BAYESIAN APPROACHES TO THE PRECAUTIONARY PRINCIPLE

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I. INTRODUCTION

A. *The Precautionary Principle*

In recent years, variations of the Precautionary Principle have been adopted in international environmental agreements and by national regulatory agencies.[1] Despite the apparent increase in its application, the Precautionary Principle remains ill-defined. Generally, it espouses the belief that under conditions of substantial scientific uncertainty environmental regulations should err on the side of caution

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* Associate, Fried Frank Harris Shriver & Jacobson, New York; J.D., New York University, 2001; B.S., University of Minnesota, 1997. The views expressed here are those of the author and should not be attributed to Fried Frank Harris Shriver & Jacobson.

1. *See, e.g.,* The Framework Convention on Climate Change, May 9, 1992 art. 3(3), 31 I.L.M. 849, 854 (1992); *The Rio Declaration on Environment and Development*, UN Conference on Environment and Development, U.N. Doc. A/CONF.151/5/Rev.1 (1992), *reprinted in* 31 I.L.M. 874 (1992) ; *The Ministerial Declaration of the Second World Climate Conference*, Nov. 7, 1990, *reprinted in* 1 Y.B. Int'l Env'l L. 473 (1990) (Ministers and representatives of 137 countries agree to "protect the ozone layer by taking precautionary measures to control . . . emission of substances that deplete it . . . ."); Comm'n of the Eur. Cmtys., Communication from the Commission on the Precautionary Principle 11,27 (Feb. 2, 2000) *available at* http://europa.eu.int/comm/dgs/health_consumer/linrary/pub/pub07_en.pdf.; *The Second International Conference on the Protection of the North Sea, Ministerial Declaration 1* (1987) (Ministers of the EEC and eight countries agree that the North Sea ecosystem should be protected through the reduction of pollution "even where there is no scientific evidence to prove the causal link between emissions and effects ('the principle of precautionary action')."); *The Framework Convention on Climate Change*, U.N. Doc. A/AC.237/18, Part III/Add. 1 (1992), *reprinted in* 31 I.L.M. 849 ("The Parties should take precautionary measures to anticipate, prevent or minimize the causes of climate change and mitigate its adverse effects. Where there are threats of serious or irreversible damage, lack of full scientific certainty should not be used as a reason for postponing such measures."); *The World Charter for Nature*, U.N. GAOR, 37th Session., Supp. No. 51, U.N. Doc. A/Res/37/7 (1982), *reprinted in* 22 I.L.M. 455 (1983) ("Activities which are likely to pose a significant risk to nature shall be preceded by an exhaustive examination; their proponents shall demonstrate that expected benefits outweigh potential damage to nature, and where potential adverse effects are not fully understood, the activities should not proceed.").

in order to prevent harm.[2] The Rio Declaration on Environment and Development states that "In order to protect the environment, the precautionary approach shall be widely applied by States according to their capabilities. Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation."[3] Advocates argue that the Precautionary Principle merely reinforces common sense notions of environmental stewardship.[4] However, opponents view it as a fundamentally unscientific rule of decision that exploits the public's fear of the unfamiliar and promotes radical environmental agendas or protectionist trade policies disguised as environmental regulations.[5]

Advocates further argue that the application of the Precautionary Principle is justified by science's demonstrated fallibility in anticipating environmental harms such as asbestosis and ozone depletion.[6] Additionally some potential environmental hazards cannot be quantified with certainty by existing scientific methods. Thus the Precautionary Principle would allow such harms to be regulated even if conclusive proof of harm has yet to be established.[7] In this sense, the Precautionary Principle may be viewed as a burden-shifting device that places the responsibility of demonstrating a product's or process's *safety* on those who would introduce it, rather than a demonstra-

2. *Id.*

3. The Rio Declaration on Environment and Development, *UN Conference on Environment and Development, U.N. Doc. A/CONF.151/5/Rev.1 (1992),* reprinted in *31 I.L.M. 874 (1992).*

4. *See Use and Abuse of the Precautionary Principle,* ISIS submission to US Advisory Committee on International Economic Policy (ACIEP) Biotech. Working Group, July 13, 2000.

5. Peter Huber, Editorial, *Fear of the Future,* WALL ST. J., Feb. 10, 2000 (arguing that the principle was "invoked to trump scientific evidence and move directly to banning things they [environmentalists] don't like—biotech, wireless technology, hydrocarbon emissions. In other words, science got in their way, so they shoved it aside.").

6. The failure of science to predict the dangers associated with a new technology or process often prevents recovery under the common law due to causation problems in the law of torts. *Cf.* ANDREWS CONTINUING EDUCATION INSTITUTE, ELECTROMAGNETIC FIELD LITIGATION: THE NEXT ASBESTOS? (1993).

7. Beyond the threshold problem of whether an environmental harm poses a threat at all, conventional environmental regulatory mechanisms which rely on assessing the costs of an environmental harm may be ineffective if there is uncertainty regarding how those costs will develop over time. *See* WORKING GROUP II TO THE SECOND ASSESSMENT REPORT OF THE INTERGOVERNMENTAL PANEL ON CLIMATE CHANGE, CLIMATE CHANGE 1995: IMPACTS, ADAPTATIONS AND MITIGATION OF CLIMATE CHANGE: SCIENTIFIC-TECHNICAL ANALYSES 78 (Robert T. Watson et al., eds., 1996) at 5. Thus, even where price reflects environmental costs, if those costs are unlikely to increase in a predictable, linear fashion, the market may undervalue the impacts of additional production and fail to allocate resources appropriately.

tion of *harm* on those who would regulate it.[8] In this vein, some commentators view the Precautionary Principle as a particular policy application of a more general "safety principle," in which people are allowed to weigh harms that they find particularly dreadful more heavily than would be the case under traditional cost-benefit methods.[9] However, environmental regulations entail real costs, both in terms of foregone social benefits and—in a world of limited regulatory resources—foregone environmental protections.[10] Thus, because of these unintended negative consequences, the Precautionary Principle is vulnerable to criticisms directed against those who would take it too far.[11]

The distinction between *uncertainty* and *ambiguity*, or what will be referred to in this paper as true uncertainty is important to understanding the scope of the Precautionary Principle.[12] *Uncertainty* generally refers to situations in which a harm is probabilistic in nature, but for which a probability distribution is known or may be assigned.[13] *True uncertainty* refers to situations in which even the probability of

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8. DAVID HUNTER ET AL., INTERNATIONAL ENVIRONMENTAL LAW AND POLICY 934 (1998) ("[T]he precautionary principle switches the burden of scientific proof necessary for triggering policy responses from those who support prohibiting or reducing a potentially offending activity to those who wants to continue.")

9. Mark Geistfeld, *Reconciling Cost-Benefit Analysis with the Principle that Safety Matters More than Money*, 76 N.Y.U. L. REV. 1 (2001) (arguing that such a safety principle is consistent with a modified cost-benefit analysis) *available at* http://www.nyu.edu/pages/lawreview/76/1/geistfeld.html.

10. *See, e.g.,* John D. Graham & Jonathan Baert Wiener, *Confronting Risk Tradeoffs, in* RISK VS. RISK: TRADEOFFS IN PROTECTING HEALTH AND THE ENVIRONMENT 1 (John D. Graham & Jonathan Baert Wiener, eds., 1995); STEPHEN BREYER, BREAKING THE VICIOUS CIRCLE: TOWARD EFFECTIVE RISK REGULATION 12-13, 22-23 (1993); Christopher H. Schroeder, *Rights Against Risk*, 86 COLUM. L.REV. 495 (1986). *See generally* PAUL R. PORTNEY, ECONOMICS AND THE CLEAN AIR ACT, *reprinted in* 136 Cong. Rec. H12911.01, *H12916 (Oct. 26, 1990) (discussing a macroeconomic approach to assessing costs of environmental regulation,) *Cf.* CASS R. SUNSTEIN, FREE MARKETS AND SOCIAL JUSTICE Ch. 12 (1997) (discussing "health-health tradeoffs").

