THE IMPACT OF REGULATION ON AN EXHAUSTIVE RESOURCE INDUSTRY: A METHODOLOGICAL APPROACH AND A MODEL*

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I

INTRODUCTION AND OVERVIEW

The study described here is part of a project financed by the National Science Foundation (NSF) to examine and evaluate the consequences of government regulation on the U.S. copper industry. There have been many attempts to understand the economy-wide consequences of particular forms of regulation, even of the family of regulations administered by a particular governmental agency.[1] There have been far fewer studies of the effects of a variety of regulations on a particular target group. A single industry can be the target of many different kinds of regulation, not only by different agencies at the same level of government, but also by agencies at federal, state and local levels. Taking the perspective of a single regulated group is important to our growing awareness of the need to evaluate regulation as searchingly as government expenditure programs. Regulatory constraints from different sources may offset or augment one another with regard to achievement of regulatory goals and to the real social costs imposed on the economy. Mutual coordination, explicit or tacit, of different types of regulation and attention to the temporal change of regulations may be critical in increasing the efficiency of such programs enough to save them from the rising tide of criticism.

Our project has attempted to provide a theoretical and quantitative framework for measuring and evaluating the consequences of different forms of regulations. The data base laboriously developed and the analytical model to which it is to be applied are intended to permit tracing various kinds of regulatory constraint. We have chosen three federal programs to illustrate how the model works—assignment and change of water rights, federal land withdrawal from mineral exploitation, and air pollution control, but a wide variety of constraints can be dealt with.

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[1] For a comprehensive treatment, *see*, A. KAHN, THE ECONOMICS OF REGULATION (1970).

One consequence to which the model is sensitive is investment in new and different facilities. Several extant technologies are available to firms wishing to replace or extend their current facilities. These technologies differ in profitability as adaptations to various kinds of regulation—including especially air pollution control among the three we illustrate. Thus, the model is capable of predicting (at least in principle) what technical modifications will be chosen from among existing technologies. Moreover, by examining the loss of profitability (and/or markets) suffered by the industry when even the most favorable currently available technology is chosen as a response to new regulation, an implicit demand for innovation in the desired direction can be inferred.

Regulatory change may induce a "defensive" or "reactive" form of innovation. It may also influence "initiatory" innovation. This type is generated to shift relative market shares or preclude potential entry or simply to enhance profitability because of the inherent malleability of technology with respect to either products or productive processes. It is usually believed that such initiatory innovation, given the special features of the industry in question, is encouraged by high accumulated retained earnings—and ultimately, of continued high rates of return. Because of their level and changes in level, regulatory constraints can affect returns adversely enough to significantly deplete research and development (R&D) budgets and so initiatory innovations. Overall innovationary activity may rise or fall depending on the extent to which reactive and initiatory innovations compete with one another for scarce resources. Our model does not explicitly treat initiatory innovation or R&D budgets. It provides an important input to this relationship by predicting the profit consequences of different kinds of regulation.

Time and resource limitations have led us to a research strategy of successive approximation in model specification. We have designated a relatively simple (but nonetheless complex) model as a first cut, to explore some of the most salient features of the real world. We hope to substitute more elaborate components in future work. The model described here should therefore be understood not as a final statement of the real relational web, but as a first entry that possesses enough richness to illuminate many of the salient issues in this area.

The methodology of the study is as follows. We seek to evaluate the positive and negative consequences of each of a number of regulatory programs, both singly and in groups. The positive aspects relate to the achievement of the goals of the regulatory program that results from the regulation-induced modification of industry behavior. The negative aspects refer to the direct and indirect adjustments made by the industry in the face of the real cost of the modifications induced by regulatory constraint. For example, if regulations call for a stack scrubber and installation of such equipment leads to sharp decrease in sulphur dioxide emissions; the improved health, agricultural productivity, etc. in the downwind area constitute regulatory benefits. The resulting increased cost to the industry of remaining in business, on the other hand, leads to input mix, output level, price and even locational adjustments, as well as a changed degree of competition with foreign copper suppliers. These input-price-net import changes are components of the real social cost of the regulation.

The scheme has four main segments:

1. a classification of degrees of stringency for each type of regulation we have chosen to illuminate;
2. a model of the copper industry (from exploration to refining) that determines current output-price decisions at all the stages ("current production module");
3. a model of the effect of each kind of industry compliance with the various regulatory levels on the (generally) externality dimensions whose control is the goal of the regulatory programs ("regulatory compliance/externalities module"); and
4. a model of induced industry investment, both of normal capacity replacements and additions and of special equipment and other compliance systems ("investment module").

We begin with an assumed change in some portion of the regulatory regime impinging on the industry. This induces a direct compliance action—purchase and installation of a new system, modified input mix or output level or temporal pattern, or early replacement of plant with one embodying a different technology. The effects of these charges on regulatory goal achievement is traced via the compliance/externality module. The new technology-cost situation facing the industry leads the industry to change its current input/output/price behavior. This is traced via the current production module.

An important consequence of these current operations impacts is a change in industry profitability. So, in addition to any investments that represent direct compliance with regulatory requirements, the changed profitability influences industry decisions about capacity additions at different stages of production. These are traced via the investment module. Thus, both short and long run effects on industry operations and on achievement of regulatory goals are predicted by the model complex.

A distinctive feature of our approach is our disaggregation of the industry into five production stages: exploration, mining, milling, smelting and refining.[2] Each stage is viewed as receiving inputs from the preceeding stage and sending its outputs to the succeeding stage. Since different kinds of regulation impact the different production stages unequally; disaggregation into

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[2] See LEGEND OF TERMS, at 146.

the processes of each stage enables us to capture the specific impacts with much greater sensitivity and subtlety than if the industry were described as a single-stage aggregate. Of course this differentiation complicates the modeling effort. To keep complexity within feasible bounds we have had to make a number of simplifications in this first-round model variant. First, we have assumed away inter-firm competition at each stage and between stages, treating the industry as a single efficiently maximizing unit. Second, we treat locational decisions as exogenous, not endogenous. Third, we omit the scrap market. Other simplifying assumptions will be presented and explained in the course of the exposition. We especially want to relax the three mentioned here in subsequent model variants since they concern interesting issues.

II

COMPONENTS OF THE MODEL

A. Firm and Industry

In modeling the U.S. copper industry, a number of simplifying assumptions must be made if the results are to be analytically tractable. For the present, we model the industry as if it were a scaled-up version of one of its major parts—a simple, vertically integrated firm. In fact, a large part of the industry is made up of a few such firms, whose operations run from exploration through mining to refining. Of course, not all of the inputs to a given facility under the firm's control arise within the firm, nor does all that facility's output pass to other facilities controlled by the firm; nevertheless, most of the important links pertaining to such firms are internal. Therefore a single-firm model seems to be a good approximation to the greater complexity of the industry as a whole.

This assumption simplifies the representation of many of the transactions between productive facilities, because a single objective function may be assumed to govern these transactions when the facilities belong to a single firm. This is not equivalent to assuming that the industry is a single monopoly. It does not prejudge the power of the industry in the market. It merely implies that, whatever the internal competitive process and governmental constraints that determine an aggregate price-output position consistent with the industry demand function, the output is produced at least cost for the industry as a whole. The relative internal stability often expected of highly concentrated oligopolies such as the copper industry makes this a fair approximation to reality.

Thus, we assume that interfacility, interstage transactions within the industry are conducted as though the industry were a single firm. We do not specify how this firm adopts a price strategy for the market. However, once a price has been established, the industry responds by maximizing aggregate industry profit: it supplies an amount of copper such that marginal cost to the industry equals market price.

For the present we assume that some production stages comprise a single facility—for instance, only one production technology at the mill or refinery, or in exploration. In reality, a variety of technologies will be available at each stage, but circumstances will generally make one of them most profitable. We assume that this technology will be adopted by the industry as a whole. Changes in regulatory stringency will typically alter the relative profitabilities of the different technologies, and thus will induce some technological shift. The present model can indicate the direction and magnitude of such shifts by determining the relative profitabilities of different technological structures under identical regulatory regimes.

The exceptions to the assumption of a single facility are the mining and smelting stages. Because we wish to allow for differences in grade and in deposit depletion, and for the resulting shutdown of old mines and replacement by new ones over time, we assume a distribution of mines of different grades and ages (i.e., percent of deposit removed) in each period, along with the existence of an inventory of different discovered, but as yet undeveloped, deposits. For smelting, we wish to permit different technologies to be adopted as adjustments to changing regulatory constraints; so in the context of high smelter durability different kinds of facilities will be in operation as a reflection of the different circumstances prevailing at the time of their construction.

The level of aggregation of the model represents a compromise. By aggregating all facilities into a single facility at each stage, we avoid the troublesome issues of relative and absolute facility size, and specific input-output relations between facilities. Nevertheless, production stages remain distinct in the model because different types of regulation may affect each stage in different ways.

Subsequent refinements of this model will introduce more than one facility at each stage, some with different technology. This will permit a more exact investigation of differential effects of regulations. A further refinement will attempt to introduce competitive relations between firms within the context of oligopoly.

B. Objective Function

The assumption that the industry can be represented by the behavior of a single firm means that industry behavior is directed toward the maximization of a single objective function. This implies not just that all facilities have a common standard of performance, but that, more particularly, a unit increase in the level of the objective function due to the operation at any one facility is as good as a unit increase anywhere else.

