

# 行政院國家科學委員會專題研究計畫 成果報告

## 廣義錯誤率公式於等化器最佳化之研究

計畫類別：個別型計畫

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計畫主持人：嚴雨田

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# 行政院國家科學委員會補助專題研究計畫成果報告

## 廣義錯誤率公式於等化器最佳化之研究

### Equalization Using Generalized Symbol Error Rate for Optimization

計畫編號：NSC 91-2219-E-032-005

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#### 中文摘要

本論文推導  $M$ -ary PAM 系統的廣義錯誤率公式，並使用最陡坡降法最小化錯誤率於線性等化器系統，此稱為 GMSE，我們比較 GMSE 與平方誤差最小化準則的優劣，以及與由 Yeh&Barry 提出的 AMSE 做比較。為了降低計算複雜度，我們亦提出 SGMSE 以隨機的方式做最陡坡降值的估計，當然亦與最小平方誤差、AMBER 做比較。最後以數學分析的方式將字符取樣速率的等化器，推廣到過取樣速率的等化器系統。

**關鍵詞：**等化器，過取樣速率等化器、最小平方誤差，字符間干擾

#### Abstract

A generalized formula for the decision error probability is derived for an  $M$ -ary PAM system. Then a GMSE algorithm based on the derived formula is proposed to minimize the symbol error rate for linear equalization. Comparisons made between the GMSE algorithm and the MMSE criterion show that the minimum symbol error rate criterion outperforms the minimum mean-squared error criterion. Comparisons between our GMSE and AMBER as proposed by Yeh and Barry is also made. Then a SGMSE algorithm is obtained by further simplification of GMSE to reduce the computational complexity in GMSE. Comparison is also made between the SGMSE algorithm, the MMSE criterion, and the AMBER algorithm as proposed by Yeh and Barry. Finally, an extension of the GMSE algorithm employing fractionally-spaced equalization is presented with mathematical formulations.

**Keywords:** Equalization, Fractionally-spaced Equalizer, MMSE, Intersymbol Interference

#### 1. Introduction

It has been shown that an equalizer that directly minimizes bit error rate (BER) or symbol error rate (SER) may outperform the minimum mean-square error (MMSE) equalizer. Chen and Mulgrew have obtained a

BER expression for a linear-combiner DFE with binary signaling under the assumption of linear separable decision regions. They then used the gradient algorithm to derive a minimum bit error rate (MBER) solution [1,2,3]. Yeh and Barry proposed a simplified stochastic algorithm called the approximate minimum-BER (AMBER) equalization for a linear equalizer with binary signaling as well as  $M$ -ary PAM transmission [4,5,6].

In this paper, a generalized decision error probability is derived for a linear equalizer using  $M$ -ary PAM transmission. The word "generalized" is used because the error probability expression is valid for any assignment of the equalizer weight coefficients. That is, the equalizer weight coefficients may not be optimized. Then, the decision error probability is minimized using gradient search to obtain the optimum weight coefficients. We shall term the method as *generalized* minimum-SER (GMSE). The drawback of this method is that the computational complexity increases significantly for high order signaling. A simplified process is thus introduced to approximate the gradient search to considerably reduce the computations. This simplified method will be termed stochastic GMSE or SGMSE. An important merit of the GMSE or SGMSE algorithm is that the learning process converges rather fast.

We have also added fractionally-spaced equalization (FSE) into consideration. It is now well known that the advantage of the FSE over SRE (symbol-rate equalizer) is its insensitivity to the receiver timing phase [7]-[9]. The structure of the over-sampling FSE receiver can be viewed as a multiple of SRE receivers connected in parallel [10], so the channel model is called SIMO (single-input multiple-output) channel.

This paper is organized as follows. Section 2 derives the generalized decision error probability for linear equalization employing  $M$ -PAM transmissions. Section 3 presents the GMSE and SGMSE algorithms. Then some numerical results are presented in Sec. 4 including comparisons made between MMSE and GMSE. The GMSE algorithm employing the fractionally-spaced equalizer is discussed in section 5. Finally, conclusions

are made in Sec. 6.

