# 行政院國家科學委員會專題研究計畫成果報告

## 加壓及無加壓模型下指數及相關模型之第一類第二類及隨機篩檢資料的貝氏抽檢設計

## Bayesian variable sampling schemes with and without Accelerated life testing for exponential and Related models with type I,II and Random censoring

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### 一、中英文摘要

本計畫主要在探討壽命模型為指數分布及相關模型下，基於篩檢資料的貝氏抽檢設計。本文利用一適合之損失函數，導出了貝氏樣本抽檢設計$(n_B, t_B, \delta_B)$，並依其貝氏風險與 Lam(1994)所提之抽檢設計做比較。由數值計算結果顯示，在相同條件下，以貝氏風險觀點而言，本計畫所提之貝氏抽檢設計優於 Lam(1994)所提之設計。

關鍵詞：貝氏樣本抽檢設計、第一類篩檢、貝氏風險、指數分布

### Abstract

In this project we study variable sampling plans for the exponential distribution based on type I censoring data. Using a suitable loss function, a. Bayesian variable sampling plan $(n_B, t_B, \delta_B)$ is derived. For certain prior distributions and loss functions, the numerical values of the Bayesian sampling plans and the associated minimum Bayes risks are tabulated. In terms of Bayes risks, comparisons between the proposed Bayesian sampling plans $(n_B, t_B, \delta_B)$ and the "Bayesian" variable sampling plans $(n_o, t_o, \delta_{To})$ of Lam (1994) have been made. The numerical results indicate that under the same conditions, the proposed Bayesian sampling plan is superior to that of Lam (1994) in the sense that the Bayes risk of

$(n_B, t_B, \delta_B)$.

**Keywords** : Bayesian variable sampling plan, Type I censoring, Bayes risk, Exponential distribution

### 二、Result and Discussion

It is well-known that quality control of products is essential to the manufacturers since it directly affects the market sale and the profits of the manufacturers. In the research area of quality control, there are a lot of sampling plans (see Wetherill (1977)) that may be applied for various criteria. Among these, the decision-theoretic approach is more reasonable and realistic because the sampling plan is determined by making an optimal decision on the basis of some realistic criterion such as maximizing the return or minimizing the risk. The research work along this approach has generated vast literature. However, most statisticians working on this problem are confined to a linear loss function (see Fertig and Mann(1974) and Wetherill and Köllerström(1979)). Hald(1967,1981) and Lam(1988a,b, 1994), among others, use polynomial loss functions. In most situations, it is more reasonable and acceptable to consider a polynomial loss since it approximates a loss more closely and it includes linear loss as a special loss. Interested readers are referred to Hald (1981) and Lam(1988a,b, 1994) for the motivation of the usage of polynomial losses.

Suppose we are given a batch of lifetime components for acceptance sampling. The quality of an item in the batch is measured by its lifetime X. In order to estimate the quality of the components of the batch, a sample of size n items are put on life test at the outset and are not replaced on failure. Because of highly developed technology in engineering, most products have high reliabilities. Thus, usually, it may take a long time to observe the complete lifetime data. Due to the time restriction, the experiment terminates at a pre-specified time t. The failure time of an item is observable if it fails before or by time t. If an item still functions at the close of the experiment its failure time is not observable. The item is said to be censored at time t. This type of time censoring is known as type I censoring. Type I censoring scheme has received much attention in the statistical literature, see Bartholomew(1963), Sparrier and Wei(1980), Mann, Schafer and Han(1982), Yang and Sirvanci(1977) and Huang and Chen(1992), among many others.

Let $X_1, \cdots, X_n$ denote the lifetimes of the n components put on a life test experiment. It is assumed that $X_1, \cdots, X_n$ are mutually independent, follow an exponential distribution having expected lifetime $\theta = \lambda^{-1}$ . We denote such an exponential distribution by $\varepsilon(\lambda)$. Let $X_{(1)} \leqq \cdots \leqq X_{(n)}$ be the order statistics of $X_1, \cdots, X_n$ . Since the sample is subject to type I censoring at time t, the true observations are: $Y_i = min(X_{(i)}, t)$, $i = 1, \cdots, n$. Then, $M \equiv M(n, t) = max\{i \mid X_{(i)} \leqq t, \ i = 1, \cdots, n\}$ is the number of failures by time t. Thus, for the given sample size n and the censoring time t, M and $\underline{Y}(M,t) = (Y_1, \cdots, Y_M)$ are the observable random variables and $Y(n,t,M) = \sum_{i=1}^{M} Y_i + (n-M)t$ is the total lifetime of the n items up to time t. It is known that :If $M > 0$, the maximum likelihood estimator(MLE) of the expected lifetime $\theta$ is given by $\hat{\theta}_{ML} = \frac{Y(n,t,M)}{M}$, and if M=0, then $\hat{\theta}_{ML} = nt$ (see Sinha( 1986)).