The objective function measures industry earnings, but the form and definition of earnings are not self-evident. As would be expected, the need to model intertemporal trade-offs leads to a formulation in terms of present discounted values, but there is an unusual complication in that the whole industry is based on the exploitation of a depletable resource, copper reserves. The basic trade-off is this: by mining a unit of reserves in the present, the industry incurs an opportunity cost because it will not be able to mine that unit (and gain the associated profits) in the future. Because significant stockpiling is costly, this trade-off can have several ramifications which significantly complicate the task of modeling. Shall a unit of reserves be mined today, at today's prices, or next year, when prices are expected to be higher? Shall a low-grade ore be mined today if doing so permits a high-grade ore to be mined, even though the low-grade falls below the minimum economical (cutoff) grade given present prices; or should the lower-grade, and the higher-grade linked to it, not be mined until a later date when higher copper prices lower the cutoff grade? (The risks associated with such decisions lie not only in the length of postponement, but in the possibility that the mine will shut down permanently before that unit or those units have been exploited.) Or, along similar lines, should lower-grades be mined whenever prices render them profitable, so as to decrease the risk that lower prices in the future will force them to remain in the ground forever?

Decisions about such intertemporal choices lead to two forms of waste. First, grades of ore may be mined that are not worth sending to the mill because the downstream processing costs exceed expected revenues. Such ores, which fall below the economical cutoff grade, are mined because knowledge of the distribution of reserves is never exact, so that such ores may be mined by chance, and because, in some cases, excavation of high-grade reserves requires that low-grade ores be moved to expose them. This form of waste could be regarded as an unavoidable cost of mining (in the same sense that clearing overburden is a cost) except that ores dumped as waste today might well have proved profitable if mined in the future under a different set of prices and production costs. So whether or not an ore is waste is determined by pervading circumstances at the time of mining.

A second form of waste can be identified when a mine shuts down permanently with part of its original reserves unexploited—even grades which currently exceed the cutoff grade. This comes about when, as a result of past mining decisions, the mine is left with a distribution of ores which, given geological constraints, cannot be mined profitably at current prices.

Both forms of waste are variable and depend upon a particular pattern of intertemporal choices, which in turn depends upon expectations of costs and selling prices. Public regulations can have a significant effect on costs over time. Because of the durability of the capital equipment required for compliance, and because of the practical irreversibility of waste dumping, the *pattern* of regulatory change over time may significantly influence the social cost of regulation. We intend to examine such effect by imposing regulatory time-profiles on the model.

Our treatment of the firm's objective function for current operations attempts to capture some of the essence of intertemporal choice by careful specification of single-period profits, rather than by expanding the domain of the function over multiple periods. We assumed that the firm attempts to maximize profits from operations in the current period alone. Profits are defined as revenues from current output, minus costs of producing that output. Costs for labor, energy, water, and capital are defined conventionally. The cost of reserves used in producing output are more difficult to determine.

The cost of using the reserves in the current period is that they are no longer available for use in the future, i.e., the opportunity cost of future exploitation of that specific amount and kind of reserve. What is lost is the profitability of that future use expressed in present value terms. We calculate the profitability by assuming that, *relative to an earlier, foregone use* the particular reserve has zero opportunity cost in the later use. The revenue and costs involved in the reserve's later use are those expected to prevail in the relevant future. Ideally, both cost and revenue expectation are assumed to be held (with perfect confidence) for all future periods. The optimization program for the whole industry is run for each of those periods (assuming zero cost for the reserve itself) and present discounted value of profits noted. The highest of such future present values becomes the opportunity cost of the reserve for the *current* period. For practical feasibility, calculation for multiple alternative future periods is generally eschewed, and in the basic model we simply calculate future profitability for one period into the future, except when exploring the behavior of the industry under an assumed highly variable future price and/or cost profile.

Since the cost of future profitability in using depletable reserves is included as an explicit cost of today's use of them, current period profits will be positive only if today's use is more profitable than expected future use. Thus, the intertemporal choice issue is directly confronted. The most complicated expected future scenarios about prices and costs can be handled by variants of the basic procedure where the maximum of an appropriate, limited, multiperiod set of future alternatives is calculated.

More complex aspects of intertemporal choice can of course be dealt with by specifying an explicit multiperiod model (for which dynamic programming must be used, instead of the static linear programming problem we formulate here). Complicated sequence patterns can be predicted. Some of the issues raised by these properties are significant. We have decided to make our first approximation model a two-period opportunity cost model (today versus the best future) for a number of reasons:

1. it is simpler;
2. it captures the substance of present exploitation versus delay issues;
3. it permits a simpler, powerful investment module specification, since gains from capacity expansion are directly generated from it;
4. it permits the modeling of industry surprises when regulatory stringency is changed, since the multiperiod model requires expectations for all future years;
5. we are less interested in a particular temporal sequence of outcomes than in the "settled-down" steady-state consequences of regulatory changes;
6. industry experts report that explicit multi-year future profiles are not in fact made. Nonetheless, we hope to employ a multiperiod formulation in the future, to compare with the properties of the simpler two-period model.

C. Overview of the System

We are concerned with the following stages of copper production: exploration, mining, milling, smelting, refining. The last four are linked on a daily basis in current production operations. Exploration is not linked with mining for current operation, but only to provide an inventory of discovered copper deposits, some of which will be developed into new mines in the future. Thus, exploration is a form of long run investment in new mine capacity and will be treated as such.

We assume the industry comprises a single multifacility firm. There are many mines and mills, fewer smelters and refineries. In this model variant, we are not concerned with how many there are, or their size distribution, or who ships what to whom to make up the inter-stage flow of throughput from mining to refined copper market. The industry has a number of interfaces with the rest of the world: it buys productive resources from resource owners for both current production operations and investment in productive capacity; it sells refined copper to the United States and world refined copper market; its operations affect the quality of the environment and the availability of certain special natural resources, like land and water, for other uses (e.g., recreation). All three interfaces are treated explicitly in the model.

1. *Current Production Module*

The characteristics of each facility are assumed to remain constant over the course of each year; all changes in capacity and other parameters take place instantaneously at the end of each period. As a result of demand and supply interaction in the market for refined copper (which depends on world demand and supplies other than those in the United States) at the end of the preceding period, the new period begins with a price of refined copper facing the U.S. industry. On the basis of this actual market price (which the market power of the U.S. industry may have helped determine), the refining facilities make an optimization decision that consists of setting an internal bid price which they will be willing to pay for each ton of blister copper "sold" to them by smelters such that they will be willing to accept total quantities as determined by the smelter facilities, but constrained by current refining capacity. Thus, refiners actively set buying prices and passively accept quantities determined earlier in the production process. The sense in which this type of price/quantity strategy is optimal will be examined below. The refinery actively sets a bid price only for the copper throughput. All other productive resources set their own prices at which they are willing to be used by the smelter.

The refiner's bid price for anode copper is based on both the refined copper price in the market and the cost of refining it. In this context of a specific production cost situation, the market price is "passed-through" the refining stage to the smelting stage. Corresponding to this backward price pass-through, operating decisions result in a throughput pass-through of blister copper into refined copper.

The same process occurs at the smelting stage. Based on the price at which the smelter can sell its output of blister copper (the refiner's bid price) and the cost of producing it, it sets its own bid prices for throughput flowing to it from the milling stage. As in refining, other productive resources used in smelting set their own selling prices. Determination of pass-through bid prices, with a willingness passively to allow quantities of throughput to be determined by the mills (subject to smelter capacity constraints) constitutes an optimization process for the smelter. Here it is more complicated than for refining because more than one grade of concentrate is subjected to transformation into blister copper. Since the real cost of producing a ton of homogeneous output differs when different grades of concentrate are used, each concentrate grade will be given a different bid price, the differences reflecting differences in cost to produce a given output. Details of optimal bid pricing will be presented below. So, blister prices are passed backward to milling and the resulting operating decisions at the mill and smelter result in a determinate forward throughput pass-through from concentrates to blister.

The same process, with a further complication, occurs at the milling stage. Here there are more than one kind of throughput input and output: different grades of ore from the mine as well as different grades of concentrate output. The marginal cost of each input grade differs depending on which output grade it is used for. Since this latter decision is made by the mill, not the mine, only a single bid price must be set for each input grade used, regardless of what specific use is made of it. At the same time the price must enable the mill to accept what quantities of ore are offered by the mine—again, as modified by the mill's capacity constraints—and allocate ore inputs optimally to output processes. Thus, bid prices are set on different ore grade by an optimization process. In response, supplying mines will bring quantities of the various ore grades to the mill—again, a backward price pass-through inducing a distinctive forward pass-through of the throughput.

Finally, at the mine, the ore bid prices (in conjunction with the different generally probabilistic, cost of producing each kind of ore and various capacity constraints) elicit a decision by the mine on a level and selection of mining activities and on the basis of this decision appropriate ores are sent to the mill: a complete backward price pass-through induces a complete forward throughput of copper. The twin movements result from optimization decisions. The optimization is modeled as linear programming maximization of the objective function for each stage. To simplify actual programming we have divided the four current operating stages into two groups: mining and downstream processing (milling, smelting, refining). Based on market prices for refined copper (however they are in fact set), group two maximizes its profits. This generates bid prices for ore outputs from mining. Taking these bid prices as selling prices, the mining stage makes production decisions to maximize its profits. The resulting mine output either just meets downstream processing capacity or not. If it does, this set of outputs/pass-through prices represents the industry's "supply response" to the refined copper price. If it does not, an iterative procedure changes bid prices until convergence is reached.

For a given set of technologies and productive capacities, the initial price will elicit a certain supply of refined copper from the industry to the market. This supply need not necessarily be equal to demand at that price. (Demand is derived from an aggregate demand function for U.S. production, scaled down by the ratio of the firm's capacity to total U.S. capacity.) If demand and supply differ, the model uses a mechanism that progressively forces convergence of the two. It involves successively trying new adjustment prices that narrow the demand-supply divergence. We obtain the first of these adjustment prices by stating at the amount supplied at the initial price and determining from the demand function what price would be required to elicit an amount demanded equal to that quantity. The new adjustment price is simply the average of this price and the initial one. Substituted into the model, the new adjustment price elicits another supply level at the market, closer to the new level of demand this time. This generates a new adjustment price. The process is continued until convergence is obtained. At this point, a short run equilibrium between supply and demand has been obtained and the results of the model are accepted as 'real' for the current period.

