-in-variables problems.

Moraux, Navatte, and Villa (1999) assess the predictive ability of the French volatility index (VX1) in the period from 1994 to 1998. They found daily VX1 has a substantial amount of information for the next day's volatility. The lagged monthly STD has no incremental information with the presence of implied volatility. Although the VX1 is biased, it is an efficient predictor of future volatility.

In addition, they create two other implied volatility series from the VX1. One has 2 week maturity; another has 2 month maturity. The 2-week implied volatility is biased but has substantially more information about future volatility than forecast by the lagged STD. This 2-week implied volatility, however, does not fully incorporate the information contained in lagged STD. With respect to the 2-month implied volatility, it has less information for future volatility and the lagged STD contains all the information embedded in this 2-month implied volatility for future volatility forecast.

In the Hong Kong option market, Fung (2007) concludes that implied volatility derived from Hang Seng (HS) index options is biased and an efficient predictor for future volatility. He uses non-overlapping monthly samples from 1993 to 2000. He constructs several alternatives of implied volatility which include lagged STD, option volume, option open interest, future volume, future open interest, and arbitrage basis of index future. With the presence of implied volatility, regression test shows that all alternatives have no incremental information for future volatility.

In the same market, Yu, Lui, and Wang (2010) come to the same conclusions. Their test covers the period of 1998 to 2005. Unlike Fung (2007), Yu, Lui, and Wang (2010) examine volatility implied from both OTC (over-the-counter) options and exchange market options. They construct two alternatives of implied volatility: lagged STD and GARCH volatility. Both implied volatilities contain all the information embedded in lagged STD and GARCH volatility.

In the Australian option market, Frijns, Tallau, and Tourani-Rad (2010a) construct Australian implied volatility index (AVX) from the S&P/AVX 200 index options in the period from 2002 to 2006. The construction of the AVX is similar to that of the CBOE VXO. Unlike the VXO that has a constant 22 trading day's maturity, the AVX has a 66 trading day's maturity. RiskMetrics and GJR-GARCH volatility are used to stand for the competing forecasts of implied volatility; STD is used as the benchmark for comparison. Their tests show that although AVX is a biased estimator, it has substantial amount of information for future volatility and is superior to its two alternatives for all forecasting horizons. The AVX, however, does not contain all the information embedded in either Riskmetric volatility (for 1-, 5-, 10-day forecast horizons) or in GJR-GARCH volatility (for 22-, 66-day forecast horizons). Thus, the AVX is not an efficient predictor for future volatility.

Frijns, Tallau, and Tourani-Rad (2010b) reassess their results with a longer sample period from 2002 to 2008. The regression test with 22-trading day forecast horizon shows that the AVX incorporates all information embedded in its two alternatives, Riskmetrics and GARCH volatility. The AVX is an efficient predictor for future volatility. The conflicting finding on forecast efficiency may arise from the different predictive power between the standard GARCH and the GJR-GARCH model. When return – volatility exhibits an asymmetric relationship, the asymmetric GARCH model outperforms the standard GARCH model for forecast purposes (Hansen and Lunde, 2005; Awartani and Corradi, 2005).

In the Taiwan option market, Hung, Tzang, and Hsyu (2009) examine the forecast power of volatility implied from Taiwan stock index options in the period from 2004 to 2007. They expand the standard GJR-GARCH model with other variables: Parkinson (1980) volatility, Taiwan VIX, and/or Taiwan VXO. The Taiwan VIX and Taiwan VXO are constructed with similar approach to that of the CBOE VIX and VXO, respectively. Their large 500-days in-sample test shows that Parkinson range-based volatility dominates the two implied volatility indexes.

Unlike Hung, Tzang, and Hsyu (2009), Wong and Tu (2007) find that the volatility implied from Taiwan index options has a substantial amount of information for future volatility, and outperforms lagged realized volatility. Their data spans from 2002 to 2004. They construct implied volatility index following the methodology of the CBOE VXO. Their realized volatility is constructed from 5-minute Taiwan index returns. Although implied volatility outperforms lagged realized volatility for forecasting purpose, the lagged realized volatility contains incremental information beyond that embedded in the volatility index. This suggests that the Taiwan option market is not efficient.

In the Indian option market, Panda, Swain, and Malhotra (2008) examine the forecast ability of implied volatility derived from both call and put of the S&P CNX Nifty index options from 2001 to 2004. With monthly non-overlapping sampling procedure, they conclude that both implied volatilities are efficient predictors for future volatility, although they all are biased. Both implied volatilities from call and put options contain all the information embedded in alternative volatility forecast, the lagged STD. Kumar (2010) comes to similar results with the Indian volatility index (IVIX) from 2007 to 2009. The daily VIX contains a substantial amount of information for forecasting the next day's volatility.

In the Danish option market, Hansen (2001) examines forecast ability of volatility implied from the Danish KFX share index options from 1995 to 1999. He uses monthly STD to stand for the true latent volatility and lagged STD as the alternative forecast. He found that the implied volatility contains a substantial amount of information for future volatility and outperforms the lagged STD. With the use of instrumental variables, he argues that implied volatility is an unbiased and an efficient indicator for future volatility.

In the UK option market, Noh and Kim (2006) document that the implied volatility derived from FTSE 100 index options is a biased and inefficient estimator for future volatility in the period from 1994 to 1999. Several comparison benchmarks are used in their tests, which include realized volatility constructed from 10-, 30-, 60-, 120-minute, and daily frequency returns on FTSE 100 futures. When comparison benchmark is built from high frequency returns, they find that the lagged realized volatility outperforms the implied volatility for monthly forecasts, although the implied volatility does contain useful information as well. Unlike Noh and Kim (2006), Wong and Tu (2007) find that FTSE 100 implied volatility outperforms lagged realized volatility for monthly forecast. One explanation is that Noh and Kim (2006) construct realized volatility from future returns, while Wong and Tu (2007) use index returns to construct realized volatility. The different comparison benchmarks may cause these two studies to come to contrary conclusions (Chang, Cheng, and Fung 2010).

Areal (2008) comes to the same conclusion to that of Noh and Kim (2006). Areal (2008) uses UK data in the period from 1993 to 2000. He constructs seven volatility indexes: one model free index; three indexes based on out-of-money options; three B-S indexes. The R-squared value and loss function (heteroskedasticity root mean squared errors) all show that the lagged realized volatility build from 5-minute returns of FTSE 100 futures outperforms various implied volatility indexes when forecasting monthly volatility. Among these indexes, the model free index performs the worst while the others produce similar results. He ascribes the low quality of the model free index to insufficient option data available in the UK option market.

In the German option market, Claessen and Mittnik (2002) evaluate the forecast quality of the implied volatility derived from German DAX index options from 1992 to 1995. The latent volatility is assumed to be the standard deviation over the remaining life of the options. They use several time series volatilities, including lagged STD, GARCH/EGARCH, EWMA, and random walk, as the alternatives of implied volatility. The in-sample fitness and out-sample tests that are based on loss functions all suggest that implied volatility dominates its diverse competitors. The information in historical index prices is contained in option prices. Thus, the German option market is efficient. Consistent with their findings, Muzzioli (2010) assesses the information content of DAX index options from 2001 to 2005. She uses STD to stand for the true volatility as well. Her results show that B-S implied volatility is a biased but efficient predictor for future volatility. The volatility information embedded in GARCH, AR, or lagged STD is incorporated in B-S implied volatility.

In the Korean option market, Maghrebi, Kim, and Nishina (2007) construct an implied volatility index based on the KOSPI200 index options in the period from 1997 to 2006.

They follow the same methodology as that of the CBOE for constructing VIX. The monthly STD is chosen as the proxy of latent volatility. By incorporating implied and/or contemporaneous volatility into standard GJR-GARCH model, they find that the implied volatility contains incremental information beyond that embedded in historical index prices. The significance of estimated parameters of GJR-GARCH specification, however, points out that the implied volatility index does not contain all the information for future volatility that is reflected in historical prices.

In the Japanese option market, Nishina, Maghrebi, and Holmes (2006) construct the Nikkei-225 model free implied volatility index in the period from 1990 to 2004. They follow the same methodology that is used by the CBOE to construct the VIX. Employing the GJR-GARCH model enlarged with Nikkei 225 volatility index, they argue that implied volatility contains a substantial amount of information about future volatility. The implied volatility, however, does not contain all the information embedded in historical prices. Their out-of-sample P value (Blair, Poon, and Taylor, 2001) suggests that the Nikkei-225 volatility index outperforms its competitors for forecasting future volatility. Yu, Lui, and Wang (2010) verify that the implied volatility derived from Nikkei-225 index options is superior to both lagged STD and GARCH volatility for forecast purposes. They use implied volatility provided by Bloomberg in the period from 1998 to 2005. In addition, they suggest that this implied volatility index is an efficient predictor for future volatility. Their conclusions are based on regression tests that use non-overlapping monthly samples.

In summary, non-US implied volatility indexes appear to be biased. These indexes, however, outperform a set of time series forecasts of volatility and appear to be efficient. The finding that the implied volatility derived from non-US index options is an efficient estimator for future volatility may be due to the test methodologies used in these studies. One main drawback of these studies is that no one constructs efficient time series forecasts of volatility. Since the S&P 500 option market is more actively traded, we argue that it may be more efficient than these non-US option markets. However, a number of studies document that the SPX option market is not efficient. Therefore, the finding of option market efficiency in non-US financial market is puzzling. Next, we review the findings on currency option markets.

2.2 Implied volatility derived from currency options

Most studies document that implied volatility derived from currency options is superior to other commonly used competitors, such as lagged STD, ARCH/GARCH, and/or EWMA. The reason seems to be that currency options are easily hedged with their underlying assets (Figlewski, 1997). The frictions in currency markets produce less effect than they do in stock market (Jorion, 1995). Whether the implied volatility fully subsumes all information contained in historical returns, however, is not conclusive. Employing high-frequency intra day data in conjunction with an efficient time series model (e.g., long memory ARFIMA model) casts dubitation.

Jorion (1995) examines the information content of three currency options traded in the CME market from 1985 to 1992. He finds that the implied volatility has a substantial amount of information for future volatility and outperforms its two time-series-volatility competitors: MA (moving average) and GARCH volatility. Although the implied volatility is biased, it is an efficient predictor for future volatility.

Xu and Taylor (1995) examine the forecasting ability of implied volatility from four exchange rates: Pound, Mark, Yen, and Franc in the PHLX market from 1985 to 1991. They find that implied volatility outperforms alternative forecasts, such as forecast from GARCH volatility. The in-sample test suggests that the implied volatility for the Pound, the Mark, and the Franc is an efficient predictor for next day's volatility. The GARCH volatility does not contain additional information for future volatility. In their out-of-sample tests, they find that the implied volatility forecast significantly outperforms time series forecasts of volatility, such as lagged STD, GARCH volatility.

Chang and Tabak (2010) examine the information content of volatility implied from the dollar-real exchange rate from 1999 to 2002. They construct three kinds of time-series volatility forecasts: GARCH, MA, and EWMA. Their results show that the implied volatility is a biased but efficient predictor. The time-series forecasts do not have incremental information for future volatility with the presence of implied volatility. Meanwhile, the univariate tests also show that implied volatility is more informative than time-series forecasts of volatility.

A number of studies document that the implied volatility has a substantial amount of information related to future volatility and outperforms alternative time series forecasts.

However, the implied volatility does not contain all the information embedded in historical returns.

Li (2002) examines the forecasting ability of OTC implied volatility from several currency options: the Mark, the Yen, and the Pound. Employing high frequency intra day data simultaneously with a long memory ARFIMA model, he contends that the long memory ARFIMA forecast contains incremental volatility information beyond that is revealed by the implied volatility across currency options and forecast horizons. The implied volatilities are biased as well. The long-memory ARFIMA volatility outperforms implied volatility when forecasting horizons are over 3 months. He suggests that the best forecast for future volatility can be obtained by combining time series forecasts and implied volatility. His results imply that the currency option market is inefficient. We, however, argue that his findings may be due to test methodology because the results are based on overlapping samples. The overlapping sample procedure favors to time series volatility in regression test (Christensen and Prabhala, 1998; Christensen, Hansen, and Prabhala, 2002)

Martens and Zein (2004) examine the information content of currency options on Yen/$US from 1996 to 2000. The proxy for the true latent volatility is the summation of high-frequency intra day squared returns; two time-series volatilities were constructed: GARCH and ARFIMA. The GARCH volatility forecast was outperformed by implied volatilities. The long memory ARFIMA model, however, has almost the same or even higher ability to forecast future volatility than the implied volatility. Furthermore, the long memory model does not contain all the information revealed by implied volatility, and vice versa. They suggest that the optimal forecast can be obtained by combining these two forecasting approaches.

Pong, Shackleton, Taylor, and Xu (2004) examine the forecasting ability of implied volatility from three currency options: the pound, mark, and yen, against the US dollar, from 1987 to 1998. The alternatives of implied volatility are ARMA, ARFIMA and GARCH volatility forecasts. The benchmark of the true latent volatility is realized volatility constructed from 5-minut returns. The time-series volatilities have incremental information beyond that contained in the implied volatility across three markets in short forecasting horizons. The ARMA and ARFIMA produce better forecasts than implied volatility for the pound and yen when forecasting one day and one week volatility. For one month and three month horizons, implied volatility outperforms both ARMA and ARFIMA forecasts. With longer horizon, the implied volatility incorporates most of the future volatility information of historical volatilities.

Charoenwong, Jenwittayaroje, and Low (2009) examine the information content of four currency options traded in OTC, the CME, and the PHLX market from 2001 to 2006. They document that implied volatility outperforms a number of time series forecasts of volatility across markets. The forecast ability, however, appears to decrease when forecast horizon increases. Furthermore, the implied volatilities derived from these four options across markets do not always contain the volatility information embedded in historical prices.

Neely (2009) assesses the forecast quality of four currency options quoted in the CME from 1987 to 1998. He constructs the latent realized volatility from 30-minutes returns. The time series forecasts of volatility are built from ARIMA, long memory ARFIMA, GARCH, and OLS regression models. Except for the Japanese Yen, there is bias in the Pound, Franc, Mark options. Regression results show that the implied volatility derived from the Pound, the Franc, or the Mark options does not incorporate all information contained in historical prices.

In summary, implied volatility derived from currency options outperforms a number of its competitors for forecasting future volatility. Whether it can incorporate all volatility information embedded in historical prices is not conclusive. Next, we review the findings on individual stock/future options.

