行政院國家科學委員會專題研究計畫 成果報告

考量不確定性資金供給之時間成本權衡模式

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行政院國家科學委員會專題研究計畫成果報告
不確定性資金供給下之時間成本權衡模式

Project time-cost tradeoff model under uncertain financial constraint

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一、摘要

時間成本權衡問題一向被視為營建專案管理中重要的課題。其主要目標乃在建立不同成本下最佳的專案工期；或在不同工期需求下最佳的成本以供專案評估人員選擇各作業時間與成本的選項。文獻回顧發現，過去的研究並沒有考慮專案進行中資金供給的不確定性。事實上，營建專案所需資金一向較龐大而取得過程受到市場、政經、與管理等風險影響。也因此，即便專案管理人員事先計算各項作業工期與成本的最佳組合，當資金供給與原先評估發生誤差時，工程往往被迫停工或產生嚴重延誤。承攬廠商將面臨龐大的財務壓力；而業主亦將陷入暫停施工或以較劣條件尋求其它融資的兩難困境。因此本研究之主要目的為採用序率限制規劃建構考量不確定性資金供給之時間成本權衡數學模式。該數學模式將於各種資金供給可靠度下求得時間成本權衡問題之最佳解並就兩部分加以驗證：實際案例探討及電腦模擬比較。

關鍵詞：時間成本權衡、序率限制規劃、排程、風險評估、可靠度分析

Abstract

Time-cost tradeoff problem has long been regarded as an important issue in project management. The main goal is to determine the minimum cost under specific project duration or the shortest project duration under a particular funding level. Literature reviews indicate none of the previous time-cost tradeoff approaches considered the uncertainty of project financing. Yet construction projects usually require significant amount of funding that is consequently subject to risks from various perspectives, such as politics, marketing, and management. Thus when the supply of funding experiences unexpected deviations from original estimates, the project is either postponed or completely shut down. Either case would bring serious financial problems to both owners and contractors. This research adopts chance-constrained programming techniques to build a time-cost tradeoff model considering the uncertainty of project financing. The proposed model helps evaluate the optimal solution under different reliability levels of project financing. This model is validated by real-life case studies and comparisons with results from computer simulations.

Keywords: Time-cost tradeoff, Chance-constrained programming, Scheduling, Risk assessment, Reliability analysis

二、研究目的

In construction projects, the acceleration of an activity entails additional resources and hence requires a higher cost, different decisions as to how the various activities are performed result in different time-cost realizations for the overall project. The process is called the time-cost tradeoff analysis.

The time-cost tradeoff problem is of practical interest when a planner has to crash activities to meet a pre-specified deadline or when he/she wants to evaluate whether such a crash is worth an additional cost. Moreover, the time-cost tradeoff analysis would be beneficial if some activities are lengthened whereas the extended project duration is still acceptable and the project cost is reduced. Since the time-cost tradeoff problem involves two objectives: minimum cost and minimum duration, the solution is said to be Pareto optimal where there is no other shorter duration under a given budget or no lower cost under a pre-specified duration. The Pareto optimal front is also called the minimum time-cost curve, which is negatively sloped and convex to the origin of time-cost coordinate axes.

Time-cost tradeoff problems have been recognized since the 1950s, almost simultaneously with the development of the critical path method (CPM). Many researchers studied the assumption of time-cost relationship of each activity, such as piecewise linear (Fondahl 1961), convex (Foldes and Sourmis 1993), concave (Falk and Horowitz 1972), neither convex nor concave (Moder and Phillips 1970), quadratic (Deckro et al. 1995), and discrete (Demeulemeester et al. 1996).

A variety of heuristic procedures were used to solve the time-cost tradeoff problem. In general, these procedures provided rule-of-thumb guidelines for crashing activities with least costs but cannot guarantee optimality. In the field of network programming, Phillips and Dessouky (1977) located a minimum cut in a flow network to solve the problem. Another large group attacked the problem by mathematical programming techniques, such as linear programming (Fulkerson 1961), mixed integer programming (Crowston and Thompson 1967), and dynamic programming (Deckro et al. 1995). Elmaghraby (1993) used the network reduction method to structure hybrid algorithms (dynamic programming and branch-and-bound) for solving the problem.