2. *The Regulatory Compliance/Externalities Module*

In addition to its linkage with earlier and later production stage each facility has various relations with the "outside" world. One consists in competing for especially scarce, even unique productive resources (water in the Southwest), or certain wilderness lands. Another consists in generating negative externalities such as polluted air and water or dangerous and unhealthy working conditions. Production stages differ markedly in their demand for special resources and in their environmental effects; the model therefore distinguishes these quantities for each stage. Given a technology and set of operating procedures, the size and character of social impacts due to each facility is a direct function of operating level. The output of the current production module provides the information necessary to assess these impacts. This assessment is to be carried out, of course, in the context of current governmental regulations. In the model this takes the form of performance standards, emission ceilings, water allocations, and land leases or withdrawals.

Compliance with regulation is dealt with in two ways. Most simply, we assume complete compliance with a regulatory regime by a particular means. Typically, this will take the form of investment in specific equipment, which changes the total cost of production and sometimes the variable (marginal) cost of production. Profitability will always be affected by regulation; optional production level and factor mix will sometimes be affected. On the other hand, externalities will be reduced to the required level. The existence of alternative acceptable compliance strategies is dealt with by applying overall consequences. The more complex treatment consists in applying each of the most relevant alternative compliance strategies to the industry supply model, selecting the one which results in the highest industry profitability as the optimal strategy, and predicting that the industry will in fact adopt it.

Different compliance strategies will not only differentially affect industry operations and profitability but also the level and character of externalities. So the regulation/externality module predicts social impacts as well as industry adjustments.

3. *Investment Module*

For given technology, regulatory compliance and initial productive capacity, the current production module predicts a demand-supply-market price equilibrium via industry-wide optimization (calculated as a linear programming problem) and market clearing adjustment. This equilibrium rests on a supply response that reflects the initial resource constraints resulting from initial productive capacities at the various stages. Our linear programming solution to the industry maximization problem generates shadow prices in each of these capacities that proved a binding constraint. These shadow prices provide the key to the third major type of interface the industry makes with the outside: investment in additional capacity.

A binding constraint generates a positive shadow price, which indicates the amount by which a unit increment in that capacity would increase the level of objective function goal achievement in each subsequent period (assuming other conditions unchanged). This indicates therefore, what such an increment would be worth each future period to the investing firm under unchanging conditions. The investment module compares that prospective flow of returns from investment with the cost of adding one unit of such capacity. No questions of minimum necessary scale of investment arise because, having aggregated all facilities at each stage to a fictional "average" or typical facility, "unit" increases at a particular stage are large relative to actual individual facilities; each can be allocated *among* individual facilities to avoid actual lumpiness constraints.[3]

For post-mining stages this is straightforward. Additional plant and equipment can be added to an existing operational site or can serve to establish a new operating site. At the mining stage, it generally means the development of a new mine physically separated from existing sites. This is possible because the shadow price process does not put an opportunity cost (or profit enhancing) valuation only on reserves in already-developed mines but also in deposits held by the industry as an inventory for future mine development. This is the analytical link between the exploration stage and all the others. Exploration creates an inventory of reserves which have a potential marginal (incremental) value for the industry given by the shadow prices or reserves of different grade. The cost of developing each into an operating mine determines

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[3] The calculation is as follows:

Let a particular productive capacity i have shadow price P_{ij} associated with it at the beginning of time period j (in effect, the end of time period j−1). Let the cost of producing an additional unit of it at j be CC_{ij}. Then the expected rate of return from the discounted gains of the investment in an extra unit of i is:

$$(1) \quad R_{ij} \equiv \left[P_{ij} \sum_{k=1}^{t} \frac{1}{(1+r)}k - CC_{ij} \right] \div CC_{ij}$$

where T is the expected lifetime of the proposed investment. We compare R_{ij} with the required (target) rate of return for the stage of production involved (we shall use the same rate for all stages), p_j, as follows:

$$(2) \quad IP_{ij} = I\ (R_{ij} - p_j)$$
$$\text{with } 0 = I\ (0) \quad I > 0$$

where IP is investment potential. So any excess of R_{ij} over p_j generates investment in capital capacity i, the amount of investment being that amount which when in place (assumed to be one period in the future j+1) will, all other things equal, reduce the rate of return on that capacity in j+1 to p: *i.e.*, capacity will be expanded to the extent that, when a new shadow price is generated in j+1, it will imply $R_{i,j+1} = p_{j+1}$. (Our greater interest in "steady-state" than in dynamic sequence consequences is shown in our willingness to assume a one-period gestation period for investment. This can easly be modified to permit longer and different gestation periods for different stages.)

marginal profitability. The rate of new mine development then depends on the same overall output supply considerations that determine how much additional *capital capacity of any sort is warranted by the market* at any time.

4. *Inter-Modular Flow*

a. *Market Clearing Scenario*

Market equilibrium determines not only copper production but also its "by product" environmental incursions, mostly externalities. The amount and character of these incursions is determined as a function of current operations-output level and input mix in the context of compliance with the existing regulatory regime by the regulatory compliance/externalities module. This module, in its short run form, assumes that a compliance strategy with existing regulations has been implemented by the industry in the form of special equipment or operational procedures, so that its current equilibrium production decisions suffice to determine the vector of externalities and other non- or quasi-market incursions in the current period.

The production equilibrium, and its externalities by products, represent only a short run resting place, however. At that equilibrium, various types of productive and compliance capacity will constrain the solution. These will have generated positive shadow (efficiency) prices. The investment module gauges which of these capacities would raise total industry profitability by being expanded through investment. Where such prospective profitability exists, new investment takes place, both for production capacity and compliance equipment. Capacity is enlarged to a degree where further additions would earn no more than the rate of return on existing capital.

The adjusted capital stock changes industry costs, thus in general the previous market price will elicit a new, generally higher amount supplied, followed by a new iterative market adjustment convergence process among price, demand and supply. The new equilibrium resulting will be a long run equilibrium in that both capacity and its utilization are tailored to market demand. Similarly, the change from short to long run operational equilibrium will generate a change in externality incursions. So all three modules are constantly interacting to produce a three way mutually compatible situation.

b. *Regulatory Change Scenario*

The main reason for developing the overall model is to predict and evaluate the effect of regulatory change. Suppose, then, a long run three way equilibrium has been achieved. Now let there be a change in the regulatory regime; a particular regulation is tightened or loosened. First, the regulatory/externality module enables us to calculate the profitability of the changed set of permissible compliance strategies, then predicts adoption of the new maximum profitability strategy. This different strategy generally changes industry costs (at particular production stages) and so elicits a different supply response than heretofore. Again, the current production module is called upon to trace these supply adjustments and resulting market adjustment convergence process, leading to a new short run and then, via the investment module, to a new long run equilibrium. At this new output-input mix-compliance technique situation, the externality module predicts the character of externality incursions. Once again, all three modules interact to form the new three way coherence.

c. *Demand-Cost-Technology*

Regulatory changes are the focus of the overall analysis. But the performance of different regulatory regimes under alternative industry contexts is an important aspect of this investigation. This is accomplished by hypothetically varying (exogenously) world and U.S. demand conditions, cost conditions and technologies over time. In a later, nonhomogeneous industry model, more than one cost and technological situation can be envisaged simultaneously by applying them to different parts of the industry, respectively.

Begin with a particular demand-cost-technological situation. This generates a long run three way equilibrium. Now let any of these components of the situation hypothetically change. Via the production and investment modules this will generate a new long run equilibrium, and via the regulatory/externality module this will in turn generate a new flow of externalities (as well as possibly changed compliance strategies).

In general, then, any exogeneous change impinging on the system calls for mutual adjustments registered by all three modules.

D. Optimization and Inter-Stage Linkages

The mining and downstream sectors optimize by maximizing their respective profits. As indicated above, this interactive optimization process results in pass-through "bid prices" being set at the interface between the two sectors that elicits a throughput flow that simultaneously satisfies capacity and other constraints in both sectors. These bid prices indicate the real opportunity cost of transforming the throughput input into output at that stage. They are an appropriate downstream profitability indicator to earlier production stages. Each type of throughput input receives a separate bid price. Differentials among them indicate differential downstream profitability for the different types of input, given input-output relations, and therefore serve to allocate mining and processing capacity efficiently among the different input grades.

In each bid price, the prices for all variable nonthroughput input are those set on appropriate factor markets and are, in this treatment, out of the control of the purchasing copper facility. The situation is different for fixed capital (capacity) "used up." The cost of capital "used up" per unit of each activity is the sum of depreciation, maintenance and repair and the opportunity cost of capital to the facility. For this version of the model, we assume very long durability; so to a first approximation we can take depreciation as zero. The cost of capital "used up" is thus measured as the cost of maintenance and repair, and the opportunity cost of capital. The former can be sensitive to total level of activity; the latter is not. Opportunity cost of capital is the "target rate of return" on the total capital in that facility. This is a given quantity with given total value. A target rate of return means applying a given percentage of this value; it is a return fixed in size in the current period. Therefore, the opportunity cost per unit of activity is a declining function of total use, and thus profitability varies inversely to this.

Thus, the bid price pass-through procedure allows for variable profits, the variability stemming from capital capacity. It thus reflects the efficiency in use of both variable and fixed inputs at any period.