2.3 Implied volatility derived from individual stock/future options

A number of studies evaluate the forecast quality of volatility implied in individual stock/future options. In general, the forecast quality relies significantly on whether stock/future options are traded actively. The implied volatility derived from options that are actively traded produces more accurate forecasts for future volatility. More over, time horizons play an important role regarding forecasting accuracy.

Lamoureux and Lastrapes (1993) examine the predictive power of implied volatility from 10 individual stock options for the period from 1982 to 1984. The implied volatility is derived from Hull and White (1987) option model. They use GARCH and lagged STD volatility as the two competitors of implied volatility. Both in-sample and out-of-sample tests show that the implied volatility outperforms time series forecasts of volatility but does not fully incorporate all the information from GARCH or lagged STD forecasts. Their results suggest that the option market is not efficient. The best forecasts can be obtained by combining implied and time series forecasts of volatility. They explain their results as the rejection of the joint hypothesis of market efficiency and the correctness of Hull and White (1987) model when variance risk is non-priced.

Taylor, Yadav, and Zhang (2007) assess the information content of implied volatilities with respect to 149 firms from 1996 to 2000. The alternative of the implied volatility is GARCH volatility. Parkinson's (1980) measure of volatility is used to stand for the true latent volatility. In one-day-ahead prediction, the forecast of time series volatility outperforms the implied volatility over one third of the firms. With the longer forecasting horizons, however, implied volatility is more informative than historical volatilities for 126 out of 149 firms.

Szakmary, Ors, Kim, and Davidson III (2003) evaluate the forecasting ability of implied volatilites derived from 35 future options traded in 8 separate exchange markets. They use 30 day moving-average STD and GARCH volatility to stand for time series forecasts of volatility. They find that implied volatility is a biased forecast for future realized volatility but contains more information than historical volatility. In their encompassing regression tests, the hypothesis that historical volatility has no information beyond that in option prices is rejected.

Brous, Ince, and Popova, (2010) examine the forecast ability of implied volatilities from 92 stock options over the period 1996 to 2006. These underlying stocks are constitutes of the S&P 100 index. They construct four types of time series forecasts of volatility which include AMAD (adjust mean absolute return), Parkinson (1980), STD, and Garman and Klass (1980) volatility. For comparison purpose, the true latent volatility is represented by STD. The Black-Scholes implied volatility is adjusted with volume information and has a constant 30-trading day's maturity. They find that on average the implied volatility produces the worst forecast among these five forecast measures. However, when they group stocks by volume or market capitalization, the scenario is changed. On the one hand, on average, the implied volatility derived from high liquid options produces the best accuracy; on the other hand, on average, all time series forecasts produce the most accurate forecast comparing to implied volatilities derived from low liquid options.

Mayhew and Stivers (2003) and Godbey and Mahar (2005) document that implied volatilities derived from high liquid options outperform the time series forecasts of volatility. The former study uses 50 firms that have the largest option trading volume on the CME market over the period 1988-1995; the latter use 460 firms that are the constitutes of the S&P 500 index over the period from 2001 to 2002.

In summary, a number of studies show that the liquidity features of stock options play a key role as to the predictive power of implied volatility. In general, the implied volatility is superior to its competitors, e.g., lagged STD, GARCH volatility, for forecasting future volatility. However, for options with low liquidity, the opposite is true.

2.4 Predictive power summary

An inspection of the findings on diverse option markets indicates that the implied volatility appears to contain more information for future volatility. The information embedded in time series volatilities may be subsumed by implied volatility. In addition, we find that most studies that come to the efficiency conclusion either use noisy daily data to construct the true latent volatility or do not attempt to build efficient time series forecasts of volatility. The simple forecasting measures based on historical prices may contribute to the finding that implied volatility is an efficient estimator for future volatility. In the next section, we discuss the theory behind the use of ATM (at-the-money) options to compute the implied volatility.

3. THEORETICAL FOUNDATION FOR THE DERIVATION OF B-S IMPLIED VOLATILITY FROM OPTION PRICES

Academics developed a variety of option pricing models. The most commonly used model to derive implied volatility is the Black and Scholes (1973)/Merton (1973) model. Merton (1973) extended Black and Scholes (1973) model with the consideration of dividend payments. Black and Scholes (1973) assumed the underlying asset follows a geometric Brownian motion with constant volatility, as shown with equation (1).

$$dS_t = \mu S_t dt + \sigma S_t dW \tag{1}$$

where S_t is the stock price at time t, μ is the mean return of stock prices, σ is a constant volatility during the life of an option, dW is a geometric Brownian motion.

It may be inappropriate to assume constant spot asset return volatility. It is well known that the asset return variance is of time varying, stochastic process. Thus, the implied volatility derived from Black and Scholes (1973) may deviate from the true spot return volatility. This can be a source of measurement error due to the option pricing specification (see Harvey and Whaley 1991, 1992, among others)

Hull and white (1987) develop an option pricing model assuming the underlying asset has a stochastic volatility. Let P refer to the option price on the underlying asset S which has the following stochastic process (equation (1) and (2) in Hull and White (1987) paper are repeated here):

$$dS_t = \emptyset(S_t, V_t, t) S_t dt + \sigma_t S_t dW \tag{2}$$

$$dV_t = \mu(V_t, t) V_t dt + \xi(V_t, t) V_t dZ \tag{3}$$

where S_t is stock price at time t, V_t is the variance, σ_t is the time t stock volatility, dW and dZ are two Winer processes. The variable $\emptyset$ depends on S_t, V_t, t; variables μ and ξ rely on V_t, t only, and $\sigma_t = \sqrt{V_t}$. With additional assumptions, they show that the option price P can be presented by Black-Scholes prices. These assumptions are: there is no correlation between two Wiener process dW and dZ; the variance risk is not priced; and the risk free rate is constant or deterministic.

$$P_t = \int_0^{\infty} BS\,(\bar{V})\, h(\bar{V}|V_\tau) d\bar{V}, \quad \text{where } \bar{V} = \frac{1}{T-t}\int_t^T V_\tau\, d\tau \tag{4}$$

In equation (4), $h(\bar{V}|V_t)$ refers to the probability density of $\bar{V}$ conditional on V_τ and $\bar{V}$ is the mean variance of underlying asset over the life of option in time interval $[t,\ T]$. The

RHS of (4) is the expected value of Black-Scholes prices conditional on the distribution of $\bar{V}$. Furthermore, Feinstein (1989) showed that Black-Scholes model has nearly linear relationship to volatility for at-the-money with short maturity options. Thus, equation (4) can be further extended as follows.

$$P_t^{ATM} = \int_0^\infty BS\,(\bar{V})\,h(\bar{V}|V_\tau)d\bar{V} = E[BS(\bar{V})|V_\tau] \approx BS_{ATM}(E[\bar{V}|V_\tau]) \qquad (5)$$

$$P_t^{ATM} = BS_{ATM}(V_{[t,T]}{}^{implied})$$

$$V_{[t,T]}{}^{implied} \approx E[(\bar{V}|V_\tau)] \qquad (6)$$

The notation $V_{[t,T]}{}^{implied}$ refers to the squared value of implied volatility derived from Black-Scholes formula for at-the-money option with short maturity. Equation (6) says that the squared value of implied volatility implied from an at-the-money option is approximately the same as the average value of spot return variance over the life of option. The error from approximation is not significant (see Poteshman 2000, Lamoureux and Lastrapes 1993, Fleming 1998, Chernov 2001, for more discussions)

4. THE CONSTRUCTION OF MVX AND VIXC

4.1 The construction of MVX

The MVX is a weighted average of implied volatilities derived from eight close to-the-money, near-by and second nearby options written on the iShare S&P/TSX 60 index fund (XIU). MVX has a constant 22 trading-day maturity and is quoted in percentage form. The calculation of MVX is as follows: first, its calculation requires identifying four near-by options, two call and two put options that have at least 8 calendar days until maturity. Let $\sigma^{up}_{call,\ near-by}$, $\sigma^{up}_{put,\ near-by}$, $\sigma^{down}_{call,\ near-by}$, and $\sigma^{down}_{put,\ near-by}$ refer to the B-S implied volatilities derived from the four near-by options by iteration method. The "up" and "down" indicate the first option that has a strike price just higher and lower than the spot price, respectively. Second, by averaging the implied volatilities of call and put options for each strike and maturity, one obtains two theoretical implied volatilities, denoted by $\sigma^{up}_{near-by}$ and $\sigma^{down}_{near-by}$. Third, one interpolates $\sigma^{up}_{near-by}$ and $\sigma^{down}_{near-by}$ for strike price, as shown by equation (7a) and then obtains the first near-by implied volatility. Let $\sigma_{near-by}$ denote this hypothetical at-the-money implied volatility. Employing the same method with another set of four second near-by options, one obtains a second near-by hypothetical volatility, $\sigma_{second\ near-by}$. Finally, by linearly interpolating these two implied volatilities for maturity, as shown by equation (7b), one estimates a 22 trading-day implied volatility.

$$\sigma_{near-by} = \sigma^{down}_{near-by} \frac{strike\ up - spot}{up\ strike - down\ strike} + \sigma^{up}_{near-by} \frac{spot - strike\ down}{up\ strike - down\ strike} \quad (7a)$$

$$\sigma_t = \sigma_{near-by} \frac{T_2 - 22}{T_2 - T_1} + \sigma_{second\ near-by} \frac{22 - T_1}{T_2 - T_1} \quad (7b)$$

where σ_t is the hypothetical ATM (at-the-money) implied volatility at time t, T_1 *and* T_2 are trading maturity days for near-by and second near-by options (see Fleming, Ostdiek, and Whaley (1995) or www.m-x.ca for more details).

4.2 The construction of VIXC

The VIXC represents a 22 trading day's risk-neutral expected volatility over the next 22 days. It is computed from real-time option prices that include out-of-the-money and close at-the-money options. Therefore, the information contained in out-of-the-money is considered. The VIXC is computed by interpolating implied volatilities from the near-by and second near-by options, as shown by equations (8-9)

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \quad (8)$$

$$VIXC = \sqrt{\frac{N_y}{N_m}\left\{T_1\sigma_1^2\left[\frac{N_{T2}-N_m}{N_{T_2}-N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_m-N_{T1}}{N_{T_2}-N_{T_1}}\right]\right\}} * 100 \quad (9)$$

where σ is the implied volatility for near-by or second near-by options, T is the time to maturity in minutes for options, F is the forward index level derived from option prices, r is the risk free interest rate, K_0 is the first strike price below F, $Q(K_i)$ is the mid price of the bid/ask spread for options with strike price K_i, ΔK_i is the difference between two consecutive strike prices, N_y is days in a year, N_m, here, is 22 days, N_{T_1} and N_{T_2} are the time to maturity for near-by and second near-by option. The near-by options have at least 5 calendar days to maturity.

4.3 The comparison between MVX and VIXC

The forecast quality of MVX partially relies on the specification of Black-Scholes (1973) formula. Also, MVX only uses close to the money options; the information embedded in out-of-the-money options has been ignored. The VIXC is independent of any option formula and uses all out-of-money options. Therefore, the VIXC is expected to be more accurate than the MVX for forecasting future volatility.

On the other hand, the computation of VIXC requires an infinite range of continuous strike prices of the S&P/TSX 60 index. In practice, it is impossible to obtain such prices series. Therefore, approximation errors arise from both truncation error (due to limited strike prices) and discretization error (due to numerical integration). The forecast accuracy of VIXC may suffer from these two sources of approximation errors.

5. DATA

In this study, we use the following data series: (1) daily closing quotes of implied volatility indexes, VIXC and MVX; (2) 5-minute S&P/TSX 60 price index returns; and (3) daily S&P/TSX 60 price index high-low range and daily closing returns. The closing quotes of volatility indexes are obtained from the ME (Montréal Exchange) (web site http://www.m-x.ca). The VIXC extends from October 1, 2009, the first day on which the ME started to compute the VIXC, to December 31, 2010 (314 daily observations). The MVX is from October 1, 2009 to October 15, 2010 (261 observations), the last day on which the ME computes the MVX. The daily and high frequency 5-min S&P/TSX 60 index quotes are obtained from the Bloomberg for the period January 3, 2006 to December 31, 2010 and December 1, 2009 to December 31, 2010, respectively.

5.1 Implied volatilities

To address our research questions, we use the daily closing levels of volatility indexes produced by the Montreal exchange. As this data is issued by an official exchange, most of the problems arising from mis-measurement are eliminated (see Harvey and Whaley, 1991, 1992). In case the daily implied variance is needed, it can be converted from the volatility index by equation 10[(1)].

$$\text{Daily implied variance} = \left(\frac{VIXC\ or\ MVX}{100\sqrt{252}}\right)^2 \qquad (10)$$

Although both MVX and VIXC are robust to mis-measurment problems, they are still biased estimators for future volatility outcomes. In the period December 1, 2009 to October 15, 2010, the mean of MVX is equivalent to an annualized volatility of 17.58%. The VIXC has an even larger number, 18.45%. In the same period, however, the average annualized volatility derived from 5-min returns is 13.92%. Both VIXC and MVX over- predict future volatility. The phenomenon that implied volatility over-predicts future realized volatility is common across a variety of options markets (Poteshman, 2000; Poon and Granger, 2003).

(1) See TMX VIXC methodology, www.m-x.ca.

5.2 Daily realized volatility

5.2.1 Daily realized volatility from intra daily returns

Variances or volatilities computed from daily returns contain much noise, which make prediction analysis based on them inapt (Anderson and Bollerslev, 1998). Andersen, Bollerslev, Diebold, and Labys (2001), among others, have theoretically proved that the summation of squared intra-day returns, defined as realized variance, is an unbiased and efficient estimator of the daily integrated variance when the sampling frequency goes to infinity. In practice, however, when the sampling frequency is too high, the market microstructure effects such as bid-ask bounce make realized variance bias and inconsistent.

We construct our measure of daily realized volatility for the S&P/TSX 60 index with intra-day returns. Following Anderson and Bollerslev (1998) and Anderson et al. (2001), we use 5-minute sampling frequency for intraday returns and select the last traded price in each interval. The daily realized volatility is computed from the square root of the summation of squared intra-day returns plus the squared overnight returns between the closing price and opening price in two consecutive trading days. For example, realized volatility on Thursday is the summation of 79 squared returns, specifically one overnight return from Wednesday 16:00 to Thursday 9:30 plus 78 squared 5-min returns starting from 9:30 to 9:35 and concluding with the return from 15:55 to 16:00 (Toronto time).