## 2. Generalized Probability of Decision Error for Linear Equalization

We consider a bandlimited channel employing  $M$ -ary PAM transmission having the discrete response  $f_k, k = -L, \dots, 0, \dots, L$ , (non-causal filter of length  $2L + 1$ ). A linear equalizer at the receiving end has tap weights  $w_k, k = -N, \dots, 0, \dots, N$  (non-causal filter of length  $2N + 1$ ). The entire system (cascade of channel and equalizer) thus has tap weight coefficients  $q_k$  (length  $2L + 2N + 1$ ) given by

$$q_k = f_k * w_k, \quad (1)$$

where  $k = -(L + N), \dots, 0, \dots, (L + N)$ ,  $*$  denotes convolution, and  $\sum_k f_k^2 = 1$ . With input data  $x_k$  and channel noise  $\eta_k$ , the equalizer output estimate  $\hat{x}_k$  is given by

$$\hat{x}_k = x_k * q_k + \eta_k * w_k. \quad (2)$$

Here,  $\eta_k$  is a Gaussian random variable with zero mean and variance  $N_0/2$ . The source data symbols  $\{x_k\}_{-\infty}^{\infty}$  takes the discrete values with equal probability given by

$$x_k = (2m - 1 - M)d, \quad m = 1, 2, \dots, M \quad (3)$$

where  $M = 2^n$  with  $n$  being the number of bits per symbol and  $2d$  is the distance between adjacent data symbols. Also, we will take the average energy of  $x_k$  to be  $E_{av} = \frac{1}{3}(M^2 - 1)d^2 = 1$  [11].

The estimate error  $e_k$  is

$$e_k = x_k - \hat{x}_k = (x_k - x_k * q_k) - \eta_k * w_k. \quad (4)$$

Now, define  $z_k$  as

$$z_k = x_k - x_k * q_k. \quad (5)$$

Since  $x_k$  is a discrete uniformly distributed random variable with  $M$  possible outcomes, it is obvious from (5) that  $z_k$  is a discrete uniformly distributed random variable with  $D = M^{2L+2N+1}$  possible outcomes, say  $\alpha_{i,k}, i = 1, 2, \dots, D$ , with probability

$$p(z_k = \alpha_{i,k}) = \frac{1}{D}, \quad i = 1, 2, \dots, D. \quad (6)$$

For a set outcome  $z_k$ , say  $\alpha_{i,k}$ , the estimate error has a p.d.f. given by

$$p(e_k | \alpha_{i,k}) = \frac{1}{\sqrt{\pi N_0} \|\mathbf{w}_k\|} \exp[-(e_k - \alpha_{i,k})^2 / \|\mathbf{w}_k\|^2 N_0] \quad (7)$$

, where  $\|\mathbf{w}_k\|^2 = \sum_{i=-N}^N w_i^2$ .

Therefore, the average estimate error probability is

$$\begin{aligned} p(e_k) &= \frac{1}{D} \sum_{i=1}^D p(e_k | \alpha_{i,k}) \\ &= \frac{1}{D} \sum_{i=1}^D \frac{1}{\sqrt{\pi N_0} \|\mathbf{w}_k\|} \exp\left[-\frac{(e_k - \alpha_{i,k})^2}{\|\mathbf{w}_k\|^2 N_0}\right]. \end{aligned} \quad (8)$$

Thus, the estimate error probability is the sum of shifted Gaussian functions. The probability of equalizer decision error  $P_M$  for  $M$ -ary PAM transmission can now be readily obtained as follows:

$$\begin{aligned} P_M &= \frac{2(M-1)}{M} \int_d^\infty p(e_k) de_k \\ &= \frac{2(M-1)}{M} \cdot \frac{1}{D} \sum_{i=1}^D Q\left(\frac{d - \alpha_{i,k}}{\|\mathbf{w}_k\| \sqrt{N_0/2}}\right) \end{aligned} \quad (9)$$

Equation (9) is in fact a generalized formula. It applies to any assignment of equalizer tap weights, whether the tap weights are in optimum condition or non-optimum

condition. It is also valid for any algorithm, whether the algorithm is zero-forcing, MSE, or least square, or others. However, this formula is not quite informative, for the  $D$  values of  $\alpha_{i,k}$  are nowhere to be determined.