E. Costs and Production Constraints

Our treatment of the technology-cost characteristics of all stages is similar, but with appropriate modifications at each stage. For each plant at each stage, the period begins with a given capital installation embodying a particular technology of production and complaince with the current regulatory regime. This installation implies certain capacity constraints on current production. We assume a stepwise linear technology and cost function; i.e., we break down the range of production possibility into a number (usually three) of operation intervals. In each of those intervals a slightly different fixed-proportions linear technology is specified. The middle (the widest-interval) is the range of normal operations, where resource requirements per unit of output are lowest; operation levels higher than that possible when using equipment beyond normal capacity rates, but they increase real resource costs of each output unit via postponing and shortening examination and maintenance periods, increasing maintenance and repair requirements, increasing labor fatigue, etc. Operation levels lower than the normal range, to a lesser extent, make production in that range more expensive than normal because they may make it harder for the different productive factors to combine in the most efficient proportions or in the best rhythm (e.g., unoccupied periods may be more numerous, making start-up and shut-down costs greater than with steady operations). We assume that input prices (except for interstage throughput) are fixed to the individual facility, so different real resource requirements (i.e., variable, not fixed inputs) are translated proportionally into differential per unit production costs. (Thus, the higher unit costs for the low level interval is *not* due to distributing fixed costs over a smaller number of output units, but to higher variable costs per unit of output.)

The technology attributed to the mining stage is special in a number of ways. We have tried to capture a number of the distinctive features of "real world" copper mining while keeping the analysis tractable, so as to make it widely applicable (at least as an approximation) through parametric variation. We have incorporated:

1. the heterogeneity of reserve grades in a given deposit and their spatial irregularity within the deposit;
2. the technical inability to excavate only the ore grades desired, having to remove grades above and often to the sides of grades one wants, even if one knows the exact location of the desired grades;
3. inadequate knowledge about the grade identity of different parts of the deposit;
4. despite these, knowledge and ability sufficient to enable some differentiation and selection of average grade excavations.

We assume that in the base period five different mine types exist, distinguished by average grade, in current operation, and a queue of five different undeveloped deposit types, also distinguished by average grade and arrayed in descending average grade order.

The quantities of these deposits are assumed as follows:

1. the total amount of each *average* grade in operating mines at the start of the base period equals the actual distribution in the copper industry in 1970, as developed in our data base;
2. the total amount of each average grade in discovered but undeveloped deposits at the beginning of the base period is our estimate of the actual amount in the industry inventory as of 1970;
3. the age-average grade distribution for the five operating mine types is based on the actual distribution in 1970;
4. amounts of the capacity in each average grade to be added by investment (either replacement or net additions) is assumed to be continuous up to the total available amount under three above.

Since our mine types are aggregates of individual mines, each type made up of several individual deposits, we avoid problems of lumpiness in investment.

Each individual mine is assumed to consist of a single deposit of given average grade, comprising a number of elementary grades distributed lognormally.

Each mine can extract material selectively for a mean grade that differs from the deposit's average grade, $\tilde{R}$, but only to a limited degree in the upward direction. By stripping additional cover, the average grade actually removed can be entirely free of metallic content. Thus, the range of average grade minable in any period t_i is from 0 to $\bar{R}$ $(1+\epsilon)$. Our practice is to take ϵ as 10 percent.

Our behavioral hypothesis is that in each period t_i the mine selects that mean grade to be extracted, $\bar{R}_i$, which will maximize profits (in the sense given in the objective function). Since ϵ is small, we assume that the constituent grades making up $\bar{R}_i$ are also distributed log-normally. In period t_i material extracted is divided into ore (to be sent to the mill) and waste (to be dumped). Waste consists of all grades below a cutoff grade that is determined by the price of refined copper and downstream processing costs, which in combination with the price of refined copper, would result in losses.

At the end of each period, the mine's remaining deposit must be adjusted for metallic material extracted. Thus, the mine's aging is in terms of the total of reserves already extracted and the part remaining.

In t_i different mines may well select a different $\bar{R}_i$ if they are all attempting to extract the highest grade possible, since they begin the period with a different initial $\bar{R}$. However, if market conditions dictate a lower-than-maximal average grade as optimal, then all mines might conceivably select the same optimal grade.

In smelting, we assume an initial set of facilities with the different technologies existing in 1970, and quantitatively distributed in the actual 1970 proportions. These technologies differ in required input mixes as well as cost and productivity. Their relative as well as absolute profitability are influenced by the character of regulatory constraints in the industry. So changes in these constraints can induce technological shifts either through supplantation or net additions to capacity.

The second major distinguishing feature of our treatment of mining technology is that real cost of extraction is time bound: the older the mine in terms of percent of deposit already mined, the higher the cost of extracting a given amount of material. This reflects the deepening of the open pit with its attendant increasing distances needed to carry materials.

In addition to embedding stepwise linearity of production functions with the noted modifications for mining, the installations at the beginning of each period embody various capacity constraints. At each stage the typical facility experiences capacity constraints. The given scale of capital implies the location of the normal, redundant and forced operations intervals. Moreover, the forced operation interval is not open ended; there is an absolute ceiling beyond which current production levels cannot go. For smelting and mining capital constraints are the only ones specified. For mining and milling additional constraints are assumed.

In mining and milling we assume that water may be subject to absolute ceilings in any period. Water can be obtained from rivers and streams or federal water project sources to the extent of water rights held; additional supplies (wells) must be developed by the facility. The former may be fixed for long periods; the latter can be varied only by additional investment: present developed capacity establishes absolute or, via recycling, approximate ceilings on production.

The other special constraint refers to the labor force. Mines are generally not located in or near large labor markets. It is difficult to change the size of the labor force quickly: geographic separation makes both significant recruitment and dismissals difficult to accomplish quickly (in mining towns, labor dependence on the mine is very strong so contractual pressures are incorporated to prevent large contractions in short periods). This gives rise to two constraints in each period: an effective ceiling and an effective floor on total labor hours worked. The mine must use at least x and no more than y hours each year.

One final constraint in the model applies to the interstage movement of copper content from the mine to the market for refined copper. The weight of material passed through by a given stage to the next as throughput is the weight of the throughput received from the preceeding stage less the waste lost through processing. Waste as a percentage of output is a function of the type of activity chosen by the given stage. Interstage throughputs must conform to these relationships. For the mining stage throughput received from prior stage is interpreted as amount of material extracted from the deposit.

F. Mine Shutdown

Copper resources are a depleting resource. Unlike facilities past the mining stage, whose plant can be replaced (or simply maintained) on the same site by investment, a given mine has an absolute termination when the deposit in which it is based is wholly excavated. As noted earlier, however, the mine's effective life can come to an end before (sometimes considerably before) its deposit is exhausted. If the remaining reserves are of too low a grade to promise profitability under processing (considering present and prospective refined copper prices), they may be left in the ground. The mine may cease operations temporarily or permanently, depending on prices and start-up costs. Thus, a mine may shut down while part of the deposit remains unexploited. On the other hand, the mine may not shut down in a period when revenues fall short of costs, when losses are incurred. Operations may continue despite current losses because of tax considerations or intertemporal calculations. Two related but different elements are involved.

First, the actual shutdown decision depends not on before-tax profits, but on after-tax profits. Tax considerations may significantly decrease the negativity of profits. Intertemporal considerations may then be great enough to convert negative after-tax profitability into positive profitability within a larger context.

1. *United States Income Tax*

In calculating tax liability for any year, positive profits (income) may be offset by 100 percent of the total of any losses sustained by the firm in the five previous years or in the five years ahead (for loss carry forward tax liability for any year is subject to revision in subsequent years as a result of losses in those years). So a prospective loss at t_0 is not necessarily to be avoided by shutting down. Since it serves to reduce the taxable income of some other year, it represents a smaller after-tax loss to the industry overall. Since the marginal tax rate on income is less than 100 percent, the loss carry over by itself cannot transform a loss to a gain, but it can decrease the net size of the loss.

The nature of the adjustment is complicated because the loss carry forward aspect involves expectations about the future (unlike loss carry back, which simply involves an accouting adjustment).

Equation (6)[4] is explained as follows:

Expression (6a) is the five year carry-back alone: i.e., the presence of posi-

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[4] a. $LC_0 = T \cdot (-\Pi_o) \text{ if } \sum_{i=1}^{r} \Pi^+_{0-i} \geq \Pi_o$

b. $LC_0 = T \cdot \sum_{i=1}^{r} \Pi^+_{0-i} + \frac{T}{(1+p)} [\delta_1 (\Pi_o - \delta_1^1 \sum_{i=1}^{r} \Pi^+_{0-1}) + \delta_1{}' \Pi_1{}^*]$

$+ \frac{T}{(1+p)^2} [\delta_2 (\Pi_o - \sum_{i=1}^{r} \Pi^+_{0-i} - \Pi^*_1) + \delta^1_2 \Pi_2{}^*] + \ldots \text{ until } t_5$

$\text{if } \sum_{i=1}^{r} < \Pi_o$

where $\delta_1 = 1 \text{ if } \Pi_o - \sum_{i=1}^{r} \Pi^+_{0-1} \leq \Pi^*_1$

$= 1 \text{ if } \Pi_o - \sum_{i=1}^{r} \Pi^+_{0-1} > \Pi^*_1$

$\delta^1_1 = 0 \text{ if } \delta_1 = 1$

$= 1 \text{ if } \delta_1 = 0$

$\delta_2 = 1 \text{ if } (\cdot) \leq \Pi^*_2$

$\delta_2 = 0 \text{ if } (*) < \Pi^*_2$

where $\delta^1_2 = 0 \text{ if } \delta_2 = 1$

$= 1 \text{ if } \delta_2 = 0$

T is the marginal Federal tax rate
ρ is the discount rate
$\Pi_j{}^*$ is the expected Π in future year t_j, if $\Pi_j{}^* > 0$
Π^+_{0-1} is each *positive* Π value within the past five years from t_j.

tive profits within the past five years totalling no less than Π_0 enables the loss at t_0 to have an immediate tax rebate in previous taxes paid. Thus, the full offset to present loss is immediate. If profits in the past five years fall short of Π_0, expression (6b) becomes relevant. Total profits in the past five years are offset for a full offset (first term). The balance is pushed to expected next year's positive profits, Π_1; any remaining balance to the following year, Π_2; and so on until either the whole of Π_0 has been distributed or the five year period ahead is exhausted. Offset from future expected profits will only come in those future years—and is only *expected*—so each such gain must be discounted back to the present. (The δ terms are simply dummies to account for whether total distribution of Π_0 has or has not occurred with the preceding year.)