Recently, with the fast growth in computer technology and advances in artificial intelligence of computer science, computational optimization techniques were used to solve the problem by means of genetic algorithms (Hegazy 1997; Li et al. 1997). Other efforts were devoted to treat time and cost options as random variables (Feng et al. 2000) or fuzzy numbers (Leu et al. 2001).

All the previous approaches solved the time-cost tradeoff problem by treating the project budget as a fixed amount without considering uncertainty. An underlying assumption is that actual funds to support the project would never deviate from the original estimate. This assumption is often unrealistic because a project is generally funded by one or several external financing sources, including financial institutions, investors, and government, as well as via various financial alternatives, such as loans, bonds, or stocks. All of these financial alternatives are associated with different levels of market, business, and political risks. Consequently, the actual amount of money distributed to the project may deviate from the original estimate. Once the money is insufficient to disburse the expenditures, the project might be forced to stop either temporarily or, even worse, permanently. In either case, the project would by no means be completed in an acceptable timeframe, leading to crucial economic losses for all parties. Thus it is necessary to quantify and incorporate budget uncertainty into the time-cost tradeoff analysis.

三、研究內容

An underlying assumption of the present model is that the duration for each activity lies in a range with a piecewise linear relationship holding between cost and duration within this range. The range is defined between two extremes: the minimum cost, called “normal point”, and the minimum project duration, called “crash point”. Since costs are assumed to vary linearly from the normal point to the crash point, the cost slope for activity *ij* (between event nodes *i* and *j*) can be expressed as

$$\alpha_{ij} = \frac{CC_{ij} - CN_{ij}}{TC_{ij} - TN_{ij}} \qquad (1)$$

where CC_{ij} = cost of activity *ij* at the crash point; CN_{ij} = cost of activity *ij* at the normal point; TC_{ij} = duration of activity *ij* at the crash point; and TN_{ij} = duration of activity *ij* at the normal point

Thus, the intercept on the cost axis is

$$\beta_{ij} = CC_{ij} - \alpha_{ij} TC_{ij} \qquad (2)$$

With both coefficients, the cost associated with any feasible duration can be expressed as

$$C_{ij} = \beta_{ij} + \alpha_{ij} T_{ij} \qquad (3)$$

The linear programming formulation of the time-cost tradeoff problem for an activity-on-arrow (AOA) network is

$$\text{Minimize} \quad \sum_{\forall(i,j)} C_{ij} = \sum_{\forall(i,j)} \beta_{ij} + \sum_{\forall(i,j)} \alpha_{ij} T_{ij} \qquad (4)$$

subject to

$$TE_i + T_{ij} - TE_j \le 0 \quad \forall(\mathrm{i, j}) \qquad (5)$$

$$T_{ij} \le TN_{ij} \quad \forall(\mathrm{i, j}) \qquad (6)$$

$$T_{ij} \ge TC_{ij} \quad \forall(\mathrm{i, j}) \qquad (7)$$

$$TE_s = 0 \qquad (8)$$

$$TE_f \le T \qquad (9)$$

$$TE_i, T_{ij} \geq 0 \qquad (10)$$

where s and f denote the start and finish events; T_{ij} is the duration of activity (i,j); TE_i represents the event time of node i; and T is the pre-specified project duration.

The linear programming formulation for an equivalent activity-on-node (AON) network is

$$\text{Minimize} \ \sum_{\forall i} C_i = \sum_{\forall i} \beta_i + \sum_{\forall i} \alpha_i T_i \qquad (11)$$

subject to

$$ES_i + T_i - ES_j \leq 0 \quad \forall \text{ (i, j)} \qquad (12)$$

$$T_i \leq TN_i \quad \forall \text{ i} \qquad (13)$$

$$T_i \geq TC_i \quad \forall \text{ i} \qquad (14)$$

$$ES_s = 0 \qquad (15)$$

$$ES_f \leq T \qquad (16)$$

$$ES_i, T_i \geq 0 \qquad (17)$$

where T_j is the duration of activity i; ES_i represents the early start time of activity i.