5.2.2 Daily realized volatility from daily index returns

Daily returns of the S&P/TSX 60 index are calculated in the normal way as the differences in the logarithm of two consecutive daily index closing prices. Let P_t and P_{t-1} denote the daily closing index price at day t and t-1, respectively. The daily returns for day t are denoted by r_t, which is computed by $\ln(\frac{P_t}{P_{t-1}})$.

We do not adjust index returns for dividends by following Blair et al. (2001), who suggest that volatilities computed from dividends adjusted returns produce the same statistical results as the volatilities from returns without dividends adjustment. The daily realized volatility is represented by the absolute value of daily index returns.

5.2.3 Daily realized volatility from daily high-low range

We also compute the Parkinson 1980) range volatility that is defined as equation (11).

$$RNG_t = \frac{\ln(hi_t/lo_t)}{\sqrt{4ln2}} \qquad (11)$$

in which hi_t and lo_t are the daily high and low index prices during day t, respectively. RNG is the daily Parkinson (1980) range volatility.

5.3 Descriptive statistics

Table 1 presents the descriptive statistics for the five volatility series: MVX, VIXC, daily realized volatility, daily squared returns, and daily range volatility. For comparison purposes, all these four volatility series have been annualized. Daily squared returns are the most volatile among these four volatility series. Their standard deviation is 3.4 times as the VIXC, 2.4 times as the MVX, and 1.5 times as the realized volatility. This statistic suggests that the daily square returns have too much noise to be used as the benchmark for forecasting analyses. It is also seen that the realized volatility is about twice as volatile as the VIXC and MVX, as judged by the respective standard deviation. This result is consistent with the notion that implied volatility represents the average volatility over the remaining life of options. Therefore, it should exhibit less volatility than realized volatility.

In Table 1 we also see that the distributions of five volatility series are skewed right and leptokurtic. The Jarque-Bera tests for normality reject the null hypothesis of normal distribution for all series at the 5% level. The results of unit-root tests with both ADF (Augmented Dickey-Fuller) and P-P (Phillips-Perron) approaches indicate that, except for the VIXC, the realized volatility, daily square returns, daily range volatility, and the MVX are stationary. We also examine the unit root for VIXC from December 1, 2009 to February 1, 2010 (295 observations). The ADF and P-P tests with trend are -2.7425 (p: 0.0682) and -2.9120 (p: 0.0452). In the same period, the unit root tests for the CBOE VIX with both approaches are -3.1754 (p: 0.0225) and -2.8204 (p: 0.0566).

Notably, among these five series, VIXC has the strongest positive serial correlation and MVX follows. Both implied volatility series, VIXC and MVX, have more long memory features than the realized volatility and range volatility, as suggested by the magnitude of serial correlations. A significant serial correlation can not be found in the squared daily returns series, possibly because of measurement errors, such as bid-ask bounce.

Regarding the autocorrelation test on series of the first-order difference, it can be seen that the first-order autocorrelations for realized volatility, range volatility, and daily squared returns are statistically significant and negative as well. This can be expected, since both time series in level are stationary. The negative first-order autocorrelation is also the evidence of mean reversion in both volatility series.

The negative serial autocorrelation on first-order difference of implied volatility can be seen as the presence of measurement errors when computing implied volatilities (see Harvey and Whaley, 1991, 1992). Therefore, it is worth mentioning that the first-order autocorrelations for VIXC series, 0.0549 (p: 0.4135), is not significantly different from zero. In comparison, for MVX, we find a significantly negative correlation on the first difference, -0.1345, which is significant at the 5% level. We conclude that the measurement errors associated to VIXC computation are less than these errors to MVX computation. From this aspect, we may expect that VIXC can better predict future volatility than MVX.

In addition, from Table 1 we find that the mean of realized volatility, 13.92, is lower than both the mean of VIXC, 18.49, and the mean of MVX, 17.57. The t-tests for the equality of means are -10.0533 and -7.4155 between realized volatility and VIXC, and between realized volatility and MVX, respectively. Equality is rejected at the 1% significance level for both tests.

Researchers have proposed several explanations for this apparent discrepancy (Poteshman 2000). For example, one explanation is that there is a non-zero price for volatility risk. MVX is derived from B-S option model, which assumes zero variance risk premium. VIXC is a model free implied volatility. Its computation is also under the risk neutral assumption. Recent research has viewed the assumption of zero variance risk premium inappropriate. For example, Poteshman (2000) derive implied volatility for SPX options from Heston (1993) option model that permits a non-zero market price of volatility risk. His results suggest that implied volatility is an almost unbiased estimate for future realized volatility. See Bakshi and Kapadia (2003), Carr and Wu (2004), Adersern, et. al. (2007), Chernov (2007), and Corsi (2009) for more discussions.

Table 1 Descriptive statistics of volatility series

Statistics	Daily realized Volatility	Squared daily Returns	Parkinson range Volatility	VIXC	MVX
Mean	13.9168	10.3554	12.0935	18.4994 (0.0000)	17.5768 (0.0000)
Median	12.3678	8.0355	10.7691	18.1081	17.4340
Maximum	45.2791	48.1778	42.0865	27.5862	32.5780
Minimum	4.5199	0.0231	2.8327	13.58900	9.8530
Std. Dev.	6.1490	9.3439	5.6457	2.773871	3.9418
Skewness	2.0738	1.2472	1.4167	0.47490	0.6476
Kurtosis	8.7576	4.2772	6.5002	2.8751	3.8254
Jarque-Bera	459.4780	71.6626	185.0658	8.3741	21.5275
Probability	0.0000	0.0000	0.0000	0.0151	0.0000
Autocorrelations					
Lag(1)	0.5476 (0.0000)	-0.0568 (0.3965)	0.4026 (0.0000)	0.9464 (0.0000)	0.8731 (0.0000)
Lag(2)	0.4735 (0.0000)	-0.0159 (0.6787)	0.3584 (0.0000)	0.8881 (0.0000)	0.7792 (0.0000)
Lag(3)	0.3604 (0.0000)	0.2304 (0.0054)	0.3306 (0.0000)	0.8242 (0.0000)	0.6985 (0.0000)
First difference	-0.4120 (0.0000)	-0.5100 (0.0000)	-0.4494 (0.0000)	0.0549 (0.4135)	-0.1345 (0.0453)
ADF statistic					
With Intercept	-5.3519 (0.0000)	-7.0924 (0.0000)	-6.2425 (0.0000)	-2.1403 (0.2292)	-3.9000 (0.0024)
With Intercept and Trend	-5.3508 (0.0001)	-7.0880 (0.0000)	-6.2303 (0.0000)	-2.1203 (0.5312)	-3.8766 (0.0146)
None	-1.8957 (0.0555)	-1.9194 (0.0526)	-1.1240 (0.2369)	-0.7220 (0.4029)	-1.0220 (0.2754)
Phillips-Perron statistic					
With Intercept	-8.5283 (0.0000)	-15.9453 (0.0000)	-10.8531 (0.0000)	-2.3563 (0.1555)	-3.7456 (0.0041)
With Intercept and Trend	-8.5197 (0.0000)	-15.9475 (0.0000)	-10.8362 (0.0000)	-2.3371 (0.4117)	-3.7198 (0.0230)
None	-2.2500 (0.0239)	-10.6438 (0.0000)	-2.9303 (0.0035)	-0.7220 (0.4029)	-0.8900 (0.3295)

Notes: The sample period extends from December 1, 2009 to October 15, 2010. Sample comprises 219 daily observations for all four time series. ADF denotes the Augmented Dickey-Fuller test of stationary. Jarque-Bera normality test reports $\mathcal{X}^2$ on the null. Autocorrelations test reports Q-stat on the null. The amounts in parentheses are p-values. Realized volatility is computed from the summation of 5-min intraday and overnight squared returns. MVX and VIXC are daily closing levels of implied volatility indexes. All volatilities have been annualized and assume 252 trading days in one year. The p values associated with mean row of VIX and MVX are for equality test between realized volatility and VIXC, and realized volatility and MVX, respectively.

6. EMPIRICAL ANALYSES

In this section we first evaluate the information content of VIXC and MVX by examining the statistic results of various GARCH specifications. Due to the mixed findings of whether implied volatility incorporates all information for future realized volatility, it is meaningful to examine whether GARCH models of market volatility contain useful information that is not reflected by implied volatility. This test provides extra evidence based on Canadian data.

We also analyze the information content of VIXC and MVX by examining out-of-sample forecasts of future realized volatility. The accuracy of forecasts between implied volatility and alternatives which are derived from historical asset prices is also tested. The out-of-sample tests will be based on regression and loss function analyses.

6.1 Examination of information content by GARCH in-sample test

In this section, we examine whether the model free implied volatility, VIXC, is superior to the model based implied volatility, MVX. Because studies in this area are sparse and inconclusive, our results will provide further evidence. Second, we examine whether implied volatility has sufficient information for conditional volatility. If so, we can conclude that the Canadian stock market is efficient.

6.1.1 Test methodology

We follow the methods of Corrado and Truong (2007) to investigate the information content of implied volatility with expanding the standard GARCH model by exogenous variables: VIXC, MVX, and/or Parkinson (1980) volatility. We can assess information content embedded in VIXC, MVX, and/or Parkinson volatility by examining the sign and significance of their GARCH model coefficients. We formulate our full GARCH model as equation (12-13)

$$r_t = \mu + \varepsilon_t \ , \ \ \varepsilon_t \sim N(0, h_t) \tag{12}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} + \omega_m MVX_{t-1} + \omega_p RNG_{t-1} \tag{13}$$

where μ and h_t denote the conditional mean and variance of returns, respectively. ε_t is the innovation process and is assumed to be normally distributed with a mean equal to zero and a conditional variance equal to h_t. I is a dummy variable that equals to one if

ε_{t-1} is negative, and zero otherwise. $VIXC_{t-1}$ is the daily model free implied variance on day t-1. MVX_{t-1} is the daily B-S implied variance on day t-1. Both daily implied variances are computed from closing levels of implied indexes. RNG_t is the Parkinson (1980) volatility that is defined by equation (11).

Equation (13) allows us to study the information content of implied volatility relative to the GARCH specification for conditional volatility as well. For example, by setting γ and ω_m equal to zero, we can evaluate the extent of conditional variance affected by past variance and implied volatility, VIXC. For different test purposes, we set various restrictions on the parameters of the equation (13). We use the following GARCH specifications for conditional variance:

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \delta h_{t-1} \qquad Model\ A \qquad (14)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_m MVX_{t-1} \qquad Model\ B \qquad (15)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} \qquad Model\ C \qquad (16)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} \qquad Model\ D \qquad (17)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_m MVX_{t-1} \qquad Model\ E \qquad (18)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} \qquad Model\ F \qquad (19a)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} + \omega_p RNG_{t-1} \qquad Model\ H \qquad (19b)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} + \omega_m MVX_{t-1} \qquad Model\ G \qquad (20)$$

It is clear that model A is the standard GARCH model and model D is the standard GJR-GARCH model. Model A assumes a symmetric response to volatility shocks. The good news and bad news have the same effects on volatility shocks. With model D, however, we differentiate the volatility shocks arising from good or bad news. The significance level and magnitude of parameter β and $\beta + \gamma$ indicate the effects of good and bad news, respectively.

Model B and C examine whether any of these two implied volatility indexes is a sufficient statistic for deriving conditional volatility. We formulate model E and F to examine whether either of the implied volatilities can capture leverage effects in index returns series. Model G examines whether either of these implied volatilities can provide extra explanatory power in addition to the information embedded in their counterpart. Finally, Model H examines the dominance between VIXC and daily range volatility with respect to conditional volatility generating process.

All model parameters are estimated by maximum likelihood method under the assumption that the errors are conditionally normally distributed. Robust standard errors are computed from Bollerslev and Wooldridge (1992), so that the inferences are robust against possible non-normality of errors.

6.1.2 **Test results**

Parameter estimates of the various GARCH models, along with their t-statistics, Durbin-Watson test, Akaike info criterion values, log-likelihood, and $\mathcal{X}^2$ are reported in Table 2. The results are based on 261 daily observations on the S&P/TSX 60 index from October 1, 2009 to October 15, 2010.

Starting from model A, the standard GARCH model, we note that the persistence estimate is β + δ=0.9485. Since its value is close to one, this persistence estimate suggests volatility clustering in the S&P/TSX 60 index prices. Further considering model D, the standard GJR-GARCH model, the persistency estimate measured by β + $\frac{1}{2}\gamma$+ δ=0.9363 indicates high volatility cluster and persistence as well over the examination period. This persistence property is consistent to most empirical results on stock index prices (see Blair et al. 2001). The positive and significant γ coefficient in model D, the GJR-GARCH model, is indicative of the asymmetric impact of news on the market volatility generating process. The excess log-likelihood for model D against model A is 10.5233, which is significant at the 1% level. The $\mathcal{X}^2$ test between model A and model D suggests that the null hypothesis of no leverage effects is rejected at the 1% significance level.

As for model B and model C, they are extended from model A by adding either VIXC or MVX to GARCH volatility specifications. The non-significance of δ coefficients and significance of ω_v and ω_v coefficients suggest that both implied volatility indexes have incremental information that is not reflected in past asset prices. However, the significance of residuals parameters, β, in both models indicates that implied volatilities do not contain all information regarding conditional volatility of index returns. The hypothesis that implied volatility is efficient is rejected based our data at the 1% level.

When leverage effects are considered, model D, is superior to model A, judging from the $\mathcal{X}^2$ values. We thus extend model D with MVX and VIXC to form model E and F. In model E, although ω_m coefficient associated with MVX is significant at the 10% level, the excess log-likelihood against model D, is only 1.3530, which is not significant at the 10% level. The $\mathcal{X}^2$ test indicates that there is no difference between model D and model E, at the 10% significance level, regarding model fitness.

The scenario changes, however, when including VIXC in GARCH specifications. The excess log-likelihood for model F with respect to model D is 6.2607, which is significant at the 1% level. The ω_v coefficient on VIXC is 0.2397 (significant at the 1% level), and ω_m for MVX is 0.0387 (significant at the 10% level) in model E and F, respectively. The δ coefficient for GARCH parameter is reduced from 0.8647 in model D to 0.6819 in model F, but reduced to 0.8262 only in model E. The larger magnitude of ω_v suggests that VIXC has more additional information than MVX.