The attractiveness of the tax carry over depends on the current unprofitable year being atypical, i.e., a positive profitability having recently occurred or expected soon. Persistent losses, or downward expectations for the future, will tend to decrease the contribution of the tax carry over in maintaining operations in the face of a prospective current loss. This "neighborhood effect" is amplified in the shutdown and start-up cost factors.

2. *Shutdown and Start-up Costs*

The tax offsets convert negative before tax income in t_0 into smaller negative after-tax income. Intertemporal choice considerations may convert these to positive gains in a larger context and thus warrant continued operation. If the mine were closed in t_0 because of after tax losses, but was expected to be reopened subsequently when profitability improved, certain shutdown and start-up costs would have to be incurred, the former in t_0, the latter when the mine was reopened. A shutdown in t_0 followed by a reopening in t_j is rational if output prices (production costs) are enough higher (lower) than at t_0 to make after-tax profits positive. Expectation at t_0 such a pattern therefore creates an additional alternative: namely, to remain open from t_0 to t_j, thereby sustaining operating losses but avoiding both shutdown and start-up costs. Since these costs can be substantial, the savings might more than offset any after-tax operating losses between t_0 and t_j, thus making continual operation over time more profitable than a stop-go pattern. This strategy will be attractive if the turn-around of profits is expected to come soon, the intervening operating losses are small, and the shutdown and start-up costs are substantial.

Our actual treatment of shutdown and start-up cost criterion simplified the above somewhat to avoid the unreality of specifying year-by-year expectation of prices and costs. At t_0 we determine whether, given basic expectation of the pattern of future prices and costs (a rising function of mine age), the present discounted value of expected future returns from continued R_0 (which adjusts each Π_i for LC), is positive or negative. If negative, a decision to close the mine at t_0 will not be reversed subsequently. Continued operation at t_0 in this case requires that *after-tax* returns at t_0 be positive: i.e., that $PD_0 + LC_0 > -\Pi_0$, which is contrary to assumption. But the present criterion is not involved, since no closure followed by reopening is envisioned. If $R_0 > 0$ although after-tax adjusted Π_0 is negative, i.e., $\Pi_0^1 < 0$, then R_1 must also be positive (where R_1 is the present discounted value of future returns starting from t_1), and the option to shut down in t_0 and reopen in t_1 is a real one. In this case, the advantage of remaining open at t_0 is the difference between the two present values:

$$(7) \qquad SSC = [\,\Pi_0^1 + R_1] - [R_1 - SC_0] = SC_0 + \Pi_0^1$$

where SSC is the shutdown and start-up cost saving criterion, SC_0 is the amount of shutdown and start-up costs, and Π_0^1 is the t_0 loss offset by LC. If $SSC > 0$, the avoidance of shutdown and start-up costs is a larger gain than the avoidance of the after-tax operational loss in t_0; so the facility should stay open and operate in t_0. $SSC < 0$ gives the opposite advice: shut down. It is clear that tax considerations and temporal discontinuities enhance one another. Large tax offsets reduce present effective operational losses and make intertemporal continuity less costly and so more attractive. Similarly, the expectation of continued operation despite temporary losses provides a more probable future against whose tax profitability tax carry forward offsets can be planned.

Mines will differ in the attractiveness of the intertemporal argument for continuity. Since mining costs rise as a function of age but all mines have to sell their output at the same prices, older mines have lower expected profitability over any given future. R_0 is less likely to be positive over even moderate future stretches. So the stop-start option is less likely to be relevant for them. Bad times that seem temporary for newer mines may result in permanent closure for old mines. Since we treat any given mine as having a life cycle, it closes permanently at some point and thereby decreases the total mining capacity in operation. Our treatment of each production stage as an industry aggregate does not mean that all mines close at the same time. Nor does it mean that the closing of any particular mine reduces aggregate operating capacity permanently.

First, we treat the mining stage as having, in any period, a distribution of mines at different stages in their life cycle and with different original deposits. So mines differ in average grades and in mining costs per unit of output of its then-average grade.

Second, previous industry exploration has established an inventory of underdeveloped deposits. When an existing mine shuts down, then, at the new lower industry capacity, the investment criterion is used to determine whether additional capacity in the form of new development of a hitherto undeveloped deposit is warranted. The average grade of this new mine is the average grade of deposits discovered since beginning the hitherto newest mine.

Thus, a regular replacement of mine shutdowns is provided for, but only to the extent warranted by the investment criterion.

Any social cost entailed by shutdowns is *not* the loss of capacity once represented by such closed mines since this will generally (or at least largely) be replaced by new mines: it is, rather, any metallic content prematurely left in the ground (i.e., which could profitably have been, or in future be, extracted).

G. New Mine Development

Decisions about new mine development are made in the model as follows. We run the operations module for period t_1 and derive the shadow price on "capital" for *each* mine. Since the current mines are arranged in terms of productivity (with differences due to original $\tilde{R}$ and age) we expect these shadow prices to be monotonically decreasing in this array. By our assumptions about declining quality of explorational output over time, the queue of discovered but undeveloped deposits arranged by declining average grade involves lower productivities than the lowest of operating mines. So we use the shadow prices of the two lowest present mine types in the investment criterion test. If either passes the test, this means that additional mining capacity of the same productivity as that mine type would be warranted (would increase the rate of return in capital). But the next best capital at the top of the undeveloped queue is less productive, so the addition of that mine might not be warranted.

We, therefore, provisionally enter additional new mining capacity (in terms of size of deposit) to the extent indicated by the investment module of the type of deposit highest in the undeveloped queue, and run the current operations module. If the objective function on the new run exceeds that for the unaugmented industry, the investment is warranted. If it falls short, no investment is undertaken. Thus, capacity is too low, but the differentiated capital does not make it worthwhile to supplement it. This "scarcity" condition continues until either market price of copper rises enough to warrant introducing inferior capital, or the productivity of the existing mines falls enough with increasing age to warrant supplantation—the substitution of a new deposit for an existing one.

H. Supplantation of Existing Mines by New Mines

In the real world each mine not only competes with all other operating mines but also, implicitly, with undeveloped deposits as well. This is because one firm may develop a new mine without another firm closing down one of its mines. The downward effect this will have in the market price of copper may result in one of the older mines becoming unprofitable and thus, closed down. In effect, the older mine has been supplanted by a new mine.

We model this by testing every five years whether or not such supplantation would increase overall industry profitability. Every five years we provisionally introduce new capacity from the top of the inventory queue, with an amount equal to the size of an individual mine of least productive currently operating type, and remove the corresponding capacity of the latter. We run the supplanted version and compare its overall profitability with the nonsupplanted state. If the test fails, no supplantation occurs. If it is successful, we test for the supplantation by the same procedure, stopping only when the overall industry profit can no longer be increased by futher substitution. This establishes the optimal degree of supplantation.

I. Exploration

The exploratory stage is integrated fully within the long run operation of the model, but only permissively in the present moderate run application we are discussing here. The function of exploration is to furnish an inventory of new deposits which can be brought into mine development when warranted by the investment criterion either to replace retired deposits or to add to existing capacity. In the long run, all of the present operating mines will be retired gradually and undeveloped deposits in the inventory will be called on to replace them as warranted. The output flow from exploration could conceivably impose constraints on the ability of the industry to have as much operational mining capacity as it would like from optimization considerations. So exploratory activity is an essential determinant of the current production of refined copper when such long term flows are involved. Our present application; however, considers only ten to twenty year impact sequences. For such a near future, it is assumed that the present inventory of discovered but undeveloped deposits resulting from previous exploration is more than enough to replace existing mine retirements and provide any additional mining capacity that is likely to be desired.

For this reason, our treatment of the relationship between exploration and postexploration operations is one way rather than two way: we treat downstream activities as affecting current exploration levels but not vice versa. The linkage comes through the valuation of reserves in developed mines as described above. Given a valuation to such reserves that depends on prospective downstream activities, the value of comparable reserves in undeveloped form can be calculated by the same principles we have used for bid price passthroughs at later production stages. The total cost of developing a deposit for mining operation must be subtracted from the overall value of such a deposit in developed form to arrive at the total undeveloped value of the deposit. This value is the implied payoff to successful exploratory activity. It enters into the prospective profitability of any incremental level of exploration in the way stipulated in our exploration optimization model. That submodel is used in the same way whether a long run two way linkage with the rest of the industry, or only the medium run one way linkage, is employed in the overall industry model.

The model contains an exploration optimization submodel. It treats exploration as a process designed to produce information about probability distributions of successful mineral discoveries. The more exploratory activity that is carried out in any area, the less intrinsic uncertainty about where and how much metal is to be found remains, and informed choices can be made about future extraction.

Increasing information about finds, whether positive or negative, can enhance the revenues of mining companies; it also incurs costs. The question of where and how much exploratory activity should be carried out is one that must be answered by balancing the expected gains from better information with the costs of obtaining it. A formal treatment of the process is included in the Appendix, at 146–48.

J. Regulation and Compliance

We will focus on three forms of federal government regulation to illustrate how the model lends itself to prediction of regulatory compliance and impacts. The first is the land withdrawal policy; second, water rights; and third, air pollution control. These have predominant impact on the exploration, mining-milling and smelter stages, respectively. For each kind of regulation, the model specifies a particular linkage between the operations at the relevant production stage and either the input or output relationship that generates a potential wider welfare impact on the society. This is the so-called "externality link." Translation of this "externality" (in the broad sense used earlier) into a social welfare impact comes via a "damage function," which connects the particular externality to various types of social cost. The damage function is part of the overall model system, but is outside the production model itself.