Both models presented above can be solved very efficiently with a guaranteed global optimum. For an AOA network, this study interchanges the objective function and constraint (9) to minimize the project duration subject to the level of budget. The new objective function is

Minimize TE_f (18)

while constraint (9) becomes

$$\sum_{\forall(i,j)} C_{ij} = \sum_{\forall(i,j)} \beta_{ij} + \sum_{\forall(i,j)} \alpha_{ij} T_{ij} \leq C \quad (19)$$

where *C* represents the level of budget.

Similarly, for an AON network, the new objective is

Minimize ES_f (20)

and constraint (19) becomes

$$\sum_{\forall i} C_i = \sum_{\forall i} \beta_i + \sum_{\forall i} \alpha_i T_i \leq C \quad (21)$$

To incorporate the budget uncertainty, the right-hand-side (RHS) of both constraints (19) and (21) is expressed as a random variable in lieu of a fixed estimate and the constraint is called the "finial constraint". The random variable, *C*, is estimated to have a certain statistical distribution, such as normal or triangular. Consequently, this constraint becomes stochastic. To solve the model, a probability level *r* for the constraint is satisfied. The stochastic constraint takes one of the following forms:

$$\Pr[\sum_{\forall(i,j)} \beta_{ij} + \sum_{\forall(i,j)} \alpha_{ij} T_{ij} \leq C] \geq r \quad \text{for AOA network} \quad (22)$$

or

$$\Pr[\sum_{\forall i} \beta_i + \sum_{\forall i} \alpha_i T_i \leq C] \geq r \quad \text{for AON network} \quad (23)$$

The prescribed probability level *r* can be viewed as the reliability of the financial constraint. Given the probability level, the stochastic constraint can be transformed into a deterministic equivalent by the application of chance-constrained programming (CCP):

$$\sum_{\forall(i,j)} \beta_{ij} + \sum_{\forall(i,j)} \alpha_{ij} T_{ij} \le F_C^{-1}(1-r) \quad \text{for AOA network} \tag{24}$$

or

$$\sum_{\forall i} \beta_i + \sum_{\forall i} \alpha_i T_i \le F_C^{-1}(1-r) \quad \text{for AON network} \tag{25}$$

where $F_C^{-1}(1-r)$ is the realization of C corresponding to probability 1-r under the inverse of cumulative distribution function (CDF) F. Because constraints (24) and (25) are slightly different, from herein only constraint (24) will be discussed further.

The normal distribution is selected because it is intuitively simple and requires only two parameters: mean and standard deviation. Moreover, by the central limit theorem, the normal distribution is particularly proper when the project budget encompasses a number of components and none of which dominates others. The disadvantage, however, is that the normal distribution cannot be skewed and therefore is not flexible enough to reflect the expert's judgment. If the normal distribution is used, constraint (24) can be derived further as follows:

$$\sum_{\forall(i,j)} \beta_{ij} + \sum_{\forall(i,j)} \alpha_{ij} T_{ij} \le m + k_{1-r} Var^{1/2} \tag{26}$$

where m = mean of C, Var = variance of C (also denotes the degree of uncertainty), and

$$k_{1-r} = \Phi^{-1}(1-r), \tag{27}$$

$$\Phi(z) = \int_{-\infty}^{z} \varphi(t)dt, \tag{28}$$

$$\varphi(t) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} t^2) \qquad (29)$$

The commonly used values of $\Phi^{-1}(1-r)$ is handily available. Notice that $\Phi^{-1}(1-r)$ =0 for r=0.50. This implies that if one uses only the mean as the RHS without considering budget uncertainty, the chance of violation would be 50%, which is obviously unacceptable. Since the standardized normal distribution value is negative for r>0.50, constraint (26) represents a "tighter" constraint for a higher variance or a higher probability level. The model would then lead to a more conservative solution (i.e., a longer project duration).