## 3. GMSER and SGMSE Algorithms for

## Linear Equalization

The  $M$ -ary symbol error probability can be minimized using the gradient method as follows:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu \frac{\partial P_M}{\partial \mathbf{w}_k}, \quad (10)$$

where  $\mu$  is the adaptive step size and the index  $k$  here denotes the iteration number. The initial value  $\mathbf{w}_0$  can be chosen to be the MMSE solution. After some mathematical manipulation, it can be shown that the  $(n+N+1)$ th component of the gradient vector  $\frac{\partial P_M}{\partial \mathbf{w}_k}$  is given by (11) as shown below, where  $n = -N, \dots, 0, \dots, N$ ,  $w_{n,k}$  is the  $(n+N+1)$ th component of the weight vector  $\mathbf{w}_k$  at the  $k$ th iteration (notice that, here we have used two subscript indices for the components in the vector  $\mathbf{w}_k$ , since now  $k$  represents the iteration index, not the sampling instant), and  $\alpha_{i,k}$  is an  $i$ th outcome of  $z_k$  at the  $k$ th iteration,  $i = 1, 2, \dots, D$ . We shall term the above method as the GMSE (generalized minimum symbol error rate) algorithm.

Equation (11) would increase computational complexity for high order signaling. When  $M$  is large,  $D$  will also become large which will result in large amount of computations. The following method of simplification is found to be very efficient and will yield satisfactory results to great accuracy.

Referring to (11), the term within the large bracket is summed over  $i$  (averaging). If instead of letting  $i$  vary from 1 to  $D$ , one can use much fewer terms as an approximation. For example, in each iteration, one randomly picks  $p$  terms to be summed,  $p$  could even be chosen to be 1. Uniform distribution is used for the random picking. In the end, after numerous iterations, an averaging effect takes place. In a sense, this is a stochastic gradient method similar to that employed by the LMS algorithm. We shall term this method of

simplification as the *stochastic* GMSE or SGMSE.

## 4. Numerical Examples

We consider a noisy binary channel having the transfer function  $H(z) = 0.2923077z^1 + 0.9105561z^0 + 0.2923077z^{-1}$ . Choosing a linear equalizer of length  $2N + 1 = 2 + 1 = 3$ , we have  $D = 2^{2L+2N+1} = 32$ . Assume the SNR is 17dB. With  $\mu = 1$ , Fig. 1 compares a learning curve using a random starting point with the curve using MMSE solution as the starting point. It is seen that a good convergence rate can be obtained. For purpose of comparison, Fig. 2 presents curves of symbol error rate vs. SNR for binary signaling ( $M=2$ ) using two different algorithms, e.g., (1) MMSE (2) GMSE. It is seen that, the GMSE gives better performance than that of MMSE. Note that, although the examples presented here are for binary signaling only, we have also performed simulations for  $M$ -ary signaling cases and found similar conclusions.

We have also compared our GMSE algorithm with the AMBER algorithm proposed by Yeh. We found that GMSE yields much faster convergence rate than AMBER, but both algorithms eventually converge to the same MSER level. This is shown in Fig. 3 with MMSE solution as the starting point for SNR=17dB. We have used a step size  $\mu = 2$  for the GMSE algorithm. While for the AMBER, we have used a step size  $\mu = 0.002$  and an update threshold  $\tau = 0.05$  [6].

