Three Essays on Firms' Financial Distress

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Abstract

Financial and output market decisions are crucial to the success or failure of an organization. These decisions are influenced by the dynamic and competitive economic environment in which firms operate and, in turn, affect the ability of firms to meet their debt obligations.

This thesis is constituted by three separate but interrelated essays which explore the impact of financial and operating decisions on the default risk. The first two essays study the equilibrium default probability, in a two-stage differentiated product duopoly model with uncertainty, where firms decide their financial structure in the first stage and their quantities in the second stage. These two essays analyze the impact of changes in the parameters of the model, on the equilibrium default probability (the first essay uses comparative statics tools while the second uses numerical simulation). The impact of changes in the uncertainty level, in the degree of product substitutability, in the marginal costs and in the default cost on the financing and output decisions and on the default risk are analyzed. The third essay tests empirical the relationship between market structure and capital structure decisions and their relationship with the default probability using a sample of eleven members of the Organization for Economic Cooperation and Development (OECD).

The three essays reach a coherent set of conclusions. In particular, they show that uncertainty, market structure and default costs influence financial and product market decisions and the probability of default. Moreover, they show that the default probability is influenced directly by the parameters, but it is also influenced by the way firms optimally adjust their financial and product market decisions when the parameters change. Therefore a less favorable environment does not necessarily imply higher default probability, as firms may respond by financing less with debt.

Keywords: Market Structure, Default Costs, Default Risk, Financial Structure, Product Market Decisions

Três Ensaios sobre as Dificuldades Financeiras das Empresas

Resumo

Decisões financeiras e no mercado do produto são cruciais para o sucesso ou falência de uma organização. Estas decisões são influenciadas pelo ambiente económico, dinâmico e competitivo em que as empresas operam e, por sua vez, afetam a capacidade das empresas cumprirem suas obrigações.

Esta tese é constituída por três ensaios distintos, mas interrelacionados que exploram o impacto das decisões financeiras e operacionais sobre o risco de incumprimento. Os dois primeiros ensaios estudam a probabilidade de incumprimento de equilíbrio, num modelo duopólio, com produtos diferenciados, com dois estágios e com incerteza, onde as empresas no primeiro estágio decidem a sua estrutura financeira, e no segundo estágio as suas quantidades. Estes dois ensaios analisam o impacto de alterações dos parâmetros do modelo na probabilidade de incumprimento de equilíbrio (o primeiro ensaio usa ferramentas de estática comparada, enquanto o segundo usa simulação numérica). É analisado o impacto de mudanças no nível de incerteza, no grau de substituibilidade do produto, nos custos marginais e no custo de incumprimento sobre as decisões de financiamento e de produção, e sobre o risco de incumprimento. O terceiro ensaio testa empíricamente a relação entre estrutura de mercado e as decisões da estrutura de capital e a sua relação com a probabilidade de incumprimento, utilizando uma amostra de onze membros da Organização para a Cooperação e Desenvolvimento Económico (OCDE).

Os três ensaios chegam a um conjunto coerente de conclusões. Nomeadamente, mostram que a incerteza, a estrutura de mercado e custos de incumprimento influenciam as decisões financeiras e no mercado do produto e a probabilidade de incumprimento. Além disso, mostram que a probabilidade de incumprimento é influenciada diretamente pelos parâmetros, mas também é influenciada pela forma como as empresas ajustam de forma ótima as suas decisões financeiras e no mercado do produto quando os parâmetros alteram. Por conseguinte, um ambiente menos favorável não significa necessariamente maior probabilidade de incumprimento, uma vez que as empresas podem responder financiando-se com menos dívida.

Palavras-chave: Estrutura de mercado, Custos de incumprimento, Risco de incumprimento, Estrutura de capital, Decisões no mercado do produto.

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Chapter 1

Introduction

Over the last decades, the financial literature has addressed the issue of financial distress and default risk. Considering its negative social and economic impact on the economy, it is not surprising that the existing literature has focused mainly on the best form to predict financial distress and default risk. However, in spite of the vast empirical literature, there is a lack of theoretical models aimed at understanding the structural factors that influence the default probability. The main objective of this thesis is to make a contribution in this direction. The thesis is divided into three separate but interrelated essays with the common aim of understanding how the market structure, financing decisions and product market decisions influence the default risk. The first two essays are the backbone of the thesis as they provide a theoretical framework to study the equilibrium default probability. The conclusions derived in these two essays, are then used in the third essay, to formulate the hypotheses which are tested empirically in this essay.

The first two essays study a two-stage differentiated product duopoly model with an uncertainty environment, where firms decide their financial structure in the first stage and their quantities in the second stage. In the first essay we consider a very general framework, where the demand and cost functions are not specified. In this framework, we analyze the impact of changing the parameters of the model (level of demand uncertainty, parameters that affect both firms and firm specific parameters) on the equilibrium default probabilities. To do this, we use analytic comparative statistics tools (in particular, the implicit function theorem is used throughout). This analysis is done both for the second stage Nash equilibrium (considering the financial structure as given but taking into account the impact on the output market decisions) as well as for the subgame perfect equilibrium (i.e., taking into account the impact on the financial structure decisions as well as on the product market decisions). We believe that both analyses are interesting as the two effects may not have the same sign and their distinction may be important for empirical work. The results, in this essay, show that both direct and indirect effects (through changes in the equilibrium capital structure and product market decisions) need to be considered and that, in some cases, the total impact of parameters' changes on the default risk may be counterintuitive.

The second essay explores a more specific two-stage model that assumes linear demands and constant marginal production costs. A novelty in this model is that it incorporates default costs into the analysis, something which has been almost unexplored in theoretical models. The literature on default costs distinguishes between direct default costs (administrative costs and legal costs) and indirect default costs (reputation effect on profit). However, to the best of our knowledge, the indirect default costs had never been incorporated in a theoretical model. Therefore, one of the main contributions of the second essay is the way it models the indirect default costs. Using numerical simulations, we then analyze the impact of changes in the level of demand uncertainty, in degree of product substitutability, in the asymmetry between the two firms' marginal production costs and in the direct (ex-post) and indirect (ex-ante) default costs parameters on the equilibrium default risk of the two firms. Furthermore we also analyze if the impact of the various parameters is the same when firms have equal marginal production costs (symmetric duopoly) and when they have different marginal costs (asymmetric duopoly). The results in this essay confirm the importance of considering both direct and indirect impacts (through the changes in the equilibrium financing and output decisions) on the default probability.

Finally, the third essay tests empirically the effect of a set of variables, that appear in theoretical studies on the subject and in the two previous essays but which have been rarely tested, on the capital structure decision and on the default risk. In particular, we evaluate the impact of the demand uncertainty, the degree of market concentration and the direct and indirect default costs. The inclusion of the indirect default cost should be noted as, due to the difficulty in their estimation, they were rarely considered in previous empirical studies. Moreover, the third essay explores the effect of the capital structure on the default risk, considering that the capital structure is endogenously determined, as shown in the first two essays. Therefore, this essay provides important contributions both to the literature on the determinants of the capital structure as well as to literature on the determinants of default risk. The first contribution is the analysis of the effect of uncertainty, degree of market concentration, direct and indirect default cost on the leverage ratio and on the default probability. The second main contribution is the methodology used in the estimation of the default probability where debt is considered as an endogenous variable. A final contribution is related to the fact that the sample includes 11 OECD countries, and we consider a year before and year after the financial crisis.

The remainder of the thesis is organized into four chapters. The next three chapters correspond to three aforementioned independent essays (each essay has its own introduction, main analysis, conclusion and bibliography). The final chapter, summarizes the main conclusions of the thesis and presents some of its limitations and suggestions for future research.

Chapter 2

Capital Structure, Product Market Competition and Default Risk

2.1 Introduction

Bankruptcy has negative social and economic consequences which explains why many researchers are interested in finding the best form to predict the default risk. Although there exists a proliferation of models to predict financial distress risk (for a survey of the empirical literature see Balcaen and Ooghe, 2006), there is a lack of theoretical models for explaining default probability. However, the development of theoretical models aimed at deriving the equilibrium default probability may provide important insights and guide future empirical work on financial distress. The main objective of this chapter is to provide a contribution in this direction.

The essay derives the equilibrium default probabilities in a model with an uncertain environment where firms first take their financing decisions and later take their product market decisions. In addition, we analyze the impact of changing certain parameters on the default probabilities. This analysis is done both for the second stage Nash equilibrium (considering the financial structure as given but taking into account the impact on the output market decisions) as well as for the subgame perfect equilibrium (i.e., taking into account the impact on the financial structure decisions as well as on the product market decisions). We believe that both analyses are interesting as the two effects may not have the same sign and their distinction may be important for empirical work.

The link between the financial structure and output market decisions has been highlighted both on the Corporate Finance literature and on the Industrial Organization literature.[1] Brander and Lewis (1986) were the first to examine the relationship between financial decisions and output market competition. They consider a two stage Cournot duopoly model with an uncertain environment. In the first stage, each firm decides the capital structure. In the second stage, taking into account their previously chosen financial structure, firms take their decisions in the output market.[2] Brander and Lewis (1986) conclude that debt tends to encourage a more aggressive behavior in the output market. Thus firms have an incentive to use their financial structure for strategic purposes. Maksimovic (1988) confirms the findings of Brander and Lewis (1986) regarding the aggressiveness of indebted firms in the output market, which is due, according to the authors, to the existence of limited liability.[3]

While Brander and Lewis (1986) present a general model, without specifying whether

[1] Riordan (2003) presents a critical survey that summarizes the existing literature on the interaction between capital structure and output market. The author argues that the capital market restrictions depend on the output market competition.

[2] Like Brander and Lewis (1986), we ignore the physical investment decision. This is equivalent to assume that the investment decision is taken before the capital structure decision. If this assumption was not made, the debt-equity mix choice would influence the investment which would have further effects on the output market. This happens in Clayton (2009) where the investment is made to reduce the marginal cost of production. As pointed out by Brander and Lewis (1986) one possible interpretation of the capital structure choice is that the firm is initially equity financed, when the loan is taken the borrowed money is fully distributed to shareholders.

[3] It should be highlighted that the existing empirical work relating financial and output market decisions clearly confirms the strategic role of debt on the output market. However the sign of the impact of greater leverage on the output market is not so clear-cut. For instance, Chevalier (1995b) examines the impact of supermarket Leveraged Buyouts (LBOs) in the product market. She concludes that the announcement of a LBO leads to an increase in the expected profit of rival firms and to a less aggressive behavior in the output market, a conclusion that goes against the results of Brander and Lewis (1986). Nishihara and Shibata (2014) support this result and conclude that high leverage leads to a "competitive disadvantage and mitigates product market competition". On the contrary, the results of Guney, Li and Fairchild (2011) support the theory of aggressive behavior by most indebted firms. Interestingly, Campos (2000) shows that limited liability firms which have higher short-term debt behave more aggressively in the output market but the long-term debt has the opposite effect, suggesting that the output market reaction may depend on the type of debt. Using a sample of Indian firms, Bandyopadhyay (2005) reaches the same conclusion.

products are homogenous or differentiated and whether uncertainty affects demand or costs, other authors have explored more specific models and analyzed the impact of changes in parameters such as the level of uncertainty and the level of substitutability among products, on the equilibrium output and debt levels. This type of approach is followed by Wanzenried (2003), Franck and Le Pape (2008) and Haan and Toolsema (2008) who analyze a two-stage differentiated goods duopoly model with demand uncertainty.[4] Franck and Le Pape (2008) only analyze Cournot competition whereas Haan and Toolsema (2008) use numerical analysis to study how the equilibrium is affected by demand uncertainty and the substitutability of products both under Cournot and Bertrand competition.[5]

Our study extends Brander and Lewis (1986) by analyzing the implications of financial structure decisions and output market decisions on the default probability and also by studying the impact, at a very general level, of changes in the parameters on the equilibrium. There are two important contributions of our work. The first is that while Brander and Lewis focus on the implications on the output market of financial structure decisions, our emphasis is in showing that the default risk depends both on financial structure and output market decisions. The second contribution is that we analyze the impact of changes in the level of demand uncertainty, changes in parameters that are common to the two firms (such as the average dimension of the market and the degree of product differentiation) and changes in parameters that are firm specific (such as the marginal costs) on the equilibrium default probabilities.