For each (n,t), a decision function $\delta(\ \mid n,t)$ is a function defined on the observed value $(m,\underline{y}(m,t))$ of the random variables $(M,\underline{Y}(M,t))$ such that $\delta(m,\underline{y}(m,t) \mid n,t)$

is the probability of accepting the batch. The determination of n,t and $\delta(\ \mid n,t)$ is called a sampling plan which is denoted by $(n,t,\delta)$. Suppose that the parameter $\lambda$ follows a prior distribution G over the parameter space $\Omega = (0, \infty)$. Then, the performance of a sampling plan (n, t, f) should be evaluated based on its associated Bayes risk $r(n,t,\delta)$.

Lam(1994) studied a type of sampling plan $(n, t, \delta_T)$ based on $\hat{\theta}_{ML}$ given by

$$(1.1) \quad \delta_T(m,\underline{y}(m,t) \mid n,t) = \begin{cases} 1, & \text{if } \hat{\theta}_{ML} \geqq T, \\ 0, & \text{otherwise;} \end{cases}$$

where $T = T(n,t)$ is some values between 0 and nt. Using the loss function

(1.2) $L(a, \lambda, n, t) = ah(\lambda) + (1-a)C_3 + nC_1 + tC_2$

with $C_2 = 0$ where $C_2$ is the cost per unit time for life test. Lam(1994) attempted to find the (optimal) sampling plans, say ($n_o$, $t_o$, $\delta_{T_o}$) such that $r(n_o, t_o, \delta_{T_o})$ =inf $r(n, t, \delta_T)$ among all sampling plans of the type $(n, t, \delta_T)$.

We have observed that Lam's(optimal) sampling plan ($n_o$, $t_o$, $\delta_{T_o}$) possesses certain defects:

(1) Though $\hat{\theta}_{ML}$ has certain optimal property, and $\delta_T$ of (1.1) is a natural decision function, it is not a Bayes decision function in general and therefore, it is not the optimal sampling plan.

(2) Since the cost of time t is not included in the loss of Lam (1994), for n being fixed, the best choice of censoring time should be $t = \infty$ from which we can observe the complete lifetime data. Based on these complete lifetime data, we can choose a suitable T value to reduce the cost of making a wrong decision.

We may consider two competitors ($n_0$, $\infty$, $\delta_{T^*}$) and ($n_{*B}$, $\infty$, $\delta_B$) that may perform better than ($n_o$, $t_o$, $\delta_{T_o}$). Here, $n_o$ is the "optimal" sample size obtained from column (3) of Tables 1-2 of Lam(1994), $T^*$ is a positive value satisfying $r(n_0, \infty, \delta_{T^*})$ =inf $r(n_0, \infty, \delta_T)$, and ($n_{*B}$, $\infty$, $\delta_B$) is the Bayesian sampling plan developed in this project for $t \to \infty$ case. In the loss $L(a, \lambda, n, t)$ of (1.2), with ($C_2 = 0$ and $h(\lambda) = a_o + a_1 \lambda + a_2 \lambda^2$, we tabulate the Bayes risks of the three sampling plans ($n_o$, $t_o$, $\delta_{T_o}$), ($n_0$, $\infty$, $\delta_{T^*}$) and ($n_{*B}$, $\infty$, $\delta_B$). Apparently the numerical results indicate that $r(n_{*B}, \infty, \delta_B) \leq r(n_0, \infty, \delta_{T^*}) \leq r(n_0, t_o, \delta_{T_o})$ with strict inequality occurring in many cases. Thus, it is clear that the "optimal" sampling plan ($n_0$, $t_o$, $\delta_{T_o}$) indeed is neither optimal nor Bayes.

As addressed previously, a reasonable sampling plan should take the cost of censoring time t into account. For finding a suitable censoring time t, the cost of the censoring time t should be included in the loss function. Therefore, in the project, it is assumed that $C_2$, the cost per unit time, is positive.

In this project, our goal is to seek an optimal sampling plan ($n_B$, $t_B$, $\delta_B$) possessing the property that $r(n_B, t_B, \delta_B)$ = inf $r(n, t, \delta)$ among the class of all sampling plan $(n, t, \delta)$. We set up a decision-theoretic formulation of the problem of acceptance sampling. A Bayesian sampling plan is derived. We provide an explicit presentation of the Bayes risk of a sampling plan $r(n, t, \delta_B(\ \mid n, t))$. Based on this expression, a numerical approximation for finding the optimal sample size $n_B$ and the optimal censoring time $t_B$ is proposed. Certain numerical results are also prepared and tabulated.