Next, SGMSE is considered. Using MMSE solution as the starting point, three learning curves are plotted in Fig. 4 with  $p = 1, 4, 32$  respectively. Adaptation step size of  $\mu = 1$  has been used. It is seen that, except for the  $p = 32$  curve (GMSE) which is rather smooth, the other two curves corresponding to  $p=1$  and 4 (SGMSE) are less smoother. But, all three curves eventually converge to the same MSER level which is below the MMSE level. Then, with  $p = 1$  and  $\mu = 1$ , Fig. 5 compares a learning curve using a random starting point with the curve using MMSE solution as the starting point. For purpose of

$$\frac{\partial P_M}{\partial w_{n,k}} = \frac{2(M-1)}{M} \frac{1}{D} \sum_{i=1}^D \left\{ \left[ -\frac{1}{\sqrt{2\pi}} e^{-(d-\alpha_{i,k})^2 / N_0 \|\mathbf{w}_k\|^2} \right] \left[ \frac{\left( \sum_{j=-L+n}^{L+n} f_{j-n} x_{k-j} \right) \sqrt{\frac{N_0}{2}} \|\mathbf{w}_k\| - (d - \alpha_{i,k}) w_{n,k} \sqrt{N_0/2}}{\|\mathbf{w}_k\|} \right] \right\} \quad (11)$$

comparison, Fig. 6 presents curves of symbol error rate vs. SNR for binary signaling ( $M=2$ ) using three different algorithms, e.g., (1) MMSE, (2) AMBER, and (3) SGMSE. It is seen that, the SGMSE and AMBER yield the same performance that is better than that of MMSE. It is also found that, although AMBER has less computational complexity than SGMSE, the latter has a much faster convergence rate. In Fig. 6, we have used a step size  $\mu = 0.2$  for the SGMSE algorithm which yields a training period of 4,000 iterations. While for the AMBER, we have used a step size  $\mu = 0.002$  and an update threshold  $\tau = 0.05$  [6]. The training period for AMBER is 1,200,000 iterations. We have also tried a smaller step size of  $\mu = 0.0002$  as done by Yeh and Barry [6], and found that an even longer training period would be required. With MMSE solution as the starting point, Fig. 7 compares the SGMSE and AMBER learning curves for SNR=17dB. It is seen that SGMSE converges much faster than the AMBER, but both algorithms eventually converge to the same MSER level.

## 5. GMSE for Fractionally-spaced Equalizers

We now have the understanding that GMSE performs better than MMSE for linear equalization. We also know that FSE outperforms SRE. But the GMSE formula for SRE, i.e., eq. (11), is not directly applicable for FSE. We therefore must derive the GMSE formula for FSE for interested researchers.

Assume the sampling rate at the input of the FSE receiver is  $P$  times the symbol rate. Thus, mathematically, we can model the system as a parallel connection of  $P$  sub-channels each of which is followed by a sub-SRE as depicted in Fig. 8[10].

The estimate error  $e_k$  is

$$e_k = x_k - \hat{x}_k = z_k - \sum_{m=1}^P \eta_{k,m} * w_{k,m} \quad (12)$$

, and define  $z_k$  as

$$z_k = x_k - \sum_{m=1}^P x_k * f_{k,m} * w_{k,m} \quad (13)$$

Assuming that the length of each sub-channel is  $2L+1$ , and the length of each sub-SRE is  $2N+1$ ,  $z_k$  is a discrete uniformly distributed random variable with  $D = M^{2L+2N+1}$  possible outcomes, say  $\alpha_{i,k}$ ,  $i = 1, 2, \dots, D$ . For a set outcome  $z_k$ , say  $\alpha_{i,k}$ , with patient manipulation of mathematics, it can be readily shown that the estimate error has a p.d.f. given by

$$p(e_k | \alpha_{i,k}) = \frac{1}{\sqrt{\pi N_0 \sum_{m=1}^P \| \mathbf{w}_{k,m} \|^2}} \exp[-(e_k - \alpha_{i,k})^2 / \sum_{m=1}^P \| \mathbf{w}_{k,m} \|^2 N_0] \quad (14)$$

$$\text{, where } \| \mathbf{w}_{k,m} \|^2 = \sum_{i=-N}^N w_{i,m}^2 .$$

The probability of equalizer decision error  $P_M$  for  $M$ -ary PAM transmission can now be obtained as follows:

$$P_M = \frac{2(M-1)}{M} \int_d^\infty p(e_k) de_k$$

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$$\frac{\partial P_M}{\partial w_{n,k,l}} = \frac{2(M-1)}{M} \frac{1}{D} \sum_{i=1}^D \left\{ -\frac{1}{\sqrt{2\pi}} e^{-\frac{(d-\alpha_{i,k})^2}{N_0 \sum_{m=1}^P \| \mathbf{w}_{k,m} \|^2}} \left[ \frac{\sum_{j=-L+n}^{L+n} f_{j-n,l} x_{k-j} \sqrt{\frac{N_0}{2} \sum_{m=1}^P \| \mathbf{w}_{k,m} \|^2} - (d - \alpha_{i,k}) \frac{w_{n,k,l} \sqrt{N_0/2}}{\sqrt{\sum_{m=1}^P \| \mathbf{w}_{k,m} \|^2}}}{\frac{N_0}{2} \sum_{m=1}^P \| \mathbf{w}_{k,m} \|^2} \right] \right\} \quad (17)$$

$$= \frac{2(M-1)}{M} \cdot \frac{1}{D} \sum_{i=1}^D Q \left( \frac{d - \alpha_{i,k}}{\sqrt{\sum_{m=1}^P \|\mathbf{w}_{k,m}\|^2 N_0 / 2}} \right) \quad (15)$$

Comparing (9) and (15), we find that the key to convert the linear SRE to FSE is simply to replace  $\|\mathbf{w}_k\|^2$  by  $\sum_{m=1}^P \|\mathbf{w}_{k,m}\|^2$ .

The gradient algorithm is

$$\mathbf{w}_{k+1,l} = \mathbf{w}_{k,l} - \mu \frac{\partial P_M}{\partial \mathbf{w}_{k,l}}, \quad (16)$$

where  $l = 1, 2, \dots, P$ .

The gradient factor for the GMSER can be obtained as in (17), where  $w_{n,k,l}$  denotes the  $(n+N+1)$ th component of the weight vector  $\mathbf{w}_{k,l}$  at the  $k$ th iteration. Again, comparing (11) and (17), it's apparent that substituting  $\sum_{m=1}^P \|\mathbf{w}_{k,m}\|^2$  for  $\|\mathbf{w}_k\|^2$  in (11) converts the GMSER for SRE to the GMSER for FSE.

## 6. Conclusions

Using a generalized formula for the decision error probability for an  $M$ -ary PAM system, a GMSER algorithm is developed for symbol-rate equalization as well as for fractionally-spaced equalization by applying the gradient algorithm on the generalized formula. It is demonstrated that the GMSER algorithm can achieve a minimum symbol error rate lower than that obtained by the MMSE criterion. Simulations also show that while the AMBER may offer less computational complexity, the GMSER offers much faster convergence rate.

Further simplification procedure similar to the stochastic gradient method can be employed to reduce the computational complexity in GMSER. Thus we obtain the SGMSE algorithm. A comparison is also made between the SGMSE and the AMBER algorithm proposed by Yeh and Barry. While the AMBER may offer less computational complexity, the SGMSE offers much faster convergence rate and lower error rate for higher order signaling.

This paper only presents the case of linear equalization. It is well known that decision feedback

equalization (DFE) will enhance the performance further. We did have performed simulations for DFE and found improved results.

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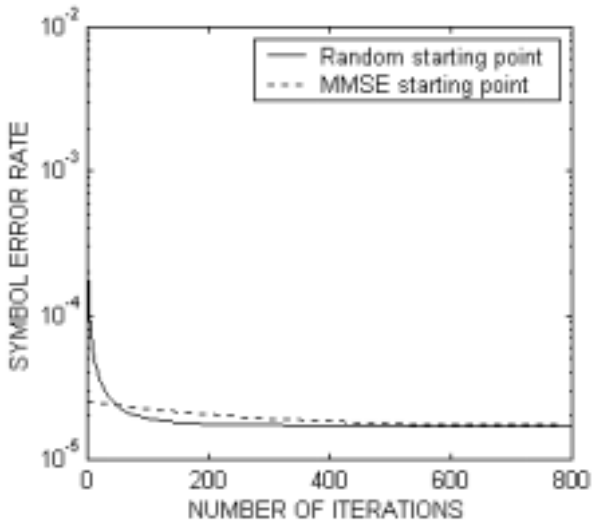