Note that the asymmetric effects of bad news are all significant with the presence of implied volatilities. Specifically the γ coefficients are significant at the 1% level for model E and F. Our results suggest that both implied indexes are unable to capture the leverage effects in the Canadian market.

To examine whether VIXC can fully incorporate all information about future volatility that have contained in MVX, we form model G by adding VIXC into the GARCH volatility specification of model E. The excess log-likelihood between the two models is 4.4785, which is significant at the 1% level. The $\mathcal{X}^2$ value of 8.9570 indicates that the null hypothesis of no difference between model E and G is rejected. Furthermore, the ω_v coefficient is 0.1742 (significant at the 1% level), and ω_m coefficient is 0.0211, which is not significant. This indicates that VIXC can incorporate the information embedded in MVX. The model free implied volatility, VIXC, is superior to the model based implied volatility, MVX.

We formulate model H by including VIXC and Parkinson range volatility into standard GJR-GARCH model. Notably, the ω_v coefficient for VIXC, 0.0166, is not significantly different from zero at the 10% level. Comparing to ω_v coefficient, the ω_p coefficient for Parkinson's volatility, 0.5512, is significant at 1% level. The latter is 33 times larger than the former. We conclude that Parkinson range volatility contains more information for conditional volatility than VIXC. In addition, the excess log-likelihood between model F and model H, 45.5539, is significant at 1% level as well. This number indicates that model H is better than model F in term of data fitness. The γ coefficient, 0.0195, which is not significant at 10% level, suggests that Parkinson's volatility can capture leverage effects.

In summary, we find that both implied volatilities, VIXC and MVX, do contain useful information about future realized volatility that is not reflected in past index prices. However, both implied volatilities do not incorporate all information regarding market volatility. Particularly, neither of implied volatilities contains much information regarding leverage effects in Canadian stock market. Our results strongly indicate that VIXC can incorporate the information contained in MVX. Finally, we find that Parkinson's range volatility has more information content about conditional volatility than VIXC.

Table 2 Estimation of GARCH models

Parameters	Model A	Model B	Model C	Model D	Model E	Model F	Model G	Model H
$\alpha * 10^5$	0.3848*	-0.0764	-4.0994**	0.5286***	0.4108***	-1.0429***	-0.6718***	-0.4521***
	(1.7507)	(-0.0744)	(-1.9604)	(6.5477)	(12.4771)	(-45.6415)	(-30.4760)	(-99.7344)
β	0.0830**	-0.0996***	-0.0944***	-0.1089**	-0.1551***	-0.1432***	-0.1385***	-0.1409***
	(2.3538)	(-3.6130)	(-6.7004)	(-2.3263)	(-4.7666)	(-4.3349)	(-3.9622)	(-8.3968)
δ	0.8655***	0.1137	0.2602	0.8647***	0.8262***	0.6819***	0.7167***	0.5195***
	(18.0302)	(0.2026)	(0.7210)	(19.2195)	(13.6931)	(7.7855)	(7.8252)	(6.3683)
γ				0.3609***	0.4435***	0.2641***	0.2458***	0.0195
				(5.0115)	(5.6926)	(4.2144)	(3.9193)	(0.9965)
ω_v			0.7475**			0.2397***	0.1742***	0.0166
			(2.1603)			(6.1098)	(3.1756)	(1.0034)
ω_m		0.6036*			0.0387*		0.0211	
		(1.7787)			(1.6724)		(0.5616)	
ω_p								0.5512***
								(5.5867)
D-W	1.9794	1.9788	1.9812	1.9793	1.9798	1.9813	1.9813	1.9771
AIC	-6.5626	-6.5945	-6.6476	-6.6356	-6.6383	-6.6759	-6.6650	-7.0173
Log-L	860.4282	865.5878	872.5212	870.9515	872.3045	877.2122	876.7830	922.7661
Excess log-L		5.1596	12.0930	10.5233	1.3530	6.2607	4.4785	45.5539
$\mathcal{X}^2$		10.3192***	24.1860***	21.0466***	2.7060	12.5214***	8.9570***	91.1078***

Notes: The sample period of daily observations spans from October 1, 2009 to October 15, 2010 (261 observations). GARCH models for daily returns on the S&P/TSX 60 index are:

$$r_t = \mu + \varepsilon_t\ ,\ \ \varepsilon_t \sim N(0, h_t)$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \delta h_{t-1} \qquad \textit{Model A}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_m MVX_{t-1} \qquad \textit{Model B}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} \qquad \textit{Model C}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} \qquad \textit{Model D}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_m MVX_{t-1} \qquad \textit{Model E}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} \qquad \textit{Model F}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} + \omega_m MVX_{t-1} \qquad \textit{Model G}$$

$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma I\varepsilon_{t-1}^2 + \delta h_{t-1} + \omega_v VIXC_{t-1} + \omega_p RNG_{t-1} \qquad \textit{Model H}$$

GARCH model parameters are estimated by maximum likelihood method under the assumption that the errors are conditionally normally distributed. The t-statistics in parentheses are computed from Bollerslev and Wooldridge (1992) to against possible non-normality errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10, 5, and 1% significance level, respectively. D-W is Durbin-Watson test. AIC is Akaike info criterion. The excess log-likelihood and $\mathcal{X}^2$ for model B, model C and model D, for model E and model F, for model G, and for model H, are computed with respect to model A, to model D, to model E, and to model F, respectively.

6.2 Out-of-sample forecast

The results reported in section 6.1.2 suggest that VIXC has more information about future volatility than MVX. However, those results are built on an in-sample test. It is useful to examine which index can produce better forecast in light of out-of-sample judgment. In addition, the forecasting ability of implied volatility can be assessed against alternative volatilities derived from historical asset prices.

6.2.1 Proxy for realized volatility over various horizons

We defined daily realized variance as the summation of intra day squared returns and overnight squared returns. We also consider realized volatility in view of time horizons longer than one day. These multiple period variance will simply be the sums of the each period's volatility.

Let $\sigma_{[t,T]}$ refer to the annualized realized volatility over an N-day horizon starting at time t and ending at time T. The daily realized variance for each day within this horizon is denoted by $RV^{(t)}$, $RV^{(t+1)}$, ..., $RV^{(T)}$. Then the realized volatility over this interval can be computed from equation (21)

$$\sigma_{[t,T]} = \sqrt{\frac{252}{T-t+1}\left(RV^{(t)} + RV^{(t+1)} + \cdots + RV^{(T)}\right)}\,, \tag{21}$$

Our intra day data extends from December 1, 2009 to December 31, 2010 (272 daily realized variance). We construct realized volatility series regarding 1-, 5-, 10-, and 22-trading days.

6.2.2 Alternatives of volatility forecasting

Becker, Clements, and White (2007) argue that it may be a statistical artifact that implied volatility incorporates all information contained in historical prices. The implied volatility may incorporate all volatility information embedded in one particular type of historical volatility, e.g., forecasts based on the previous month's sample standard deviation. It is also possible that a more complicated time-series forecast contains information beyond that contained in implied volatility. Thus, in our study, we construct three types of time-series volatility forecasts: random walk, GJR-GARCH (1,1), and

Riskmetrics EWMA. These three kinds of volatility series are constructed over various forecast horizons: 1, 5, 10, and 22 trading days, respectively.

6.2.2.1 Random walk

A simple forecast of next period's volatility is to use the volatility of the previous period. We construct the forecast series with the lagged realized volatility, denoted by y_{t-1} (N), in which N indicates the days in a forecast horizon. The series were constructed as:

$$\hat{y}_t\,(N) = y_{t-1}(N) \tag{22}$$

where $y_{t-1}(N)$ is the realized volatility in previous period. In our study, the out-of-sample forecast of volatility is from December 1, 2009 to December 31, 2010 (272 observations). With this methodology, we have 272 forecasts of 1, 5, 10, and 22-day volatility.

6.2.2.2 GJR-GARCH(1,1)

A popular forecasting measure is GARCH (1,1) which uses daily returns series (see Jorin 1995, Pong, Shackleton, Taylor, and Xu, 2004, Siu and Okunev, 2008, Yu, Lui, and Wang, 2010). Comparing with GARCH(1,1) forecast, GJR-GARCH(1,1) forecast is better when the underlying asset returns have leverage effects. Hansen and Lunde (2005) found that GARCH(1,1) is inferior to models that incorporate leverage effects in their analyses of IBM returns. A number of papers use GJR-GARCH(1,1) to forecast future volatility (e.g., Simon, 2003; Corrado and Miller,2005; Frijns, Tallau, and tourani-Rad, 2010a).

Our study uses GJR-GARCH(1,1) model to construct the series of forecast volatility. In particular, we formulate four time-series forecasts, with respect to 1, 5, 10, and 22 trading days. The GJR-GARCH(1,1) specification is as follows:

$$r_t = \mu + \varepsilon_t \quad \varepsilon_t \sim N(0, h_t) \tag{23}$$

$$h_t = \alpha + \beta h_{t-1} + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 I \varepsilon_{t-1}^2 \tag{24}$$

where r_t is the daily log return, $\ln(\frac{P_t}{P_{t-1}})$, and P_t is the daily closing price of the S&P/TSX 60 index at day t. μ is the mean daily return. ε_t is the innovation of mean daily return, which assumes a normal distribution. ε_t is conditional on variance h_t as well. I is an indicator variable which takes a value of one if the lagged innovation, ε_{t-1} , is negative, and zero otherwise. The leverage effects can be captured by the value of γ_2. A positive significant value indicates that the same amount of negative return has larger effects on conditional variance than that of positive return.

The daily volatility forecasts exceeding one day are constructed by:

$$h_{t+n} = \alpha + (\beta + \gamma_1 + 0.5\gamma_2)h_{t+n-1} \tag{25}$$

We follow the method of Frijns, Tallau, and Tourani-Rad (2010a) to construct the total variance k-day forward by using the following equation:

$$h_{k,t} = \sum_{n=1}^{k} h_{t+n} \tag{26}$$

We initially estimate the GJR-GARCH(1,1) model using 983 daily observations, from January 3, 2006 to November 30, 2009. After the initial parameters estimation, we compute out-of-sample volatility forecasts for the next 1, 5, 10, 22 days. The one day forecast comes from equation (24), whereas 5, 10, and 22-day forecasts are computed from equation (25) and (26). The first forecast of volatility is constructed for December 1, 2009. After we construct the first day's forecast, we roll the data window forward one day and delete the oldest observation. We then re-estimate the model and make the next forecast. We iterate this procedure until the end of out-of-sample forecast period, which is December 31, 2010. In this way, we construct forecast series with 272 data regarding 1, 5, 10, and 22 trading days.

6.2.2.3 Riskmetrics EWMA

EWMA forecasts volatility for the next day as equation (27)

$$V_{t+1} = (1-\lambda)\sum_{t=1}^{N} \lambda^t r_{T-t+1}^2 \tag{27}$$

where V_{t+1} is the variance forecasted for time T+1. T denotes the present time. N denotes the length of trading days used in computation. This forecast measure places more weight on recent observations and less weight on early ones. Because Riskmetrics (1996) use 0.94 for λ, we use this value for our estimations. By following Simon (2003), we set N equal to 75. After obtaining one day ahead variance, V_{t+1}, the K day ahead forecasted variance is the multiple of K and V_{t+1}. Our out-of-sample forecast period for EWMA is from December 1, 2009 to December 31, 2010. Within this period, we construct forecast series with 272 data regarding 1, 5, 10, and 22 trading days.

6.2.3 Out-of-sample forecast comparison

In this section we compare the forecasting performance of implied volatility for future realized volatility. We use two measures: regression test and accuracy test based on four loss functions. Following the Jorion (1995), we use our regression tests to assess the "information content" of daily implied volatilities for volatility over the next day, next week, and the next two weeks. With respect to loss functions, MSE (mean squared error), MAE (mean absolute error), HMSE (heteroskedasticity-adjusted mean square error), and HMAE (heteroskedasticity-adjusted absolute error) will be computed.

6.2.3.1 Regression-based tests

We examine the information content of implied volatility in levels with regression equations (28a-30a). For robust test purpose, we also test both model free and B-S model-based implied volatility in terms of the first difference of realized volatility with equation (28b-30b), because our preliminary examination reveals that the VIXC series exhibit unit root properties.

$$\sigma_{[t,t+n]} = \alpha + \beta \hat{\sigma}_{[t,t+n]}^{IM} + \varepsilon_t \quad (28a)$$

$$\sigma_{[t,t+n]} = \alpha + \gamma \hat{\sigma}_{[t,t+n]}^{TS} + \varepsilon_t \quad (29a)$$

$$\sigma_{[t,t+n]} = \alpha + \beta \hat{\sigma}_{[t,t+n]}^{IM} + \gamma \hat{\sigma}_{[t,t+n]}^{TS} + \varepsilon_t \quad (30a)$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \beta(\hat{\sigma}_{[t,t+n]}^{IM} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t \quad (28b)$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \gamma(\hat{\sigma}_{[t,t+n]}^{TS} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t \quad (29b)$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \beta(\hat{\sigma}_{[t,t+n]}^{IM} - \sigma_{[t-n-1,t-1]}) + \gamma(\hat{\sigma}_{[t,\ t+n]}^{TS} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t \quad (30b)$$

The notation $\sigma_{[t,t+n]}$ refers to the ex post realized volatility over n-day forecasts horizon [t, t+n], in which n is equal to 1, 5, and 10, respectively. $\sigma_{[t-n-1,t-1]}$ is the lag of $\sigma_{[t,t+n]}$. $\hat{\sigma}_{[t,t+n]}^{IM}$ denotes the implied volatility at time t. $\hat{\sigma}_{[t,t+n]}^{TS}$ denotes alternative forecasts of volatility derived from historical index returns, forecasts which include random walk, GJR-GARCH (1,1), and Riskmetrics EWMA.

Many studies show that sampling procedure affects the results of regression tests mentioned above. For example, Canina and Figlewski (1993) use overlapping samples to test the relationship between implied volatility and the subsequent volatility outcome. They find that lagged standard deviation dominates implied volatility for forecasting purpose. The overlapping samples favor the historical volatility in regression tests (Christensen and Prabhala, 1998; Christensen, Hansen, and Prabhala, 2002). Following the suggestions of Christensen and Prabhala (1998), we use non-overlapping sample to reduce possible problems for statistical inference.