It should be noted that the default risk has been addressed in the work of Franck and Le Pape (2008) and Haan and Toolsema (2008) using numerical simulations. However, these authors only analyzed the impact of demand uncertainty and the degree of product

[4] As pointed out by Franck and Le Pape (2008) and Haan and Toolsema (2008), the work of Wanzenried (2003) has a technical mistake when, in the second stage of the game, considers the default risk as given instead of considering the debt levels as given. In fact, the default risk depends on the output market decisions and therefore it should be endogenously determined in the second stage of the game.

[5] Socorro (2007) analyzes merger profitability in a Cournot oligopoly with linear and uncertain demand, fixed costs and constant marginal costs. She concludes that demand uncertainty and the limited liability effect lead merged firms to compete more aggressively and increase their profit.

differentiation on the probability of default risk in a symmetric duopoly model with linear demands and constant marginal costs. The aim of this essay is the generalization of the previous work by analyzing the explicit impact of parameters that affect all the firms and the impact of parameters that only affect one firm. The aim is to analyze how these parameters affect the equilibrium financial structure, the equilibrium level of output, and the corresponding default risk.[6]

The remainder of the chapter is organized as follows. In the next section we present the model. Section 2.3 analyzes the second stage of the game. In this section we also study how changes in the parameters influence the equilibrium default risk in the second stage of the game, assuming fixed debt levels. The next section derives the subgame perfect equilibrium and studies how changes in the parameters affect the equilibrium financial and output market decisions and the equilibrium default risk. Finally, section 3.6 summarizes the main conclusions of the study. The Appendix contains the proofs of all lemmas and propositions.

2.2 Model

Based on the formalization presented by Brander and Lewis (1986), we consider a two stage duopoly Cournot model.[7] In the first stage each firm (firm i and firm j) decides the financial structure, i.e., the level of debt and equity in the capital structure. In the second stage each firm takes its decision in the output market. Figure 2.1 shows the timing of the game.

Let q_i and q_j be the output of firms i and j, respectively and $R^i(q_i, q_j, z_i, \gamma, \alpha_i)$ be the operating profit for firm i. $R^i(q_i, q_j, z_i, \gamma, \alpha_i)$ is defined as the difference between revenue and variable cost and depends on the random variable z_i , parameter γ which affects both

[6]Most of the previously mentioned works consider quantity competition (strategic substitutes). With regard to price competition (strategic complements) we highlight the work of Showalter (1995) and Haan and Toolsema (2008). Showalter (1995) argues that the strategic use of debt is advantageous only if there is uncertainty in demand. Haan and Toolsema (2008) conclude that the increase in debt leads to an increase on equilibrium prices.

[7]We consider a Cournot duopoly model for the sake of simplicity. In our general context, extending the results for n firms would be possible but complex.

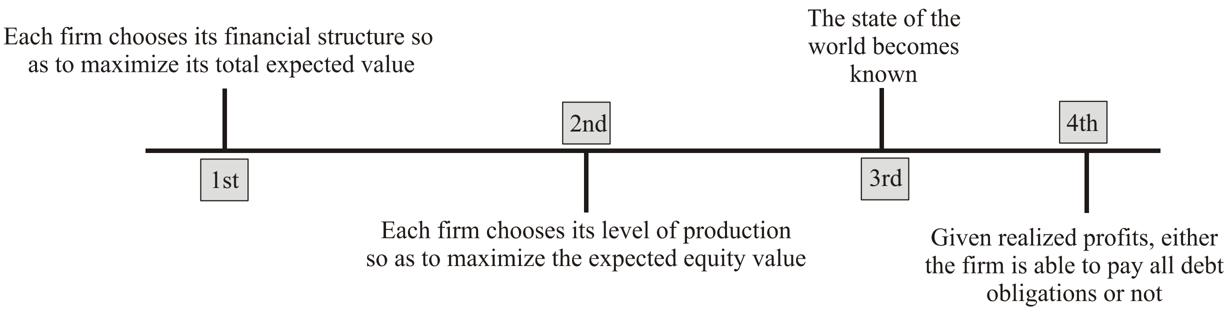

Figure 2.1: Timing of the game: first financial decisions are taken, next output decisions are taken. Output decisions are taken before the uncertainty is resolved.

firms (such as the degree of product differentiation or the average dimension of the market) and parameter α_i which affects only firm i (such as the firm's marginal cost).[8] It should be highlighted that our formalization considers explicitly the impact of the parameters on R^i so as to allow us to analyze the impact of changes in these parameters, an issue which was not explored by Brander and Lewis (1986).

The random variable z_i represents the uncertainty in the output market demand, i.e., the deviation from the average market demand (this deviation can be positive or negative). It is assumed that this variable is distributed on the interval $[-\overline{z}, \overline{z}]$ according to density function $f(z_i)$, which we assume to be positive for all $z_i \in [-\overline{z}, \overline{z}]$. We assume that z_i and z_j are independent and identically distributed.

We assume that $R^i(q_i, q_j, z_i, \gamma, \alpha_i)$ follows some standard proprieties: $R^i_{ii}(q_i, q_j, z_i, \gamma, \alpha_i) < 0$ and $R^i_{ij}(q_i, q_j, z_i, \gamma, \alpha_i) < 0$. Condition $R^i_{ii}(q_i, q_j, z_i, \gamma, \alpha_i) < 0$ indicates that the marginal profit function is negatively sloped or, equivalently, the profit function of the firm is concave on its own quantity. Condition $R^i_{ij}(q_i, q_j, z_i, \gamma, \alpha_i) < 0$ implies that we have strategic substitutes, that is, when firm j increases its quantity the optimal quantity of firm i decreases. In addition, we assume that $R^i_{z_i} > 0$ and $R^i_{iz_i} > 0$. The assumption $R^i_{z_i} > 0$ means that high values of z_i contribute to higher operating profit. That is, higher values of z_i correspond to better states of the world. Condition $R^i_{iz_i} > 0$ indicates that

[8] We could consider a more general formalization where γ and α_i are vectors of parameters. However the qualitative results would be the same and thus, to simplify notation, we consider the case where γ and α_i are scalars.

the marginal profit is higher in better states of the world. The two last assumptions are consistent with considering z_i as the deviation from the average market demand, where higher values of z_i correspond to higher demand.

In the first stage of the game each firm chooses the financial structure that maximizes the value of the firm, taking into account that this choice will affect the equilibrium in the second stage of the game. While the financial structure choice is done so as to maximize the sum of the equity value and the debt value, the quantity choice in the second stage of the game is done so as to maximize the expected value of equity.[9]

In order to find the subgame perfect equilibrium of the game we solve the game backwards. We start by computing the Nash equilibrium of the second stage game as a function of the debt level chosen by the firms in the first stage. Next we solve the first stage game. In this stage firms take their financing decisions considering their impact on the output market equilibrium.

To better follow the model resolution, table 2.1 summarizes the variables used.

2.3 Nash equilibrium in the second stage game

This section examines the second stage of the game, considering the debt levels D_i and D_j chosen by the firms in the first stage of the game. In the second stage of the game, each firm chooses the output level that maximizes the expected value of the firm to the shareholders. We start by analyzing the equilibrium in the output market and investigate how the output market decisions change with the debt levels D_i and D_j chosen by the firms in the first stage of the game as well as with changes on the parameters of the model. Next, we determine the second stage equilibrium default probabilities and again investigate how they change with the debt levels D_i and D_j chosen by the firms in the

[9] It is assumed that appropriate incentive schemes guarantee that management acts so as to maximize shareholders's value.

Table 2.1: Variables of the model

Variables	Meaning
q_i, q_j	Output of firms i and j
D_i, D_j	Debt obligation of firms i and j
R^i	Operating profits of firm i
R^i_i	Marginal operating profits of firm i
z_i	Random variable that represents the uncertainty
$\widehat{z}_i$	Critical value of z_i
α_i	Parameter which affect only firm i
θ_i, θ_j	Default probability of firms i and j
γ	Parameter which affect both firms
Y^i, Y^j	Firm value of firms i and j
V^i, V^j	Expected equity value of firms i and j
W^i, W^j	Expected value of debt of firms i and j
Wel	Welfare

first stage of the game as well as with changes in the uncertainty level and the other parameters of the model.

2.3.1 Output Market Equilibrium

In the second stage of the game the manager maximizes the expected equity value which is given by:

$$V^i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) = \int_{\widehat{z}_i(q_i, q_j, D_i, \gamma, \alpha_i)}^{\overline{z}} (R^i(q_i, q_j, z_i, \gamma, \alpha_i) - D_i) f(z_i) dz_i \qquad (2.1)$$

where D_i represents the debt obligation of firm i, and $\widehat{z}_i(q_i, q_j, D_i, \gamma, \alpha_i)$ is the critical value of z_i such that the operating profit of firm i is just enough for the firm to meet its debt obligations. This critical state of the world is implicitly defined by:

$$R^i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) - D_i = 0 \qquad \text{for} \quad -\overline{z} \leq \widehat{z}_i \leq \overline{z}. \qquad (2.2)$$

Hence $V^i(q_i, q_j, D_{i,}\overline{z}, \gamma, \alpha_i)$ corresponds to expected profit net of debt obligations in good states of the world ($z_i \geq \widehat{z}_i$). In bad states of the world, $z_i < \widehat{z}_i$, shareholders earn zero as all operating profit is paid to debtholders.[10] The existence of limited liability means that, if there are financial difficulties, only the assets and their returns, will serve as collateral for the debt fulfillment. So, when we are dealing with the bad states of nature, equityholders will not receive any income, but they do not have to pay their debt obligations with personal property.

Brander and Lewis (1986) showed that the critical state of nature, $\widehat{z}_i$, is increasing with firm i's debt, D_i, and with firm j's quantity, q_j. Moreover, the critical state of nature, $\widehat{z}_i$, is increasing with q_i if and only if $R^i_i(\widehat{z}_i) < 0$ (which holds under our assumptions as we will show later).

Similarly, it is useful to determine how the critical state of nature, $\widehat{z}_i$, changes with parameters γ and α_i:

Lemma 2.1 *The impact of γ and α_i on $\widehat{z}_i$ has the opposite sign of $R^i_\gamma(\widehat{z}_i)$ and $R^i_{\alpha_i}(\widehat{z}_i)$, respectively.*

Consider, for instance, the impact of an increase in the marginal costs of firm i. Since increasing the marginal costs has a negative impact on firm i's profit in all states of nature, $R^i_{\alpha_i} < 0$. By the previous lemma, this implies that the critical state of nature increases, which means that there are fewer states of nature where the firm is able to meet its debt obligations.

The optimal output for firm i is given by the first order condition that the partial derivative of V^i with respect to q_i is equal to zero. By Leibniz rule this is equal to:

$$V^i_i(q_i, q_j, D_{i,}\overline{z}, \gamma, \alpha_i) = \int\limits_{\widehat{z}_i(q_i,q_j,D_i,\gamma,\alpha_i)}^{\overline{z}} R^i_i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i - (R^i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) - D_i) f(\widehat{z}_i) \frac{\partial \widehat{z}_i}{\partial q_i} = 0$$

[10] If condition (2.2) does not hold for any $-\overline{z} \leq \widehat{z}_i \leq \overline{z}$ that means that either the firm is always able to meet its debt obligations or that it is never able to do so, which is equivalent to consider $\widehat{z}_i = -\overline{z}$ or $\widehat{z}_i = \overline{z}$, respectively. In the remaining of the paper we focus on the case where the critical state is in the interior of $[-\overline{z}, \overline{z}]$.

Where V_i^i denotes the partial derivative of V^i with respect to q_i. However, by definition of $\widehat{z}_i$, the second term is equal to zero. Thus the first order condition is given by:

$$V_i^i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) = \int\limits_{\widehat{z}_i(q_i, q_j, D_i, \gamma, \alpha_i)}^{\overline{z}} R_i^i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i = 0 \tag{2.3}$$

It should be noted that the previous condition takes into account the endogeneity of $\widehat{z}_i$ which depends on the quantities chosen by the two firms. Condition (2.3) tells us that, the optimal quantity is such that the expected marginal profit in good states of the world is equal to zero. Note that since $R_{iz_i}^i > 0$, marginal profit (R_i^i) is increasing with z_i, thus marginal profit is negative at $\widehat{z}_i$ but positive at $\overline{z}$. Figure 2.2 shows the expected marginal profit in good states of the world for the optimal quantity.