Fig.1 Learning Curves for GMSE Using Different Starting Points

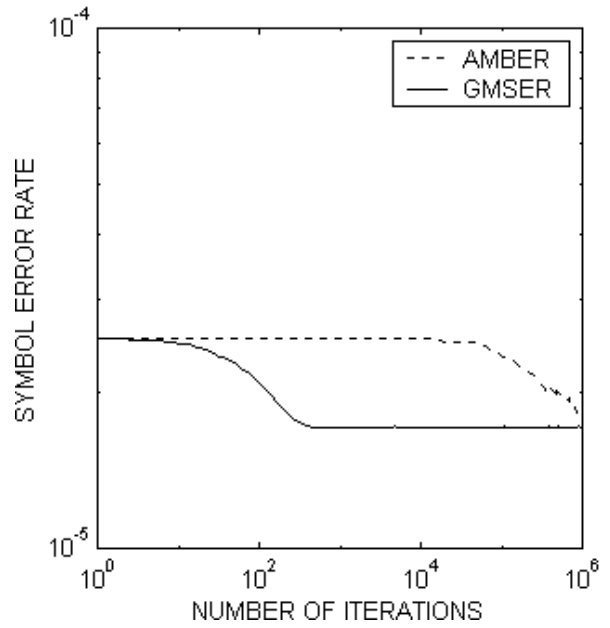


Fig.3 Comparison of Convergence Rate Between GMSE and AMBER with MMSE Starting Point

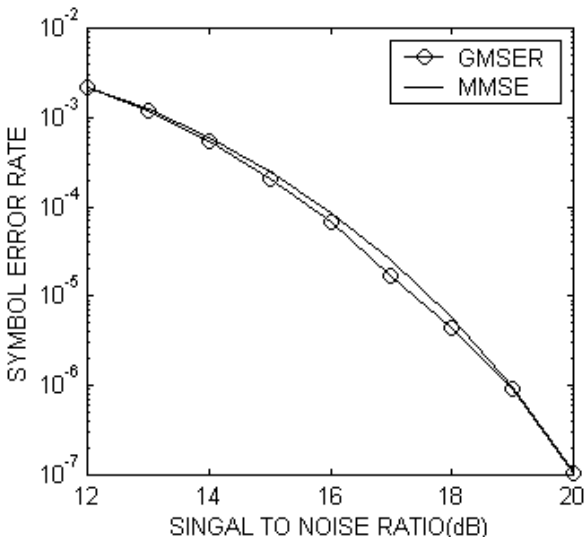


Fig.2 Symbol Error Rate VS. SNR Curves Using Binary Signaling for 2 Different Algorithms

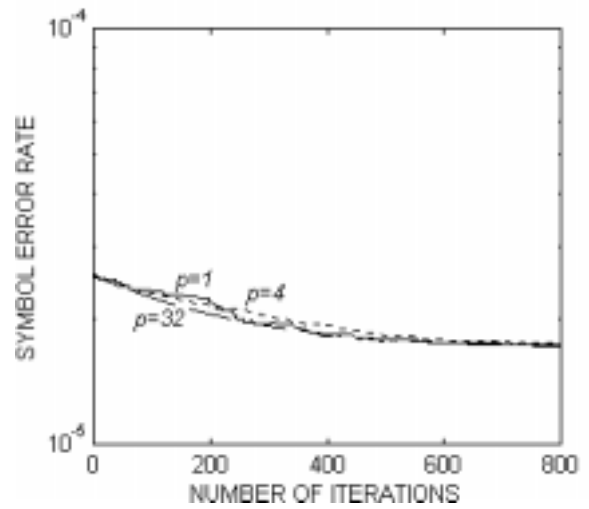


Fig.4 Learning curves for GMSE and SGMSE.