In addition to the activity externality linkage, there is a link between the regulatory regime and a set of constraints upon externality generation. Each form of regulation specifies a ceiling or floor in usage of some input or generation of some inadvertent by product output. This "regulatory constraint link" compares a set of constraints on industry operations which, along with technical and capacity constraints, determine constrained maximization behavior.

Finally, there is a link between the particular devices and operational procedures adopted by each facility to comply with regulation and the consequences on generation of externalities. This is the "compliance impact link." With the addition of information on the acquisition and operating cost of each kind of compliance equipment, the cost of each procedural modification used for compliance, and the cost of the capital equipment needed for each major technology variant, where such variants influence externality generation, we can derive the industry's responses to adoption of each compliance strategy both in terms of industry output and market price, and of changed externality flow.

In a more elaborate application of the model we can predict which compliance strategy will be adopted by the industry, because, since the consequences of each strategy include a level of objective function achievement (industry profits), we can predict that the industry will adopt, out of the set of all strategies normalized to meet given regulatory standards, one that leaves the industry with highest profits.

1. *Land Withdrawal*

Some federal lands have been withdrawn from mineral exploration and extraction by the National Park Service, the U.S: Forest Service, the Bureau of Land Management and other agencies. Since exploration and mining have been linked under this policy, the regulation is binding on the exploratory stage: i.e., it makes a distinction between where exploration may and may not take place. Once exploration has occurred, and successfully discovered a deposit, mining may proceed. Where exploration is not permitted, no mining will occur. Thus, the constraint on mining is still only the presence of a known deposit, not land withdrawals.

III

FEDERAL LAND WITHDRAWAL POLICY: THE COSTS AND BENEFITS

Federal land withdrawals primarily affect the exploration stage. The cost of this policy that we treat involves a decrease in expected profitability from exploration and thus a decrease in the overall level of exploration carried out. The benefit of the policy we treat in a somewhat more open way. It involves preserving additional land for special alternative uses. We do not attempt to place a money value on these uses, but provide a classification system that better permits policymakers to gauge their preference trade-offs between these alternative land uses and the costs that have to be paid for them.

A. Cost of Land Withdrawal Policy

The cost consequence of land withdrawals is subtle. Withdrawal removes land from exploration that has about as high a probability of containing commercial grade deposits as any remaining open to exploration. But this in itself is not a direct cost to the copper industry or the public, since a great deal of land remains open to exploration that has as high a probability of success as the land foreclosed and that has not yet been subject to exploration. So no actual diminution of exploration is forced because of lack of more than enough exploration options. The cost is indirect rather than direct.

We treat the adverse impact of land withdrawal as contributing to a form of "crowding" of exploration activity. Deposits are often found spatially clustered. Moreover, discovering the spatial extent of a single deposit requires considerable explorative activity. If a first strike becomes known quickly, other teams may enter the area and prevent the successful prospector from either claiming the entire deposit or leisurely discovering additional deposits nearby. The probability that such hasty incursions will occur depends on how spatially clustered the explorational activities are. The more spatially concentrated they are, the quicker and more surely information about successful strikes is likely to spread, with competitive incursions following. So we can speak of a "crowding" of exploratory activity in terms of this spatial concentration. Greater crowding implies a greater likelihood of incursion and an earlier one. Thus, it implies that a small percentage of total yield of any successful strike can be appropriated by the first finder. Therefore, the extent of crowding is inversely related to the percentage of the total value of any successful find that can be realized by exploration.

Since explorational crowding is a function of spatial concentration of exploring behavior, land withdrawal affects crowding by influencing the degree of exploratory concentration. The more high probability land that is withdrawn from exploration, the more exploration is likely to become spatially narrowed. Since very little exploration occurs in such land withdrawal of low probability land will not have this effect. Our treatment of the costs of land withdrawal focuses therefore only on land where exploration might take place.

We propose an "exploration crowding function" (ECF), which indicates the percentage of total value of any successful find that the finder can expect to appropriate (i.e., the capture percentage). This capture percentage depends on explorational concentration. Since we consider only one exogenous source of variation in that concentration in the time interval being considered, namely, amounts of land withdrawal; we use either amount of relevant land available for exploration or percent of relevant land withdrawn, as the explanatory variable.[5]

Our data base permits the rough calculation that present federal withdrawls have reduced available relevant exploration land by 40 percent. For our analysis, we intend to examine the effects of five different withdrawal levels. The crowding variable influences the amount of explorational activities by plugging into the exploration model.[6] Variations in capture percentages as a function of land withdrawal affects the expected capturable net earnings, and thus the expected profitability of exploration. It thus should affect the total exploratory activity carried on.

B. Benefits of Land Withdrawal Policy

The ostensible purpose of federal land withdrawals is to preserve additional land for uncorrupted wilderness use. The benefits of the program are the values obtained from the uses of the land when exploration and mining do not take place. We assume that alternative commercial development is not

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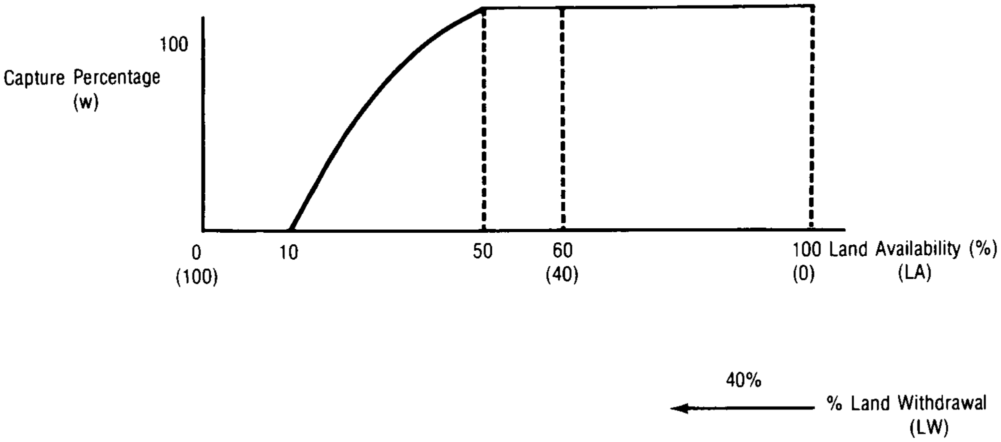
[5] Thus (1) w = f (Relevant Land Availability) = g (% of Relevant Land Withdrawn)



w = f(LA) = f(1-LW) = g(LW)

Where LA = percentage of land in exploration potential grades 1 and 2 which is available for exploration

Where LW ≡ percentage of land grades 1 and 2 which have been withdrawn from exploration

w ≡ % of full value of successful find which finder expects to be able to capture

The shape and position of this function have been estimated impresstionistically.

[6] In very simple terms, let ϕ be probability of a successful find; S_t be mean size to find at t_i; $\bar{R}_i$ be mean grade of find at t_i; $\bar{P}_i^R$ be expected *net* mean (shadow) price of copper reserves over the expected lifetime of the mine (net of mine development costs). Then:

(2) $S_j\bar{R}_i\bar{P}$ = average lifetime net earning of deposit reserves (ignoring time discount discounting);

(3) $\phi S_i\bar{R}_i\bar{P}$ = expected net earning from unit exploratory effort at t_i;

(4) $w\phi S_i\bar{R}_i\bar{P}$ = expected capturable net earnings from unit exploratory effort at t_i (where $0 \leq w \leq 1$).

TABLE I

	Policy Variant	*LW (=1−LA)*	*Variable Name*
1.	Present level	A_0	40
2.	RARE II (Forest Service Plan)	A_1	50
3.	Bureau of Land Management Roadless Plan	A_2	65
4.	Tight Plan (Hypothetical)	A_3	75
5.	Open Land (except national parks)	A_4	0

involved and that the alternative use is genuinely a wilderness use. Since no market type bidding for the land is involved, there is no direct money value in which the benefits from the wilderness use can be reflected. Moreover, various forms of indirect measurement suffer because current active "users" of the wilderness do not exhaust the use of the land; they may not even comprise the major part of its overall "social use."

For these reasons we do not attempt to give a monetary value of the land uses preserved and enhanced via land withdrawal. We attempt to measure in terms of a more "natural dimension," that is, units of the kinds of wilderness service being preserved.

What exactly is a widerness use? It seems to consist of at least two distinguishable components: a recreational-aesthetic use and an archival-educational use. The recreational-aesthetic use is close to being measureable in implicit monetary terms, since there are substitute areas in which similar activities are carried out. Where commercial areas are involved, we obtain a revealed value of the services in terms of admission and camping charges. To compare the quality of the services in these commercial areas with those at issue in withdrawal, a technique that has been employed for recreational services of this sort is to use the differential travel costs that tourists are willing to spend to visit this rather than the former areas as a measure of the differential quality. This is a questionable technique. It assumes, for one thing, that differential travel costs are a good measure of differential values. This is not even a fair approximation unless most tourists regularly consume both types of recreation as close substitutes so that their relative frequency of visits can be taken to represent an optimal balance between the two where there is a marginal equivalence between an extra commercial trip at its overall cost and an extra noncommercial wilderness trip at its overall cost. There is reason to believe that these lands are not generally treated as close substitutes for private recreational areas.