11. *See* Frank B. Cross, *Paradoxical Perils of the Precautionary Principle*, 53 WASH. & LEE L., REV. 851 (1996) (discussing the legal problems associated with the precautionary principle and its unintended consequences).

12. *See* Paul Davidson, *Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective*, 5 J. ECON. PERSP. 129, 130-31 (discussing the "objective probability environment").

13. Stochastic risks are those for which a reliable probability distribution is available but there is uncertainty about the results in each particular instance. *See generally* K. RADFORD, MANAGERIAL DECISION MAKING 64-65, 78 (1975).

harm is not known.[14] It is this latter situation with which advocates of the Precautionary Principle are primarily concerned.[15] An example of an uncertain harm might be the probability of a product malfunctioning, especially if that product is subject to testing. Several trials may be performed, to determine the frequency with which a malfunction will occur, which in turn may be used to formulate a probability distribution. Truly uncertain harms often arise when controlled testing is impossible and there is no experience from which to construct a probability distribution. Unfortunately, this is precisely the type of uncertainty—true uncertainty—which is characteristic of many environmental problems. It would be highly impractical, for example, to try to construct a controlled experiment for global warming.

B. *Statistical Analysis of Risks in the Regulatory Context*

In determining environmental policies, regulators must make decisions about the desired level of protection from a perceived risk, an inherently precautionary action.[16] Even in the face of true uncertainty regulatory priorities must be set due to limited economic and enforcement budgets.[17] For example, regulations for truly uncertain risks might usurp scarce resources that could more effectively be allocated to regulating known risks. Thus, the uncritical application of the Precautionary Principle could cause a result similar to a regulatory Gresham's Law.[18] Regulations for truly uncertain risks might

14. *See* Davidson, *supra* note 12, at 131 (discussing the "true uncertainty environment," in which the decision-maker believes that unforeseeable changes will dictate future outcomes and that no reliable estimate of probabilities can be made).

15. *See, e.g.,* Rachel Clark, *Dealing With Uncertainty*, RACHEL'S ENVIRONMENT & HEALTH WEEKLY, Sept. 5, 1996, *available at* http://www.dieoff.org/page31.htm.

16. Admittedly, most people are unfamiliar with the basic conceptual framework of statistics and decision theory; it might be argued that analyzing behavior using such stylized models is highly unrealistic. But even in areas such as the law of evidence, in which people tend to evaluate new information in a fairly informal way, legal scholars have begun to apply the lessons of Bayesian statistics with interesting results. Richard Posner wrote that Bayes' Theorem offers "the most influential model of rational decision-making under conditions of ineradicable uncertainty[.]" Richard A. Posner, *An Economic Approach to the Law of Evidence*, 51 STAN. L. REV. 1477, 1479 (1999). *But see* Richard D. Friedman, *Presumption of Innocence, Not of Even Odds*, 52 STAN. L. REV. 873 (1999) (criticizing some of Posner's starting assumptions).

17. *See* John D. Graham, *Legislative Approaches to Achieving More Protection Against Risk at Less Cost*, 1997 U. CHI. LEGAL F. 13 (1997) (arguing that priority setting is a necessary part of environmental regulation, even when it's done implicitly).

18. *See* Richard B. Stewart, *Implementing the Precautionary Principle* (1999) *at* http://www.cserge.ucl.ac.uk/Stewart.pdf**.** Gresham's Law is commonly understood to suggest that "bad money drives out good"; in other words, too much concern with improbable risks consumes resources that could better be applied to the regulation of harms with a higher probabil-

usurp scarce resources that could more effectively be allocated to regulating known risks.[19] Indeed, part of the visceral appeal of the Precautionary Principle might be that it is consistent with the tendency of people to assume familiar risks (e.g., smoking causing lung cancer) over unfamiliar risks (e.g., cell phone use causing brain cancer) even if the former pose a greater threat.[20]

Statistical analysis is the means by which regulators are informed about the degree or 'magnitude of harm' of a risk. The most common methods are the Classical "frequentist" approach or the Bayesian approach. The differences in these approaches are further explored in Part II.A of this paper. Regulators generally acknowledge that a Bayesian approach is a more sensible method than a classical approach when performing a cost-benefit analysis for a risk with a known probability distribution; an "uncertain" risk.[21] When dealing with a truly uncertain risk in which there is substantial disagreement among experts about the probability or magnitude of harm of such a risk, regulators must decide what view to take of the probability of harm in their analysis.[22] That is, a regulator must make an informed assump-

ity—that bad regulation (which can include good regulation of improbable harms) drives out good regulation.

19. *Id*.

20. *See, e.g.,* Commission on Risk Assessment and Risk Management, *Risk Assessment and Risk Management in Regulatory Decision-Making,* Draft Report for Public Review and Comment, at 45 ("The public-health consequences of exposing patients and workers to ionizing radiation and of exposing the general population to infectious agents are so well established that they might be in the category of "familiar" risks, which psychologists have shown are far less frightening to the general public than "unfamiliar" or "dreaded" risks, even when the estimated magnitudes of the former are much higher.").

21. A number of environmental regulations require regulators to use cost-benefit analysis in conjunction with some sort of subjective assessments of the risk of harm—which might be termed a Bayesian approach. *See, e.g.,* 40 C.F.R. § 1502.22. (1983). providing that: "When an agency is evaluating reasonably foreseeable significant adverse effects on the human environment in an environmental impact statement and there is incomplete or unavailable information the agency shall always make clear that such information is lacking.

> (a) If the incomplete information relevant to reasonably foreseeable significant adverse impacts is essential to a reasoned choice among alternatives and the overall costs of obtaining it are not exorbitant, the agency shall include the information in the environmental impact statement.
>
> (b) If the information relevant to reasonably foreseeable significant adverse impacts cannot be obtained because the overall costs of obtaining it are exorbitant or the means to obtain it are not, the agency shall include within the environmental impact statement (1) a statement that such information is incomplete or unavailable [and] (2) a statement of the relevance of the incomplete or unavailable information to evaluating reasonably foreseeable significant adverse impacts to the human environment..

22. *See* SEAS AT RISK, *The Final Declaration of the First European SEAS AT RISK Conference* (1994) (herinafter Seas at Risk Declaration), *synopsis available at* http://www.coastalguide.org/code/direct/html, document available at the Common Wadden Sea Secretariat, Vir-

tion about the probability of harm. Some advocates of the Precautionary Principle argue that regulators should adopt a "worst case scenario" view in making this assumption to determine the probability distribution of harm.[23] Others argue that regulators should use a "best-guess" Bayesian distribution,[24] and proceed in much the same way as would be adopted in the case of a risk with a known probability.[25] Thus, even under a generally precautionary approach to regulation, one might distinguish between different levels of caution: a "strong" Precautionary Principle under a "worst case" view and a "less strong" Precautionary Principle under a "best guess" view.

C. *Methodological Advantages of a Bayesian Approach*

In order to retain its credibility as a legitimate principle of environmental policymaking and not degenerate into an instrument of veiled protectionism, the Precautionary Principle must be implemented in a way that is as transparent as possible, does not deter further research and information gathering, and characterizes risk in a way amenable to regulatory decision-making and ongoing scientific debate. This paper argues that a Bayesian approach is not only the appropriate method to assess truly uncertain risks, but that it is uniquely suited to promoting intellectual due process principles. The latter characteristic is an important counterargument to the criticism that the Precautionary Principle is a fundamentally "unscientific" decision rule.[26]

Part II.A explains the theory behind Bayesian analysis and provides a mathematical example employing the methodology. Part II.B compares the Bayesian Approach with the Classical approach in risk assessment highlighting features of the Bayesian analysis which make it a superior risk assessment tool under conditions of substantial sci-

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chowstrabe 1, D-26382 Wilhelmshaven, Germany (articulating assumption of a "worst case view" when applying the Precautionary Principle) The Final Declaration of the First European SEAS AT RISK Conference, *Seas at Risk*, Drieharingstraat 25, 3511 BH Utrecht, The Netherlands, *Seas at Risk*, 36 –SC (1995) (herinafter Seas at Risk Declaration).