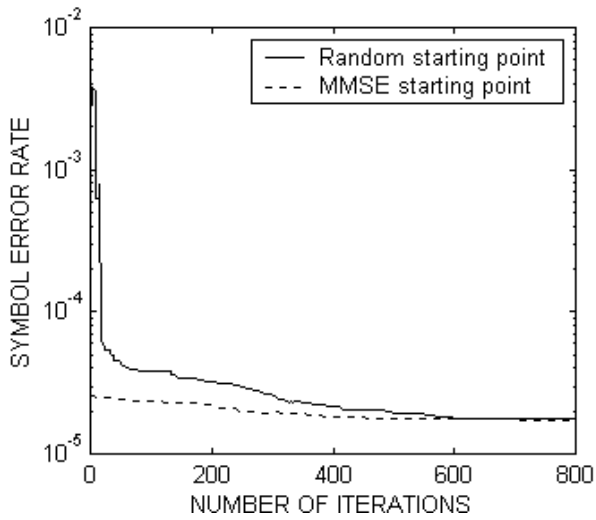


Fig.5 Learning curves for SGMSE using different starting points.

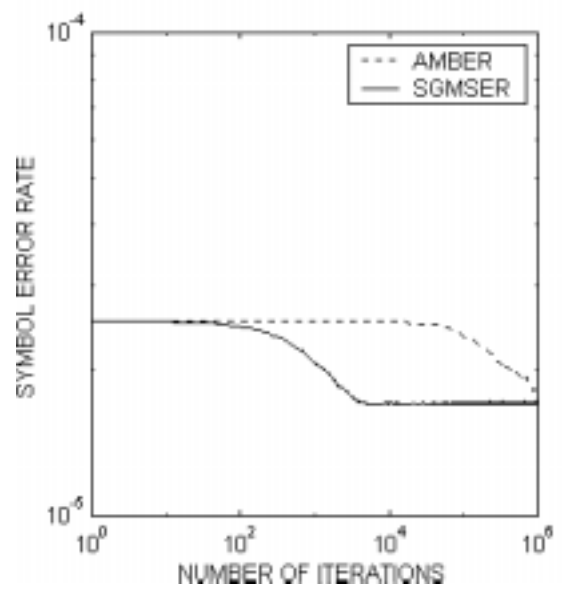


Fig.7 Comparison of convergence rate between SGMSE and AMBER with MMSE starting

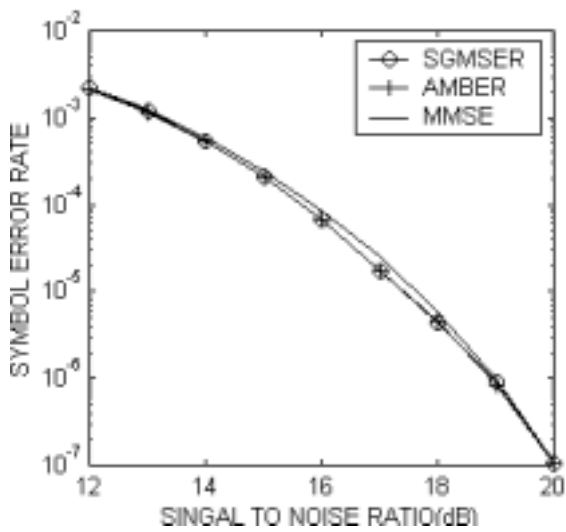


Fig.6 Symbol error rate vs. SNR curves using binary signaling for 3 different algorithms.

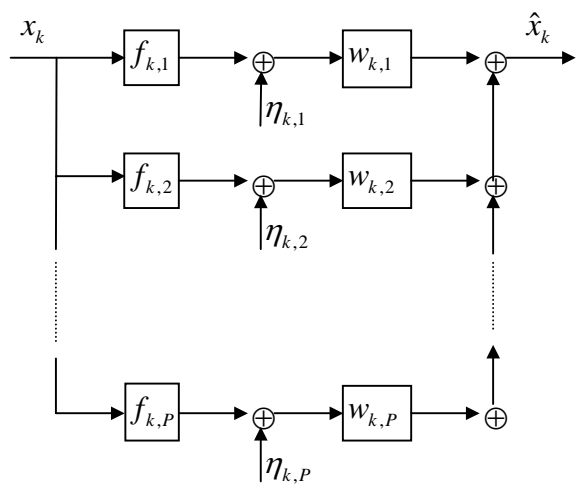


Fig.8 Equivalent Model for an FSE System.