The proposed procedure is as follows:

1. For an AOA network, formulate a linear programming model by including the object function in Eq.(18) which is subject to constraints (5)-(8), (10), and (24). For an AON network, the model should include objective function in Eq. (20) and constraints (12)-(15), (17), and (25).
2. If the budget uncertainty has a normal distribution, substitute constraint (24) by constraint (26). If the budget uncertainty has either a beta or triangular distribution, assess three estimates (minimum, mode, and maximum), based on which find the inverse value of the CDF.
3. Solve the linear programming model by any efficient LP algorithm or software.
4. Repeat Step 3 iteratively for different levels of budget. Each iteration corresponds to a point in the scatter time-cost diagram.
5. Establish the minimum time-cost curve by linking the points in the diagram.

四、結果與討論

The proposed model is implemented to solve the time-cost tradeoff problem for a construction project. This project involves remodeling two rooms into a general work area and offices for the purchasing department. The work consists of removing an existing partition between the two rooms, building new partitions, and building two floor-to-ceiling bookcases along one wall in the work area.

Table 1 contains the times and costs for each of the activities, whose precedence is depicted in an AOA network shown in Fig. 1. Note that Activity 10 can only be performed at 5 days and $500. Besides, Activity 52 has two different cost slopes: first is -$250/day between 4 and 6 days while another is -$100/day between 6 and 8 days. This reflects the practical situation that crashing more days may cost extra money; therefore Activity 52 has multiple cost slopes. A roundabout for this situation is to split Activity 52 into two sub-activities: Activities 52a and 52b. This roundabout demonstrates how the proposed model treats the activity with multiple cost slopes.

A project manager was asked to provide his opinions on the underlying distribution of C, which are listed in Table 2 that consists of estimated means and standard deviations for different budget levels if C is normally distributed.

A LP model is constructed according to the aforementioned procedure and solved by commercial optimization software, LINGO 8.0. The global optimal solutions (minimum project durations) are concluded in Table 2. Further sensitivity analysis is carried out below to illustrate that the proposed model can help quantify the importance of the stability of budget and evaluate the financial risks in project management.

For the normal distribution, **Fig. 2** plots minimum time-cost curves, showing the relation between project duration and the mean value of budget, each of which is associated with different degrees of uncertainty in terms of the standard deviation: $200, $500, and $1,000. For a higher degree of uncertainty, the minimum time-cost curve moves against the origin; this represents a longer project duration for the same mean value of budget. For instance, when the mean of budget is $19,000, reducing the uncertainty level from $1,000 to $200 can decrease the project duration from 31 to 29 days, a 6.5% save in time.

One can also observe the minimum time-cost curves in **Fig. 2** from a different angel to see how the budget should be set to achieve the same project duration under different degrees of uncertainty. For example, to complete the project within 32 days, it takes extra $7,500 when the budget is exposed to a higher degree of uncertainty (standard deviation increases from $200 to $1,000). The analyses above help quantify the negative impact for higher budget uncertainty.

This study applies chance-constrained programming to incorporate the budget uncertainty into the time-cost tradeoff analysis. The proposed model generates the minimum time-cost curve based on different levels of budget uncertainty, which is estimated in the form of statistical distributions. With the time-cost curve, one can relate the minimum direct cost associated with any given project duration and determine the crashing strategy accordingly. The minimum time-cost curve can be used to determine the optimal project schedule with the minimum total project cost, which includes direct and indirect costs as well as liquidated damage or early bonus as enforced in the contract. The proposed model, along with commercial optimization tools, can be applied in practice since it requires relatively cheap and fast computations. Moreover, the solutions are guaranteed to be globally optimal.

The significance of the proposed model stems from the fact that it has accomplished the following. First, it incorporates the budget uncertainty into the time-cost tradeoff analysis, which allows the evaluation of the impact of financial risk on the time-cost tradeoff analysis. Second, previous approaches, by using the mean (expected value) of budget as the RHS in the financial constraint, would have only a rough 50-50 chance to complete the project without violating the financial constraint. This would trigger serious problems when the money allocated to the project is not sufficient to support costs. The proposed model, in contrast, avoids the problem by taking the uncertainty into consideration. Third, the solutions of the proposed model automatically include the optimal start time and duration of each activity, which can be used to establish the project schedule conveniently.