23. *Id*.

24. *See* CHARLES A. HOLLOWAY, DECISION MAKING UNDER UNCERTAINTY: MODELS AND CHOICES (1979) (presenting a compelling technical case in favor of such a "best estimate" approach).

25. *See* STEVEN MILLOY, CHOICES IN RISK ASSESSMENT - THE ROLE OF SCIENCE POLICY IN THE ENVIRONMENTAL RISK MANAGEMENT PROCESS, i, xiii (1994) (discussing "best guess" approach, some of its assumptions and its relationship to the assumptions in risk assessment).

26. *See, e.g.*, Frank B. Cross, *Paradoxical Perils of the Precautionary Principle*, 53 WASH. & LEE L. REV. 851 (1996).

entific uncertainty and disagreement among experts. Particular attention is paid to how the transparent nature of the Bayesian approach adds to its legitimacy. Also, Bayesian techniques' consistency with intuitive ideas of how to approach risk creates a more appropriate division of power between experts and non-experts in environmental policymaking, thereby promoting democratic considerations that are sometimes put forth as a justification for the Precautionary Principle. Part II.C explores how a Bayesian approach is consistent with the well-established scientific principles of falsifiability and simplicity. Therefore, casting environmental risks in Bayesian terms offers many advantages in framing scientific controversies in a way that promotes ongoing debate and is subject to scientific scrutiny. As will be discussed, however, institutional pressures might exist in opposition to the widespread use of Bayesian techniques.

The Bayesian approach is not a panacea, however, and Part III. discusses some issues that should be kept in mind when applying Bayesian methods to the Precautionary Principle. Perhaps most significantly, policymakers' conclusions will often be a function of the prior beliefs they adopt regarding the risk of an environmental harm. A simple example is used to illustrate a situation in which two groups engaged in negotiation over the extent of an environmental harm might never reach an agreement. While the degree of harm is resolvable from an objective perspective, the groups' stalemate is a result of each group adopting inappropriately extreme initial positions combined with factors of overconfidence and a tendency to underestimate the value of further information gathering. This is particularly interesting since the scientific opinions of experts from different disciplines over risks such as the extent of global warming have failed to converge—and in fact have in some cases *diverged* further—even as these experts are exposed to a common set of new data.[27] The persistence of such disagreements are examined from a Bayesian perspective, and possible solutions are suggested. Ultimately, however, even these sometimes perverse results that can arise when two groups adopt different belief functions will be mitigated to some extent by the Bayesian requirement of a choice of prior beliefs which explicitly quantifies the assumptions that go into making a regulatory decision—explicit assumptions about prior beliefs that can themselves be

27. *See* ROGER M. COOKE, EXPERTS IN UNCERTAINTY: OPINION AND SUBJECTIVE PROBABILITY IN SCIENCE (1991).

scrutinized and might mitigate any temptation to adopt inappropriately extreme positions.

II. THE MERITS OF BAYESIAN APPROACH

A. *Background*

Bayes' Theorem[28] is an extremely important development in the history of social science, as well as a somewhat controversial area within statistics.[29] Bayes' original theory states that "the probability of an event is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the value of the thing expected upon its happening."[30] The Theorem therefore provides a way to combine prior information, or a *prior belief* about a risk or a population parameter with new information obtained through sampling or other experiments to guide a person's inferences in a rational way. It allows decision and game theorists to examine how rational actors would assimilate new information with their existing subjective beliefs.

Its appeal to statisticians and social scientists is that its rules conform to mathematical principles that reflect a certain amount of internal consistency.[31] The basic idea is that a person should not accept wholesale the results of new information that disconfirm their prior beliefs. The weight assigned to the new data versus the prior beliefs depends upon the person's degree of confidence in both their beliefs and the sampled data, which is graphically reflected in how "tightly" the probability distribution is centered around certain values.

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28. The Theorem was first proposed by an English theologian and mathematician by the name of Thomas Bayes, in a 1763 essay which was posthumously published in 1964 and titled Bayes, T. (1764) *An Essay Towards Solving a Problem in the Doctrine of Chances*. Philosophical Transactions of the Royal Society of London 53:370-418. Reprinted in facsimile in W.E. Deming, ed.,Facsimiles of Two papers by Bayes (Washington, D.C.: U.S. Dept. of Agriculture, 1940).

29. *See generally* William H. Jefferys & James O. Berger, *Sharpening Ockham's Razor on a Bayesian Strop,* Purdue University Technical Report #91-44C (August 1991) *available at* http://quasar.as.utexas.edu/Papers.html. Later published in edited form in 89 AMERICAN SCIENTIST, Jan-Feb. 1992 64-72.

30. Will Hively, *The Mathematics of Making Up Your Mind*, DISCOVER, May 1996, at 93-94.

31. Consistent in the everyday sense, meaning in harmony, as opposed to the technical term used in Bayesian statistics.

Mathematical Definition and Example

In technical terms, Bayes' Theorem states that the subjective posterior odds (odds after being exposed to new data)[32] that a hypothesis is true can be determined by multiplying the prior odds (or odds before exposure to the new data)[33] by the ratio of (1) the probability that the data would have been observed if the hypothesis were true to (2) the probability that the data would have been observed if the hypothesis were *not* true. The ratio of (1) to (2) above is referred to as the *likelihood ratio*.[34] So, using L to represent this likelihood ratio, Bayes' Theorem is

$$\text{P(hypothesis is true } \textit{after}\text{)} = \text{P(hypothesis is true } \textit{before}\text{)} \times \text{L}.$$

Alternatively, since the probability of obtaining the data and the parameters can be written as:

$$\text{P(data and parameters)} = \text{P(data} \mid \text{parameters)} \times \text{P(parameters)}$$

Or:

$$\text{P(data and parameters)} = \text{P(parameters} \mid \text{data)} \times \text{P(data)}.$$

Equating these expressions and rearranging gives us Bayes' Theorem:

$$\text{P(parameters} \mid \text{data)} = [\text{P(data} \mid \text{parameters)} \times \text{P(parameters)}] / \text{P(data)}.$$ [35]

In this case, the prior hypothesis is the probability of the parameters prior to being exposed to the data.

In practice, consider two groups, A and B, who are trying to anticipate the magnitude of a truly uncertain environmental harm. This will eventually be a real number, of course, but because of their uncertainty the best that A and B can do is assign a prior *subjective probability distribution*. The prior subjective probability distribution assigned by A and B is based on their current beliefs, given every-

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32. *See, e.g.,* Richard D. Friedman, *Presumption of Innocence, Not of Even Odds*, 52 STAN. L. REV. 873, 875 (2000) (defining the posterior as "the odds that the proposition is true as assessed after receipt of the new evidence").

33. *Id.* (describing the prior as "the odds as assessed before receipt of the new evidence").

34. *Id.* ("Simply defined, the likelihood ratio of a given body of evidence with respect to a given proposition is the ratio of the probability that the evidence would arise given that the proposition is true to the probability that the evidence would arise given that the proposition is false.").