## 三、Conclusions and Self-Evaluation

In this project, we reconsider the Lam's(1994) model under a general Bayes set up. We take the censoring time t into consideration as one of main factors and introduce cost of unit time in loss function. A Bayes sampling plan has been proposed under general setting and an explicit Bayes sampling plan has been proposed while a quadratic loss is under consideration. Usual discretization method and some algorithms to find the optimal Bayes sampling plan are also addressed. Some optimal Bayes plans and its Bayes risks are tabulated. When $t = \infty$ , i.e.

under complete date, an alternative explicit Bayes risk is also derived. Two competitors of sampling plans ($n_0$, $\infty$, $\delta_{I^*}$) and ($n_{*B}$, $\infty$, $\delta_B$) against the Lam's sampling plan are also proposed and it is found that ($n_{*B}$, $\infty$, $\delta_B$) dominates ($n_0$, $\infty$, $\delta_{I^*}$) and the latter denominates the Lam's ($n_0$, $t_o$, $\delta_{I_o}$) nontrivially in the sense of Bayes risk. Some optimal sampling plans and its Bayes risks are also computed.

It is to be noted that the proposed competitors ($n_0$, $\infty$, $\delta_{I^*}$) and ($n_{*B}$, $\infty$, $\delta_B$) are just for the theoretical study and comparisons against the Lam's ($n_0$, $t_o$, $\delta_{I_o}$), since for practical applications the two former sampling plans need complete data and this is not situation of our primary interest.

Finally, it should point that the Bayes risk is not a smooth function of those variables, particularly, there is a big fluctuation. Therefore numerical computations for finding optimal $n_B$ and $t_B$ should take a special care. Also, we will consider the same problem with accelerated life testing schemes based on some types of censored data in the near future.

## 四、References


[1] Bartholomew, D.J. (1963). The sampling distribution of an estimate arising in life testing. Technometrics , 5, 361-374.

[2] Fertig, K. W. and Mann, N. R. (1974). A decision-theoretic approach to defining variables sampling plans for finite lots: single sampling for exponential and Gaussian process. J. Amer. Statist. Assoc. 69, 665-671.

[3] Hald, A. (1967). Asymptotic properties of Bayesian single sampling plans. J. Roy. Statist. Soc. Ser. B 29,162-173.

[4] Hald. A (1981). Statistical Theory of sampling Inspection by Attributes. Academic, New York.

[5] Huang, W.T. and Chen, H.S. (1992). Estimation of the exponential mean under type I censored sampling. J. Statist. Plann. Inference. 33, 187-196.

[6] Lam, Y. (1988a). A decision theory approach to variable sampling plans. Scienta, Sinica Ser. A 31, 129-140.

[7] Lam, Y. (1988b). Bayesian apporach to single variable sampling plans. Biometrika 75,387-391.

[8] Lam, Y. (1994). Bayesian variable sampling plans for the exponential distribution with type I censoring. Ann. Statist. Vol. 22, No.2, 696-711.

[9] Mann, N.R., Schafer, R.E. and Han, M.C. (1982). Confidence bounds for the exponential mean in time-trancated life tests. Survival Analysis. (Eds. J. Crowley and R.A. Johnson), 152-165, Lecture Notes-Monograph Series, Vol. 2, Institute of Mathematical Statistics.

[10] Spurrier, J.D. and Wei, L.J.(1980). A test of the parameter of the exponentical distribution in the type I censoring case. J. Amer. Statist. Assoc., 75, 405-409.

[11] Sinha, S.K.(1986). Reliability and Life Testing. Wiley, New York.

[12] Wetherill, G. B.(1977). Sampling Inspection and Quality Control, 2nd edition, Chapman and Hall, London.

[13] Wetherill G. B. and Köllerström, J. (1979). Sampling inspection simplified (with disscussion). J. Roy. Statist. Soc. Ser. A, 142,1-32.

[14] Yang, G. and Sirvanci, M. (1977). Estimation of a time-truncated exponential parameter used in life testing. J. Amer. Statist. Assoc. 72, 444-447.

[15] Yeh, L. and Choy, S. T. B. (1995). Bayesian variable sampling plans for the exponential distribution with uniformly distributed random censoring. J. Stat. Plan. Infer. 47, 277-293.