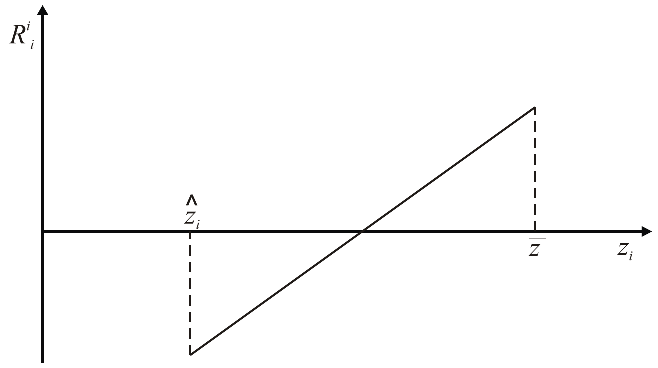

Figure 2.2: Marginal profit of firm i in good states of the world when $R_{iz_i}^i > 0$. For the optimal quantity, the expected marginal profit in good states of the world is equal to zero (the area with negative marginal profit is equal to the area with positive marginal profit).

The second order conditions are satisfied if (using Leibniz rule again):

$$V_{ii}^i = \int\limits_{\widehat{z}_i}^{\overline{z}} R_{ii}^i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i - R_i^i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) f(\widehat{z}_i) \frac{\partial \widehat{z}_i}{\partial q_i} < 0$$

It should be noted that, under the assumption that $R_{iz_i}^i > 0$, the term $-R_i^i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) f(\widehat{z}_i) \frac{\partial \widehat{z}_i}{\partial q_i}$ is positive since $R_i^i(\widehat{z}_i) < 0$ and $\frac{\partial \widehat{z}_i}{\partial q_i} = -\frac{R_i^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)} > 0$. This implies that the previous condition is harder to satisfy than in traditional games where imposing the concavity of the profit function is enough. In what follows we assume $V^i_{ii} < 0$. In addition we assume that $V^i_{ij} < 0$, which means that quantities are strategic substitutes. Finally we assume that $V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji} > 0$, which guarantees that the Nash equilibrium of the quantities game is unique.

The Nash equilibrium is given by the solution of the system of first order conditions:

$$\begin{cases} V^i_i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) = 0 \\ V^j_j(q_i, q_j, D_j, \overline{z}, , \gamma, \alpha_j) = 0 \end{cases} \Leftrightarrow \begin{cases} \displaystyle\int\limits_{\widehat{z}_i(q_i,q_j,D_i,\gamma,\alpha_i)}^{\overline{z}} R^i_i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i = 0 \\ \displaystyle\int\limits_{\widehat{z}_j(q_i,q_j,D_j,\gamma,\alpha_j)}^{\overline{z}} R^j_j(q_i, q_j, z_j, \gamma, \alpha_j) f(z_j) dz_j = 0 \end{cases} \tag{2.4}$$

where $\widehat{z}_i$ and $\widehat{z}_j$ are implicitly defined by $R^i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) - D_i = 0$ and $R^j(q_i, q_j, \widehat{z}_j, \gamma, \alpha_j) - D_j = 0$, respectively. Let $q^*_i(D_i, D_j, \gamma, \alpha_i, \alpha_j, \overline{z})$ and $q^*_j(D_i, D_j, \gamma, \alpha_i, \alpha_j, \overline{z})$ be the solution of this system. In other words, q^*_i and q^*_j are the equilibrium quantities in the output market.

Brander and Lewis (1986) analyzed the change in the equilibrium quantities when there is a unilateral increase in firm i's debt, D_i. They proved that when $R^i_{iz_i} > 0$, a unilateral increase in firm i's debt, D_i, leads to an increase in the equilibrium quantity of firm i, q^*_i, and to a decrease in the equilibrium quantity of firm j, q^*_j. This means that debt financing leads the firm to behave more aggressively in the output market. Intuitively, when a firm has a higher debt level, the firm will be able to repay its obligations in a smaller set of states of the world ($\widehat{z}_i$ increases). Since equityholders only care about good states of the world, $z_i > \widehat{z}_i$, an increase in the firm's debt increases the expected marginal profits conditional on $z_i > \widehat{z}_i$, which leads to an increase in the optimal quantity. A graphical explanation for this result is given by figure 2.3.

It is also interesting to know how the equilibrium quantities change when the level of uncertainty (measured by $\overline{z}$) increases or with changes in parameters γ and α_i, for given levels of D_i and D_j. This analysis was not done by Brander and Lewis (1986). However it is helpful to have a more complete characterization of the output market decisions when

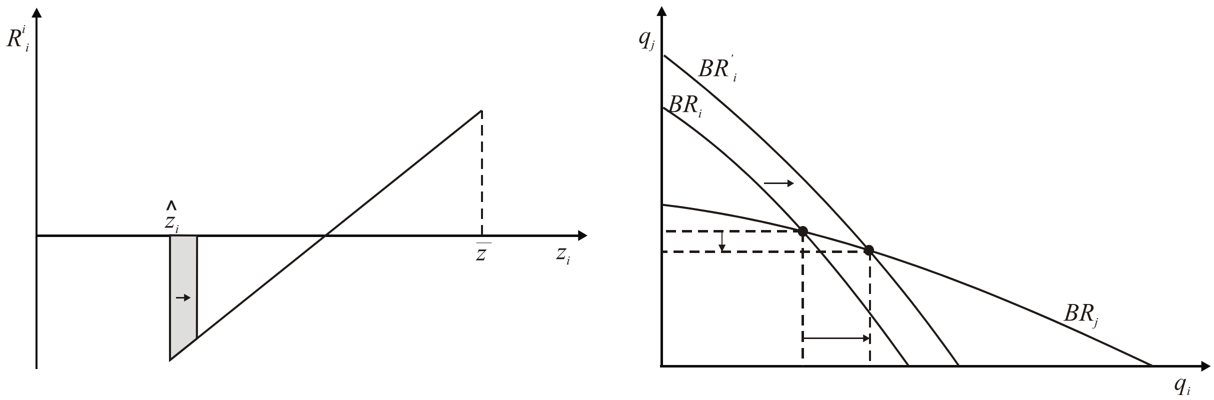

Figure 2.3: When D_i increases, $\widehat{z}_i$ increases, which increases the expected marginal equity of firm i (grey area on left panel), leading to an increase in firm i optimal quantity. This implies that the best response function of firm i shifts to the right. Thus, q_i^* increases and q_j^* decreases.

the financial structure is fixed.

Let us start by analyzing the impact of changes in the level of uncertainty. One interpretation of this exercise, would be to consider a change in the uncertainty level occurring after the first period financing decisions were taken but before the output decisions.

Lemma 2.2 *If $R^i_{iz_i} > 0$, for fixed debt levels, an increase in the level of uncertainty, $\overline{z}$, causes an increase in firm i's equilibrium quantity, q_i^*, if and only if $V^j_{jj}V^i_{i\overline{z}} - V^i_{ij}V^j_{j\overline{z}} < 0$. Moreover, if firms are symmetric $\left(V^i_{i\overline{z}} = V^j_{j\overline{z}}\right)$ an increase in the level of uncertainty leads to an increase in the symmetric Nash equilibrium quantities.*

This means that when firms are symmetric, for fixed debt values, the higher is the level of uncertainty, the more aggressive will firms be in the output market. Intuitively, the increase in the uncertainty level implies that there are more good states with positive marginal profit, thus the expected marginal profit conditional on $z_i > \widehat{z}_i$ increases, hence it is optimal to produce a higher quantity (note that increasing $\overline{z}$ also means that there are states of the world with more negative marginal profit, but equityholders do no care about these states of the world, unless the firm is all equity financed). Figure 2.4 illustrates the impact of increasing the uncertainty level on the expected marginal equity.

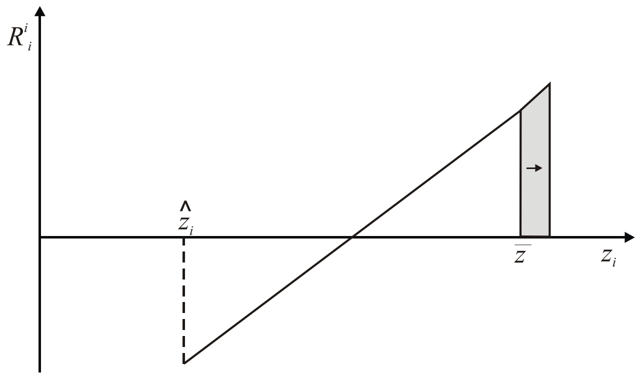

Figure 2.4: When $\overline{z}$ increases there is an increase in the expected marginal equity (area in grey), which leads to an increase in the optimal quantity. This implies that the best response functions of the two firms shift to the right when $\overline{z}$ increases.

Let us now study the impact of changes in parameters that affect the two firms on the output market equilibrium, for given D_i and D_j.

Lemma 2.3 *If $R^i_{iz_i} > 0$, for fixed debt levels, an increase in the common parameter γ, causes a change in firm i's equilibrium quantity, q_i^*, with the opposite sign of $\left(V^j_{jj}V^i_{i\gamma} - V^i_{ij}V^j_{j\gamma}\right)$. Moreover, if firms are symmetric and $V^j_{jj} < V^i_{ij}$, $\frac{\partial q_i^*}{\partial \gamma}$ has the same sign as $V^i_{i\gamma}$. Thus q_i^* increases if and only if the expected marginal equity value is increasing with γ. The sign of $V^i_{i\gamma}$ is ambiguous if γ affects in the same direction the profit and the marginal profit, i.e., if R^i_γ and $R^i_{i\gamma}$ have the same sign. If R^i_γ and $R^i_{i\gamma}$ have opposite signs, the sign of $V^i_{i\gamma}$ is the same as the sign of $R^i_{i\gamma}$.*

This result tells us that, when firms are symmetric, the impact of increasing γ on firm i equilibrium quantity depends on the way γ influences the expected marginal equity value, i.e. depends on $V^i_{i\gamma}$, which is given by (see the proof of Lemma 2.3 in the appendix):

$$V^i_{i\gamma} = \int_{\widehat{z}_i}^{\overline{z}} R^i_{i\gamma} f(z_i)dz_i + R^i_i(\widehat{z}_i)\frac{R^i_\gamma(\widehat{z}_i)}{R^i_{z_i}(\widehat{z}_i)} f(\widehat{z}_i) \tag{2.5}$$

Thus the sign of $V^i_{i\gamma}$ depends both on the effect of γ on profit, R^i_γ, and the impact of γ on marginal profit $R^i_{i\gamma}$. If R^i_γ and $R^i_{i\gamma}$ have the same sign, the two terms in (2.5) have opposite signs since $R_i^i(\widehat{z}_i) < 0$. Thus, if R_γ^i and $R_{i\gamma}^i$ have the same sign, $V_{i\gamma}^i$ has an ambiguous sign. It should be noted that this is the most natural case. For instance, if γ is the average dimension of the market, in a model with linear demands, increases in γ lead to higher profit and to higher marginal profit, thus, $R_{i\gamma}^i > 0$ and $R_\gamma^i > 0$. Since $R_i^i(\widehat{z}_i) < 0$, the second term in (2.5) is negative while the first is positive. Thus the sign of $V_{i\gamma}^i$ depends on which of these two effects dominates. Figure 2.5 illustrates the two effects of changing γ on the expected marginal profit, conditional on $z_i > \widehat{z}_i$, when $R_{i\gamma}^i > 0$ and $R_\gamma^i > 0$. The first effect is represented in light grey, while the second effect is represented in dark grey. In the figure the first effect dominates (area in light grey is larger than area in dark grey). Thus, in the case illustrated in the figure, an increase in γ leads to an increase in the equilibrium quantity levels.

It is interesting to explore a little bit further the two effects when $R_{i\gamma}^i$ and R_γ^i have the same sign. For a firm without debt, only the first effect is present and thus, when $R_{i\gamma}^i > 0$, the optimal quantity increases. For an indebted firm, the equityholders only care about good states of nature and consequently the first effect has a smaller magnitude. Moreover the second term is negative, which implies that the impact of γ is always lower for an indebted firm than for a firm without debt. In addition, when the second effect dominates, the impact of changes in γ on the equilibrium quantities is precisely the opposite of what happens in standard oligopoly models. The second effect is more likely to dominate when the parameter changes have a big impact on the firm profit (R_γ^i is larger) and when uncertainty is higher ($R_i^i(\widehat{z}_i)$ has a larger absolute value).

Finally, let us determine the change on the equilibrium quantities with changes in α_i.

