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行政院國家科學委員會補助專題研究計畫成果報告

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※ 多輸入多輸出線性離散系統之低階強健控制器設計 ※

※ Robust Low-Order Controller Design for Linear Discrete MIMO Systems ※

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多輸入多輸出線性離散系統之低階強健控制器設計

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Abstract

This research develops a reliable and systematic low-order controller design method for solving model-matching problem of linear discrete time-invariant multi-input multi-output system. Using the coprime factors and properties of discrete outer function, the low-order controller design is reformulated as a convex optimization problem. The solutions are obtained using linear matrix inequality techniques. An LV100 turbine engine is used to illustrate the model-matching design algorithm.

Keywords – low-order controller, coprime factorization, LMI, model-matching

摘要

本研究探討多輸入多輸出線性離散系統滿足模式匹配需求之低階控制器的設計方法。利用互質因子及 outer 函數的性質，將低階控制器的設計轉為 convex 之最佳化問題，並用線性矩陣不等式的技術來求解。本文所提供的設計法則不需重覆疊代求解，最後以渦輪引擎 LV100 為例來驗證模式匹配低階控制器的設計法則。

關鍵詞：線性矩陣不等式、互質因子、強健控制、低階控制器

1 Introduction

The design of a robust low-order controller to achieve a desired closed-loop transfer function is popular for practical control applications. The model-matching design approach is attractive because classical design specifications can be readily translated into a desired closed-loop transfer function. The model-matching design problem is usually formulated as an optimization problem with certain H_2 or H_∞ constraints. The design of low-order controller to optimize certain H_2 or H_∞ performance involves a bi-affine matrix inequality(BMI), which is non-convex and cannot be solved using the existing convex programming software. Instead of solving directly the BMI problem, several researchers have shown that low-order controllers can be obtained by solving iteratively LMI subproblems, which are convex. These approaches include alternating projection method[4]，rank condition minimization method[5] and successive substitution method [3,7,8]. However, global convergence has not been established for any of these iterative methods.

In [9], a low-order controller design method using coprime factors, strictly positive real function (SPR) and LMIs was developed for continuous-time single-input single-output (SISO) systems. This method is expanded to solve the model-matching problem for continuous-time MIMO systems [10].

For discrete time case, low-order robust controller design algorithms using coprime factors、discrete outer functions and LMIs were developed. The results are presented in [6]. This report summarizes the results of the development of the low-order controller design for the model-matching problems for discrete MIMO systems. An LV100 engine model-matching design is used to demonstrate the proposed design algorithm.

2 Coprime factorization

Consider a discrete linear time-invariant system $G(z)$ with the state-space realization

$$x(k+1) = Ax(k) + Bu(k) \qquad (1a)$$

$$y(k) = Cx(k) \qquad (1b)$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$. Assume that the system (1) is stabilizable and detectable。In the packed matrix notation, $G(z)$ is represented by

$$G(z) \leftrightarrow \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \qquad (2)$$

Since the system (1) is stabilizable and detectable, we perform a left coprime factorization of $G(z)$ to obtain

$$G(z) = G_D^{-1}(z) G_N(z) \qquad (3)$$

The state-space realization of $G_D(z)$ and $G_N(z)$ is

$$[G_D(z) \quad G_N(z)] \leftrightarrow \left[\begin{array}{c|cc} A-LC & -L & B \\ \hline C & I_{P\times P} & 0 \end{array}\right] \qquad (4)$$

where L is a stabilizing observer gain such that all the eigenvalues of $A\text{-}LC$ are in the unit circle. In contrast to a full-order stabilizing controller K(z) for $G(z)$, whose coprime factorization can be readily defined in terms of A, B, C, and a stabilizing full-state feedback gain F, for a reduced-order controller we first need to define its structure before performing coprime factorization. Select the reduced-order controller $K(z)$ with p inputs and m outputs to have the structure

$$K(z) = \begin{bmatrix} \dfrac{k_{11}(z)}{d_1(z)} & \cdots & \dfrac{k_{1p}(z)}{d_p(z)} \\ \vdots & \ddots & \vdots \\ \dfrac{k_{m1}(z)}{d_1(z)} & \cdots & \dfrac{k_{mp}(z)}{d_p(z)} \end{bmatrix} \qquad (5)$$