Even in recreational use, these areas are likely to have unique features. In addition, there are educational uses for which this type of comparison is entirely beside the point. It is significant that we speak of the wilderness use as being "preserved," not so much for recreation or even aesthetic pleasures, but simply to be preserved as a wilderness area. This is similar to the interest in preserving historically significant documents or species of wildlife. It represents the critical role of humans as the custodians of their own and their planet's history, critical because human actions threaten to change it and to obliterate traces of the past. It is a genuine museum or archive interest and appears to have deep roots and to be important. A significant part of this interest is that it facilitates an important kind of education about our natural habitat. This archive interest does not seem to have any close substitute in a market-type activity.

Our approach is to express land withdrawal consequences in terms, simply, of amounts of additional acres that will be preserved for wilderness use. To use wilderness acres in an undifferentiated way would be too crude. We can create a scale of acres in terms of the presumed *quality* of the wilderness function that they are capable of performing. We make use of gradations in both the recreational and educational elements of the wilderness function to construct this scale. It is subjective, but we believe that there is likely to be a high degree of consensus on where individual tracts of land fit.

Our scale consists of the following wilderness gradations:

7 spectacular and unique in both archival and aesthetic values
6 high level uniqueness in both archival and aesthetic values
5 significant uniqueness in both (but other comparable examples exist)
4 high in archival *or* aesthetic values but not both
3 moderate degrees of both—many other examples exist
2 modest level of either and there is little scarcity of examples
1 no archival interest, low aesthetic value, and no scarcity.

In registering the gains from some variant of the land withdrawal policy, the number of each kind of acre preserved is listed. But there is an implicit second kind of weighting procedure: the implicit value of each acre preserved depends both on the wilderness grade (WG) and on the number of acres of each grade already preserved.

The key to a benefit-cost analysis here is those withdrawn acres that have a high exploration prospect. Benefits *and* costs refer primarily to these. Since little or no exploratory activity is likely to take place on lower exploration grade land, land withdrawal does not change their present or prospective use much, if at all (so little, if any, benefits, regardless of their WG, will be created). Similarly, since little exploratory activity would be likely to have occurred there, withdrawal does not change the spatial concentration of exploration or anything else (so there are little, if any, costs associated with this kind of withdrawal).

The empirical magnitude of the issue is suggested by the following tables.

TABLE II

PERCENT OF ORIGINAL LAND IN WILDERNESS GRADES 1-7

1	*2*	*3*	*4*	*5*	*6*	*7*
30	20	20	10	10	7	3

TABLE III

PERCENT OF WG 1-7 IN ALL ARIZONA WHICH ARE INCLUDED IN DIFFERENT WITHDRAWAL POLICY VARIANTS

Policy/Grade	*1*	*2*	*3*	*4*	*5*	*6*	*7*
A_0	—	—	—	—	—	50	100
A_1	—	—	—	10	20	60	100
A_2	—	—	—	15	25	70	100
A_3	—	—	—	—	—	2	100

TABLE IV

PERCENT OF WG 1-7 IN TOP TWO EXPLORATION PROSPECT GRADES UNDER PRESENT WITHDRAWAL POLICY (A_0)

1	*2*	*3*	*4*	*5*	*6*	*7*
—	—	—	—	—	100	—

TABLE V

PERCENT OF WITHDRAWN LAND WITHIN TOP EXPLORATION PROSPECT GRADES BY WG 1-7, BY POLICY VARIANTS

Policy/Grade	*1*	*2*	*3*	*4*	*5*	*6*	*7*
A_0	—	—	—	—	—	100 (16,000)	—
A_1	—	—	—	3 (500)	16 (3,200)	82 (16,300)	0
A_2	—	—	—	12 (3,000)	23 (6,000)	65 (17,000)	0
A_3	—	—	—	0	0	0	0

Source for Tables II–V: Harbridge House Project, *Applied Research on the Benefits and Costs of Public Regulation of the Copper Wire Industry*, NAT'L SCIENCE FOUNDATION CONTRACT ARP 77-19752 (1978).

1. *Water Rights*

Water is an important input in mining and milling. It is obtained by two major sources: (1) streams and rivers, and water development projects, (2) wells. In the arid Southwest, water from running sources is so scarce relative to user wants that it is allocated among users by formula. This is especially true of water from state or federal water projects. Allocations are made on the basis of water rights.[7]

Water from running sources carry prices considerably lower than the cost of water from wells. It is essentially free from natural sources and purchasable at subsidy-level prices from government development projects. The price differential exists because of express public policy. Given the substantial differential, well water is used nevertheless because there is not enough water available from running sources.

Water from wells on their own land is an internally developed resource for mine-mill facilities. No externality is involved in *obtaining* this water. There is, however, in *using* it: namely pollution of water runoff since this water flows into or under other land and future use by others is affected by pollution. Runoff pollution is also an externality for water obtained from running sources. But *obtaining* water from running sources involves an "externality" in the extended sense used here. The allocation of water from these sources is not based on a price system that would guarantee use by most productive users and reflect a balance between use benefits and the social costs of providing the water. The allocations have important political considerations; moreover, the prices fall far short of the social costs of providing the water. So money paid by each mine-mill facility for running water does not necessarily reflect the social opportunity cost of the water: either some other potential user may have a higher use productivity or the value of resources used up in generating it exceeds the value of what it produces, or both. Similarly, the amount of water available to the facility via water rights is not necessarily the amount for which its productivity is no less than for an alternative use (it might well be the most productive user for considerably more).

Our treatment does not have to calculate adjusted values (shadow prices) to compensate for these potential inefficiencies, however. We may treat both the overall governmental subsidization and the "arbitrary" allocations as embedding implicit social valuations (public decisions) that supersede the private valuations involved in free market prices. Moreover, since generally water right allocations to mine-mill facilities fall significantly short of their water needs, they supplement this source with privately owned well water. Thus, the amount of water available from running sources and the price of that water, are not responsible for marginal decisions at the mine-mill about

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[7] These are especially characteristic of the U.S. Bureau of Reclamation water projects.

operation levels and character. It is supplies of well water that enter into these marginal decisions. Operational decisions are not very sensitive to allocations and price subsidization, so large social benefits or costs are not involved in the system vis à vis mining. On both scores, therefore, we accept water rights allocations as reflecting consistent social valuations of running water use.

Though existence of the water right system necessitates no separate calculation of allocational inefficiency, a change in governmental allocation of water rights, either through redistribution or newly developed water resources, can generally influence industry operations by changing input costs at the mining-milling stages. We follow these via the operations model just as changes stemming from any other form of regulation. But we do not, at the same time, change the industry impacted externality vector since the changed water rights allocation is itself the changed "externality" flow in this case. Similarly, no "damage" function needs to be used to transform a new allocation into social benefits or costs.

Runoff pollution is a different story. This is a genuine industry-generated externality. Its size is assumed to be a direct function of the level of mining-milling operations. Each particular linkage is based on the character of the compliance strategy to meet the current regulatory stringency. Change in the latter leads to a change in the former, and thus to a changed relationship between the level and character of operation on the one hand, and the degree of runoff pollution on the other. Of course, a change in regulations leads, through a change in compliance strategy, to changed industry operations via changed cost conditions. Thus the operations model follows.

In our present application of the model, we do not treat runoff pollution, but only water rights distributions. The effect of changes in water rights distributions is complex. Suppose a mine-mill facility receives a smaller water rights quota in period t_1 than in t_0. Then the same activity level will require a larger percentage of input water to come from wells than from running sources. Because of the two price system this will result in both a higher marginal cost for water and higher total water costs. The first means a change in marginal costs for the facility, and hence a likely decrease in planned operations. Both this decision and the higher total costs for any amount of water that includes well water production should reduce facility profits and thus, industry profits. An increase in water rights distribution reverses the process.

2. *Air Pollution*

Practically speaking, air quality regulations[8] have had the most serious regulatory impact on the copper industry. They affect primarily the smelting stage. Sulphur dioxide emissions from the smelter are a classic form of narrow externalities. They presumably have important damage potential. Meas-

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[8] Clean Air Act and amendments.

ures necessary to control emissions are expensive and they do not decrease such emissions to zero, so regulations prohibiting any degradation of air quality in pure airsheds (as opposed to regulations requiring an improvement in already highly fouled air) have the practical effect of outright prohibitions against new smelter capacity in the former such areas.

To deal with both the emissions side and the impact of regulatory compliance in the industry we use both the externalities and the operations models. For the first we relate sulphur dioxide emissions to function of average activity levels, emissions control equipment, smelter technology, and timing of production activity. Different types of control equipment have different effects on emissions as a function of activity levels. Different basic forms of smelter technology also influence this externaltiy generation relationship. Finally, while emissions as such are not affected by the timing of production, ambient air quality has a variable, not fixed, relationship to emissions levels; the determinant of this relationship is weather conditions. A strategy of deliberate concentration of production into periods when weather quickly disperses emissions and so improves air quality (thereby decreasing the emissions damage potential) coupled with significant production cutbacks under unfavorable weather conditions, is a possible alternative to installation of control equipment. (Essentially, it trades off high marginal operations costs for high capital equipment costs.)

To investigate the impact of these alternatives we add the capital cost of compliance equipment (or the differential capital cost of the technological variant selected) to total operating cost, and any variable cost additions (via extra energy, labor, maintenance, etc. per unit of output). This will affect profitability and, where variable costs are impacted, the optimization level of output. The application is straightforward.

K. Simulation Scenarios

The operative model has been developed primarily to help predict the consequences of changes in a number of different kinds of regulatory programs. It does, however, have a much wider potential than that. For one thing, it can be used to examine the consequences of a wider variety of regulatory programs than the few we have considered. In addition to this, it can help trace various impacts of changes in different exogenous variables-technologies, temporal profiles of world copper demand, or copper supplies other than those in the United States. We intend to illustrate some of the properties of the model by simulating several variants along these dimensions.

1. *Mine Distribution*

In each variant, there is a distribution of mine age and deposit quality. The various simulations will vary the proportions of different mine characteristics in the distribution.