Lemma 2.4 *If $R_{iz_i}^i > 0$, for fixed debt levels, an increase in firm i's parameter α_i, causes a change on firm i's equilibrium quantity, q_i^*, with the same sign as $V_{i\alpha_i}^i$ and a change on q_j^* with the opposite sign of $V_{i\alpha_i}^i$. Thus q_i^* increases (and q_j^* decreases) if and only if the expected marginal equity value is increasing with α_i. The sign of $V_{i\alpha_i}^i$ is ambiguous if α_i affects in the same direction the profit and the marginal profit, i.e., if $R_{\alpha_i}^i$ and $R_{i\alpha_i}^i$ have the same sign. If $R_{\alpha_i}^i$ and $R_{i\alpha_i}^i$ have opposite signs, the sign of $V_{i\alpha_i}^i$ is the same as the*

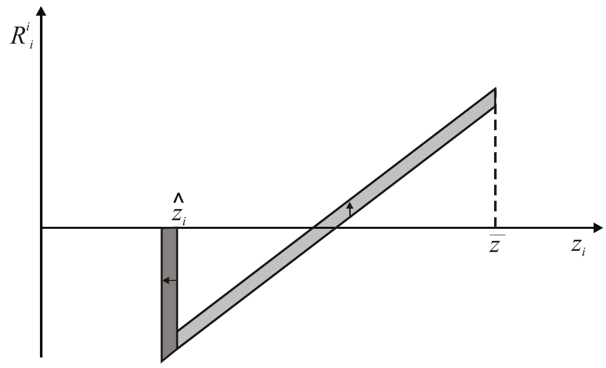

Figure 2.5: The impact of an increase in γ on the expected marginal equity when $R^i_{i\gamma} > 0$ and $R^i_\gamma > 0$. Since $R^i_{i\gamma} > 0$, an increase in γ increases the expected marginal equity (area in light grey). Since $R^i_\gamma > 0$, $\widehat{z}_i$ decreases, which leads to a decrease in the expected marginal equity (area in dark grey).

sign of $R^i_{i\alpha_i}$.

The previous results implies that a change in firm i's parameter, α_i, always has impacts with opposite signs on q_i^* and q_j^*.

One example where the previous results applies is when α_i is the marginal cost of firm i. In this case, profit and marginal profit are both decreasing with the firm's marginal cost. Thus the impact of a change in marginal cost in the firm own production is ambiguous. On the one hand the fact that expected marginal profit in good states of the world becomes lower when the marginal cost increases, tends to decrease the optimal quantity. On the other hand an increase in the marginal costs decreases the profit in all the states of the world and thus it increases the critical state of nature $\widehat{z}_i$, which leads to a more aggressive behavior by the firm. If the last effect dominates, an increase in the marginal costs of firm i leads to a higher q_i^* (this case is illustrated in figure 2.6), which is the opposite of what happens in standard oligopoly model where the limited liability effect is not considered.

2.3.2 Equilibrium default probabilities

In this subsection we analyze the equilibrium default probabilities in the second stage of the game and how they change with the financial structure chosen in the first stage of the

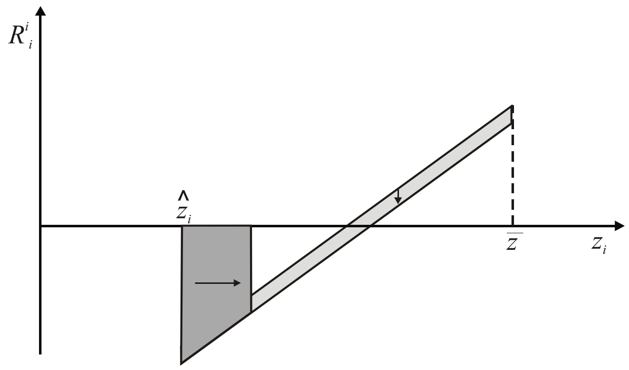

Figure 2.6: Impact of an increase in α_i on the expected marginal equity when $R^i_{\alpha_i} < 0$ and $R^i_{i\alpha_i} < 0$. Since $R^i_{i\alpha_i} < 0$, an increase in α_i decreases expected marginal equity (area in light grey). Since $R^i_{\alpha_i} < 0$, $\widehat{z}_i$ increases, which leads to an increase in the expected marginal equity (area in dark grey).

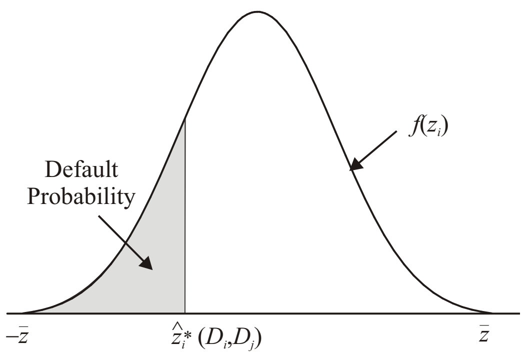

Figure 2.7: Equilibrium default probability for firm i considering the debt levels, D_i and D_j, chosen in the first stage.

game, with the level of uncertainty and with common and firm specific parameters.

The default probability of firm i (illustrated in figure 2.7) is given by (for firm j computations would be similar):

$$\Pr\left(R^i(q_i,q_j,D_i,\gamma,\alpha_i) < D_i\right) = \Pr\left(z_i < \widehat{z}_i\right) = \int\limits_{-\overline{z}}^{\widehat{z}_i(q_i,q_j,D_i,\gamma,\alpha_i)} f(z_i)dz_i = F(\widehat{z}_i(q_i,q_j,D_i,\gamma,\alpha_i))$$

where $F(z_i)$ is the cumulative density function. Thus, to compute the equilibrium default probability one needs to know the equilibrium critical state of nature, $\widehat{z}_i$. To obtain $\widehat{z}_i^*$

we just need to substitute the Nash equilibrium quantities in $\widehat{z}_i(q_i,q_j,D_i,\gamma,\alpha_i)$:

$$\widehat{z}_i^*(D_i,D_j,\gamma,\alpha_i,\alpha_j,\overline{z}) = \widehat{z}_i(q_i^*(D_i,D_j,\gamma,\alpha_i,\alpha_j,\overline{z}), q_j^*(D_i,D_j,\gamma,\alpha_i,\alpha_j,\overline{z}), D_i,\gamma,\alpha_i) \quad (2.6)$$

Consequently, the equilibrium default probability is given by:

$$\theta^*(D_i,D_j,\gamma,\alpha_i,\alpha_j,\overline{z}) = \Pr(z_i < \widehat{z}_i^*) = \int_{-\overline{z}}^{\widehat{z}_i^*(D_i,D_j,\gamma,\alpha_i,\alpha_j,\overline{z})} f(z_i)dz_i = F\left(\widehat{z}_i^*(D_i,D_j,\gamma,\alpha_i,\alpha_j,\overline{z})\right)$$

Note that since $F(z_i)$ is increasing, the default probability is increasing with the equilibrium critical state of nature. Let us now analyze how this probability changes with D_i and D_j:

Proposition 2.5 *If $R^i_{iz_i} > 0$, an increase in firm i's debt, D_i, causes an increase in the equilibrium default probability of firm i, θ_i^*, if and only if $\left(R_i^i(\widehat{z}_i)V^j_{jj} - R^i_j(\widehat{z}_i)V^j_{ji}\right)V^i_{iD_i} + V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji} > 0$. Moreover, a sufficient condition for $\frac{\partial\theta_i^*}{\partial D_i}$ to be positive is that $R_i^i(\widehat{z}_i)V^j_{jj} - R^i_j(\widehat{z}_i)V^j_{ji} > 0$. Finally, an increase in D_j causes an increase in θ_i^* if and only if $R^i_j(\widehat{z}_i)V^i_{ii} > R^i_i(\widehat{z}_i)V^i_{ij}$.*

The previous result indicates that the effect of changes in the debt level of a firm on the equilibrium default probabilities is ambiguous. The intuition is that an increase in D_i has opposite effects on the two firms equilibrium quantities as q_i^* increases but q_j^* decreases, which in turn have opposite effects on the equilibrium default probability. However, the sign of $\frac{\partial\theta_i^*}{\partial D_i}$ is very likely to be positive as a debt increase has a positive direct effect on the default probability and the impact of D_i on the own firm's quantity is expected to have a larger magnitude than the impact of D_i on the rival's quantity (this last effect is captured in the sufficient condition, $R_i^i(\widehat{z}_i)V^j_{jj} - R^i_j(\widehat{z}_i)V^j_{ji} > 0$). Thus, under standard assumptions, when a firm increases its debt, its default probability increases.

The sign of $\frac{\partial\theta_i^*}{\partial D_j}$ is harder to determine. In this case, there is no direct impact, so everything depends on how D_j changes the equilibrium quantities, q_j^* and q_i^*, and how that affects $\widehat{z}_i$. When D_j increases, firm j becomes more aggressive in the output market (produces more) whereas firm i becomes more conservative (produces less). The fact that j increases its quantity implies a lower profit for firm i in every state of nature, thus increasing the probability of default of firm i. However, firm i optimal response is to produce less, which lowers its probability of default. Consequently, the impact of D_j on θ_i^* is ambiguous.

The sign of $\frac{\partial \theta_i^*}{\partial D_j}$ depends on the marginal profits in the critical state of the world (which depends on the level of uncertainty and firm i level of debt). In particular, for small levels of uncertainty and/or large levels of debt, $R_i^i(\widehat{z}_i)$ is close to zero, thus it is very likely that $R_j^i(\widehat{z}_i)V_{ii}^i > R_i^i(\widehat{z}_i)V_{ij}^i$, in which case $\frac{\partial \theta_i^*}{\partial D_j}$ is positive. On the other hand, for large levels of uncertainty and/or low levels of debt, $|R_i^i(\widehat{z}_i)|$ might be large enough to imply that $R_i^i(\widehat{z}_i)V_{ij}^i > R_j^i(\widehat{z}_i)V_{ii}^i$ and thus $\frac{\partial \theta_i^*}{\partial D_j}$ may be negative.

One can also analyze the impact of changes in the level of uncertainty (measured by $\overline{z}$) and the impact of changes in parameters γ and α_i on the equilibrium default probabilities.

Proposition 2.6 *If $R_{iz_i}^i > 0$ and firms are symmetric, for fixed debt levels, an increase in the level of uncertainty, $\overline{z}$, causes an increase in the symmetric equilibrium default probabilities.*

This means that, for fixed debt levels, if there is an increase in the level of uncertainty, the default probability increases. The reason is that, in the second stage of the game, firms behave more aggressively when uncertainty is higher, i.e., equilibrium quantities are higher. This leads to an increase in the critical state of nature which consequently increases the default probability.

Proposition 2.7 *If $R_{iz_i}^i > 0$, for fixed debt levels, an increase in the common parameter γ, causes an increase in the equilibrium default probability of firm i, θ_i^*, if and only if $R_i^i(\widehat{z}_i)\left(V_{jj}^j V_{i\gamma}^i - V_{ij}^i V_{j\gamma}^j\right) + R_j^i(\widehat{z}_i)\left(V_{ii}^i V_{j\gamma}^j - V_{ji}^j V_{i\gamma}^i\right) - R_\gamma^i\left(V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j\right) > 0$. The impact of γ on θ_i^* is ambiguous if R_γ^i and $R_{i\gamma}^i$ have the same sign. If R_γ^i and $R_{i\gamma}^i$ have opposite signs the impact of γ on θ_i^* has the same sign as $R_{i\gamma}^i$.*

Intuitively, when we analyze the impact of γ on the equilibrium default probability we need to consider both the direct impact of γ on the critical state of nature, and the indirect effects through the changes in the equilibrium quantities. The sign of the direct effect is straightforward: if γ has a positive impact on profits, then this means that the firm will be able to repay its debt for worse states of the world, $\widehat{z}_i$ decreases, which leads to a decrease in the default probability. However, since for most parameters the impact on the profit and the impact on the marginal profit have the same sign, the indirect effect is ambiguous, as the effect of γ on the equilibrium quantities is ambiguous. Thus, the total effect of increasing γ on the default probability is, in general, ambiguous.