35. PETER KENNEDY, A GUIDE TO ECONOMETRICS 217 (4th ed. 1998).

thing they know about the harm.[36] This is their 'subjective prior belief.' For this example, let us use the bell-shaped curve of a normal distribution.[37] Assume that A and B will be able to reach an agreement on a regulation if the magnitude of harm lies somewhere between 1.8 and 2.2 million. This is the hypothesis for which they are testing. Further assume that A initially thinks that there is a 15% chance that the number lies between 1.8 and 2.2 million. This is a function of his *prior beliefs*. When A receives new information regarding the environmental harm, her belief about the magnitude of environmental harm will likely change. Bayes' Theorem tells us that her *posterior belief* that the hypothesis (in this case, that the magnitude of harm lies between 1.8 and 2.2 million) will be determined by multiplying her prior odds that it was true (15%) by the *likelihood ratio*. To determine the likelihood ratio, A asks herself two questions. First, what are the chances I would have encountered the results I did in the sample if my hypothesis (again, that the value lies between 1.8 and 2.2 million) were true? Second, what are the chances I would have received the results I did if my hypothesis were not true? The ratio of the odds in the first question to the odds in the second question forms the likelihood ratio. If her answer to the first question is twenty percent chance (0.2) and her answer to the second were eighty percent chance (0.8), her likelihood ratio would be 0.2/0.8, or 0.25. Therefore A's *posterior belief* that the hypothesis is true is: 0.15 x 0.25 or 0.0375.[38]

B. *Bayesian Versus Classical Approaches*

As mentioned before, the classical or frequentist methodology is another statistical approach to determining the probability of harm of

36. A prior distribution represents the Bayesian actor's subjective probability distribution or beliefs prior to exposure to new information. *See generally* D.H. Kaye, *What is Bayesianism?, in* PROBABILITY AND INFERENCE IN THE LAW OF EVIDENCE: THE USES AND LIMITS OF BAYESIANISM 1 (Peter Tillers & Eric D. Green eds., 1988) (discussing Bayesian terminology).

37. Bayesian statistics applies to other families of probability distributions as well, but this paper will only discuss normal (symmetrical, bell-shaped) probability distributions. A normal distribution is a bell-shaped probability curve with a probability density function $f(z) = 1 / (2\Pi)^{1/2} \exp(-1/2\ z^2)$. *See generally* BERNARD W. LINDGREN, STATISTICAL THEORY, 178-80 (4th ed. 1993).

38. Although a regulator might interpret the 0.0375 figure as being strong evidence against the hypothesis, it must be emphasized that this cannot be interpreted as the probability of the hypothesis being true. The likelihood ratios represent the ratios of the probabilities of the data being observed assuming the truth of the hypotheses, but the classical method does not give any way to relate this figure to the probability of the hypothesis itself being true.

an environmental risk. Classical statisticians are sometimes called frequentists because they view probabilities as representing the frequency with which something occurs. Classical or frequentist statisticians perform hypothesis tests by *assuming* the *null hypothesis* to be true, and then asking with what probability these results could occur by chance *given that the hypothesis were true*. A low probability of the results occurring by chance, which is measured by the *p-value*, is generally taken as evidence to refute the null hypothesis,[39] but it does not always correspond well to what one would consider the probability of harm. The frequentist perspective is not compatible with the Bayesian view of probabilities as representing subjective beliefs, since the frequentists would argue that either the hypothesis is true or it is not true.[40]

The choice between classical and Bayesian methods is not a trivial one. Consider a hypothetical example put forward by James Berger in which clinical trials are being performed to determine whether a series of drugs are more effective than a placebo.[41] The null hypothesis, that the drug is no more effective than the placebo, is known to be true in about half the cases based on past experience.[42] After testing twelve drugs, two have p-values of about 0.05 and two have p-values of about 0.01.[43] The resulting p-values for these four drugs strongly suggest that the drug is more effective than the placebo, thereby refuting the null hypothesis.[44] But using robust Bayesian analysis techniques, Berger demonstrates that the lower bounds for the percentage of drugs with p-values of 0.05 that are actually ineffective is 24% and the lower bound for the 0.01 group would be about 7%; the actual values (remembering that the robust Bayesian techniques gives lower bounds) being about 50% and 15% respectively.[45] As Bayesians are fond of saying, p-values give the right answers to the wrong questions. When classical statisticians try to bridge the gap between this right answer to the wrong question (the probability of

39. *See* David H. Kaye & David A. Freedman, *Reference Guide on Statistics*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE, 331, 378-79 (Federal Judicial Center 1994).

40. Jefferys & Berger, *supra* note 29.

41. *See* James Berger, *An Overview of Robust Bayesian Analysis*, Purdue University Technical Report #93-53C (1993), *available at* http://www.isds.duke.edu/~berger/papers/overview.html.

42. *Id*.

43. *Id*.

44. *Id*.

45. *Id*.

observing the data assuming the hypothesis is true) to the right question (whether or not the hypothesis is actually true) in the regulatory context, they will often give the wrong advice if not the wrong answer in a narrow sense.

This disconnect between *p-values* and *posterior probabilities* of ineffectiveness demonstrates the dangers of ignoring Bayesian techniques. In the context of the Precautionary Principle, classical methods might dramatically mischaracterize risks that a Bayesian approach, or at least a Bayesian robustness analysis in conjunction with classical tests – would reveal.

Classical hypothesis testing may be used to guide policy decisions in some cases, however it is not always useful in cases where there is an unknown risk. For example, in the context of Genetically Modified Organisms, regulators may want to know the probability that a particular foodstuff will pose health risks of a certain magnitude.[46] Because this is an unknown risk, for which there are no existing objective frequencies, classical statistics is unable to assign a probability to this proposition.[47] Bayesian statistics can assign such a probability through the use of prior subjective beliefs. Thus, the Bayesian approach is more useful in directly addressing questions about which regulators are concerned. Yet many argue that in the face of true uncertainty, a Bayesian approach should be abandoned[48] since any attempt to quantify risks that are unknown seems rather arbitrary.[49]

46. *See generally* Graham, *supra* note 17 (comparing classical with Bayesian approaches in environmental regulation).

47. *See* Jefferys & Berger, *supra* note 29 (discussing the philosophical differences between classical and Bayesian statisticians).

48. *See, e.g.,* Rachel Clark, *Dealing With Uncertainty*, RACHEL'S ENVIRONMENT & HEALTH WEEKLY, Sept. 5, 1996, *available at* http://www.dieoff.org/page31.htm.

49. One approach to addressing a truly unknown risk that might seem less arbitrary on its face would be to weigh the probability of each possible outcome equally. This is problematic for several reasons. First, such an approach to risk assessment is vulnerable to how the question is framed. For example, in the role of a die one event might be that the numbers are 1 or 2, with the second event being that the numbers rolled are 3, 4, 5, or 6. If a 1 or a 2 is the event being tested for, the problems associated with assigning equally weights to the hypothesis and the alternative hypothesis become clear. Also, it is unclear how one would deal with an event situation with an infinite tail, a tail in which some small probability extends out forever, or there are an infinite number of outcomes. This can happen when the possible outcomes include all real numbers, or even line on a continuous interval of real numbers. While these problems might seem self-evident, Richard Posner's otherwise interesting paper on a Bayesian approach to the law of evidence was criticized on this basis in a recent article. Because weighing the risks equally is influenced by how the question is framed. In the role of a die, it might be natural to weigh each number 1-6 evenly, because we happen to know that that is the case. But if we didn't know

Decisions ultimately must be made, however, and in the absence of some formal risk assessment method other rules of thumb will likely be adopted, such as the worst case "strong" precautionary principle approach discussed earlier, with all its shortcomings.

However, given that environmental policy decision-makers must still decide whether to act in the face of true uncertainty, and that classical statistics is unable to address unknown risks, abandoning Bayesian approaches does not make sense. Only under a "strong" version of the Precautionary Principle, where a perceived risk is regulated until it is proved harmless, could regulators act prior to quantifying the magnitude of the risk. However, the "strong" version is unlikely to be adopted not only because of budgetary constraints but also because the (overly) precautionary regulations could prevent benefits from being realized.[50] Thus, without either Bayesian or classical statistics, or an extremely preventative regulatory regime, it is likely that regulators will turn to other, perhaps less formal methods to assess the risk of a truly uncertain harm. A more informal method of risk assessment might be adopted, for example, in which regulators deliberate with experts as to the degree of risk. However, this deliberation method does not avoid the arbitrary assumptions associated with assigning a Bayesian probability distribution to a truly unknown risk. Instead, regulators may simply substitute another set of less-explicit assumptions and biases based on expert opinions.[51]

Transparency, and the Division of Power Between Experts and Non-experts

One of the arguments put forward in favor of the Precautionary Principle is a democratic ideal that people should be able to weigh more heavily risks that they consider particularly odious or dreadful.[52] This view requires that regulators, who presumably take the polity's interests into account, properly weigh various risks in their decision-

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this, we might define event 1 as the die coming up 1 and the event 2 as the die coming up 2-6. The probabilities would be completely a function of how we define the events.