We can perform a right coprime factorization of $K(z)$ as

$$K(z) = K_N(z) K_D^{-1}(z) \qquad (6)$$

The coprime factors K_N(z) and K_D(z) are stable transfer function matrices with

$$K_N(z) = \begin{bmatrix} \dfrac{k_{11}(z)}{d_{c1}(z)} & \cdots & \dfrac{k_{1p}(z)}{d_{cp}(z)} \\ \vdots & \ddots & \vdots \\ \dfrac{k_{m1}(z)}{d_{c1}(z)} & \cdots & \dfrac{k_{mp}(z)}{d_{cp}(z)} \end{bmatrix} \qquad (7)$$

$$K_D(z) = \begin{bmatrix} \dfrac{d_1(z)}{d_{c1}(z)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \dfrac{d_p(z)}{d_{cp}(z)} \end{bmatrix} \qquad (8)$$

where $d_{cj}(z)$，$j=1,\cdots,p$，are predetermined stable monic polynomials. The order of $d_{cj}(z)$ and $d_j(z)$ are the same. The order of the controller *K(z)* is the sum of the degree of $d_1(z)\ldots d_p(z)$. For reduced-order controller, the order of the controller is limited to be smaller then n (the order of the plant (1)). The state-space realization of K_N(z) can be represented in observer canonical form

$$K_N(z) \leftrightarrow \left[\begin{array}{c|c} A_{kn} & B_{kn} \\ \hline C_{kn} & D_{kn} \end{array}\right] \qquad (9)$$

where A_{kn} and C_{kn} are constant matrices determined from the pre-selected denominators $d_{cj}(z)$. The unknown coefficients of the numerators $k_{ij}(z)$ are included in B_{kn} and D_{kn}. Similarly, the state-space realization of $K_D(z)$ is expressed as

$$K_D(z) \leftrightarrow \left[\begin{array}{c|c} A_{kd} & B_{kd} \\ \hline C_{kd} & D_{kd} \end{array}\right] \qquad (10)$$

where A_{kd} and C_{kd} are constant matrices, and $D_{kd} = I$. The unknown coefficients of the denominators $d_j(z)$ are included in B_{kd}.

3 Low-order stabilizing controller design

Consider the closed-loop regulation system in Figure 1

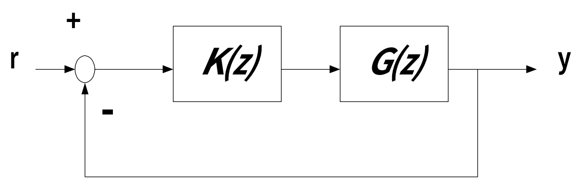

Figure 1 : closed-loop regulation system

The closed-loop transfer function from the command r to the output y, denoted as $T(z)$, is

$$T(z) = (I + G(z)K(z))^{-1} G(z)K(z) \qquad (11)$$

$$= I - (I + G(z)K(z))^{-1} \qquad (12)$$

Using coprime factorization of $G(z)$ and $K(z)$, the closed-loop transfer function $T(z)$ is

$$T(z) = I - K_D(z)Q(z)^{-1} G_D(z) \qquad (13)$$

where $Q(z)$ is defined as

$$Q(z) = G_D(z)K_D(z) + G_N(z)K_N(z) \qquad (14)$$

Using the state-space realizations (4), (9) and (10), a state-space realization of $Q(z)$ can be written as

$$Q(z) \leftrightarrow \left[\begin{array}{ccc|c} A - LC & -LC_{kd} & BC_{kn} & -L + BD_{kn} \\ 0 & A_{kd} & 0 & B_{kd} \\ 0 & 0 & A_{kn} & B_{kn} \\ \hline C & C_{kd} & 0 & I_{p\times p} \end{array}\right] \qquad (15)$$

$$\triangleq \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & I \end{array}\right] \qquad (16)$$

where A_q is stable. We note that the design parameters appear linearly in B_q. The following results are crucial to the development of the design method proposed in this report.