## 計畫成果自評

本計畫執行結果充分符合當出原計畫的研究內容與執行進度，執行期間配合研究進度，並多次赴國外發表最新研究成果，充分與國外之專家學者與相關領域之權威人士討教，使研究成果相當完整與豐富。

計畫執行期間出席之國際學術會議暨發表之論文如下：

- [1] Hong-Yu Liu, Rainfield Y. Yen, and Jin-Qun Chen, "Generalized Minimum Symbol Error Rate for Symbol-rate And Fractionally-spaced Equalizations," *IASTED International Conference on Signal Processing, Pattern Recognition, And Applications, SPPRA 2002*, Crete, Greece, pp.450-454, June 2002.
- [2] Rainfield Y. Yen, Hong-Yu Liu, and Jin-Qun Chen, "Generalized Decision Error Probability for Linear Equalization Using  $M$ -ary PAM Transmission," *European Signal Processing Conference*, EUSIPCO-2002, Toulouse, France, pp. 361-364, Sep. 2002.
- [3] Rainfield Y. Yen, Hong-Yu Liu, and Yi-Tun Yang, "Generalized Minimum Symbol Error Rate for Linear Equalization," *IEEE International Symposium on Intelligent Signal Processig and Communication Systems, ISPACS 2002*, Kaohsiung, Taiwan, R.O.C., pp. 448-451, Nov. 2002.

# 可供推廣之研發成果資料表

可申請專利

可技術移轉

日期：93年2月16日

<b>國科會補助計畫</b>	計畫名稱：廣義錯誤率公式於等化器最佳化之研究 計畫主持人：嚴雨田 計畫編號：NSC 91-2219-E-032-005 學門領域：電信
<b>技術/創作名稱</b>	GMSER/SGMSER
<b>發明人/創作人</b>	嚴雨田
<b>技術說明</b>	中文： 推導M-ary PAM系統的廣義錯誤率公式，並使用最陡坡降法最小化錯誤率於線性等化器系統，此稱為GMSER，我們比較GMSER與平方誤差最小化準則的優劣，以及與由Yeh&Barry提出的AMSER做比較。為了降低計算複雜度，我蠅亦提出SGMSER以隨機的方式做最陡坡降值的估計，當然亦與最小平方誤差、AMBER做比較。並以數學分析的方式將字符取樣速率的等化器，推廣到過取樣速率的等化器系統。  ( 100~500 字 )
	英文： A generalized formula for the decision error probability is derived for an $M$ -ary PAM system. And GMSER algorithm based on the derived formula is proposed to minimize the symbol error rate for linear equalization. Comparisons made between the GMSER algorithm and the MMSE criterion show that the minimum symbol error rate criterion outperforms the minimum mean-squared error criterion. Comparisons between our GMSER and AMBER as proposed by Yeh and Barry is also made. Then a SGMSER algorithm is obtained by further simplification of GMSER to reduce the computational complexity in GMSER. Comparison is also made between the SGMSER algorithm, the MMSE criterion, and the AMBER algorithm as proposed by Yeh and Barry. Further, an extension of the GMSER algorithm employing fractionally-spaced equalization is presented with mathematical formulations.
<b>可利用之產業及可開發之產品</b>	通訊接收器之等化器設計
<b>技術特點</b>	錯誤率比 MMSE 等化器還低
<b>推廣及運用的價值</b>	改進通訊品質

1. 每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送 貴單位研發成果推廣單位（如技術移轉中心）。

**2. 本項研發成果若尚未申請專利，請勿揭露可申請專利之主要內容。**