We use several forecasting horizons: 1-, 5-, 10-, and 22-trading days. Let "N" be the length of a particular forecast horizon. We select the last day's closing price of both volatility indexes in each horizon to construct implied volatility series. The VIXC and the MVX series will be compared to the eventual realized volatility in next N trading days. For example, regarding the 5-day horizon forecast, we obtain the first VIXC observation on November 30, 2009. This implied volatility will be compared with the realized volatility in the next 5 trading days, which is from December 1, 2009 to December 7, 2009. We use the second VIXC observation on December 7, 2009 and compare this amount with the realized volatility in the period from December 8, 2009 to December 14, 2009. We iterate this sampling procedure until the end of our sample. Therefore, we construct both implied volatility series and realized volatility series one by one. For forecast horizons of 1-, 5-, 10-, and 22-trading days, we obtain 220, 44, 22, and 11 observations(2). Figure 1 illustrates the sampling procedure.

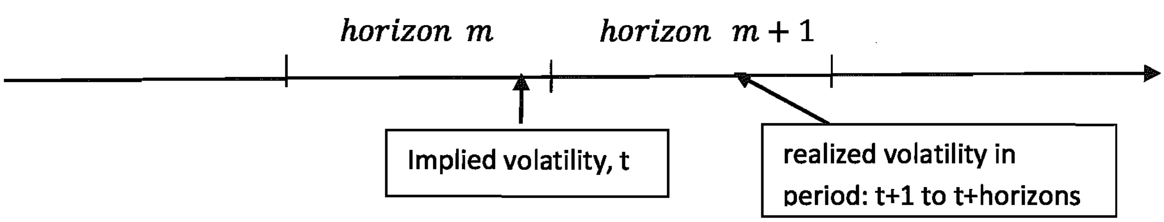

Figure 1. The non-overlapping Sampling procedure

(2) The Montréal Exchange stopped computing the MVX on October 15, 2010. Because we compare the predicting power of both the VIXC and the MVX, we use observations in the period from December 1, 2009 to October 15, 2010, during which VIXC and MVX coexisted.

Both VIXC and MVX are market estimates of average future volatility over the next 22 trading days. Therefore, with the forecast horizons of 1-, 5-, and 10-trading days, the regression equations (28a-30b) can not test the forecasting accuracy of implied volatilities. Jorion (1995) calls such tests "information content of implied volatilities". These tests can answer the question whether implied volatilities have some useful information for predicting the future volatilities.

We only have 11 observations for 22-day horizon. Such a small number of observations make statistical inference difficult. Thus, we focus on forecast test regarding the 1, 5, and 10 days horizon. With respect to the test for 22-trading day horizon, we use loss functions to examine the superiority among VIXC, MVX, and time series of volatility forecasts.

We formulate two hypotheses based on regression equations (28a-30b).

Hypothesis 1: VIXC incorporates all information that is contained in MVX for predicting the next 1-, 5-, and 10-trading day's realized volatility.

Hypothesis 2: VIXC incorporates all information that is contained in alternative forecasts of time series volatility for predicting the next 1- and 5-trading day's volatility.

6.2.3.1.1 Hypothesis 1 test results

Hypothesis 1 can be examined by analyzing log likelihood ratio between an unrestricted model, which uses both VIXC and MVX as explanatory variables, and a restricted model, which uses only either VIXC or MVX as a regressor. In addition, the R-squared values from regression tests indicate the fitness of models. The ranking of R-squared values thus represents the order of forecasting ability.

We start examining the information contents of MVX and VIXC for the next 1, 5, and 10 day's realized volatility. Results are displayed in Table 3 and Table 4 for regression tests in levels and first difference, respectively. All parameters are estimated by standard OLS with Newey-West (1987) corrected errors for heteroskedasticity and serial correlation. The results in Table 3 and Table 4 are very similar for statistical inference, thus we only discuss the results in Table 4. We focus on Table 4 because most Durbin-Watson statistics are far away from two in Table 3. For example, for 1-day prediction, the Durbin-Watson values in Table 3 are 1.32, 1.08, and 1.33, respectively. Such low

Durbin-Watson values indicate that the residuals from regression tests in levels are serially correlated and thus the regression model is not a good fit with the data.

Starting with Panel A in Table 4, it is clear that both VIXC and MVX contain information about the next day's realized volatility, because in univariate regression, both slope coefficients are significant at the 1% level. In addition, the R-squared value for VIXC, 0.3265, is larger than that for MVX, 0.2463. This suggests that VIXC predicts the next day's volatility more accurately than MVX. All Wald-F tests indicate that the intercept is not equal to zero and the coefficient is not equal to one. Therefore, VIXC and MVX are biased forecasts for the next day's volatility. This result is expected, because both VIXC and MVX anticipate the average volatility in one month, not in one day. The likelihood ratio test, 0.5818, indicates that VIXC can incorporate all information contained in MVX. Further evidence can be seen from the coefficients of the VIXC and MVX. When both VIXC and MVX are included as explanatory variables, the coefficients for VIXC and MVX are 0.5990 and 0.0793, with the former being significant at the 1% level, whereas the latter is not significantly different from zero at the 10% level.

We now turn to Panel B and C in Table 4. The statistical implications inferred from 5- and 10-day horizon are similar to those for the 1-day horizon test. The likelihood ratio and magnitude of coefficients for VIXC and MVX indicate that VIXC incorporates all information embedded in MVX for the next 5 and 10 day's volatility forecasts. In addition, when VIXC is of a regressor in univariate regression, we obtain a higher R-squared value in 5-day horizon test, 0.3532, than in 1-day horizon test, 0.3265. This suggests that when forecasting a longer horizon, VIXC can better forecast future volatility. However, in 10- day horizon test, the R-squared values are close to zero for all regression tests. These surprising results may be due to our limited number of observations. For a horizon of 10 days, we only have 22 observations. Without considering the R-squared values, we still can come to the same statistical inference from the likelihood ratio test and the magnitude of coefficients for VIXC and MVX. In the 10-day horizon test, the likelihood ratio test for MVX, 1.6750, clearly indicates that MVX is a redundant explanatory variable when both VIXC and MVX are present in regression model. According to the results in Table 3 and 4, we can not reject the hypothesis one and hence VIXC does incorporate all information that is reflected in MVX for the future 1, 5, and 10 day's realized volatility.

As a robustness check, we follow Jorion (1995) to use daily squared returns as the dependent variable and retest the relationship between the next day's volatility, represented by $\sqrt{R_{t+1}^2}$, which is calculated as a daily return of SP/TSX 60 index, and implied volatility, VIXC and/or MVX. Results are presented in Table 5. We do not test the forecast horizon with 5 and 10 days, because a noisy benchmark, $\sqrt{R_{t+1}^2}$, combined with small samples will make statistic inference unreliable. From the univariate regression, we find that the coefficients for both MVX and VIXC are significant at the 1% level. This suggests that both implied volatilities contain information for the next one day's volatility. The R-squared value for the VIXC forecast, 0.0838, is higher than the one for MVX, 0.0321. We conclude that VIXC is superior to MVX for predicting the next day's volatility. Our R-squared values are consistent to empirical results when using $\sqrt{R_{t+1}^2}$ as the comparison benchmark, e.g, Moraux et al. (1999) report R-squared value of 0.0359; Frijns et al. (2010) report R-squared value of 0.1214 when forecasting the next day's volatility;

Considering the results from the bivariate regression test in Table 5, we note that the coefficient for MVX is not significant at the 10% level. The coefficient of 0.9678 for VIXC is about 30 times larger than the coefficient of 0.0333 for MVX. This suggests that VIXC almost incorporates all the information that is contained in MVX for predicting the next one day's volatility. In addition, the likelihood ratio test between restricted model, which sets γ equal to zero, and unrestricted model indicates that MVX is a redundant explanatory variable when VIXC is included.

To summarize, results from Table 3, 4, and 5 suggest that both VIXC and MVX do contain information about 1-, 5- and 10-day forward volatility. However, VIXC subsumes all information that is reflected in MVX with respect to predicting future volatility. We confidently accept hypothesis 1. Therefore, in the next subsection, we only examine the relationship between VIXC and alternative forecasts of time series volatilities which include GJR-GARCH, EWMA, and random walk.

Table 3 Information content regression tests for VIXC and MVX in levels

	Comparison in levels	α	β	γ	D-W	Adj. R^2	Wald F test	Likelihood ratio
Panel A (220 obs)	Forecast horizon=1 day							
		-10.1955**	1.3007***		1.3224	0.3412	75.3721	50.3375
		(-2.4849)	(5.5525)				(0.0000)	(0.0000)
		2.0426		0.6736***	1.0822	0.1817	19.3443	2.6343
		(0.6944)		(3.7520)			(0.0000)	(0.1046)
		-10.4478**	1.1478***	0.1753	1.3323	0.3460		
		(-2.5697)	(4.9734)	(1.2147)				
Panel B (44 obs)	Forecast horizon=5 day							
		-3.6159	0.9651***		1.5474	0.2386	11.5373	10.7707
		(-0.7953)	(3.8891)				(0.0001)	(0.0010)
		8.8039**		0.3106	1.2730	0.0328	11.5507	0.2470
		(2.1217)		(1.4301)			(0.0001)	(0.6192)
		-3.3685	1.0503***	-0.1036	1.5528	0.2244		
		(-0.7296)	(2.8849)	(-0.3751)				
Panel C (22 obs)	Forecast horizon=10 day							
		6.6971	0.4194		1.0682	0.0154	11.7213	3.5662
		(1.1707)	(1.5098)				(0.0004)	(0.0832)
	-	16.8017***		-0.1333	0.9254	-0.0390	14.1765	2.3807
		(2.9073)		(-0.4467)			(0.0001)	(0.1570)
		8.3866	0.8025**	-0.5033	1.0878	0.0699		
		(1.3803)	(2.0066)	(-1.2774)				

Notes:

$$\sigma_{[t,t+n]} = \alpha + \beta\hat{\sigma}_{[t,t+n]}^{VIXC} + \varepsilon_t$$

$$\sigma_{[t,t+n]} = \alpha + \gamma\hat{\sigma}_{[t,t+n]}^{MVX} + \varepsilon_t$$

$$\sigma_{[t,t+n]} = \alpha + \beta\hat{\sigma}_{[t,t+n]}^{VIXC} + \gamma\hat{\sigma}_{[t,t+n]}^{MVX} + \varepsilon_t$$

$\sigma_{[t,t+n]}$ denotes the future realized volatility series computed from 5-min returns. It is regressed against the volatility forecast $\hat{\sigma}_{[t,t+n]}^{VIXC}$ and/or $\hat{\sigma}_{[t,t+n]}^{MVX}$. The test period starts on November 30, 2009 and ends on October 15, 2010. 220, 44, 22 observations are used in regressing test for the next one, five, and ten days forecast. The numbers in parentheses for intercept and slope coefficients are Newey-West (1987) standard errors of the estimated parameters. D-W is Durbin-Watson test. The Wald test reports the test of whether the intercept is equal to zero and the coefficient is one. P values are listed in parentheses. The likelihood ratio tests the unrestricted model which includes both MVX and VIXC against the restricted model which include either VIXC or MVX as a regressor. P values are reported in parentheses. *, **, and *** indicate that the coefficient is significantly different from zero at the 10, 5, and 1% level, respectively.

Table 4 Information content regression tests for VIXC and MVX in the first difference

Comparison in the first difference	α	β	γ	D-W	Adj. R^2	Wald test	Likelihood ratio
Forecast horizon=1 day							
	-3.1198***	0.6722***		2.0214	0.3265	137.5210	25.3307
	(-4.4942)	(6.8932)				(0.0000)	(0.0000)
Panel A	-1.9558***		0.5260***	2.1472	0.2463	75.6350	0.5818
(220 obs)	(-3.4160)		(5.4198)			(0.0000)	(0.4456)
	-3.0739***	0.5990***	0.0793	2.0101	0.3252		
	(-4.4913)	(5.0819)	(0.6577)				
Forecast horizon=5 day							
	-3.8839***	0.9082***		1.7385	0.3532	15.7524	13.3486
	(-5.0784)	(5.3486)				(0.0000)	(0.0005)
Panel B	-1.6457*		0.4707**	2.3821	0.1299	11.3313	0.3003
(44 obs)	(-1.7492)		(2.3709)			(0.0001)	(0.5837)
	-3.9309***	1.0083***	-0.1137	1.7730	0.3419		
	(-4.7633)	(2.8412)	(-0.4610)				
Forecast horizon=10 day							
	-1.6292	0.3710		1.7192	0.0061	34.0603	2.8385
	(-0.8716)	(1.3584)				(0.0000)	(0.0920)
Panel C	-0.0454		-0.0518	1.9674	-0.0479	51.3551	1.6750
(22 obs)	(-0.0346)		(-0.2686)			(0.0000)	(0.1956)
	-1.9278	0.7446**	-0.4099	1.7781	0.0304		
	(-1.3132)	(2.3759)	(-1.3336)				

Notes:

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \beta(\hat{\sigma}_{[t,t+n]}{}^{VIXC} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \gamma(\hat{\sigma}_{[t,t+n]}{}^{MVX} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \beta(\hat{\sigma}_{[t,t+n]}{}^{VIXC} - \sigma_{[t-n-1,t-1]}) + \gamma(\hat{\sigma}_{[t,t+n]}{}^{MVX} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t$$

$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]}$ denotes the future realized volatility differential series computed from 5-min returns. It is regressed against the volatility differential forecasts $\hat{\sigma}_{[t,t+n]}{}^{VIXC} - \sigma_{[t-n-1,t-1]}$ and/or $(\hat{\sigma}_{[t,t+n]}{}^{MVX} - \sigma_{[t-n-1,t-1]}$. The test period starts on November 30, 2009 and ends on October 15, 2010. 220, 44, 22 observations are used in regressing test for the next one, five, and ten days forecast. The numbers in parentheses for intercept and slope coefficients are Newey-West (1987) standard errors of the estimated parameters. D-W is Durbin-Watson test. The Wald test reports the test of whether the intercept is equal to zero and the coefficient is one. P values of Wald test are listed in parentheses. The likelihood ratio test is the test between an unrestricted model which includes both MVX and VIXC and the restricted model which include either VIXC or MVX as a regressor. P values of likelihood ratio test are listed in parentheses. *, **, and *** indicate that the coefficient is significantly different from zero at the 10, 5, and 1% level, respectively.