Lemma 1 : *If there exist a symmetric positive definite matrix P, such that the following matrix inequalities are satisfied*

$$\begin{bmatrix} A_q P A_q^T - P & A_q P C_q^T - B_q \\ C_q P A_q^T - B_q^T & -I \end{bmatrix} < 0 \qquad (17)$$

$$I - C_q P C_q^T \geq 0 \qquad (18)$$

then all zeros of $Q(z)$ are inside the unit circle of the z-plane。

Theorem 1 *If there exist matrices B_{kn}, D_{kn}, and K_{kd} having the observer canonical realization structure defined in (9) and (10), such that the LMIs (17) and (18) are satisfied, then u=-K(z)y is a stabilizing controller.*

Theorem 1 gives a practical method for finding a low-order stabilizing controller. The LMIs (17) and (18) together with the pre-determined structure of B_{kn}, D_{kn}, and K_{kd} can be solved as a feasibility problem using a convex programming toolbox such as [2]. Theorem 1 can be used as a building block for more complex design problems. In this paper, we will concentrate on the design of low-order controller for the model-matching optimization problems.

4 Model Matching Optimization Problem

The model-matching optimization problem, as shown in Figure 2, discussed in this section is to find a low-order controller K(z) for system (1) so that the closed-loop transfer function T(z) (11) matches as closely as possible, in the frequency domain, to a desired stable transfer function T_d(z), which is usually a low-order transfer function incorporating the features of the control specifications.

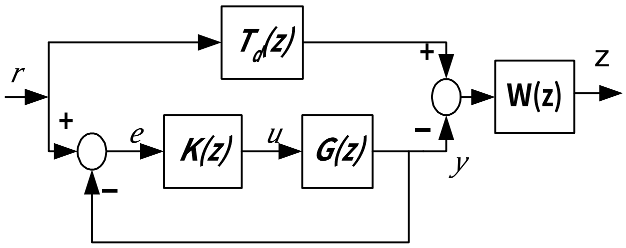

Figure2 : Model-Matching Formulation

Using the coprime factors of the plant G(z) and the controller $K(z)$, this problem can be defined as

$$\min_{k(z)} \left\| W(z)\left(T_d(z) - T(z)\right)\right\|_\infty \qquad (21)$$

where $W(z)$ is a stable weighting function characterizing the emphasized frequency domain requirement. The closed-loop transfer function T(z) can be written as

$$T(z) = (I + G(z)K(z))^{-1} G(z)K(z)$$

$$= (K_D(z)(G_D(z)K_D(z) + G_N(z)K_N(z))^{-1} G_D(z))(G_D^{-1}(z)G_N(z)K_N(z)K_D^{-1}(z))$$

$$= G_D^{-1}(z)G_N(z)K_N(z)(Q(z))^{-1} G_D(z) \qquad (22)$$

Therefore, the optimization problem can be expressed as

$$\min_{k(z)} \left\| W(z)\left(T_d(z) - G_D^{-1}(z)G_N(z)K_N(z)(Q(z))^{-1} G_D(z)\right)\right\|_\infty \qquad (23)$$

Obviously, (23) cannot be directly set up as a convex optimization problem. To circumvent this difficulty, we consider the ideal case in which T(z) perfectly matches the desired transfer function T_d(z), that is

$$T_d(z) = G_D^{-1}(z)G_N(z)K_N(z)(Q(z))^{-1} G_D(z) \qquad (24)$$

Assume that the plant G(z) is stable , that is, $G_D^{-1}(z)$ is stable. From (24)

$$\hat{T}_d(z)Q(z) = G_N(z)K_N(z) \qquad (25)$$

where $\hat{T}_d(z) = G_D(z)T_d(z)G_D^{-1}(z)$. Thus we can reformulate the optimization problem (23) as a suboptimal problem of

$$\min_{k(z)} \left\| W(z)\left(\hat{T}_d(z)Q(z) - G_N(z)K_N\right)\right\|_\infty \qquad (26)$$

Construct the state-space realization of (26) as

$$\left\{W(z)\left(\hat{T}_d(z)Q(z) - G_N(z)K_N\right)\right\}^T \leftrightarrow \left[\begin{array}{c|c} A_m & B_m \\ \hline C_m & D_m \end{array}\right] \qquad (27)$$

We note that in (27), the design parameters B_{kn}, D_{kn} and B_{kd} only appear in matrix B_m. The matrices A_m, C_m and D_m in (27) are known.