50. *See* Cartagena Protocol on Biosafety to the Convention on Biological Diversity, January 29, 2000, arts. 10.6, 11.8., *available at* http://www.unep.ch/biosafety/cartagena-protocol-en.pdf (adopting a relatively strong version of the Precautionary Principle). *See also* John H. Adler, *More Sorry Than Safe: Assessing the Precautionary Principle and Proposed International Biosafety Protocol*, 35 TEX. INT'L L.J. 173 (2000) (analyzing problems associated with an overly precautionary approach).

51. *See* Berger, *supra* note 41.

52. *See generally* Richard H. Pildes & Cass R. Sunstein, *Reinventing the Regulatory State*, 62 U. CHI. L. REV. 1, 8 (1996).

making and prioritization. While expert opinion will often be necessary to guide these regulations and give them credibility,[53] ideally normative judgments should be left to regulators.[54] Thus, the role of the expert should be to present regulatory decision-makers with the probabilities and magnitudes of the risks that are to be regulated and allow the regulator to decide how to regulate based on the public's will.[55]

The Bayesian approach is more responsive to these democratic concerns in two ways. First, as alluded to earlier, the Bayesian approach is more transparent. Bayesian analysis requires an explicit assignment of a subjective probability prior to exposure to new information. This differs greatly from classical statistics where experts make assumptions regarding the risk in the course of the hypothesis testing. Thus, the assumptions are effectively hidden from the decision-maker, although they should be recorded in the statistical report. Also, the Bayesian approach frames the regulator's question about a perceived risk in the way in which most people intuitively think of risk assessment. Bayesian statisticians ask what is the probability of harm, whereas classical statisticians ask what the probability is that a set of data would occur by chance if a particular hypothesis were assumed to be true. Bayesian assumptions are explicitly made at the beginning of the process of formulating prior beliefs. As a result, this method reduces the gap between the results obtained and their regulatory interpretation. By contrast, the classical approach requires a much larger interpretive role in bridging the gap between the results and their regulatory implications, and includes assumptions that may be subtler but which are just as significant.

Second, the Bayesian approach decreases the reliance on experts in the regulatory process thereby alleviating concerns that experts might inappropriately influence the regulator decision-making process with their personal beliefs. The Bayesian approach allows the *decision-maker*, who is responsible for expressing the polity's will, to as-

53. *See generally* SHEILA JASANOFF, THE FIFTH BRANCH: SCIENCE ADVISORS AS POLICYMAKERS (1990).

54. This view isn't universally accepted, however, and "rule by experts" isn't a bad characterization of the development of the regulatory state in post-War America. *See* Mark Seidenfeld, *A Civil Republican Justification for the Administrative State*, 105 HARV. L. REV. 1516, 1518 (1992) ("[T]he New Deal contemplated that Congress should identify an area in need of regulatory control and turn the expert agency loose to regulate.").

55. At least if one accepts that normative judgments should be decided in a more democratic fashion.

sign a subjective probability to a perceived risk, which reflects his uncertainty regarding the magnitude of the risk.[56] As we have seen, classical methods do not allow this. Assigning a probability to a population parameter makes no sense to a classical statistician, because to him probabilities reflect objective frequencies rather than subjective beliefs.[57] Therefore, instead of presenting the regulatory authorities with the probabilities they need in order to institute environmental regulations, it is likely that an expert using classical statistical methods will require a far more active role in what will likely be a deliberative, collective decision. While such a process might have its virtues, it does offer far more opportunity for experts to use their superior knowledge to impose their own biases in the decision-making process.[58]

Adopting a Bayesian approach in a regulatory context would likely lead to greater accountability on the part of experts who would have to commit to actual assessments of risk that are testable through subsequent experience and subject to peer review. After all, even assuming the expert is disinterested, it seems unreasonable to expect him to internalize the many conflicting societal interests and opinions that may guide him to some sort of opinion. A far more sensible approach would be to limit the expert's role to his area of technical competence, with the expert serving as a guide for popular will rather than the instrument of its expression.

Greater transparency in risk assessment benefits the implementation of the Precautionary Principle. As has been suggested, there are democratic concerns that experts might not fully represent the interests of the polity.[59] Experts focusing on the quantitative aspects of an environmental harm might undervalue outcomes that people particularly dread from a qualitative perspective. Also, it is not unreasonable to imagine that experts, particularly in the environmental field, are to some extent self-selecting, motivated by personal passions that do not necessarily reflect popular will in terms of risk tolerance and other factors. Transparency in the regulatory process may also hedge against claims that the Precautionary Principle is being employed as a trade protectionist measure. As the issue has been

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56. *See* Jefferys & Berger, *supra* note 29, at 12.

57. *Id*.

58. *See* UNDERSTANDING RISK: INFORMING DECISIONS IN A DEMOCRATIC SOCIETY 111-17 (Paul C. Stern & Harvey V. Fineberg eds., 1996) (summarizing the studies on expert bias).

59. *See* Pildes & Sunstein, *supra* note 52.

framed here, adoption of Bayesian techniques would entail increased accountability and diminished influence for experts, thus some resistance to adopting Bayesian methods may be encountered. Since experts generally decide what methods they will use in making risk assessments, it seems unlikely that they would spontaneously adopt such techniques in the absence of external pressures. By adopting an overly deferential view of expert opinion for fear of overstepping their institutional competences, courts and tribunals may be deferring to experts not just on purely technical questions, but also on a range of judgments that are actually more properly left to the courts and other institutions.

C. *Bayesian Techniques and Intellectual Due Process Considerations*

In implementing and applying the Precautionary Principle, administrative agencies and tribunals will be called upon to decide among several competing hypotheses that purport to explain a set of data. Although some administrative law scholars, such as Stephen Breyer,[60] have at times seemed to advocate using the hypothesis that enjoys the greatest degree of acceptance within the scientific community,[61] courts have been understandably concerned about overstepping their institutional competencies in judging the validity of rival scientific hypotheses.[62] Stepping outside the area of administrative and regulatory law for a moment, the line of cases following *Daubert*, for example, allows even minority scientific opinions to be considered as evidence so long as they enjoy some degree of acceptance within the scientific community.[63] Particularly in the context of the Precaution-

60. *See* STEPHEN BREYER, BREAKING THE VICIOUS CIRCLE: TOWARD EFFECTIVE RISK REGULATION 12-13 (1993).

61. Frye v. United States, 293 F. 1013, 1014 (D.C. Cir. 1923) (articulating the "general acceptance" standard for the admissibility of scientific evidence as "while courts will go a long way in admitting expert testimony deduced from a well-recognized scientific principle or discovery, the thing from which the deduction is made must be sufficiently established to have gained general acceptance in the particular field in which it belongs.").

62. *See, e.g.*, Daubert v. Merrill Dow Pharm. Inc., 509 U.S. 579, 600-601 (Rehnquist, J., concurring in part and dissenting in part) ("I do not doubt that Rule 702 confides to the judge some gatekeeping responsibility . . . . But I do not think it imposes on them either the obligation or the authority to become amateur scientists in order to perform that role."). *See also* Ruiz-Troche v. Pepsi Cola, 161 F.3d 77, 85 (1st Cir. 1998) ("*Daubert* neither requires nor empowers trial courts to determine which of several competing scientific theories has the best provenance.").