五、計畫成果自評

The output of this study has been published in (Yang 2005) and has since attracted a significant amount of attentions. Future research may be conducted in the following directions. First, it may be necessary to address the stochastic nature of activity duration and cost. This can be attained by using Monte Carlo simulations to sample the stochastic duration and cost of each activity. The simulation result can then be used to evaluate if the pre-specified probability level is satisfied. Second, a mix of continuous and discrete time-cost options would lead to a much more difficult combinatorial optimization problem, which may not be solved efficiently by classic programming approaches and require the application of computational optimization techniques, such as genetic algorithms, simulation annealing, and tabu search.

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七、圖表

Table 1. Activity times and costs

Act. ID	Act.	Normal		Crash		Cost slope ($/day)	Intercept ($)
		Time (days) TN	Cost ($) CN	Time (days) TC	Cost ($) CC		
10	Clear rooms	5	500	5	500	-	500
20	Remove partition	3	900	1	1200	-150	1350
30	Rough mechanical and electrical	7	3250	4	4150	-300	5350
32	Sand floors	5	1000	3	1300	-150	1750
40	Partition I	6	1400	3	2750	-450	4100
42	Partition II	5	1100	3	1900	-400	3100
44	New bookcases	7	1500	4	2400	-300	3600
50	Finish mechanical and electrical	10	4200	5	7200	-600	10200
52a	Paint	6	0	4	1500	-250	1500
52b	Paint	2	800	0	1000	-100	1000
60	Carpet	2	1100	1	1300	-200	1500
70	Furnish	3	1300	2	1500	-200	1900
						Sum	35850

Table 2. Project duration under different budget levels

Budget Levels	Mean ($)	Std. Dev. ($)	Deterministic Equivalent of *C* *r*=0.90	Project Duration (days)
C	m	$Var^{1/2}$	$C = m + k_{1-0.90} Var^{1/2}$	
C1	18000	500	17360	33.95
C2	19000	500	18360	29.98
C3	20000	500	19360	27.84
C4	21000	500	20360	26.41
C5	22000	500	21360	25.05
C6	23000	500	22360	24.17
C7	24000	500	23360	23.00
C8	25000	500	24360	22.13

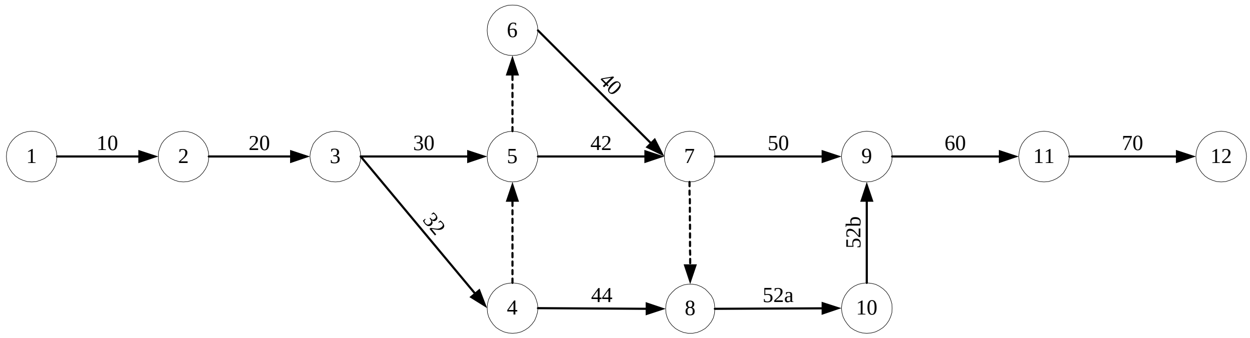

Fig. 1. Precedence network (AOA)

