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Decomposition of $2K_{p,q,r}$ into most cycles

一、中文摘要

令 $K_{p,q,r}$ 表示一個完全三分圖，C_r 表示一個長度為 r 的基本迴圈又 $2K_{p,q,r}$ 表示一個完全三分圖其每一邊均出現兩次．一個圖 G 為可分解成迴圈表示 G 可被分割成邊均相異的迴圈， 在這計劃中我們得到的結論是當 $p \le q \le r$ 時，以下為產生最多迴圈之分解情形：

(a) 若 $(p+q)r$ 為偶數時，$2K_{p,q,r}$ 可分解成 2pq 個 C_3 及 $[p(r-q)+q(r-p)]/2$ 個 C_4；

(b) 若 $(p+q)r$ 為奇數時，$2K_{p,q,r}$ 可分解成 2pq 個 C_3，$[p(r-q)+q(r-p)-1]/2$ 個 C_4 及一個 C_6.

關鍵詞： 完全三分圖，分解

二、英文摘要

Let $2K_{p,q,r}$ be the 2-fold complete tripartite graph. In this paper, we will show that for each triple p,q,r of positive integers, $p \le q \le r$, $2K_{p,q,r}$ can be decomposed into most cycles as follows:

(a) if $(p+q)r$ is even, decompose $2K_{p,q,r}$ into 2pq triangles, and $[p(r-q)+q(r-p)]/2$ 4-cycles, and

(b) decompose $2K_{p,q,r}$ into 2pq triangles, $[p(r-q)+q(r-p)-1]/2$ 4-cycles, and one 6-cycles, otherwise.

Keywords: complete tripartite graph, decomposition.

Introduction.

In [2], A. T. White studied the

relationship between block designs and graph embeddings and he pointed out a BIBD on v objects with k = 3 and $\lambda = 2$ (a 2-fold triple system) determines a triangular embedding of K_v in some generalized pseudo-surfaces : each block becomes a triangle with vertices labeled by the objects of the block; since $\lambda = 2$, each pair of vertices appears exactly twice - so that a 2-manifold results from the standard identification procedure of combinatorial topology.

Then he extended the study to group divisible design GDD, thus a balanced complete multipartite graph $K_{n(m)}$ is considered. But, not every $2K_{n(m)}$ can be decomposed into triangles. Therefore, the work of Hanani on GDD [1] completes the generalized pseudo-characteristic for $K_{n(m)}$ in 7/9 of the possible cases. For other cases, we have to decompose $2K_{n(m)}$ into as many cycles as possible (not all triangles).

Instead of considering the cases left in $K_{n(m)}$, in this note, we consider a general complete tripartite graph and we are able to decompose it into most cycles in two different cases.

Let $K_{n,n,n}$ denote the complete tripartite graph with the partite sets $\{r_1, r_2, ..., r_n\}$, $\{c_1, c_2, ..., c_n\}$ and $\{e_1, e_2, \ldots, e_n\}$ and $L = [l(i,j)]$ be a latin square of order n. Then corresponds to a decomposition of $K_{n,n,n}$ into n^2 triangles. Each entry $l(i,j)$ of L corresponds to a triangle $(r_i, c_j, e_{l(i,j)})$ of $K_{n,n,n}$ for each $1 \le i, j \le n$. Now, we are ready to decompose $2K_{p,q,r}$, the 2-fold complete tripartite graph. Let the partite sets of $K_{p,q,r}$ be $\{r_1, r_2, ..., r_p\}$, $\{c_1, c_2, ..., c_q\}$ and $\{e_1, e_2, \ldots, e_r\}$. We assume throughout the paper that $p, q, r \in N$ and $p \le q \le r$.

Theorem 1. $2K_{p,q,r}$ can be decomposed into (a) 2pq triangles, and [p(r-q)+q(r-p)]/2 4-cycles, if (p+q)r is even, and

(b) 2pq triangles, [p(r-q)+q(r-p)-1]/2 4-cycles, and one 6-cycle, otherwise.

Corollary 2. Let $\chi''(G)$ denote the generalized pseudo-characteristic of G. Then

$\chi''(D(K_{p,q,r})$

$= (p+q+r) - (p+q)r/2$ if (p+q)r is even

$(p+q+r) - [(p+q)r - 1]/2$ if (p+q)r is odd.

Reference.

1. H. Hanani, Balanced incomplete block designs and related designs, Discrete Math., 11 (1975) 255-369.
2. A. T. White, Block Design and Graph Imbeddings, J. of Combinatorial Theory, (1975),166-183.