63. *See* Daubert v. Merrell Dow Pharm., Inc., 43 F.3d 1311, 1319 (9th Cir. 1995) (stating that methods used by "a recognized minority of scientists in their field" still could be acceptable).

ary Principle, legal scholars have sometimes advocated using the hypothesis that adopts the worst-case scenario view in determining remedies.[64] Recent literature on the relationship between Bayesian techniques and the scientific method, however, suggests that Bayesian techniques may provide a framework to assist regulators' choices of models, and may actually illuminate the way towards a more objective standard for judging rival scientific hypotheses.[65] The renowned Austrian philosopher of science Sir Karl Popper argued that *falsifiability* was the quality that distinguished genuinely scientific theories from pseudo-science.[66] Although widely accepted among philosophers of science and scientific practitioners alike,[67] the practical application of this standard to the courtroom has perplexed as formidable a jurist as Chief Justice William Rehnquist, who wrote in regard to his fellow federal judges that "I am at a loss to know what is meant when it is said that the scientific status of a theory depends on its 'falsifiability,' and I suspect some of them will be, too."[68] In general terms, falsifiability means that a hypothesis makes assertions that can disprove, or at least disconfirm, the hypothesis if they fail to come about, making it vulnerable to testing.[69] A classic example of claims that lack falsifiability are psychics and other paranormalists who claim that the skepticism of researchers or the artificiality of laboratory conditions limit their ability to exercise their "gifts." That being the case, there is no way to test the hypothesis that a psychic's ability is not genuine without interference; the claim can not be disproved. Another example of a lack of falsifiability are disciplines whose hypotheses have

64. *See, e.g.,* Chris W. Backes & Jonathan M. Verschuuren, *The Precautionary Principle in International, European, and Dutch Wildlife Law*, 9 COLO. J. INT'L ENVT'L L & POL'Y 43, 56 (1998) (interpreting the Precautionary Principle such that a severe enough "worst-case" scenario justifies prohibition of an activity).

65. *See generally* Jefferys & Berger, *supra* note 29.

66. *See* KARL R. POPPER, THE LOGIC OF SCIENTIFIC DISCOVERY 279 (rev. ed. 1992) (arguing that falsifiability was the quality that distinguished genuinely scientific theories from dogmatic, non-scientific views not subject to constant testing and revision). *See also* Daubert, 509 U.S. at 593 (citing Popper's view of falsifiability in the scientific evidence context).

67. *See generally* IAN HACKING, REPRESENTING AND INTERVENING (1983) (assessing Popper's influence on the philosophy and practice of science).

68. *Daubert*, 509 U.S. at 600-601 (Rehnquist, J., concurring in part and dissenting in part). Although *Daubert* did not deal with a regulatory decision, it is not unreasonable to think that regulatory decision-makers might adopt many of the same standards in judging the relative merits of rival scientific hypotheses in regulatory matters as have been adopted under the common law.

69. *See* Popper, *supra* note 66, at 42 (The principle of falsification is distinguished by "its manner of exposing to falsification, in every conceivable way, the system to be tested.").

such vague predictive power that virtually any data can be interpreted as being consistent with those claims. Such a discipline is astrology, where nearly any event could fulfill a horoscope.[70] Popper himself included the disciples of Freud and Marx in this category as well.[71]

In 1991, James Berger and William Jefferys wrote a paper on the relationship between Bayesian techniques and Ockham's Razor—the scientific principle which asserts that the simpler hypothesis is usually the better.[72] In the paper, they compared the ability of two rival hypotheses (Einstein's theory of general relativity and a "fudged Newtonian" hypothesis) to explain observed perturbations in the orbit of Mercury around the sun in about 1920.[73] The discussion of this example demonstrates not only Bayesianism's close relationship to Ockham's Razor, but also to more general principles of falsifiability.[74]

In order to compare the relative likelihood that the rival hypotheses ("E" for Einstein's, "F" for the "fudged Newtonian" hypothesis) explained the data that had actually been observed, Berger and Jefferys compared the ratio of the probability that the data would be observed if the Einstein hypothesis were true to the probability that the data would be observed given that the "fudged Newtonian" hypothesis were true (this ratio is noted as "B").[75] They then made some other fairly weak assumptions,[76] with the end result being that

70. *See* KARL POPPER, CONJECTURES AND REFUTATIONS 34, 37-38 (1962).

71. *See* Adina Schwartz, *A "Dogma of Empiricism" Revisited:* Daubert v. Merrell Dow Pharmaceuticals, Inc.*, and the Need to Resurrect the Philosophical Insight of* Frye v. United States*,* 10 HARV. J.L. & TECH. 149, 165 (1997) (citing that Popper sought to distinguish empirical science from what he viewed as the pseudoscientific excesses of Karl Marx's economic theories and much of Freudian psychology).

72. *See* Jefferys & Berger, *supra* note 29. ("Ockham's Razor, an established principle used every day in science, has deep connections with Bayesian reasoning, which traces directly back to Sir Harold Jeffreys' pioneering work on statistics during the 1920s and 30s.").

73. *See id* at 5-8.

74. *See* notes 86-104 and accompanying text.

75. *See* Jefferys & Berger, *supra* note 29. They represent Bayes' factor (B) as (with (a) as the observed data; (E) for Einstein's hypothesis, (F) for the Fudged Newtonian" hypothesis): $B = P(a \mid E) : P(a \mid F)$. Jefferys and Berger assume that the error is normal, so the probability of observing data a given that the true value is α can be written $P(a|\alpha) = 1/((2\pi)^{½}\sigma) \exp(-(a-\alpha)^2/2\sigma^2)$.

76. Since Einstein's theory predicted a value for $\alpha = \alpha_E = 42.9$", Jefferys and Berger simply substituted 42.9 for α, giving

$$P(a \mid E) = 1/((2\pi)^{½}\sigma) \exp(-(a-42.9)^2/2\sigma^2)$$

as the probability of observing the data a assuming the Einstein hypothesis to be correct.

In order to estimate the probability of observing the data under the "fudged Newtonian" hypothesis F, Jefferys and Berger made some additional assumptions. As they pointed out in their paper, if a prior density for the true value of α under F could be determined (represented as $P(\alpha|F)$), the conditional density of the data under F would be the Einstein hypothesis was about 28 times as likely an explanation of the perturbations as the "fudged Newtonian" explanation.

In this example, however, even though the Einstein hypothesis was 28 times as *probable* an explanation of the data as the "fudged Newtonian" hypothesis, virtually none of this is attributable to the fact that one model *more accurately explained* the data than the other.[77] The exponential factors in the equation of the Bayes' factor represent the respective fit of the Einstein hypothesis to the data (D_E) and the "fudged Newtonian" hypothesis, both of which approximately equal 1.[78] Instead, virtually the entire value of B which accounts for the greater probability assigned to Einstein's hypothesis can be attributed to the factor $(1 + \tau)^{½}$, which represents the ratio of the spread of the prior distribution for the "fudged Newtonian" hypothesis to the spread of the data.[79] The size of this factor, the *Ockham factor*,[80] reflects the fact that the "fudged Newtonian" hypothesis must spread its prior probabilities over a large number of values that

$$P(a \mid F) = \int P(a \mid \alpha)\, P(\alpha \mid F) d\alpha.$$

Jefferys and Berger chose the value of $P(\alpha|F)$ to be a normal distribution centered around 0, represented as

$$P(\alpha \mid F) = 1/((2\pi)^{½}\tau \exp \{-\alpha^{½}/2\tau^{½}\}.$$

This assumption was motivated by a number of factors. First, it was necessary that the value of $P(\alpha|F)$ be independent of the data, and only reflect information known about F. Jefferys and Berger made the rather weak assumptions that smaller deviations would be more common than larger ones and the distribution would be symmetrical (since positive or negative variations were equally likely), so a normal distribution with mean 0 and standard deviation τ, which for various reasons they estimate to be around 50 (τ=50).