Table 5 Information content regression tests for VIXC and MVX with daily squared returns

Comparison in levels	α	β	γ	D-W	Adj. R^2	Wald test	Likelihood ratio
Forecast horizon=1 day							
	-8.1129**	0.9968***		2.2457	0.0838	141.6798	12.1032
	(-2.3082)	(5.0093)				(0.0000)	(0.0006)
	2.3704		0.4534***	2.1605	0.0321	77.7965	0.0296
	(1.0896)		(3.6437)			(0.0000)	(0.8634)
	-8.1608**	0.9678***	0.0333	2.2474	0.07974		
	(-2.2794)	(4.2922)	(0.2061)				

Notes:

$$\sqrt{R_{t+1}^2} = \alpha + \beta\hat{\sigma}_{[t]}{}^{VIXC} + \varepsilon_t$$

$$\sqrt{R_{t+1}^2} = \alpha + \gamma\hat{\sigma}_{[t]}{}^{MVX} + \varepsilon_t$$

$$\sqrt{R_{t+1}^2} = \alpha + \beta\hat{\sigma}_{[t]}{}^{VIXC} + \gamma\hat{\sigma}_{[t]}{}^{MVX} + \varepsilon_t$$

$\sqrt{R_{t+1}^2}$ denotes the future realized volatility of SP/TSX 60 index returns. It is regressed against the volatility forecast $\hat{\sigma}_{[t]}{}^{VIXC}$ and/or $\hat{\sigma}_{[t]}{}^{MVX}$. The test period starts on November 30, 2009 and end on October 15, 2010. 220 observations are used in regressing test for the next one day forecast. The numbers in parentheses for intercept and slope coefficients are Newey-West (1987) standard errors of the estimated parameters. D-W is Durbin-Watson test. The Wald test reports the test of whether the intercept is equal to zero and the coefficient is one. P values of Wald test are listed in parentheses. The likelihood ratio test is the test between a unrestricted model which includes both MVX and VIXC and the restricted model which include either VIXC or MVX as a regressor. P values of likelihood ratio test are listed in parentheses. *, **, and *** indicate that the coefficient is significantly different from zero at the 10, 5, and 1% level, respectively.

6.2.3.1.2 Hypothesis 2 test results

We report results of regression tests in Table 6 for an independent variable in levels and in Table 7 for an independent variable in the first differences. For comparison purpose, we re-present test results for the VIXC in Table 6 and 7. The results in both tables are exactly the same in terms of statistical implications. Therefore, we discuss the results reported in Table 7 only, because most Durbin-Watson values in Table 6 are far away from two, suggesting that the regression model in levels does not fit the data. The test period starts on November 30, 2009 and ends on October 15, 2010. There are 220, 44, and 22 observations corresponding to a forecasting horizon of 1-, 5-, and 10-day, respectively.

Starting from Panel A, the univariate regression with a 1-day horizon, in Table 7, we note that VIXC, GJR-GARCH, EWMA, and random walk volatility forecasts have predictive power, because coefficients associated with each forecast measure are all significant at the 1% level. Ranking the R-squared values for these four forecasting approaches, we see that VIXC contains the most information with regard to the next 1 day's volatility, followed by random walk, GJR-GARCH, and EWMA forecasts.

Considering the bivariate regression test, the VIXC continually shows a significant and positive coefficient. Except for the GJR-GARCH forecast, both Random walk and EWMA forecast show a significant coefficient. The R-squared values increase in all bivariate tests. The increase of R-squared value suggests that all alternative forecasts contain incremental information that is already reflected in VIXC. Furthermore, the likelihood ratio test statistically indicates at the 5% level that all alternative forecasts make the bivariate model a better fit with the data. Therefore, we conclude that although VIXC contain more information about the next day's volatility, it can not subsume all information contained in alternative forecast measures.

Next we turn to Panel B, the 5-day horizon. It is seen again that all forecasts contain information for predicting the future 5-day's volatility. The VIXC ranks first among these four forecast measures. With the bivariate test, however, only EWMA forecasts have incremental information beyond that contained in VIXC. The R-squared value increases from 35% to 40% when EWMA is included in regression model. The likelihood ratio test statistically indicates that the hypothesis that EWMA has no explanatory power in the bivariate model is rejected at the 5% significance level.

In Panel C, when we set the forecast horizon equal to 10 days, only the coefficient for random walk is significantly different from zero. All other forecasts, including the VIXC, have non-significant coefficients. This evidence indicates that the random walk has the best predicting power with 10-day horizon. The R-squared values are pretty low for all cases but the random walk. However, these results are based on only 22 observations. Such a small samples probably make statistical inference questionable.

Overall, our results from both regression test in levels and first differences indicate that the VIXC has predicate power for the next 1- and 5-day's volatility. However, VIXC can not subsume all information contained in historical index prices. Based on our results, we reject hypothesis 2. Our conclusion is consistent to our previous in-sample examination in section 6.1, in which we extend the standard GJR-GARCH model with VIXC as an explanatory variable and we find that the VIXC can not incorporate all information embedded in historical index prices.

Due to the limited data, the statistical implications from regression tests may be questionable when examining the forecast performance between the VIXC and its three alternatives for 10 and 22-day horizon. Thus, in the following section, we use loss functions to examine such research questions.

Table 6 Information content regression tests for VIXC and historical volatility in levels

Comparison in levels		Intercept	VIXC	GARCH	EWMA	Random Walk	D-W	Adj. R^2	Wald F test	Likelihood ratio
Panel A (220 obs)	Forecast horizon= 1 day									
		-10.1955**	1.3007***				1.322	0.3412	75.3721	
		(-2.4849)	(5.5525)						(0.0000)	
		2.0461		0.7839***			1.1114	0.1464	2.2689	3.5389
		(0.9223)		(4.9878)					(0.1059)	(0.0599)
		7.4029***			0.4782**		0.9690	0.0415	3.7471	19.7289
		(2.6516)			(2.5091)				(0.0251)	(0.0000)
		6.2335***				0.5500***	2.2592	0.2976	12.7936	21.0827
		(4.9546)				(5.5432)			(0.0000)	(0.0000)
		-10.3770***	1.5758***	-0.3252			1.3154	0.3487		
		(-2.6538)	(3.7267)	(-1.1193)						
		-9.2788***	1.7709***		-0.7100**		1.3918	0.3949		
		(-3.0217)	(5.2200)		(-2.4754)					
		-6.9020**	0.8918***			0.3073***	1.9994	0.3986		
		(-2.2907)	(4.9317)			(3.4855)				
Panel B (44 obs)	Forecast horizon= 5 days									
		-3.6159	0.9651***				1.5474	0.2386	11.5373	
		(-0.7953)	(3.8891)						(0.0001)	
		6.5568		0.5008**			1.4093	0.0672	4.3708	0.9301
		(1.5821)		(2.1254)					(0.0189)	(0.3348)
		10.9295**			0.2456		1.2442	-0.0053	3.9335	5.4706
		(2.2984)			(0.8349)				(0.0272)	(0.0193)

Table 6 continued

Comparison in levels		Intercept	VIXC	GARCH	EWMA	Random Walk	D-W	Adj. R^2	Wald F test	Likelihood ratio
		8.3354***				0.4127***	2.2550	0.1467	33.7740	0.3146
		(3.8503)				(3.8757)			(0.0000)	(0.5748)
		-3.7406	1.2359**	-0.3175			1.5058	0.2363		
		(-0.8384)	(2.1351)	(-0.5892)						
		-3.3500	1.4978***		-0.7433		1.5778	0.3112		
		(-0.8333)	(3.1904)		(-1.3790)					
		-2.5568	0.8274*			0.1036	1.7586	0.2255		
		(-0.4198)	(1.6827)			(0.4866)				
Panel C (22 obs)	Forecast horizon= 10 day									
		6.6971	0.4194				1.0682	0.0154	11.7213	
		(1.1707)	(1.5098)						(0.0004)	
		10.3882*		0.2649			1.0401	-0.0226	9.2594	0.1258
		(1.8359)		(0.8729)					(0.0014)	(0.7228)
		15.1087**			-0.0469		0.9413	-0.0491	6.9624	2.5186
		(2.6065)			(-0.1337)				(0.0051)	(0.1125)
		6.8359***				0.5207***	1.5966	0.2170	6.0716	7.0133
		(3.1581)				(3.7870)			(0.0087)	(0.0081)
		6.6807	0.5929	-0.2076			1.0527	-0.0304		
		(1.1346)	(1.0463)	(-0.3271)						
		6.2949	1.0107*		-0.7796		1.0867	0.0757		
		(1.1488)	(2.2361)		(-1.2293)					
		14.8298*	-0.7041			0.8659*	1.8704	0.2465		
		(1.7665)	(-1.0120)			(2.2594)				

Notes:

$$\sigma_{[t,t+n]} = \alpha + \beta \hat{\sigma}_{[t,t+n]}{}^{VIXC} + \varepsilon_t$$

$$\sigma_{[t,t+n]} = \alpha + \gamma \hat{\sigma}_{[t,t+n]}{}^{TS} + \varepsilon_t$$

$$\sigma_{[t,t+n]} = \alpha + \beta \hat{\sigma}_{[t,t+n]}{}^{VIXC} + \gamma \hat{\sigma}_{[t,t+n]}{}^{TS} + \varepsilon_t$$

$\sigma_{[t,t+n]}$ denotes the future realized volatility series computed from 5-min S&P/TSX 60 index returns. It is regressed against the volatility forecast $\hat{\sigma}_{[t,t+n]}{}^{VIXC}$ and/or $\hat{\sigma}_{[t,t+n]}{}^{TS}$ which include GJR-GARCH, EWMA, and random walk. The period is from November 30, 2009 to October 15, 2010. 220, 44, and 22 observations are used in regression test for the next 1, 5, and 10 day's forecasts. The numbers in parentheses for intercept and coefficients are Newey-West (1987) standard errors of the estimated parameters. D-W is Durbin-Watson test. The Wald test reports the test of whether the intercept is equal to zero and the coefficient is equal to one. P values for Wald test are reported in parentheses below the statistic. The likelihood ratio test is reported between an unrestricted model which includes both time series volatility and VIXC and the restricted model which include only VIXC as a regressor. P values are reported in parentheses as well. *, **, and *** indicate that the coefficient is significantly different from zero at the 10, 5, and 1% level, respectively.

Table 7 Information content regression tests for VIXC and historical volatility in the first difference

Comparison in the first difference	Intercept	VIXC	GARCH	EWMA	Random Walk	D-W	Adj. R^2	Wald F test	Likelihood ratio
Forecast horizon=1 day									
	-3.1198***	0.6722***				2.0214	0.3265	137.5210	
	(-4.4942)	(6.8932)						(0.0000)	
	-0.6513		0.5258***			2.1882	0.2298	33.7592	6.1870
	(-1.4025)		(4.3895)					(0.0000)	(0.0129)
Panel A	0.1322			0.4412***		2.2729	0.1955	15.8207	18.2714
(220 obs)	(0.3904)			(3.9785)				(0.0000)	(0.0000)
	6.2335***				0.5500***	2.2592	0.2976	12.7936	21.0827
	(4.9546)				(5.5432)			(0.0000)	(0.0000)
	-4.4899***	1.0738***	-0.40162			2.0147	0.3421		
	(-6.2900)	(4.6843)	(-1.5557)						
	-6.4355***	1.3451***		-0.6122**		1.9747	0.3773		
	(-5.7094)	(5.3862)		(-2.5477)					
	-6.9020**	0.8918***			0.3073***	1.9994	0.3986		
	(-2.2907)	(4.9317)			(3.4855)				
Forecast horizon=5 day									
	-3.3389	0.9082***				1.7385	0.3532	15.7524	
	(- 0.5.0783)	(5.3485)						(0.0001)	
	-0.7717		0.6347***			2.2042	0.2002	4.1591	0.9346
	(-0.9638)		(4.8844)					(0.0225)	(0.3337)
Panel B	0.2613			0.5119***		2.3072	0.1566	7.7809	4.3215
(44 obs)	(0.3108)			(3.8224)				(0.0013)	(0.0376)
	8.3354***				0.4127***	2.2550	0.1467	33.7740	0.3146
	(3.8503)				(3.8757)			(0.0000)	(0.5748)
	-4.8009	1.2066*	-0.3125			1.7288	0.3513		
	(-2.5668)	(1.9089)	(-0.6435)						
	-7.1647**	1.5863**		-0.6270		1.6904	0.3994		
	(-2.3845)	(2.2121)		(-1.2271)					
	-2.5568	0.8274*			0.1036	1.7586	0.2255		
	(-0.4198)	(1.6827)			(0.4866)				

Table 7 continued

Comparison in the first difference	Intercept	VIXC	GARCH	EWMA	Random Walk	D-W	Adj. R^2	Wald F test	Likelihood ratio
Forecast horizon=10 day									
	-1.6292	0.3710				1.7192	0.0061	34.0603	
	(-0.8716)	(1.3585)						(0.0000)	
	-0.4041		0.2836			1.7644	-0.0066	8.7045	0.0030
	(-0.3625)		(1.0215)					(0.0019)	(0.9563)
Panel C	-0.0177			0.1551		1.8411	-0.0330	12.0998	0.3882
(22 obs)	(-0.0227)			(0.7045)				(0.0004)	(0.5332)
	6.8359***				0.5207***	1.5966	0.2170	6.0716	7.0133
	(3.1581)				(3.7870)			(0.0087)	(0.0081)
	-1.5358	0.3410	0.0305			1.7175	-0.0461		
	(-0.8779)	(1.2182)	(0.0852)						
	-3.2316*	0.6995**		-0.2950		1.7408	-0.0279		
	(-2.0079)	(2.2462)		(-0.9697)					
	14.8298*	-0.7041			0.8659**	1.8704	0.2465		
	(1.7665)	(-1.0120)			(2.2594)				

Notes:

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \beta(\hat{\sigma}_{[t,t+n]}{}^{VIXC} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \gamma(\hat{\sigma}_{[t,t+n]}{}^{TS} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t$$

$$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]} = \alpha + \beta(\hat{\sigma}_{[t,t+n]}{}^{VIXC} - \sigma_{[t-n-1,t-1]}) + \gamma(\hat{\sigma}_{[t,t+n]}{}^{TS} - \sigma_{[t-n-1,t-1]}) + \varepsilon_t$$

$\sigma_{[t,t+n]} - \sigma_{[t-n-1,t-1]}$ denotes the future realized volatility differential series computed from 5-min returns. It is regressed against the volatility forecast differentials $\hat{\sigma}_{[t,t+n]}{}^{VIXC} - \sigma_{[t-n-1,t-1]}$ and/or $\hat{\sigma}_{[t,t+n]}{}^{TS} - \sigma_{[t-n-1,t-1]}$ which include GJR-GARCH, EWMA, and Random Walk. The test period starts on November 30, 2009 and ends on October 15, 2010. 220, 44, and 22 observations are used in regressing test for the next one, five, and ten day's forecast. The numbers in parentheses for intercept and slope coefficients are Newey-West (1987) standard errors of the estimated parameters. D-W is Durbin-Watson test. In univariate regression, the Wald test reports the test of whether the intercept is equal to zero and the coefficient is one. p-values are reported in parentheses. The likelihood ratio test is reported between an unrestricted model which includes both time series volatility and VIXC and the restricted model which include only VIXC as a regressor. The corresponding p-values are reported in parentheses as well. *, **, and *** indicate that the coefficient is significantly different from zero at the 10, 5, and 1% level, respectively.