2. *Smelter Technologies*

Presently a number of different smelter technologies are available. It is believed that the cost and effectiveness of each basic compliance strategy differ for the different technologies. We intend to trace the consequence of each regulatory change or model variants where: (a) smelting is successively represented by each of the technologies exclusively; and (b) in each period there is a mix of technologies.

3. *Market Price Trajectories Over Time*

Because of the critical importance of intertemporal tradeoffs, expectations about the future, and temporal irreversibilities, we intend to examine the importance of the temporal shape of exogenous changes. To this end we plan to simulate the industry responses to exogenous changes that combine into refined market price effects that have different temporal patterns: monotonic increases, monotonic decreases, alternations, etc.

4. *Regulatory Regimes*

We have been forced by project scope to concentrate on a few types of regulation. For each of these; however, we intend to examine not only the effect on industry operations of different degrees of stringency, but also the impact of the way regulatory requirements change over time, e.g., a single change at the beginning of a ten year period, or the same change divided into two equal installments. Each regulatory stance will be traced through its industry impacts, and each regulatory change as well, and some changes will be traced in variant temporal patterns.

IV

CONCLUSION

This has been an account of the analytic framework, and some details of the empirical model, used in a research project designed to measure and evaluate the benefits and costs of public regulation of the United States copper inudstry. No attempt has been made to give an exhaustive picture, but rather only the flavor of the chief theoretical emphasis, and an idea of how more specific issues are dealt with under the approach.

Neither the model nor the larger project is explicitly addressed to the question of innovation. Yet innovationary impacts are not only consistent with the kinds of consequence chiefly addressed, but some aspects of innovation can be illuminated by the model. Two aspects have been alluded to in our introduction. The first concerns the direct compliance adjustment to changed regulatory requirements, especially at the smelting stage various available technologies differentially suit different regulatory constraints. Decisions about net additions to existing facilities or replacement of these facilities may involve selection of a different technology. Systematic investment tendencies toward one technology also constitute a form of excess demand for additional technical elaboration or progression in the particular directions that induced the tilt toward that technology over the other available ones. So both modest and more extreme innovation may result from these "defensive" adjustments to regulation.

The second aspect refers to possible tradeoffs between this induced "defensive" innovationary activity and "initiatory" innovation—more general enhancement of productivity or commodity quality—not directly related to the substance of regulation. In the copper industry, productivity goals are more likely. The tradeoffs stem from common budgetary constraints. Where defensive adjustments come from R&D budgets and general capital funds, and both are positively tied to retained earnings, rivalry with initiatory innovation activity derives from the latter dependence on the same budgetary constraints. Changing R&D and capital budget stringency can be predicted from the model's predictions about changes in overall industry profitability resulting from regulatory changes.

While a general multipurpose model, not specialized directly to innovationary issues, can throw this kind of modest, rather indirect light on innovationary activity, its contribution to the area may be somewhat broader. Innovation is not really a completely separable activity or dimension of the activity of business firms. It is related to other facets of business action. A general model of the firm or industry can provide a larger context of business adjustment within which to view innovationary activity. Tradeoffs, symbiotic linkages, and whatever degree of genuine independence from other dimensions, may be more effectively understood in the larger context. In addition to this, innovation itself as a social goal does not stand isolated from other social goals. Its technical interrelation with other kinds of goals generates some appreciation of both its social opportunity cost of incremental achievement and also of its substitutability with other instrumentalities towards the same more ultimate social objectives. Though the particular general model presented here is not centrally concerned with these larger issues, it can be used to develop an awareness of these considerations and is offered in this spirit.

LEGEND OF TERMS

anode copper — refined copper on the positive terminal of an electrolytic cell
milling — the shaping and dressing part of the copper industry
mining — the process or business of working mines
refining — processing to make the copper free of impurities
smelter — that process that melts and fuses the ore

APPENDIX

Exploration Optimization Model

While we attempt to explain and predict the behavior of individal facilities or types of facilities at other production stages, we predict only the aggregate exploratory activity of the industry as a whole. This difference derives from the fact that some exploratory behavior is performed by newcomers to the industry who have no facilities at any other production stage and are attempting to enter the industry by means of exploration. Unlike the other stages, new entry is a matter of engaging in what can be considered current operations, not new investment.

a. *Inputs*

The inputs at this stage are various types (and locations) of land, A_1, A_2, A_3, . . . , A_n, and labor L, and capital, K. (We abstract from the several types of labor and capital employed.) We distinguish two forms of exploration; an early, superficial reconnaissance, and exploration carried beyond some minimal level and designed actually to discover deposit bodies. Reconnaissance is not designed to discover deposits but only to narrow down alternative sites as candidates for real exploration. We consider reconnaissance an overhead activity and its inputs as a lumpy, fixed form of overhead characterized for convenience as a particular kind of input.

b. *Activities*

There is initial reconnaissance, followed by a level of activity (including zero) on each different type of land X_i.

c. *Outputs*

The output of reconnaissance is detailed below for a number of land areas and an initial winnowing out of areas for which the probability is so very low that, in effect, no conceivable market and production conditions make real exploration worthwhile at a given point in time (for a given period).

The outputs of exploration proper are additions of proven (i.e., sufficiently drilled-out) reserves of different types, ${}^{P}R_1$, ${}^{P}R_2$, . . . Proven reserves are deposit bodies of given character known to exist in particular areas but without an actual mine being set up to exploit them in the following period. Development of these reserves into an operating mine requires an additional "development investment," an action which depends on whether or not the gain to industry profitability from such action exceeds its cost. The profitability of development is calculated in terms of the shadow prices on reserves generated as part of mining stage optimization.

These different reserves are treated as forms of vintage capital at the mining stage. Expected additions equal in number the activity level on ech: χ_i (i.e., some level of L, K and total acres) times the probability of each find.

d. *Output Values*

Assume that at time t, the average expectted new deposit has a frequency distribution of reserve grades, $R = (R_i, R_2, \ . \ . \ . \ )$ with density function equal to $G^t(R)$, and mean grade $\bar{R}(t)$, where $G^t(R)$ is a function of the total accumulated copper discoveries up to time t. Call this the "sandard find."

$\theta \equiv$ probability of finding $G^t(R)$,

Production function for activity i:

(8) $\theta_{it} = \theta \ (\chi_{it})$ all i

where

(9) $\chi_i = \min({}^{E}a_iA_i, {}^{E}a_LL, {}^{E}a_KK)$ all i

i.e., each exploratory activity requires fixed input proportions. Land of types 1, 2, . . . , n are graded in order of declining Θ for equal acres explored.

$\Theta(A_1,L_0,K_0) > \Theta(A_2,L_0,K_0) > \ldots > \Theta(A_n,L_0,K_0)$

where $A_i = A_2 = A_3 = \ldots = A_n$

e. *Profits*

The profit of each exploratory activity is the present discounted value of *expected* increase in reverse wealth less the cost of producing that increase:

$$(10) \quad {}^{E}\Pi_{it} = \left[(w\Theta_{it}G^{t}P^{R}) - (P_{it} \frac{Ai}{{}^{E}a_i} + P_{Lt} \frac{L}{{}^{E}a_{Li}} + P_{Kt} \frac{K}{{}^{E}a_{Ki}}) \right] {}^{P}\chi_{it} \div \frac{1}{(1+r)}g$$

all i

$o \leq w \leq 1$

where P^R is the vector of shadow prices (values) of reserves at the proven state:

r is the industry discount rate

g is the gestation period for obtaining proven reserves

w is the earnings capturability parameter: percent of potential gains appropriable by original discoverer-impairment due to competitive "crowding" of exploratory gain exploitation (to be explained below in discussion of land withdrawal policy)

The prices of lands 1, 2, . . . , n—whether explicit purchase or lease prices or the implicit competitive information, search and queueing costs—are a function of the supply function of these lands, any absolute land availability constraints, and the opportunity cost in terms of competitive uses:

(11) $P_i = A(S_i,V_i)$

where S_i is the supply function of land type i

V_i is the opportunity cost of land type i in terms of other users.

f. *Objective Function and Linear Programming Optimization*

The objective function for the industry as a whole at the exploration stage is an aggregate exploration profit function. The behavioral goal is assumed to be to maximize this function:

$$(12) \quad \max {}^{E}\Pi_t = \sum_{i=1}^{n} {}^{E}\Pi(\chi_{it})$$

subject to:

$$(13) \quad \frac{A_i}{{}^{E}a_i} \chi_{it} \leq A_i^* \quad \text{all i}$$

$$(14) \quad {}^{E}\Pi(\chi'_{it}) \geq 0 \quad \text{all i}$$

$$(15) \quad \Sigma_i \left(\frac{K}{{}^{E}a_{Ki}}\right) \chi_{it} \leq K_t^*$$

$$(16) \quad \Theta_{it} = \Theta(\chi_{it}) \quad \text{all i}$$

$$(17)\quad \begin{aligned} &(a)\ P_{It} &&= A(S_{It}, V_{It}) \\ &(b)\ P_{Lt} &&= P_L{}^{o} \\ &(c)\ P_{Kt} &&= P_K{}^{0} \end{aligned}$$

$$(18)\quad \chi_{It} \geq 0$$

where * refers to a given capacity availability

Equation (13) stipulates land availability constraints for each type of land. Equation (14) requires that no activity be undertaken that has a negative profit. Equation (15) specifies an overall capital availability limitation at time t. Finally, equation (16) specifies the technology to be used, equation (17) the relations determining input prices, and equation (18) the requirement for non-negative activities.

The capital availability constraint is employed in lieu of an explicit treatment of disaggregated differences in risk preference and opportunity costs of capital, non-linearities in input-output relations, and actual rising shadow prices as well as rationing of capital availability. Alternative formulations involving non-linearities may be explored subsequently.