Incorporating these values and assumptions into $P(a|F) = \int P(a|\alpha)\, P(\alpha|F)d\alpha$, the conditional density of $P(a|F)$ was determined to be

$$P(a \mid F)=1/[2\pi(\sigma^2 + \tau^2)] \exp (- a^2 / 2(\tau^2 + \sigma^2)).$$

Plugging in the values $a = 41.6$, $\sigma = 2.0$, and $\tau = 50$, the relative probability of observing the data given the truth of the Einstein hypothesis compared to the probability of observing the data if the "fudged Newtonian" hypothesis were true can be given by

$$B = P(a \mid F)/P(a \mid E) = (1 + \tau'^{2})^{½}\exp(-D_E^{\,2}/2)\exp[D_F^{\,2}/2(1 + \tau'^{2})].$$

In this case, $D_E = (a - \alpha_E) / \sigma = -0.65$, $D_F = a/\sigma = 20.8$, and $\tau' = \tau / \sigma = 25.0$. When all the numbers are plugged in we get the value

$$B = 28.6.$$

This means that under the rather weak assumptions made, the Einstein hypothesis is over 28 times as likely an explanation of the data compared to the "fudged Newtonian" hypothesis.

77. *Id*. at 8.

78. *Id*. at 8.

79. *Id.* at 8.

80. *See* S. Gull, *Bayesian Inductive Reference and Maximum Entropy, in* 1 MAXIMUM ENTROPY AND BAYESIAN METHODS IN SCIENCE AND ENGINEERING, at 53-74 (C.J. Erickson and C.R. Smith eds., 1988).

are actually never observed, compared to the rather sharp predictions of the Einstein model.[81]

It is the ability of the Bayesian method to make sharper predictions that relates to falsifiability. Even though both the Einstein and "fudged Newtonian" models fit the data about equally well,[82] the additional free parameters and the resulting extra degrees of freedom[83] in the fudged Newtonian models serve as a double-edged sword. In order to accommodate such a large range of possible values, **which could be observed and still be consistent with the hypothesis**, the fudged Newtonian model must spread its prior distribution over this range of values thereby decreasing its predictive sharpness.[84] Essentially the Newtonian model attempts to "hedge it bets" through a dispersed prior distribution and additional degrees of freedom.[85]

Like the palm-reader whose overly vague predictions can accommodate a wide range of possible outcomes, the Newtonian hypothesis fails to make the sharp predictions of the simpler Einstein hypothesis, also making it difficult to disconfirm. The Bayesian method properly penalizes the Newton hypothesis for this "hedge,"[86] consistent with principles of falsifiability. The fudged Newtonian hypothesis loses out not because it is more *false*, but because it is less *falsifiable*. It is in this sense that the Bayesian method gives form to and in some sense quantifies the principle of falsifiability.

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81. Jefferys & Berger, *supra* note 29, at 8.

82. *Id.* at 8. Additional degrees of freedom make a model less 'simple.'

83. Degrees of freedom represent the number of linearly independent observations used in order to calculate a statistic. PETER KENNEDY, A GUIDE TO ECONOMETRICS at 65, (4th ed. 1998). A model with fewer parameters has a smaller degree of freedom, and is therefore simpler. A line, for example, has fewer parameters than a parabola. But because the parabola can "bend", and a line is really just a kind of parabola that happens to be straight, the parabola will always fit a scatter plot at least as well as the line. The idea of Ockham's razor is that goodness of fit is not the only criteria of what makes a model good, and that there must be a tradeoff between fit and simplicity. Models are meant to reflect reality, not data. Since there is usually some degree of observation error, making an excessively complicated model that bends to exactly fit the data would be mistaking the observation error in the data for the actual underlying phenomenon that is meant to be modeled. Although in this example the Bayesian approach penalizes the more complicated model, in the case of nested models, such as the line and parabola (nested in the sense that a line is a type of parabola), the posterior probabilities will always favor the more complicated model. *See* KARL POPPER, THE LOGIC OF SCIENTIFIC DISCOVERY (1959).

84. Jefferys & Berger, *supra* note 29, at 8.

85. *Id.*

86. *Id.*

III. DRAWBACKS OF THE BAYESIAN METHOD

A. *Problems in the Context of Disagreement Among Experts*

Although we have seen that the Bayesian method is in many ways a more transparent decision rule that may provide guidance in judging the validity of two rival scientific hypotheses, a number of perverse situations can arise when groups adopt significantly different prior beliefs about the risks associated with an environmental harm. This is an issue of particular concern with regard to implementation of the Precautionary Principle, since debate over the risks of a number of environmental harms have often been characterized by an increasing divergence of opinion among experts, even as more data has come to light.[87] Intuitively, one might think that the beliefs of two groups in disagreement would tend to converge as they are exposed to a common set of data—one might even take increasing disagreement as evidence of some hidden or undiscovered motives on the part of the experts. But when two groups adopt inappropriately extreme initial positions, exposure to a common set of data may well increase disagreement just because of the manner in which they rationally assimilate the new information under a Bayesian framework, aside from hidden motive. The transparency required in a choice of prior beliefs might go a long way towards mitigating bad-faith adoption of extreme priors to manipulate outcomes. However, Bayesian techniques do have the potential for creating some bizarre outcomes even when two groups disagree in good faith.

For an example of this somewhat perverse Bayesian phenomenon, assume that two groups, Z and Y, initially adopt sharply divergent prior beliefs about the magnitude or probability of an environmental harm, X. Both groups' prior beliefs can be represented by the familiar, bell-shaped curve of a normal probability distribution.[88]

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87. The phenomenon of increasing divergence of positions during negotiations has also been attributed to internal group dynamics, the assumption being that members of a group with a particular ideological disposition embolden one another to adopt even more extreme positions when they engage in dialogue with other members of the group. As this section suggests, however, even formal Bayesian approaches might not be adequate in countering this phenomenon. CASS SUNSTEIN, THE LAW OF GROUP POLARIZATION, JOHN M. OLIN LAW & ECONOMICS WORKING PAPER NO. 91, UNIVERSITY OF CHICAGO LAW SCHOOL (1999).

88. Bayesian statistics may extend to the more general area of negotiations among sophisticated actors. This is especially true considering that such discussions are frequently conducted with the assistance of policy advisors employing formal quantitative modeling techniques. Rather than offering a loosely heuristic view of how rational actors might act under very par-

Group Z has a normal prior distribution with mean m and standard deviation s. Let group Y have a normal prior distribution for X with mean m´ and standard deviation s.

Both Z and Y are then exposed to a common set of data that lies exactly between their prior beliefs. The likelihood function for the data is normal with mean n and standard deviation t, with the condition that t is assumed to be known.

When exposed to information characterized by such a likelihood function, group A's posterior belief distribution will have mean[89]

$$[m/s^2 + n/t^2] / [1/s^2 + 1/t^2]$$ [90]

with variance

$$[1/s^2 + 1/t^2].$$ [91]

Group B will have a posterior mean of

$$m'[1/s^2 + 1/t^2]$$

with the same variance as group A.[92]

Intuitively, one might think that the data will cause the two groups' posterior belief functions to creep closer together, thereby promoting a degree of consensus.

However, Z and Y actually disagree more vigorously than before because although the mean values of the belief distributions moved closer together after exposure to new information, the functions simultaneously became "tighter" thereby increasing the degree of confidence of each group in their own beliefs. So, even though the mean value of a group's posterior belief function will be closer to the true value after exposure to good information, the increased "tightness" of the group's posterior beliefs might actually cause that decision-maker to assign a lower probability to some region surrounding the true value of X. In other words, the increased tightness of the function mitigates the fact that the medians have moved closer together such

ticular conditions, Bayes' Theorem might well describe negotiation dynamics as they actually occur in areas such as international environmental disputes.

89. Equations and example provided in response to question presented to James K. Hammit in e-mail correspondence of 7/6/99.

90. *Id*.

91. *Id*.

92. *Id*.

that, when all the dust settles, there is actually less overlap between the two groups beliefs.

This result might not in itself be so unsettling, since eventually the two groups' belief functions will converge in a more meaningful way as they are exposed to more information. The problem is that a decision to pursue more information is dependent on the decision-maker's expected value of information, which in turn is sensitive to the degree of confidence the decision-maker has in his beliefs. Generally there is an inverse relationship between the degree of confidence and the expected value of information: the more confident you are, the less likely you are to think that more information will change your beliefs. This can lead to the development of information deadzones, in which a stalemate can persist even though additional information would eventually resolve it.