Proposition 2.8 *If $R^i_{iz_i} > 0$, for fixed debt levels, an increase in firm i's parameter α_i causes an increase in the equilibrium default probability of firm i, θ_i^*, if and only if $R^i_i(\widehat{z}_i)V^j_{jj}V^i_{i\alpha_i} - R^i_j(\widehat{z}_i)V^j_{ji}V^i_{i\alpha_i} - R^i_{\alpha_i}\left(V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}\right) > 0$.*

Like before, in order to analyze the impact of changes in α_i on the firm's default probability, we need to consider both the direct effect of α_i on θ_i^* and indirect effects through the equilibrium quantities. Since the indirect effect has an ambiguous sign, the impact of changing α_i on the firm's default probability is, in general, ambiguous. However, since α_i has opposite effects on q_i^* and q_j^*, it seems quite likely that the direct effect dominates as the two effects through the equilibrium quantities tend to cancel each other. If the direct effect dominates the indirect effects and parameter α_i influences positively the profit of firm i, $R^i_{\alpha_i} > 0$, then an increase in α_i leads to a decrease in the default probability θ_i^*.

2.4 Subgame perfect equilibrium

2.4.1 Equilibrium debt levels

In the first stage firms choose simultaneously their debt levels so as to maximize the value of the firm. The value of the firm $Y^i(q_i, q_j, D_{i,}\overline{z}, \gamma, \alpha_i)$, is equal to the sum of the equity value $V^i(q_i, q_j, D_{i,}\overline{z}, \gamma, \alpha_i)$ and the debt value $W^i(q_i, q_j, D_{i,}\overline{z}, \gamma, \alpha_i)$, where the equity value is defined by (2.1) and the debt value is equal:

$$W^i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) = \int_{-\overline{z}}^{\widehat{z}_i(q_i, q_j, D_i, \gamma, \alpha_i)} R^i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i + (1 - F(\widehat{z}_i)) D_i$$

The first term is the value that creditors receive in the worst states of the world (where expected operating profit is not sufficient to meet debt obligations). The second term is the amount received in the good states of the world, $z_i > \widehat{z}_i$.

Considering the equity and debt values, it is easy to show that the value of the firm is equal to its expected operating profits:

$$\begin{aligned} Y^i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) &= V^i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) + W^i(q_i, q_j, D_i, \overline{z}, \gamma, \alpha_i) \\ &= \int_{\widehat{z}_i(q_i, q_j, D_i, \gamma, \alpha_i)}^{\overline{z}} (R^i(q_i, q_j, z_i, \gamma, \alpha_i) - D_i) f(z_i) dz_i + \\ &\quad \int_{-\overline{z}}^{\widehat{z}_i(q_i, q_j, D_i, \gamma, \alpha_i)} R^i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i + (1 - F(\widehat{z}_i)) D_i \\ &= \int_{-\overline{z}}^{\overline{z}} R^i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i \end{aligned}$$

Taking into account the second stage Nash equilibrium, firm i chooses D_i so as to maximize the total value of the firm.

$$\max_{D_i} \int_{-\overline{z}}^{\overline{z}} R^i(q_i^* (D_i, D_j, \cdot), q_j^* (D_i, D_j, \cdot), \cdot) f(z_i) dz_i$$

The first order condition, $Y^i_{D_i} = 0$, is:

$$\left[\int_{-\overline{z}}^{\overline{z}} R_i^i(q_i^*\left(D_i,D_j,\cdot\right),q_j^*\left(D_i,D_j,\cdot\right),\cdot)f(z_i)dz_i\right]\frac{\partial q_i^*}{\partial D_i}+$$
$$\left[\int_{-\overline{z}}^{\overline{z}} R_j^i(q_i^*\left(D_i,D_j,\cdot\right),q_j^*\left(D_i,D_j,\cdot\right),\cdot)f(z_i)dz_i\right]\frac{\partial q_j^*}{\partial D_i}=0$$

which can also be written as:

$$\begin{aligned}
Y_{D_i}^i &= \left[\int_{-\overline{z}}^{\widehat{z}_i} R_i^i(q_i^*\left(D_i,D_j,\cdot\right),q_j^*\left(D_i,D_j,\cdot\right),\cdot)f(z_i)dz_i\right]\frac{\partial q_i^*}{\partial D_i} \\
&\quad+\left[\int_{\widehat{z}_i}^{\overline{z}} R_i^i(q_i^*\left(D_i,D_j,\cdot\right),q_j^*\left(D_i,D_j,\cdot\right),\cdot)f(z_i)dz_i\right]\frac{\partial q_i^*}{\partial D_i} \\
&\quad+\left[\int_{-\overline{z}}^{\overline{z}} R_j^i(q_i^*\left(D_i,D_j,\cdot\right),q_j^*\left(D_i,D_j,\cdot\right),\cdot)f(z_i)dz_i\right]\frac{\partial q_j^*}{\partial D_i}=0
\end{aligned}$$

By the first order condition of the second stage game, the second term is equal to zero. The first term captures the impact of the second stage induced change in q_i on the firm's debt value. Assuming $R_{iz_i}^i > 0$, $R_i^i(z)$ is increasing and we already know that $R_i^i(\widehat{z}) < 0$, hence $R_i^i(z) < 0$ for all $z < \widehat{z}$, which implies that the first term is negative (since $\frac{\partial q_i^*}{\partial D_i} > 0$). A higher D_i induces firm i to choose higher quantity levels in the second stage of the game, which hurts debtholders. The third term is the strategic effect of debt. When firm i increases its debt that induces firm j to reduce its output in the second stage game, $\frac{\partial q_j^*}{\partial D_i} < 0$. The reduction in q_j^* benefits firm i as $R_j^i < 0$. Thus, the strategic effect is positive.

To summarize the subgame perfect Nash equilibrium (SPNE) debt choices are the solution of the system:

$$\begin{cases} Y^i_{D_i} = 0 \\ Y^j_{D_j} = 0 \end{cases} \Leftrightarrow \begin{cases} \left[\int_{-\overline{z}}^{\widehat{z}_i} R^i_i(q^*_i, q^*_j, \cdot) f(z_i) dz_i\right] \frac{\partial q^*_i}{\partial D_i} + \left[\int_{-\overline{z}}^{\overline{z}} R^i_j(q^*_i, q^*_j, \cdot) f(z_i) dz_i\right] \frac{\partial q^*_j}{\partial D_i} = 0 \\ \left[\int_{-\overline{z}}^{\widehat{z}_j} R^j_j(q^*_i, q^*_j, \cdot) f(z_j) dz_j\right] \frac{\partial q^*_j}{\partial D_j} + \left[\int_{-\overline{z}}^{\overline{z}} R^j_i(q^*_i, q^*_j, \cdot) f(z_j) dz_j\right] \frac{\partial q^*_i}{\partial D_j} = 0 \end{cases} \tag{2.7}$$

In order to have a well behaved game, we assume that $Y^i_{D_iD_i} < 0$ (that is the firm's value function is concave in D_i, which implies that the point that satisfies the first order condition is a maximum), that $Y^i_{D_iD_j} < 0$ and $Y^i_{D_iD_i}Y^j_{D_jD_j} - Y^i_{D_iD_j}Y^j_{D_jD_i} > 0$. Let $D^{**}_i(\overline{z}, \gamma, \alpha_i, \alpha_j)$ and $D^{**}_j(\overline{z}, \gamma, \alpha_i, \alpha_j)$ be the solution of this system.[11]

Brander and Lewis (1986) showed that, in the subgame perfect equilibrium, firms choose a positive level of debt. In other words, D^{**}_i and D^{**}_j, are strictly positive.

Let us now analyze the impact of changes in the parameters $\overline{z}$, γ and α_i on the SPNE. We start by analyzing the impact of changes in the uncertainty level:

Lemma 2.9 *If $R^i_{iz_i} > 0$ and firms are symmetric, an increase in the level of uncertainty, $\overline{z}$, causes a change on firm i equilibrium debt level, D^{**}_i, with the same sign as $Y^i_{D_i\overline{z}}$. Thus D^{**}_i increases if and only if the firm's marginal value (with respect to its debt) is increasing with $\overline{z}$. The sign of $Y^i_{D_i\overline{z}}$ is ambiguous.*

It should be highlighted that, although in our general framework one cannot say whether the equilibrium debt levels are decreasing or increasing with the uncertainty level, in the linear demand case, with constant marginal costs, and z uniformly distributed, it has been shown that the equilibrium debt levels are decreasing with the uncertainty level (Franck and Le Pape, 2008; Haan and Toolsema, 2008). Intuitively, when the uncertainty level increases, for given debt levels, firms tend to be more aggressive in the output market, as expected demand conditional on $z_i > \widehat{z}_i$ is higher. Considering this, firms can get the

[11] We use two stars (**) to denote the subgame perfect equilibrium variables' levels so as to distinguish from the notation used for the second stage Nash equilibrium.

same strategic effect with a lower level of debt. Therefore firms act in a more conservative manner in the debt market when uncertainty increases.

Let us now study the impact of changes in the common parameter, γ, on the SPNE.

Lemma 2.10 *If $R^i_{iz_i} > 0$ and firms are symmetric, an increase in the common parameter γ, causes a change on the equilibrium debt level, D_i^{**}, with the same sign as $Y^i_{D_i\gamma}$. Thus D_i^{**} increases if and only if the firm's marginal value of debt increases with γ. The sign of $Y^i_{D_i\gamma}$ is ambiguous.*

For many common parameters, such as the average dimension of the market, the impact of the parameter on profits and on marginal profits are likely to have the same sign. Thus the parameter has an ambiguous influence on the second period market equilibrium, which in turn implies that the impact on the equilibrium debt levels is also ambiguous. However it should be noted that the impact on the equilibrium debt levels is also influenced by the way the parameter affects the firm marginal profit, $R^i_{i\gamma}$, as well as the way it influences the marginal effect of the rival quantity, $R^i_{j\gamma}$. An increase in a parameter with a positive impact on the marginal profits (like the average dimension of the market) is quite likely to lead to higher equilibrium debt levels due to the direct impact of the parameter on the marginal profits of the firm.

Lemma 2.11 *If $R^i_{iz_i} > 0$, an increase in firm i's parameter α_i causes a change on the firm i's equilibrium debt level, D_i^{**}, with the same sign as $Y^i_{D_i\alpha_i}$ and a change on D_j^{**} with the opposite sign of $Y^i_{D_i\alpha_i}$. The sign of $Y^i_{D_i\alpha_i}$ is ambiguous.*

One important feature of the impact of changes in firm i's specific parameter, α_i, is that the impact on the equilibrium debt level of the firm has always the opposite sign of the impact on the equilibrium debt level of the rival firm. In the most likely case, where parameter α_i affects in the same direction the profit and the marginal profit of the firm, the impact of changes of α_i on the second period market equilibrium quantities is ambiguous, which also leads to an ambiguous impact of the parameter on the equilibrium debt levels. However the way the parameter affects the marginal profits is quite important to determine the effect on D_i^{**}. An increase in a parameter with a negative impact on the marginal profits (like the marginal cost of the firm) is quite likely to lead to a lower equilibrium debt level by the firm and to a higher equilibrium debt level by the rival. Thus, it seems likely that a less efficient firm (higher marginal cost) to be more conservative in the debt market (having a smaller equilibrium debt level).

2.4.2 SPNE default probabilities

Considering the SPNE, the equilibrium critical state of nature, $\widehat{z}_i^{**}$, can be obtained by substituting $D_i^{**}(\overline{z},\gamma,\alpha_i,\alpha_j)$ and $D_j^{**}(\overline{z},\gamma,\alpha_i,\alpha_j)$ and the corresponding SPNE quantities in $\widehat{z}_i(q_{i,}q_{j,}D_{i,}\gamma,\alpha_i)$

$$\widehat{z}_i^{**}(\gamma,\alpha_i,\alpha_j,\overline{z}) = \widehat{z}_i(q_i^{**}(D_i^{**},D_j^{**},\gamma,\alpha_i,\alpha_j,\overline{z}),q_j^{**}(D_i^{**},D_j^{**},\gamma,\alpha_i,\alpha_j,\overline{z}),D_i^{**},\gamma,\alpha_i) \quad (2.8)$$

Consequently, the equilibrium default probability is given by:

$$\theta^{**}(\gamma,\alpha_i,\alpha_j,\overline{z}) = \Pr(z_i < \widehat{z}_i^{**}) = \int_{-\overline{z}}^{\widehat{z}_i^{**}(\gamma,\alpha_i,\alpha_j,\overline{z})} f(z_i)dz_i = F\left(\widehat{z}_i^{**}(\gamma,\alpha_i,\alpha_j,\overline{z})\right)$$

Let us analyze the impact of changes in the level of uncertainty $\overline{z}$ and the impact of changes in the parameters γ and α_i on the subgame perfect Nash equilibrium default probabilities.