6.2.3.2 Forecast accuracy assessment with loss functions

In addition to using regression test results as evaluation criterion, we use other forecast criteria for comparing forecast accuracy, the criteria which include MSE (mean squared error), MAE (mean absolute error), HMSE (heteroskedasticity-adjusted mean squared error), and HMAE (heteroskedasticity-adjusted mean absolute error).

6.2.3.2.1 Test methodology

Hansen and Lunde (2006) and Patton (2006), among others, note that different loss functions are sensitive to the proxy of the unobserved latent volatility. Meanwhile, different loss functions give different weights to "surprising" observations. For example, unlike the MAE, the MSE gives greater weight to outlier observations. Furthermore, both MSE and MAE place more weight on the errors associated with the greater realized volatilities, but the HMAE and HMSE put less weight on such errors. Therefore, we use various loss functions to try to obtain consistent conclusions. With various loss functions, the best predicting measure will be expected to stand out in most cases.

At time t, let $\hat{y}_t$ be the estimated volatility from a particular forecasting approach and y_t be the realized volatility outcome. The MSE, MAE, HMSE, and HMAE are defined:

$$MSE: \quad f(\hat{y}_t, y_t) = \frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t - y_t)^2 \tag{31}$$

$$MAE: \quad f(\hat{y}_t, y_t) = \frac{1}{n}\sum_{t=1}^{n}|\hat{y}_t - y_t| \tag{32}$$

$$HMSE: \quad f(\hat{y}_t, y_t) = \frac{1}{n}\sum_{t=1}^{n}(\frac{\hat{y}_t}{y_t} - 1)^2 \tag{33}$$

$$HMAE: \quad f(\hat{y}_t, y_t) = \frac{1}{n}\sum_{t=1}^{n}|\frac{\hat{y}_t}{y_t} - 1| \tag{34}$$

To test the equality between two forecasts measures, we use Diebold and Mariano (1995) test for such a purpose.

Diebold and Mariano (1995) devise a test statistic that evaluates whether two competing forecast measures are significantly different. They use a loss function, $f(e)$, to test the null hypothesis of equal accuracy:

$$H_0: \ E[f(e_{1,t}) - f(e_{2,t})] = 0 \tag{35}$$

In our study, we use the four loss functions mentioned above for the Diebold and Mariano (DM) test. The Diebold and Mariano test follows a normal distribution and is calculated as following:

$$DM = \frac{\bar{d}\sqrt{n}}{\sqrt{\widehat{var}(d_t)}} \tag{36}$$

In the above equation, $d_t = f(e_{1,t}) - f(e_{2,t})$. $f(e_{i=1\ or\ 2,t})$ is one of four loss functions. $\widehat{var}(d_t)$ is the asymptotic variance of the differential series of d_t. $\bar{d}$ is the sample mean of loss differential, $\bar{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$.

6.2.3.2.2 Adjustment of implied volatility indexes

Previous literature finds that implied volatility over-estimates future volatility (Poon and Granger, 2003, 2005). The statistics in Table 1 show that the average of VIXC is greater than that of realized volatility by about 0.045 in the period from December 1, 2009 to October 15, 2010. This difference can be seen as an average risk premium. See more discussion form Bakshi and Kapadia, 2003, Andersen, Frederiksen, and Staal, 2007, Corsi, 2009, Jiang and Tian, 2005, Poteshman, 2000.

Researchers propose several methods to correct the obvious bias of implied volatility indexes when using it to forecast future volatility. For example, Pong et al. (2004) adjust implied volatilities derived from currency options by a regression approach. In a pre-forecast period, they regress realized volatilities against the implied volatilities; then in out-of-sample period, they use the estimated coefficients to adjust the implied volatilities. Blair et al. (2001) use an ARCH model to adjust the CBOE VXO as well. In our study, as we only have one year data available, these approaches can't be applied. We follow the approach used by Whaley et al. (1995) to adjust the VIXC and the MVX series. We assume that the risk premium is constant monthly. Then the risk premium is computed as an average of daily biases from the most recent month observations. The VIXC in the next month will be adjusted by this risk premium. For example, for VIXC observations in January 2010, we adjust each observation by the risk premium that is the average difference between the VIXC and realized volatility in December 2009. We then adjust VIXC in February 2010 according the risk premium of January of 2010. We repeat this procedure until October 2010. Since we only use historical information at the time we observe VIXC, the adjusted VIXC and MVX are the out-of-sample forecasts of market volatility.

6.2.3.2.3 Sampling data

Observations in the period, from January 4, 2010 to December 31 2010 (251 observations), is used to evaluate the out-of-sample forecast accuracy. We follow Blair et al. (2001) to construct overlapping samples. Therefore, with respect to each horizon of 1, 5, 10, and 22 day, we have the same number of 251 observations. For example, considering a forecast horizon of 22 days, the five forecast series are constructed as follows. Starting at January 4, 2010, we obtain the first forecast of volatility based on the method described in section 6.2.2 for GJR-GARCH, EWMA, and Random walk series. On January 4, 2010, the closing quotes of VIXC and MVX are adjusted by the risk premium in December 2009. We then compute the realized volatility in next 22 trading days. The five forecasts of volatility will be compared to this future realized volatility. We then move to January 5, 2009 and use the same approach to construct the second observations for each of five forecasts series. We compute the realized volatility anew in the next 22 trading days starting from January 5, 2010. We iterate this procedure until December 31, 2010.

6.2.3.2.4 Test results with loss functions

To examine which forecasting measure produces the most accurate prediction for future volatility, we examine the loss of accuracy computed from each loss function. Since the MVX ends on October 15, 2010, we use data from January 4, 2010 to October 15, 2010 (198 observations) to assess the forecasting ability for our five measures. Table 8 contains the results with the four loss functions. We note that with all forecast horizons and under all loss criteria, the VIXC consistently produces less loss comparing to the MVX. This evidence indicates that the VIXC is superior to the MVX for predicting future volatility in all horizons.

We also see that when the forecast horizon increases from 1 day to 5 and 10 days, the forecast accuracy of VIXC consistently increases as shown by the decreased amounts from each loss measure. However, as the forecast horizon increases to 22 days, three of four loss amounts increases. This may suggest that the VIXC has poor performance when forecasting longer horizons than short horizons.

We turn to the results when forecasting horizon is equal to one day. Table 8 shows that the VIXC produces less loss comparing to all other measures. But when forecasting horizon increases to 10 days, the four loss measures consistently show that random walk produces a less loss than the VIXC. Further considering the 22-day horizon, we find that EWMA and GJR-GARCH now consistently outperforms the VIXC based on the four loss functions. This evidence indicates that with longer forecasting horizon, time series forecasts of volatility outperform the VIXC and contain incremental information that is not reflected in VIXC.

Since each loss function treats forecast errors in different ways, it may make more sense to consider the overall performance of each predicting measure. We rank each forecast approach where one means the best and five means the worst. Table 9 presents the ranking results based on the data reported in Table 8. We rank five forecast measures according to their performance under each loss criterion. In Table 9, the row of "total" denotes the sum of rankings order for each forecast measure. The least number associated with forecast measure indicates the best performance among five measures.

Starting from the forecast of the next 1 day's volatility, the VIXC ranks the highest among all forecast approaches. This finding is consistent with our univariate regression tests that show that the VIXC produces the largest R-squared value compared to other forecasts. When the forecasting horizon increases to 5 days, random walk ranks first then followed by the VIXC. However, we should note that the forecast loss from the VIXC is close to loss from the random walk. The DM tests in Table 14 (the first number in each group) indicate that VIXC has the same forecast accuracy as the random walk under MSE and HMSE criteria.

With the 10-day horizon, random walk ranks the first. The VIXC slides to the third place. While considering the rankings with the 22-day horizon, we find that the GJR-GARCH forecast stands out among these five measures. The VIXC is just better than the MVX. This evidence suggests that with a longer forecast horizon (over 10 days), time series forecasts seem to predict future volatility better than the implied volatilities.

Many empirical researches do find that implied volatility is not always superior to volatilities computed from historical index prices for predicting future volatility (e.g., Poon and Granger, 2003). On the other hand, our naïve adjusting approach for the VIXC may be responsible for such a poor performance regarding longer horizons. Therefore, we follow Blair et al. (2001) to adjust our VIXC data. However, we have to sacrifice one third of our observations. This makes our out-of-sample contain only 137 observations.

We use observations from October 1, 2009 to March 31, 2010 (125 observations) to initially estimate the ARCH model. After the first estimation, we adjust the observed VIXC based on the estimated parameters. The sample for estimation is increased by including one more observation, and the second adjusted VIXC is generated. Through this way, we obtain 137 adjusted VIXC. We then compute the loss functions again. The results are displayed in Table 10.

Table 10 contains the results for observations from March 31, 2010 to October 15, 2010 (137 observations). The associated rankings of prediction measures are listed in Table 11. The results presented in Table 10 and 11 reinforce that with short horizon, the VIXC does produce the most accurate forecast for predicting future volatility with short horizons. With forecast horizon equal to 1 and 5 days, the VIXC is consistently ranked the first. However, with longer horizons, the results are similar to those in Table 8, in which time series forecasts of volatility produce less loss for 10 and 22 day horizons. This consistent evidence suggests that the VIXC does not subsume all information embedded in historical index prices. Furthermore, with longer horizons, the VIXC does not outperform the time series forecasts of volatility.

Table 8 Out-of-sample forecasting comparison for the S&P/TSX 60 by loss functions (198 observations)

	Forecast loss functions			
	MSE	MAE	HMSE	HMAE
Forecast Horizon=1 day				
VIXC	29.9659	3.9860	0.1337	0.2896
MVX	32.7465	4.3928	0.1820	0.3362
GJR-GARCH	34.3781	4.1872	0.1766	0.3188
EWMA	39.5088	4.1174	0.1346	0.2858
Random Walk	34.3481	4.0504	0.1582	0.2899
Forecast Horizon=5 day				
VIXC	23.4234	3.5763	0.0846	0.2348
MVX	29.7833	4.2027	0.1250	0.2919
GJR-GARCH	24.1653	3.6205	0.0856	0.2437
EWMA	30.0739	3.5695	0.0769	0.2198
Random Walk	23.6192	3.1522	0.0719	0.1975
Forecast Horizon=10 day				
VIXC	27.4646	3.9206	0.0981	0.2538
MVX	36.0728	4.5984	0.1405	0.3108
GJR-GARCH	23.6893	3.8140	0.0918	0.2566
EWMA	29.7040	3.7295	0.0798	0.2256
Random Walk	20.6118	3.1949	0.0746	0.2085
Forecast Horizon=22 day				
VIXC	36.4016	4.5386	0.1299	0.2882
MVX	47.2850	5.5236	0.1813	0.3663
GJR-GARCH	25.3437	4.1849	0.1124	0.2879
EWMA	31.5764	3.9695	0.0917	0.2431
Random Walk	32.7169	4.3974	0.1362	0.2914

Notes: The sample period starts from January 4, 2010 and ends October 15, 2010. Within this period, 198 observations are included in computation. The MSE, MAE, HMSE, and HMAE are defined in section 6.2.3.2.1.

Table 9 Forecasting efficiency rankings of prediction measures based on four loss functions (198 observations)

	VIXC	MVX	GJR-GARCH	EWMA	Random Walk
Forecast horizon=1 day					
MSE	1	2	4	5	3
MAE	1	5	4	3	2
HMSE	1	5	4	2	3
HMAE	2	5	4	1	3
Total	**5**	17	16	11	11
Forecast horizon=5 day					
MSE	1	4	3	5	2
MAE	3	5	4	2	1
HMSE	3	5	4	2	1
HMAE	3	5	4	2	1
Total	10	19	15	11	**5**
Forecast horizon=10 day					
MSE	3	5	2	4	1
MAE	4	5	3	2	1
HMSE	4	5	3	2	1
HMAE	3	5	4	2	1
Total	14	20	12	10	**4**
Forecast horizon=22 day					
MSE	4	5	1	2	3
MAE	4	5	1	3	2
HMSE	3	5	2	1	4
HMAE	3	5	2	1	4
Total	14	20	**6**	7	13

Notes: This table lists the overall performance of each forecast measure. The rankings are based on the data in Table 8. The sample period is from January 4, 2010 to October 15, 2010. Within this period, 198 observations are used to compute each loss function. The numbers with the least amount in each total row indicate the best performance.