From bounded real lemma [5], the inequality

$$\left\| W(z)\left(\hat{T}_d(z)Q(z) - G_N(z)K_N\right)\right\|_\infty < \gamma \qquad (28)$$

is satisfied for a prespecified constant $\gamma > 0$ if there exist constant matrices B_m, satisfing the following LMIs

$$\begin{bmatrix} A_m X A_m^T - X & A_m X C_m^T & B_m \\ C_m X A_m^T & C_m X C_m^T - \gamma I & D_m \\ B_m^T & D_m^T & -\gamma I \end{bmatrix} < 0 \qquad (29)$$

$$X > 0 \qquad (30)$$

Therefore we can formulate the low-order model-matching control problem as

$$\min_{K(z)} \gamma \qquad (31)$$

subject to the LMIs (17), (18), (29) and (30). The problem is convex and the solution can be obtained using semidefinite programming software such that MATLAB LMI toolbox [2]. The design variables are B_{kn}, D_{kn}, B_{kd} (which appear linearly in B_q and B_m) P, X and γ. We further note that the solution space of the optimization problem (26) is not the same as the optimization (21). This may result in suboptimal design.

If the plant $G(z)$ is unstable, $\hat{T}_d(z)$ is also unstable so that the problem cannot be used as an optimization objective. However, we will use a left coprime factorization of $\hat{T}_d(z)$ to obtain

$$\hat{T}_d(z) = \hat{T}_{dD}(z)^{-1}(z)\hat{T}_{dN}(z) \qquad (31)$$

The state-space realization of $\hat{T}_{dD}(z)$ and $\hat{T}_{dN}(z)$ is

$$\begin{bmatrix} \hat{T}_{dD}(z) & \hat{T}_{dN}(z) \end{bmatrix} \leftrightarrow \left[\begin{array}{c|cc} A_{tdh} - L_{tdh}C_{tdh} & -L_{tdh} & B_{tdh} \\ \hline C_{tdh} & I_{P\times P} & 0 \end{array}\right] \qquad (32)$$

where A_{tdh}, B_{tdh}, C_{tdh}, and D_{tdh} are state matrices of $\hat{T}_d(z)$ and L_{tdh} is a stabilizing observer gain, that is all eigenvalues of A_{tdh}-$L_{tdh}C_{tdh}$ are inside the unit circle of the z-plane. Substituting (28) into (25), we have

$$\hat{T}_{dN}(z)Q(z) = \hat{T}_{dD}(z)G_N(z)K_N(z) \qquad (33)$$

Therefore, this problem can be defined as

$$\min_{K(z)} \left\| w(z)\left(\hat{T}_{dN}(z)Q(z) - \hat{T}_{dD}(z)(G_N(z)K_N(z))\right)\right\|_\infty \qquad (34)$$

The state-space realization of (34) is

$$\{w(z)\left(\hat{T}_{dN}(z)Q(z) - \hat{T}_{dD}(z)(G_N(z)K_N(z))\right)\}^T$$

$$\leftrightarrow \left[\begin{array}{c|c} A_{mu} & B_{mu} \\ \hline C_{mu} & 0 \end{array}\right] \qquad (35)$$