Proposition 2.12 *If $R_{iz_i}^i > 0$ and firms are symmetric, an increase in the level of uncertainty, $\overline{z}$, has an ambiguous effect on the equilibrium default probability of firm i, θ_i^{**}.*

The impact of the uncertainty level on the default probability can be decomposed on the impact of the uncertainty level on the second period market equilibrium and the impact on the equilibrium debt levels, which in turn influence the second period equilibrium and the default probabilities. By proposition 2.8 the first effect is positive whereas by lemma 2.9 the second effect is ambiguous, which explains the previous result.

It is interesting to notice that if $\frac{\partial D_i^{**}}{\partial \bar{z}} < 0$, the impact of $\bar{z}$ on the default probability may be negative.[12] The fact that there is larger uncertainty leads firms to behave in a more aggressive manner in the output market for fixed debt levels. This effect tends to increase the default probability. However, the greater uncertainty may lead firms to be more conservative in the debt market, thus issuing less debt. A lower debt, lowers the default probability directly and indirectly, through its influence on the second period equilibrium quantities. As a consequence we may obtain a counterintuitive result where more uncertainty leads to lower equilibrium default probabilities. This result is explained by the fact that, firms behave less aggressively in the debt market when uncertainty is higher, which leads to lower equilibrium default probabilities.

Similarly, in our general framework we cannot determine the sign of the impact of γ and α_i on the equilibrium default probability.

Proposition 2.13 *If $R^i_{iz_i} > 0$, an increase in γ has an ambiguous effect on the equilibrium default probability of firm i, θ_i^*. Similarly, an increase in α_i has an ambiguous effect on the equilibrium default probability of firm i, θ_i^*.*

Although we are unable to determine the sign of the effects of changes in γ and α_i on the equilibrium default probabilities, we would like to emphasize the possibility of having counterintuitive results. For instance, a firm with higher marginal costs, for fixed debt and quantities, has a higher probability of default, as profit decreases for all states of nature, which increases the critical state of nature and thus the default probability. However a less efficient firm may also have an incentive to issue less debt in equilibrium, which leads to a less aggressive behavior in the output market and, eventually to a lower default probability.

[12] In the linear demand case, with symmetric firms and constant marginal costs, it has been shown numerically by Frank and Le Pape (2008) and Haan and Toolsema (2008) that increasing uncertainty decreases the equilibrium default probability.

2.5 Conclusion

This essay extends Brander and Lewis (1986) by analyzing the implications of financial structure decisions and output market decisions on the default probability and also by studying the impact of changes in the parameters of the model on the equilibrium. This analysis is done both for the second stage Nash equilibrium (considering the financial structure as given but taking into account the impact on the output market decisions) as well as for the subgame perfect equilibrium (i.e., taking into account the impact on the financial structure decisions as well as on the product market decisions).

By analyzing the second stage of the game we conclude that, under quite reasonable conditions, increasing the level of demand uncertain has a positive effect on the equilibrium quantities; i.e., firms behave in a more aggressive way in the output market. In addition, the impact of changing either common parameters or firm specific parameters on the equilibrium quantities, for fixed debt levels, is generally ambiguous and it depends on how the parameter affects both the profit and the marginal profit. Moreover, changing a firm specific parameter always has effects with opposite signs on the firm and the rival's equilibrium quantities.

The analysis of the impact of changes in the model parameters on the second stage equilibrium quantities revealed the possibility of some non-standard results. For instance, it is possible that an increase in the marginal costs, for fixed debt levels, leads to an increase on the firm's equilibrium quantity, which is the opposite of what happens in standard oligopoly models where the limited liability effect is not considered. The intuition is that higher marginal costs imply that the set of states of the world where the firm is able to repay its debt becomes smaller, which leads the firm to behave in a more aggressive manner in order to maximize the expected equity value.

The analysis of the second stage equilibrium default probabilities also reveals some interesting conclusions. First, the effect of changes in the debt level of a firm on its equilibrium default probability is very likely to be positive. This happens because increasing debt has a positive direct effect on the firm default probability and the positive indirect impact through the increase in the firm's quantity is likely to outweigh the negative indirect impact through the decrease in the rival's quantity. Second, the effect of increasing the debt level of a firm on the equilibrium default probability of the rival firm is ambiguous. The intuition is that an increase in a firm's debt has opposite effects on the two firms equilibrium quantities, which in turn have opposite effects on the rival's equilibrium default probability. Third, we show that increasing the level of demand uncertainty, for fixed debt levels, implies higher default probabilities as firms become more aggressive in the output market. Finally, the impact of changes in the common parameter as well as in the firm specific parameter on the default probabilities is generally ambiguous.

Considering our general framework, the sign of the impact of changes of the parameter values on the subgame perfect equilibrium debt values and default probabilities cannot be determined, which is a somewhat disappointing result. However the direct impact of the parameter on the default probability and the indirect impact of the parameter on the default probabilities through the equilibrium debt levels and the equilibrium quantity levels may not all have the same sign. Consequently, one may obtain unexpected results, when the indirect effects outweigh the direct effect. For instance, a less efficient firm may have a lower probability of default than a more efficient one or default probabilities may be lower in markets with higher uncertainty. Intuitively, although higher marginal costs or higher uncertainty imply higher default risk, for fixed debt and quantity levels, the firm may have an incentive to decrease its debt level, which leads to less aggressive behavior in the output market and a lower default probability.

In order to have a more complete analysis of the equilibrium default probabilities it would be very interesting to extend the current model so as to incorporate default costs as well as the impact of taxes on the analysis. We believe these extensions would provide important insights for empirical work on default risk.

Appendix

Proof of Lemma 2.1. Applying the implicit function theorem to (2.2) we get:

$$\begin{aligned}\frac{\partial \widehat{z}_i}{\partial \gamma} &= -\frac{R^i_\gamma(\widehat{z}_i)}{R^i_{z_i}(\widehat{z}_i)};\\ \frac{\partial \widehat{z}_i}{\partial \alpha_i} &= -\frac{R^i_{\alpha_i}(\widehat{z}_i)}{R^i_{z_i}(\widehat{z}_i)}.\end{aligned}$$

Since $R^i_{z_i} > 0$, $\frac{\partial \widehat{z}_i}{\partial \gamma}$ and $\frac{\partial \widehat{z}_i}{\partial \alpha_i}$ have the opposite signs of $R^i_\gamma(\widehat{z}_i)$ and $R^i_{\alpha_i}(\widehat{z}_i)$, respectively.■

Proof of Lemma 2.2. Applying the implicit function theorem to the system of equations (2.4) that define the Nash equilibrium, we get:

$$\begin{bmatrix} \frac{\partial q_i^*}{\partial \overline{z}} \\ \frac{\partial q_j^*}{\partial \overline{z}} \end{bmatrix} = -\begin{bmatrix} V^i_{ii} & V^i_{ij} \\ V^j_{ji} & V^j_{jj} \end{bmatrix}^{-1} \begin{bmatrix} V^i_{i\overline{z}} \\ V^j_{j\overline{z}} \end{bmatrix}$$

which is equivalent to:

$$\begin{bmatrix} \frac{\partial q_i^*}{\partial \overline{z}} \\ \frac{\partial q_j^*}{\partial \overline{z}} \end{bmatrix} = -\frac{1}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}} \begin{bmatrix} V^j_{jj}V^i_{i\overline{z}} - V^i_{ij}V^j_{j\overline{z}} \\ -V^j_{ji}V^i_{i\overline{z}} + V^i_{ii}V^j_{j\overline{z}} \end{bmatrix}$$

Thus the sign of $\frac{\partial q_i^*}{\partial \overline{z}}$ is:

$$\text{sign}\left(\frac{\partial q_i^*}{\partial \overline{z}}\right) = \text{sign}\left(-\frac{V^j_{jj}V^i_{i\overline{z}} - V^i_{ij}V^j_{j\overline{z}}}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}}\right)$$

We already assumed that $V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji} > 0$. Thus $\frac{\partial q_i^*}{\partial \overline{z}} > 0$ if and only if $V^j_{jj}V^i_{i\overline{z}} - V^i_{ij}V^j_{j\overline{z}} < 0$, which shows the first part of the result.

If we consider a symmetric game and restrict our attention to symmetric Nash equilibria, we have $V^i_{i\overline{z}} = V^j_{j\overline{z}}$. Moreover if $V^j_{jj} < V^i_{ij}$ the sign of $\frac{\partial q_i^*}{\partial \overline{z}}$ is equal to the sign of $V^i_{i\overline{z}}$. However, in a symmetric equilibrium, $V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji} > 0$ implies that $V^j_{jj} < V^i_{ij}$, thus the sign of $\frac{\partial q_i^*}{\partial \overline{z}}$ is equal to the sign of $V^i_{i\overline{z}}$. Looking at the V^i_i function, we see that $\overline{z}$ only appears in the upper integration limit. Thus, by Leibniz rule:

$$V^i_{i\bar{z}} = R^i_i(q_i, q_j, \bar{z}, \gamma, \alpha_i) f(\bar{z})$$

Since $R^i_{iz_i} > 0$, marginal profit (R^i_i) is positive at $\bar{z}$. Hence $V^i_{i\bar{z}}$ is positive and consequently $\frac{\partial q_i}{\partial \bar{z}} > 0.$■

Proof of Lemma 2.3. Once again, if we apply the implicit function theorem to (2.4), we get

$$\begin{bmatrix} \frac{\partial q_i^*}{\partial \gamma} \\ \frac{\partial q_j^*}{\partial \gamma} \end{bmatrix} = - \begin{bmatrix} V^i_{ii} & V^i_{ij} \\ V^j_{ji} & V^j_{jj} \end{bmatrix}^{-1} \begin{bmatrix} V^i_{i\gamma} \\ V^j_{j\gamma} \end{bmatrix}$$

which is equivalent to:

$$\begin{bmatrix} \frac{\partial q_i^*}{\partial \gamma} \\ \frac{\partial q_j^*}{\partial \gamma} \end{bmatrix} = - \frac{1}{V^i_{ii} V^j_{jj} - V^i_{ij} V^j_{ji}} \begin{bmatrix} V^j_{jj} V^i_{i\gamma} - V^i_{ij} V^j_{j\gamma} \\ -V^j_{ji} V^i_{i\gamma} + V^i_{ii} V^j_{j\gamma} \end{bmatrix}$$

Hence the sign of $\frac{\partial q_i^*}{\partial \gamma}$ is:

$$\operatorname{sign}\left(\frac{\partial q_i^*}{\partial \gamma}\right) = \operatorname{sign}\left(- \frac{V^j_{jj} V^i_{i\gamma} - V^i_{ij} V^j_{j\gamma}}{V^i_{ii} V^j_{jj} - V^i_{ij} V^j_{ji}}\right)$$

Thus $\frac{\partial q_i^*}{\partial \gamma}$ has the opposite sign of $V^j_{jj} V^i_{i\gamma} - V^i_{ij} V^j_{j\gamma}$, which proves the first part of the result.

Considering now the case where firms are symmetric, $V^i_{i\gamma} = V^j_{j\gamma}$ and $V^j_{jj} < V^i_{ij}$, the sign of $\frac{\partial q_i^*}{\partial \gamma}$ is the same than the sign of $V^i_{i\gamma}$. Noting that γ appears both in the integrand function and in the lower integration limit and applying Leibniz rule, we get:

$$\begin{aligned} V^i_{i\gamma} &= \int_{\widehat{z}_i}^{\bar{z}} R^i_{i\gamma}(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i - R^i_i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) f(\widehat{z}_i) \frac{\partial \widehat{z}_i^*}{\partial \gamma} \\ V^i_{i\gamma} &= \int_{\widehat{z}_i}^{\bar{z}} R^i_{i\gamma} f(z_i) dz_i + R^i_i(\widehat{z}_i) \frac{R^i_\gamma(\widehat{z}_i)}{R^i_{z_i}(\widehat{z}_i)} f(\widehat{z}_i) \end{aligned}$$

As a consequence the sign of the impact depends both on the effect of γ on profit, R^i_γ, and the impact of γ on marginal profit $R^i_{i\gamma}$. If R^i_γ and $R^i_{i\gamma}$ have the same sign, the two terms will have opposite signs since $R^i_i(\widehat{z}_i) < 0$. Thus, if R^i_γ and $R^i_{i\gamma}$ have the same sign, $V^i_{i\gamma}$ has an ambiguous sign.