Table 10 Out-of-sample forecasting comparison for the S&P/TSX 60 by loss functions (137 observations)

	Forecast loss functions			
	MSE	MAE	HMSE	HMAE
Forecast Horizon=1 day				
VIXC	27.0600	3.7598	0.1156	0.2621
MVX	34.7004	4.6757	0.1973	0.3520
GJR-GARCH	37.0972	4.3354	0.1455	0.3035
EWMA	43.9682	4.3566	0.1202	0.2802
Random Walk	35.2448	4.2495	0.1729	0.3013
Forecast Horizon=5 day				
VIXC	21.1874	3.2370	0.0652	0.2036
MVX	33.4697	4.4162	0.1366	0.3001
GJR-GARCH	27.4247	3.6267	0.0756	0.2234
EWMA	34.6863	3.7074	0.0765	0.2151
Random Walk	27.3681	3.3701	0.0757	0.2036
Forecast Horizon=10 day				
VIXC	24.8710	3.5430	0.0768	0.2182
MVX	40.1354	4.7532	0.1473	0.3110
GJR-GARCH	26.2515	3.7810	0.0784	0.2311
EWMA	33.9831	3.8212	0.0776	0.2148
Random Walk	17.9734	2.9388	0.0552	0.1850
Forecast Horizon=22 day				
VIXC	34.7494	4.2027	0.1033	0.2522
MVX	53.2834	5.6288	0.1741	0.3508
GJR-GARCH	27.6907	3.4982	0.0915	0.2668
EWMA	37.4856	3.4833	0.0873	0.2239
Random Walk	32.7296	2.7022	0.0935	0.2446

Notes: we rescale the VIXC by following Blair et al. (2001). The sample period is from March 31, 2010 to October 15, 2010. Within this period, 137 observations are included in computation. The MSE, MAE, HMSE, and HMAE are defined in section 6.2.3.2.1.

Table 11 Forecasting efficiency ranking of prediction measures based on four loss functions (137 observations)

	VIXC	MVX	GJR-GARCH	EWMA	Random Walk
Forecast horizon=1 day					
MSE	1	2	4	5	3
MAE	1	5	3	4	2
HMSE	1	5	3	2	4
HMAE	1	5	4	2	3
Total	**4**	17	14	13	12
Forecast horizon=5 day					
MSE	1	4	3	5	2
MAE	1	5	3	4	2
HMSE	1	5	2	4	3
HMAE	1	5	4	3	1
Total	**4**	19	12	16	8
Forecast horizon=10 day					
MSE	2	5	3	4	1
MAE	2	5	3	4	1
HMSE	2	5	4	3	1
HMAE	3	5	4	2	1
Total	9	20	14	13	**4**
Forecast horizon=22 day					
MSE	3	5	1	4	2
MAE	4	5	3	2	1
HMSE	4	5	2	1	3
HMAE	3	5	4	1	2
Total	14	20	10	**8**	**8**

Notes: This table lists the overall performance of each forecast measure. The rankings are based on the data in Table 10. The sample period is from March 31, 2010 to October 15, 2010. Within this period, 137 observations are used to compute each loss function. The numbers with the least amount in each total row indicate the best performance.

Our sample size may have influences on our statistical results. We further use full year data that starts from January 4, 2010 and ends on December 31, 2010 (251 observations). The results are presented in Table 12 for loss functions and in Table 13 for rankings sequences. The results in Table 12 have the same patterns with those in Table 8. With short horizons, 1 and 5 days, the VIXC produces better forecasts. However, with longer horizons, 10 and 22 days, the time series volatilities produce the most accurate forecasts. The absolute amount of loss and associated rankings can not differentiate whether forecasting measures are significantly different from each other. Thus, we resort to the DM tests. The results are displayed in Table 14 for both sample periods.

Table 12 Out-of-sample forecasting comparison for the S&P/TSX 60 by loss functions (251 observations)

	Forecast loss functions			
	MSE	MAE	HMSE	HMAE
Forecast Horizon=1 day				
VIXC	26.0940	3.6628	0.1517	0.2891
GJR-GARCH	30.7822	4.0165	0.2464	0.3461
EWMA	34.4366	3.8801	0.1831	0.3041
Random Walk	31.1097	3.8909	0.1782	0.3056
Forecast Horizon=5 day				
VIXC	19.4615	3.1903	0.0762	0.2216
GJR-GARCH	20.9223	3.3903	0.0955	0.2522
EWMA	25.1179	3.3006	0.0768	0.2234
Random Walk	19.5929	2.8484	0.0672	0.1931
Forecast Horizon=10 day				
VIXC	22.8392	3.5032	0.0891	0.2413
GJR-GARCH	20.6375	3.431	0.0971	0.2606
EWMA	24.6492	3.3895	0.0747	0.2222
Random Walk	16.9153	2.8316	0.0659	0.1964
Forecast Horizon=22 day				
VIXC	30.1788	4.0269	0.1177	0.2725
GJR-GARCH	22.6850	3.9385	0.1212	0.2957
EWMA	25.9268	3.5361	0.0827	0.2324
Random Walk	26.4908	3.8015	0.1154	0.2646

Notes: we use data in the period from January 4, 2010 to December 31, 2010, 251 observations in this period. The MSE, MAE, HMSE, and HMAE are defined in section 6.2.3.2.1.

Table 13 Forecasting efficiency ranking of prediction measures based on four loss functions (251 observations)

	VIXC	GJR-GARCH	EWMA	Random Walk
Forecast horizon=1 day				
MSE	1	2	4	3
MAE	1	4	2	3
HMSE	1	4	3	2
HMAE	1	4	2	3
Total	**4**	14	11	11
Forecast horizon=5 day				
MSE	1	3	4	2
MAE	2	4	3	1
HMSE	2	4	3	1
HMAE	2	4	3	1
Total	7	15	13	**5**
Forecast horizon=10 day				
MSE	3	2	4	1
MAE	4	3	2	1
HMSE	3	4	2	1
HMAE	3	4	2	1
Total	13	13	10	**4**
Forecast horizon=22 day				
MSE	4	1	2	3
MAE	4	3	1	2
HMSE	3	4	1	2
HMAE	3	4	1	2
Total	14	12	**5**	9

Notes: This table lists the overall performance of each forecast measure. The rankings are based on the data in Table 12. The sample period is from January 4, 2010 to December 31, 2010. Within this period, 251 observations are used to compute each loss function. The numbers with the least amount in each total row indicate the best performance.

6.2.3.2.5 **Diebold and Mariano test for equal accuracy**

We report DM test results in Table 14. The first number in each group is based on observations from January 1, 2010 to October 15, 2010 (198 observations) and the second one from January 4, 2010 to December 31, 2010 (251 observations). We first discuss the results with 198 observations. Starting from comparison between MVX and VIXC, we find that in14 of 16 tests the VIXC produces less loss than the MVX at the 5% significance level. This evidence indicates that the prediction power of VIXC is superior to that of MVX. This finding is consistent to the findings from both regression tests and GJR-GARCH in-sample examination.

Overall, the VIXC produces less loss when forecasting the one and five day volatility. However, most DM results indicate that the difference of loss between VIXC and alternatives is not consistently significant at the 1% level. This evidence suggests that the VIXC outperforms its counterparts for predicting short-horizon volatility but may not subsume information embedded in time series forecasts. When the forecast horizon is beyond ten days, the time series forecasts of volatility produce less loss. Particularly, with ten-day horizon, random walk consistently produces a smaller loss than the VIXC and the difference in loss is significantly different from zero at the 1% significance level. In twenty-two-day test, the loss from EWMA is smaller than the loss from VIXC and also the difference in loss is significant at the 5% level. This evidence indicates that time series forecasts of volatility outperform the VIXC for forecasting volatility with longer horizons. Volatility derived from index prices contains incremental information that is not reflected in the VIXC.

Next, we discuss the results from 251 observations. It is seen that sample size affects the statistical results of GJR-GARCH volatility. When sample size increases from198 to 251, most DM test results between the VIXC and GJR-GARCH change. For example, under MSE criteria and with one day horizon, the DM changes from -1.5902 for small sample to -2.1006 for larger sample. However, these changes can not distort our conclusions based on 198 observations. Because we find that with 10-day horizon, the DM results for random walk are consistent with four measures. With 22-day horizon, the DM results for EWMA are consistent as well. In both cases, the significance levels from DM tests do not change. These DM tests show clearly that with longer horizons, the time series forecasts of volatility produce more accurate forecasts of future volatility. Particularly, the random walk seems to forecast better than the VIXC. Our findings are consistent with Noh and Kim (2006) for FTSE 100 implied volatility and Koopman, Jungbacker, and Hol (2005) for the S&P 100 implied volatility. They all indicate that the historical volatilities derived from high frequency intra daily data have incremental information and outperform implied volatility for predicting future volatility.

Table 14 DM test results

	DM test results based on four loss functions			
	MSE	MAE	HMSE	HMAE
Forecast Horizon=1 day				
VIXC vs MVX	-1.1735/	-2.1057**/	-2.5943***/	-2.5658**/
VIXC vs GJR-GARCH	-1.5902/-2.1006**	-1.2392/-2.5449**	-1.9692**/-2.7190***	-2.1046**/-4.0054***
VIXC vs EWMA	-2.0700**/-2.2743**	-0.6721/-1.3349	-0.0650/-1.3802	0.2849/-1.1870
VIXC vs Random Walk	-0.6369/-0.9179	-0.2008/-0.8710	-0.7311/-0.8786	-0.0175/-0.8335
Forecast Horizon=5 day				
VIXC vx MVX	-3.6239***/	-3.6223***/	-4.1015***/	-3.9297***/
VIXC vs GJR-GARCH	-0.3559/-0.8740	-0.2583/-1.3526	-0.1271/-2.3222**	-0.7179/-2.4711**
VIXC vs EWMA	-2.0372**/-2.1845**	0.0371/-0.7048	1.0198/-0.0895	1.2927/-0.1692
VIXC vs Random Walk	-0.0628/-0.0533	1.7920*/1.7871*	1.1219/0.9750	2.4372**/2.2169**
Forecast Horizon=10 day				
VIXC vs MVX	-4.5128***/	-3.6961***/	-4.4728***/	-3.8583***/
VIXC vs GJR-GARCH	1.7824*/1.2895	0.5842/0.2557	0.8397/0.8397	-0.2242/-1.1131
VIXC vs EWMA	-0.8566/-0.8690	1.0051/0.7030	2.3797**/2.3795**	2.4166**/2.1553**
VIXC vs Random Walk	3.4135***/3.7083***	4.1774***/4.7452***	3.2377***/3.2377***	4.0872***/3.8701***
Forecast Horizon=22 day				
VIXC vs MVX	-4.7574***/	-5.1278***/	-4.4730***/	-0.0527/
VIXC vs GJR-GARCH	4.3104***/3.5691***	1.7530*/0.5059	2.0379**/-0.4210	0.0246/-1.8017
VIXC vs EWMA	2.1066**/2.3234**	3.1668***/3.2167***	4.5258***/4.7752***	3.9275***/3.7336***
VIXC vs Random Walk	1.7252*/2.1687**	0.8861/1.6933*	-0.6848/0.2985	-0.2785/0.7706

Notes: DM denotes Diebold and Mariano (1995) test. It is computed with the equation: $DM = \frac{\bar{d}\sqrt{n}}{\sqrt{\widehat{var}(d_t)}}$, where $d_t = f(e_{1,t}) - f(e_{2,t})$. $f(e_{i=1,2,t})$ is one of four loss functions: MSE, MAE, HMSE, HMAE. $\widehat{var}(d_t)$ is the asymptotic variance of the differential series of d_t. $\bar{d}$ is the sample mean of loss differential, $\bar{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$. For the first number in each column, it is based on observations in sample period from January 4, 2010 to October 15, 2010, in which 198 observations are included. For the second number in each column, it is based on observations in sample period from January 4, 2010 to December 31, 2010, in which 251 observations are included. Because the ME stops computing MVX on October 15, 2010, we can not compute DM tests for VIXC vs. MVX for the second period. *, **, and *** indicate that the DM tests is significant at the 10, 5, and 1% levels, respectively. A negative amount indicates that VIXC make less loss comparing to its counterpart.

7. Conclusions

In this thesis, we examine information content of Canadian implied volatility indexes: VIXC – a model free implied volatility and MVX – a model-based implied volatility. Both indexes are computed and disseminated by the Montréal Exchange; therefore the error-in-variables problems make the least influence on our statistical inferences. To compare the information content of each index, we use the values of VIXC and MVX in the period from October 1, 2009 to October 15, 2010, during which both indexes coexisted.

Our GARCH in-sample test indicates that VIXC subsumes all information embedded in MVX for predicting future volatility. In addition, we find that the residual coefficient of the GARCH model, expanded with VIXC or MVX, is significant at the 1% level. This evidence implies that the Canadian stock market is inefficient. Furthermore, when we expand GJR-GARCH model with VIXC or MVX, we find that both volatility indexes cannot fully capture the leverage effect in daily index returns. We conclude that both VIXC and MVX do not incorporate all information contained in historical index prices for predicting future volatility.

In addition to GARCH in-sample comparisons, we use both regression and forecast accuracy tests to compare the prediction ability between the VIXC and MVX in light of out-of-sample tests. With 1-, 5-, and 10-day horizons, our regression tests indicate that VIXC incorporates all information that is reflected in MVX. Under four loss criteria, we find that VIXC consistently produces more accurate prediction than MVX in all time horizons. The DM tests indicate that in 14 of 16 cases, the loss resulting from VIXC prediction is significantly less than the loss from MVX prediction. We conclude that VIXC is superior to the MVX for predicting future volatility. Our conclusion is consistent with Jiang and Tian (2005) who find that model free implied volatility derived from the CBOE SPX options, is superior to B-S implied volatility derived from the same index options.

We also compare the forecasting power of VIXC with alternative forecasts of volatility derived from historical index prices. We use realized volatility computed from high frequency 5-minute index returns as our comparison benchmark. With respect to the prediction ability for future volatility, we find that in time horizons lesser than 10-trading days, VIXC provides the most accurate forecasts. On the other hand, with longer horizons, the historical volatilities derived from index prices, particularly the random walk, provide better forecasts. Our findings are consistent with Charoenwong, Jenwittayaroje, and Low (2009) who find that the prediction power of implied volatilities decreases with the increase of forecast horizons. We conclude that VIXC can not subsume all information that is reflected in historical index prices. The time series forecasts of volatility do have incremental information that is not reflected in the VIXC.

Our results have strong practical implications, because our results indicate that in Canadian stock market, the implied volatility index, VIXC, alone cannot provide the most accurate forecast regarding future realized volatility. The combination of VIXC and other forecasting measures of volatility such as random walk and EWMA may produce better results.

Our results indicate that VIXC does not forecast future 22-trading day's volatility better than alternative forecasts based on historical index prices. This finding is consistent to some empirical findings across option markets (Poon and Granger, 2003). We suggest that future research is needed to investigate what factors make VIXC less accurate to predict monthly volatility.

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