If Z and Y were engaged in negotiations over a regulatory standard, this phenomenon could disrupt negotiations. If a group's decision to continue negotiating is influenced only by the mean value of their belief function, exposure to new information will probably lead towards consensus. But it is not implausible to imagine that other factors might influence the decision to continue negotiating.

1. The Standardized Difference in Means

One of these factors is the standardized difference in means between group Z and Y's belief functions. Graphically, the standardized difference in means is represented by the area of overlap between the two groups' probability distributions. If the standardized difference in means decreases after exposure to new information, this suggests that the region of values of X for which both groups assign a positive probability is decreasing. It also means that it would become increasingly surprising[93] for the two groups to reach consensus.

The standardized difference in means is the difference in means between the belief functions for groups Z and Y divided by the standard deviation of one of the distributions. To determine whether this area of overlap has decreased after exposure to information, one must determine whether the posterior standardized difference can exceed the prior standardized difference. Since the prior distributions differ by

93. Surprising in the sense that each assigned a lower degree of probability to the values which could lead to agreement.

$$2m/s$$ [94]

and the posteriors differ by

$$(2m/s^2) / (1/s^2 + 1/t^2)^{3/2},$$ [95]

this can be the case in our example if the values of s and t are sufficiently large. As a numerical example, if t = s and n = 0 (with the data centered midway between the two priors), the posterior standardized difference will exceed the prior standardized difference if s is greater than $2^{3/4}$, which is approximately 1.68.[96] Significantly, this result does not depend on the values of m and m´, so the data need not be extreme. Each group interprets the data as confirming its own prior beliefs, with the result being that the area of overlap between the posterior beliefs decreases.

2. . . .And the Expected Value of Information

Of course, if information gathering were free there would be no problems. The two groups would continue gathering information until all uncertainty was effectively eliminated. But information gathering is usually not free, and a large part of what influences the decisions of negotiators to continue a dialogue is the relative costs and benefits of gathering more information in hopes of overcoming a factual disagreement.[97]

Decision theorists formalize the expected benefits of further data gathering by using the *expected value of information*. In assigning a number to the value of more information, decision theorists ask two distinct questions. First, how is more data likely to affect my belief function? Second, what will be the increase in value I am likely to realize if I alter my decision in a way consistent with my expected new beliefs?[98]

In our example, the problems with this approach are self-evident. Both groups have been exposed to data that from an objective per-

94. Hammit, *supra* note 89.

95. *Id*.

96. *Id*.

97. One substantial cost, for example, is the risks associated with inaction while further research is conducted. *See* Adam M. Finkel & John S. Evans, *Evaluating the Benefits of Uncertainty Reduction in Environmental Health Risk Management*, 37 J AIR POLLUTION CONTROL ASSN 1164 (1987); GRANGER M. MORGAN & MAX HENRION, UNCERTAINTY: A GUIDE TO DEALING WITH UNCERTAINTY IN QUANTITATIVE RISK AND POLICY ANALYSIS (1991).

98. *See* James K. Hammitt & Alexander I. Shlyakhter, *The Expected Value of Information and the Probability of Surprise,* 19 RISK ANALYSIS, 135 (Feb. 1999).

spective seems like it should disconfirm their prior beliefs. After all, the data is centered midway between their prior belief functions; one might think that they would realize that they have adopted inappropriately extreme positions. From a purely intuitive perspective, one would think that this would lead them to be more open to questioning their prior beliefs.

As we have seen, however, it is entirely possible that the groups will both become more rigid in the sense that they will have "tighter" belief functions. Both groups become even more confident in their posterior beliefs if they assimilate the information in a way consistent with Bayes' Theorem. From their own subjective viewpoint, however, this is the most rational way for them to behave.

But such overconfidence dramatically decreases the chances that they will pursue further information gathering.[99] By adopting posterior belief functions in which too little probability is assigned to outcomes further away from the center of the probability distribution, a negotiator will believe that it is unlikely that more information will dramatically alter his beliefs. Because he believes that it is unlikely that more information will change his beliefs, he will also tend to underestimate the extent to which more information gathering will alter his decisions. Since he believes that more information gathering will only further confirm his current beliefs, he will dramatically underestimate the value of more information gathering.

The extent to which such overconfidence affects an actor's expected value of information is a complicated question that depends on a number of factors, but work by prominent decision theorists suggests that even a small degree of overconfidence can lead people to dramatically underestimate the expected value of information.[100] If negotiators underestimate the value of further information gathering when there is a factual dispute, it becomes less likely that they will pursue the dialogue in hopes of resolving the question. They are far more likely to agree to disagree, and the negotiations will end in a stalemate.

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99. *Cf.* Bruce L. Beron, *Litigation Risk Management Analysis: A Comprehensive, Logical Approach to Litigation Decision-Making in* LITIGATION RISK ASSESSMENT FOR INSURANCE COUNSEL 1996, at 27, 34 (PLI Litig. & Admin. Practice Course, handbook Series No. 550 1996). If our expert on a particular question has had much experience in similar cases, we would naturally be more confident of the resulting probabilities and would not feel compelled to research the topic more thoroughly.

100. Hammitt & Shlyakhter, *supra* note 98.

At this point, it becomes apparent how the interplay of factors can lead to such stalemates. Two groups begin with widely divergent prior beliefs, and are exposed to a little information. From an objective "bird's eye" perspective, it seems like the data lies midway between their beliefs and should open them up to questioning their existing beliefs and engaging in further negotiations. From a subjective perspective, however, the data can cause them to become even more confident in their posterior beliefs. They will also "disagree" more in the sense that they will have less overlap in their belief functions. Finally, their overconfidence will make them less likely to gather more information in the hopes of overcoming any disagreement. Stalemate.

In all likelihood, the best method of overcoming such problems would be to provide a systematic bias in favor of more information gathering when two regulatory agencies are in disagreement over the extent of an environmental harm. This could be accomplished through a number of legal devices, such as a heightened burden of persuasion for the party in the best position to conduct further research and information gathering in disputes over application of the Precautionary Principle. It would be hoped, however, that the very use of Bayesian methods would discourage regulators and their experts from adopting inappropriately extreme prior distributions. If they were to do so, however, Bayesian techniques at least make explicit the assumptions that go into a choice of prior.

Such outcomes might suggest that Bayesian decision-making, with its emphasis on subjective beliefs, might not be a helpful decision tool used in isolation when groups of experts from different disciplines, for example, hold dramatically different beliefs about the probability of an environmental harm. The problem might be most acute in the international context, where coordination among nations is often necessary for effective regulation. The fact that Bayesians must explicitly adopt prior beliefs might discourage bad-faith posturing, but good-faith disagreement might be difficult to overcome if two groups begin with radically different positions. Any systematic bias in favor of more information gathering holds the potential for wasting of resources. Moreover, when further information gathering takes the form of observing the effects of an environmental threat, might lead to dangerous delays in regulation.

IV. CONCLUSION

In adopting environmental regulations under conditions of true scientific uncertainty, many advocates of the Precautionary Principle have suggested that the use of Bayesian methods to assess probability distributions may be inappropriate. But as this paper demonstrates, even in cases of true uncertainty the Bayesian approach has characteristics that make it a more appropriate means of assessing even truly uncertain risks than its methodological rivals. Bayesian techniques are more transparent than classical hypothesis testing, and are consistent with well-established scientific principles such as falsifiability and simplicity when used to compare rival scientific hypotheses. Although Bayesian techniques can lead to perverse results when groups adopt inappropriately extreme prior assessments of risk, such problems can be discouraged through institutional innovations. Moreover, the Bayesian requirement that regulators adopt an explicit prior subjective distribution for an environmental risk subjects regulatory judgment to scrutiny through subsequent experience, making the choice of "doctored" priors chosen to influence or defend policy or trade decisions less likely. Its transparency and consistency with well-established scientific principles such as simplicity and falsifiability demonstrate that the Bayesian approach possesses many attributes that make it an appealing risk assessment tool in operationalizing the sometimes controversial ideas behind the Precautionary Principal.