On the other hand if R^i_γ and $R^i_{i\gamma}$ have opposite signs, the sign of $V^i_{i\gamma}$ is the same as the sign of $R^i_{i\gamma}$, since the sign of $R^i_i(\widehat{z}_i)R^i_\gamma(\widehat{z}_i)$ is the same than the sign of $R^i_{i\gamma}$ as $R^i_i(\widehat{z}_i) < 0$.■

Proof of Lemma 2.4. By the implicit function theorem we know that:

$$\begin{bmatrix} \frac{\partial q_i^*}{\partial \alpha_i} \\ \frac{\partial q_j^*}{\partial \alpha_i} \end{bmatrix} = - \begin{bmatrix} V^i_{ii} & V^i_{ij} \\ V^j_{ji} & V^j_{jj} \end{bmatrix}^{-1} \begin{bmatrix} V^i_{i\alpha_i} \\ 0 \end{bmatrix}$$

Which is equivalent to:

$$\begin{bmatrix} \frac{\partial q_i^*}{\partial \alpha_i} \\ \frac{\partial q_j^*}{\partial \alpha_i} \end{bmatrix} = - \frac{1}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}} \begin{bmatrix} V^j_{jj}V^i_{i\alpha_i} \\ -V^j_{ji}V^i_{i\alpha_i} \end{bmatrix}$$

Hence the sign of $\frac{\partial q_i^*}{\partial \alpha_i}$ is:

$$\text{sign}\left(\frac{\partial q_i^*}{\partial \alpha_i}\right) = \text{sign}\left(-\frac{V^j_{jj}V^i_{i\alpha_i}}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}}\right) = \text{sign}(V^i_{i\alpha_i})$$

while the sign of $\frac{\partial q_j^*}{\partial \alpha_i}$ is:

$$\text{sign}\left(\frac{\partial q_j^*}{\partial \alpha_i}\right) = \text{sign}\left(\frac{V^j_{ji}V^i_{i\alpha_i}}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}}\right) = \text{sign}(\,-\,V^i_{i\alpha_i})$$

Noting that α_i appears both in the integrand function and in the lower integration limit of V_i^i and applying Leibniz rule, we get:

$$
\begin{aligned}
V_{i\alpha_i}^i &= \int_{\widehat{z}_i}^{\overline{z}} R_{i\alpha_i}^i(q_i, q_j, z_i, \gamma, \alpha_i) f(z_i) dz_i - R_i^i(q_i, q_j, \widehat{z}_i, \gamma, \alpha_i) f(\widehat{z}_i) \frac{\partial \widehat{z}_i^*}{\partial \alpha_i} \\
V_{i\alpha_i}^i &= \int_{\widehat{z}_i}^{\overline{z}} R_{i\alpha_i}^i f(z_i) dz_i + R_i^i(\widehat{z}_i) \frac{R_{\alpha_i}^i}{R_{z_i}^i(\widehat{z}_i)} f(\widehat{z}_i)
\end{aligned}
$$

As a consequence the sign of the impact depends both on the effect of α_i on profit, $R_{\alpha_i}^i$, and the impact of α_i on marginal profit $R_{i\alpha_i}^i$. If $R_{\alpha_i}^i$ and $R_{i\alpha_i}^i$ have the same sign, the total effect of α_i on V_i^i will be ambiguous as $R_i^i(\widehat{z}_i) < 0$. On the other hand if $R_{\alpha_i}^i$ and $R_{i\alpha_i}^i$ have opposite signs, the sign of $V_{i\alpha_i}^i$ is the same as the sign of $R_{i\alpha_i}^i$, since the sign of $R_i^i(\widehat{z}_i) R_{\alpha_i}^i(\widehat{z}_i)$ is the same than the sign of $R_{i\alpha_i}^i$ as $R_i^i(\widehat{z}_i) < 0$.■

Proof of Preposition 2.1. By Leibniz rule $\frac{\partial \theta_i^*}{\partial D_i}$ is given by:

$$
\begin{aligned}
\frac{\partial \theta_i^*}{\partial D_i} &= f(\widehat{z}_i^*) \frac{\partial \widehat{z}_i^*}{\partial D_i} \\
\frac{\partial \theta_i^*}{\partial D_j} &= f(\widehat{z}_i^*) \frac{\partial \widehat{z}_i^*}{\partial D_j}
\end{aligned}
$$

Since $f(\widehat{z}_i^*) > 0$ the sign of these derivatives are equal to the sign of $\frac{\partial \widehat{z}_i^*}{\partial D_i}$ and $\frac{\partial \widehat{z}_i^*}{\partial D_j}$, respectively. Applying the chain rule to (2.6) we get:

$$
\frac{\partial \theta_i^*}{\partial D_i} = f(\widehat{z}_i^*) \left(\frac{\partial \widehat{z}_i}{\partial q_i^*} \frac{\partial q_i^*}{\partial D_i} + \frac{\partial \widehat{z}_i}{\partial q_j^*} \frac{\partial q_j^*}{\partial D_i} + \frac{\partial \widehat{z}_i}{\partial D_i} \right) \tag{2.9}
$$

$$
\frac{\partial \theta_i^*}{\partial D_j} = f(\widehat{z}_i^*) \left(\frac{\partial \widehat{z}_i}{\partial q_i^*} \frac{\partial q_i^*}{\partial D_j} + \frac{\partial \widehat{z}_i}{\partial q_j^*} \frac{\partial q_j^*}{\partial D_j} \right) \tag{2.10}
$$

These expressions clearly indicate that the total impact of D_i on θ_i^* includes a direct effect, given by $f(\widehat{z}_i^*) \frac{\partial \widehat{z}_i}{\partial D_i}$, and indirect effects through the influence of D_i on the equilibrium quantities which in turn affect $\widehat{z}_i$. On the other hand, D_j does not influence $\widehat{z}_i^*$ directly but it has indirect impacts as it affects the equilibrium quantities. Considering the signs of the partial derivatives computed before, we can immediately see that the first term and the third term in (2.9) are positive while the second term is negative. Similarly, in (2.10) the first term is negative while the second term is positive. Thus we need to investigate which effect dominates.

$$\begin{aligned}\frac{\partial \theta_i^*}{\partial D_i} &= f(\widehat{z}_i^*)\left(\frac{R_i^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{jj}^j V_{iD_i}^i}{V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j} - \frac{R_j^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{ji}^j V_{iD_i}^i}{V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j} + \frac{1}{R_{z_i}^i(\widehat{z}_i)}\right)\\ \frac{\partial \theta_i^*}{\partial D_j} &= f(\widehat{z}_i^*)\left(-\frac{R_i^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{ij}^i V_{jD_j}^j}{V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j} + \frac{R_j^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{ii}^i V_{jDj}^j}{V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j}\right)\end{aligned}$$

The sign of $\frac{\partial \theta_i^*}{\partial D_i}$is positive if $R_i^i(\widehat{z}_i)V_{jj}^j V_{iD_i}^i + V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j > R_j^i(\widehat{z}_i)V_{ji}^j V_{iD_i}^i$. Since $V_{jj}^j < V_{ji}^j$, $V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j > 0$ and $V_{iD_i}^i > 0$, the previous condition is likely to be satisfied. A sufficient condition, for $\frac{\partial \theta_i^*}{\partial D_i}$ to be positive is $R_i^i(\widehat{z}_i)V_{jj}^j > R_j^i(\widehat{z}_i)V_{ji}^j$. If this condition holds, an increase in the debt of firm i increases the default probability of firm i.

On the other hand, the sign of $\frac{\partial \theta_i^*}{\partial D_j}$is positive if and only if $R_j^i(\widehat{z}_i)V_{ii}^i > R_i^i(\widehat{z}_i)V_{ij}^i$.■

Proof of Preposition 2.2. The impact of changes in the level of uncertainty on the default probability is (applying the chain rule to (2.6)):

$$\frac{\partial \theta_i^*}{\partial \overline{z}} = f(\widehat{z}_i^*)\frac{\partial \widehat{z}_i^*}{\partial \overline{z}} = f(\widehat{z}_i^*)\left[\frac{\partial \widehat{z}_i}{\partial q_i^*}\frac{\partial q_i^*}{\partial \overline{z}} + \frac{\partial \widehat{z}_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial \overline{z}}\right]$$

We have shown before that, if firms are symmetric, $\frac{\partial q_i^*}{\partial \overline{z}}$ and $\frac{\partial q_j^*}{\partial \overline{z}}$ are both positive and we also know that $\frac{\partial \widehat{z}_i}{\partial q_i^*} > 0$ and $\frac{\partial \widehat{z}_i}{\partial q_j^*} > 0$. Thus, if firms are symmetric $\frac{\partial \theta_i^*}{\partial \overline{z}}$ is positive.■

Proof of Preposition 2.3. When parameter γ changes, the impact on the default probability is:

$$\frac{\partial \theta_i^*}{\partial \gamma} = f(\widehat{z}_i^*)\frac{\partial \widehat{z}_i^*}{\partial \gamma} = f(\widehat{z}_i^*)\left[\frac{\partial \widehat{z}_i}{\partial q_i^*}\frac{\partial q_i^*}{\partial \gamma} + \frac{\partial \widehat{z}_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial \gamma} + \frac{\partial \widehat{z}_i}{\partial \gamma}\right]$$

These expression clearly indicates that the total impact of γ on θ_i^* includes a direct effect, given by $f(\widehat{z}_i^*)\frac{\partial \widehat{z}_i}{\partial \gamma}$, and indirect effects through the influence of γ on the equilibrium quantities which in turn affect $\widehat{z}_i$. The previous expression can be written as follows:

$$\frac{\partial\theta_i^*}{\partial\gamma} = f(\widehat{z}_i^*)\left[\frac{R_i^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{jj}^jV_{i\gamma}^i - V_{ij}^iV_{j\gamma}^j}{V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j} + \frac{R_j^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{ii}^iV_{j\gamma}^j - V_{ji}^jV_{i\gamma}^i}{V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j} - \frac{R_\gamma^i}{R_{z_i}^i(\widehat{z}_i)}\right]$$

Since $R_{z_i}^i(\widehat{z}_i) > 0$ and $V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j > 0$ one concludes immediately that $\frac{\partial\theta_i^*}{\partial\gamma} > 0$ if and only if

$$R_i^i(\widehat{z}_i)\left(V_{jj}^jV_{i\gamma}^i - V_{ij}^iV_{j\gamma}^j\right) + R_j^i(\widehat{z}_i)\left(V_{ii}^iV_{j\gamma}^j - V_{ji}^jV_{i\gamma}^i\right) - R_\gamma^i\left(V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j\right) > 0.$$

The rest of the result is a direct consequence of Lemma 2.3 and the fact that the direct impact has the opposite sign of R_γ^i.■

Proof of Preposition 2.4. The impact of changes in α_i is given by:

$$\frac{\partial\theta_i^*}{\partial\alpha_i} = f(\widehat{z}_i^*)\frac{\partial\widehat{z}_i^*}{\partial\alpha_i} = f(\widehat{z}_i^*)\left[\frac{\partial\widehat{z}_i}{\partial q_i^*}\frac{\partial q_i^*}{\partial\alpha_i} + \frac{\partial\widehat{z}_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial\alpha_i} + \frac{\partial\widehat{z}_i}{\partial\alpha_i}\right]$$

The total impact of α_i on θ_i^* includes a direct effect, given by $f(\widehat{z}_i^*)\frac{\partial\widehat{z}_i}{\partial\alpha_i}$, and indirect effects through the influence of α_i on the equilibrium quantities (q_i^* and q_j^*) which in turn affect $\widehat{z}_i$. Substituting the results that were obtained previously:

$$\frac{\partial\theta_i^*}{\partial\alpha_i} = f(\widehat{z}_i^*)\left[\frac{R_i^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{jj}^jV_{i\alpha_i}^i}{V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j} - \frac{R_j^i(\widehat{z}_i)}{R_{z_i}^i(\widehat{z}_i)}\frac{V_{ji}^jV_{i\alpha_i}^i}{V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j} - \frac{R_{\alpha_i}^i}{R_{z_i}^i(\widehat{z}_i)}\right]$$