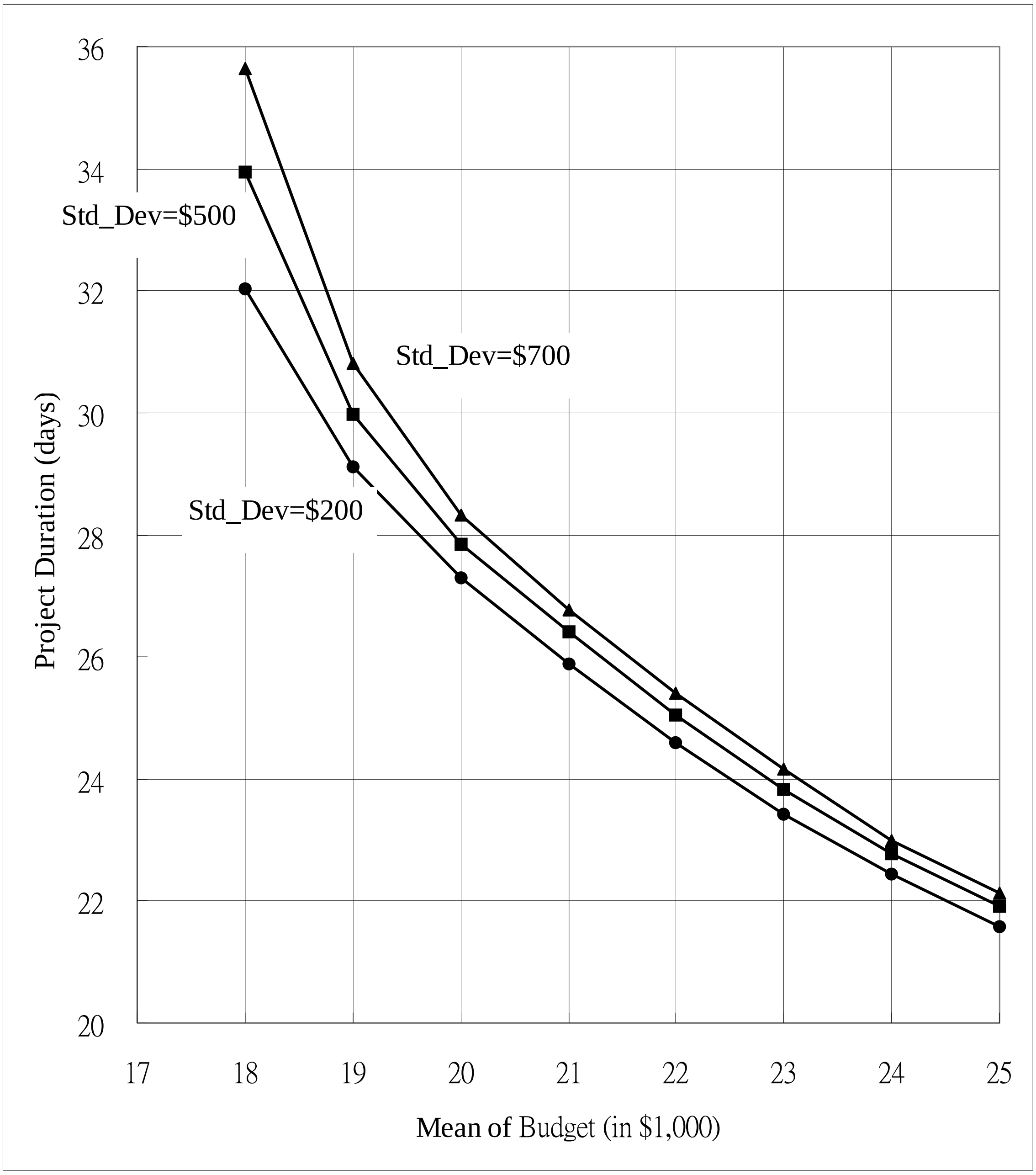

Fig. 2. Minimum time-cost curves for various standard deviation= $200, $500, and $1,000