5 GE LV100 engine control

The first example we used to illustrate the proposed model-matching design algorithm is the GE LV100 turbine engine model [11]. This engine differs from standard turbine engines in that a recuperator is inserted into the airstream in order to preheat the air entering the combustor, resulting in a higher efficiency. The main control objective is to design a controller to regulate the shaft speed N_p and a temperature T_6, related to the engine internal temperature, via modulation of the fuel flow W_f and the variable area turbine nozzle $VATN$. The linearized GE LV100 engine model at one operating point is of 6th order and has two inputs, the fuel flow (W_f) and the variable nozzle ($VATN$), and two outputs , the shaft speed (N_P) and temperature (T_6) related to the engine internal temperature. The design objectives are to regulate the two outputs and to achieve input-output decoupling. The desired closed-loop transfer function T_d(s) is specified to be

$$Td(s) = \begin{bmatrix} \dfrac{9}{s^2 + 4.2s + 9} & 0 \\ 0 & \dfrac{25}{s^2 + 7s + 25} \end{bmatrix} \qquad (67)$$

We aim at designing a fourth order controller for the system. To illustrate the model-matching design, we first discretize the engine model and the desired transfer function $T_d(s)$ with a sampling rate of 100 Hz ($W(z) = I$ is assumed in the design). Following the design procedure proposed in the previous section, a fourth order controller is obtained at γ=0.3636 as

$$K(z) = \begin{bmatrix} \dfrac{k_{11}(z)}{d_1(z)} & \dfrac{k_{12}(z)}{d_2(z)} \\ \dfrac{k_{21}(z)}{d_1(z)} & \dfrac{k_{22}(z)}{d_2(z)} \end{bmatrix} \qquad (39)$$

$$k_{11}(z) = 0.629(z-0.998)(z-0.709)$$
$$k_{12}(z) = 1.12(z^2-1.967z+0.959)$$
$$k_{21}(z) = 40.753(z-1.015)(z-0.999)$$
$$k_{22}(z) = -40.77(z^2-1.959z-0.96)$$
$$d_1(z) = (z-1)(z-0.961)$$
$$d_2(z) = (z-1)(z-0.92)$$

In order to achieve zero steady-state error, we set a root of $d_j(z)$, $j=1,2$ at 1. The step responses of the resulting closed–loop system are shown in figures 3 and 4.

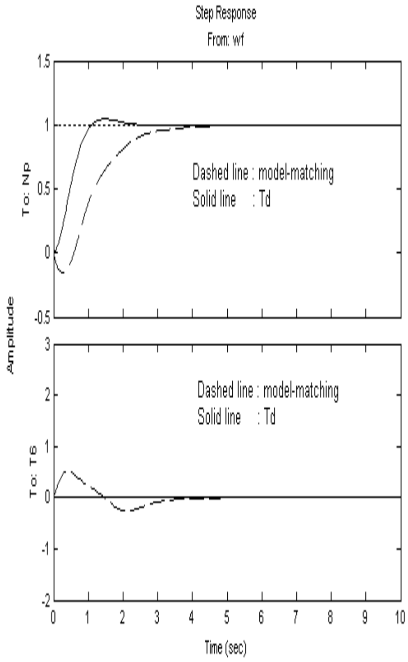

Figure 3 Step responses from W_f

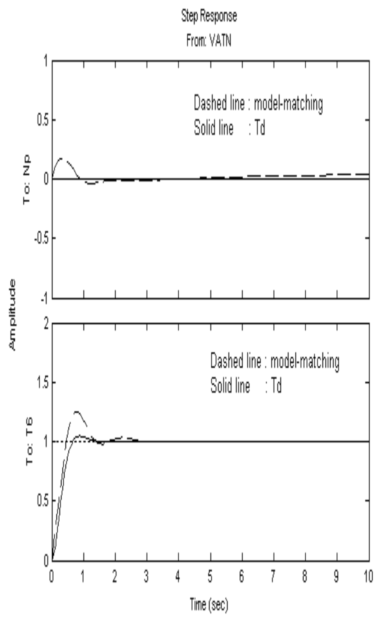

Figure 4 Step responses from *VATN*

6 Conclusions

This research develops a reliable and systematic low-order controller design method for linear discrete time-invariant multi-input multi-output system. Using the coprime factors and properties of discrete outer function, the low-order controller design for model-matching optimization problem is formulated as convex optimization problem subject to several LMI constraints. The solutions are obtained using LMI techniques. The design algorithm is successfully applied to an LV100 turbine engine design.

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