Since $R_{z_i}^i(\widehat{z}_i) > 0$ and $V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j > 0$ one concludes immediately that $\frac{\partial\theta_i^*}{\partial\alpha_i} > 0$ if and only if $R_i^i(\widehat{z}_i)V_{jj}^jV_{i\alpha_i}^i - R_j^i(\widehat{z}_i)V_{ji}^jV_{i\alpha_i}^i - R_{\alpha_i}^i\left(V_{ii}^iV_{jj}^j - V_{ij}^iV_{ji}^j\right) > 0$.■

Proof of Lemma 2.5. Applying the implicit function theorem to equation (2.7) which defines the subgame perfect Nash equilibrium, we get:

$$\begin{bmatrix} \frac{\partial D_i^{**}}{\partial\overline{z}} \\ \frac{\partial D_j^{**}}{\partial\overline{z}} \end{bmatrix} = -\begin{bmatrix} Y_{D_iD_i}^i & Y_{D_iD_j}^i \\ Y_{D_jD_i}^j & Y_{D_jD_j}^j \end{bmatrix}^{-1}\begin{bmatrix} Y_{D_i\overline{z}}^i \\ Y_{D_j\overline{z}}^j \end{bmatrix}$$

which is equivalent to:

$$\begin{bmatrix} \frac{\partial D_i^{**}}{\partial \overline{z}} \\ \frac{\partial D_j^{**}}{\partial \overline{z}} \end{bmatrix} = -\frac{1}{Y^i_{D_iD_i}Y^j_{D_jD_j} - Y^i_{D_iD_j}Y^j_{D_jD_i}} \begin{bmatrix} Y^j_{D_jD_j}Y^i_{D_i\overline{z}} - Y^i_{D_iD_j}Y^j_{D_j\overline{z}} \\ -Y^j_{D_jD_i}Y^i_{D_i\overline{z}} + Y^i_{D_iD_i}Y^j_{D_j\overline{z}} \end{bmatrix}$$

Let us evaluate the signs of these derivatives, taking into account that $Y^i_{D_iD_i} < 0$, $Y^j_{D_jD_j} < 0$, $Y^i_{D_iD_j} < 0$ and $Y^i_{D_iD_i}Y^j_{D_jD_j} - Y^i_{D_iD_j}Y^j_{D_jD_i} > 0$. Assuming that $\left|Y^j_{D_jD_j}\right| > \left|Y^i_{D_iD_j}\right|$ and considering that firms are symmetric, $Y^i_{D_i\overline{z}} = Y^j_{D_j\overline{z}}$, the sign of $\frac{\partial D_i^{**}}{\partial \overline{z}}$ is:

$$\text{sign}\left(\frac{\partial D_i^{**}}{\partial \overline{z}}\right) = \text{sign}\left(-\frac{Y^j_{D_jD_j}Y^i_{D_i\overline{z}} - Y^i_{D_iD_j}Y^j_{D_j\overline{z}}}{Y^i_{D_iD_i}Y^j_{D_jD_j} - Y^i_{D_iD_j}Y^j_{D_jD_i}}\right) = \text{sign}(Y^i_{D_i\overline{z}})$$

Applying Leibniz rule, $Y^i_{D_i\overline{z}}$ is given by (we need to consider all the impacts of $\overline{z}$ on Y_{D_i} except the ones through D_i and D_j):

$$\left[\int_{-\overline{z}}^{\overline{z}} R^i_{ii}(q_i^*, q_j^*, \cdot)\frac{\partial q_i^*}{\partial \overline{z}} f(z_i)dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{ij}(q_i^*, q_j^*, \cdot)\frac{\partial q_j^*}{\partial \overline{z}} f(z_i)dz_i + R^i_i(\overline{z})f(\overline{z}) - R^i_i(-\overline{z})f(-\overline{z})\right]\frac{\partial q_i^*}{\partial D_i} +$$
$$\left[\int_{-\overline{z}}^{\overline{z}} R^i_{ji}(q_i^*, q_j^*, \cdot)\frac{\partial q_i^*}{\partial \overline{z}} f(z_i)dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{jj}(q_i^*, q_j^*, \cdot)\frac{\partial q_j^*}{\partial \overline{z}} f(z_i)dz_i + R^i_j(\overline{z})f(\overline{z}) - R^i_j(-\overline{z})f(-\overline{z})\right]\frac{\partial q_j^*}{\partial D_i} +$$
$$\left[\int_{-\overline{z}}^{\overline{z}} R^i_i(q_i^*, q_j^*, \cdot)f(z_i)dz_i\right]\frac{\partial^2 q_i^*}{\partial D_i \partial \overline{z}} + \left[\int_{-\overline{z}}^{\overline{z}} R^i_j(q_i^*, q_j^*, \cdot)f(z_i)dz_i\right]\frac{\partial^2 q_j^*}{\partial D_i \partial \overline{z}}$$

Consider the expression inside the first parentheses. Since $R^i_{ii} < 0$ and $\frac{\partial q_i^*}{\partial \overline{z}} > 0$, the first term is negative. Similarly, since $R^i_{ij} < 0$ and $\frac{\partial q_j^*}{\partial \overline{z}} > 0$, the second term is also negative. However, the third term, $R^i_i(\overline{z})f(\overline{z}) - R^i_i(-\overline{z})f(-\overline{z})$, is positive. Thus the sign of the expression inside the first parentheses is ambiguous (note that this expression is multiplied by $\frac{\partial q_i^*}{\partial D_i} > 0$). Similarly, the sign of the expression inside the second parentheses is also ambiguous as the two first terms are negative whereas the last term is positive (note that this expression is multiplied by $\frac{\partial q_j^*}{\partial D_i} < 0$). Finally, the sign of the last two terms in the expression is also not clear as the terms inside parentheses are negative but the sign of $\frac{\partial^2 q_i^*}{\partial D_i \partial \overline{z}}$ and $\frac{\partial^2 q_j^*}{\partial D_i \partial \overline{z}}$ are not known. Therefore, without further restrictions, the impact of $\overline{z}$ on the equilibrium debt levels is ambiguous.■

Proof of Lemma 2.6. Applying the implicit function theorem to equation (2.7) we get:

$$\begin{bmatrix} \frac{\partial D_i^{**}}{\partial \gamma} \\ \frac{\partial D_j^{*}}{\partial \gamma} \end{bmatrix} = - \begin{bmatrix} Y^i_{D_i D_i} & Y^i_{D_i D_j} \\ Y^j_{D_j D_i} & Y^j_{D_j D_j} \end{bmatrix}^{-1} \begin{bmatrix} Y^i_{D_i \gamma} \\ Y^j_{D_j \gamma} \end{bmatrix}$$

which is equivalent to:

$$\begin{bmatrix} \frac{\partial D_i^{**}}{\partial \gamma} \\ \frac{\partial D_j^{*}}{\partial \gamma} \end{bmatrix} = - \frac{1}{Y^i_{D_i D_i} Y^j_{D_j D_j} - Y^i_{D_i D_j} Y^j_{D_j D_i}} \begin{bmatrix} Y^j_{D_j D_j} Y^i_{D_i \gamma} - Y^i_{D_i D_j} Y^j_{D_j \gamma} \\ -Y^j_{D_j D_i} Y^i_{D_i \gamma} + Y^i_{D_i D_i} Y^j_{D_j \gamma} \end{bmatrix}$$

Taking into account that $Y^i_{D_i D_i} < 0$, $Y^j_{D_j D_j} < 0$, $Y^i_{D_i D_j} < 0$, $Y^i_{D_i D_i} Y^j_{D_j D_j} - Y^i_{D_i D_j} Y^j_{D_j D_i} > 0$, $\left| Y^j_{D_j D_j} \right| > \left| Y^i_{D_i D_j} \right|$ and considering that firms are symmetric, $Y^i_{D_i \gamma} = Y^j_{D_j \gamma}$, the sign of $\frac{\partial D_i^{**}}{\partial \gamma}$ is:

$$\text{sign}\left(\frac{\partial D_i^{**}}{\partial \gamma} \right) = \text{sign}\left(- \frac{Y^j_{D_j D_j} Y^i_{D_i \gamma} - Y^i_{D_i D_j} Y^j_{D_j \gamma}}{Y^i_{D_i D_i} Y^j_{D_j D_j} - Y^i_{D_i D_j} Y^j_{D_j D_i}} \right) = \text{sign}(Y^i_{D_i \gamma})$$

Where $Y^i_{D_i \gamma}$ is given by

$$\begin{aligned}
&\left[\int_{-\overline{z}}^{\overline{z}} R^i_{ii}(q_i^*, q_j^*, \cdot) \frac{\partial q_i^*}{\partial \gamma} f(z_i) dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{ij}(q_i^*, q_j^*, \cdot) \frac{\partial q_j^*}{\partial \gamma} f(z_i) dz_i \right] \frac{\partial q_i^*}{\partial D_i} + \\
&\left[\int_{-\overline{z}}^{\overline{z}} R^i_{ji}(q_i^*, q_j^*, \cdot) \frac{\partial q_i^*}{\partial \gamma} f(z_i) dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{jj}(q_i^*, q_j^*, \cdot) \frac{\partial q_j^*}{\partial \gamma} f(z_i) dz_i \right] \frac{\partial q_j^*}{\partial D_i} + \\
&\left[\int_{-\overline{z}}^{\overline{z}} R^i_{i}(q_i^*, q_j^*, \cdot) f(z_i) dz_i \right] \frac{\partial^2 q_i^*}{\partial D_i \partial \gamma} + \left[\int_{-\overline{z}}^{\overline{z}} R^i_{j}(q_i^*, q_j^*, \cdot) f(z_i) dz_i \right] \frac{\partial^2 q_j^*}{\partial D_i \partial \gamma} + \\
&\int_{-\overline{z}}^{\overline{z}} R^i_{i\gamma}(q_i^*\left(D_i, D_j, \cdot\right), q_j^*\left(D_i, D_j, \cdot\right), \cdot) f(z_i) dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{j\gamma}(q_i^*\left(D_i, D_j, \cdot\right), q_j^*\left(D_i, D_j, \cdot\right), \cdot) f(z_i) dz_i
\end{aligned}$$

In the symmetric case $\frac{\partial q_i^*}{\partial \gamma} = \frac{\partial q_j^*}{\partial \gamma}$ and the sign of the expressions inside the first and the second parentheses have the opposite sign of $\frac{\partial q_i^*}{\partial \gamma}$ (note that the first expression is multiplied by $\frac{\partial q_i^*}{\partial D_i} > 0$ whereas the second expression is multiplied by $\frac{\partial q_j^*}{\partial D_i} < 0$, thus the first and second line have opposite signs). By lemma 2.3 we know that if γ affects profits and marginal profits in the same direction (R_γ^i and $R_{i\gamma}^i$ have the same sign) then $\frac{\partial q_i^*}{\partial \gamma}$ has an ambiguous sign. This implies that the sign of $Y_{D_i\gamma}^i$ is also ambiguous. In addition notice that the sign of $Y_{D_i\gamma}^i$ is also influenced by the sign of $R_{i\gamma}^i$ and the sign of $R_{j\gamma}^i$. If R_γ^i and $R_{i\gamma}^i$ have opposite sign, the sign of $\frac{\partial q_i^*}{\partial \gamma}$ is the same sign as $R_{i\gamma}^i$. Thus the sign of the expression inside the first parentheses has the opposite sign of $R_{i\gamma}^i$. However the sign of the second line and also the sign of the penultimate term is the same sign as $R_{i\gamma}^i$. Thus the sign of $Y_{D_i\gamma}^i$ is ambiguous.■

Proof of Lemma 2.7. Applying the implicit function theorem to equation (2.7), we get:

$$\begin{bmatrix} \frac{\partial D_i^{**}}{\partial \alpha_i} \\ \frac{\partial D_j^*}{\partial \alpha_i} \end{bmatrix} = - \begin{bmatrix} Y_{D_iD_i}^i & Y_{D_iD_j}^i \\ Y_{D_jD_i}^j & Y_{D_jD_j}^j \end{bmatrix}^{-1} \begin{bmatrix} Y_{D_i\alpha_i}^i \\ 0 \end{bmatrix}$$

which is equivalent to:

$$\begin{bmatrix} \frac{\partial D_i^{**}}{\partial \alpha_i} \\ \frac{\partial D_j^*}{\partial \alpha_i} \end{bmatrix} = - \frac{1}{Y_{D_iD_i}^i Y_{D_jD_j}^j - Y_{D_iD_j}^i Y_{D_jD_i}^j} \begin{bmatrix} Y_{D_jD_j}^j Y_{D_i\alpha_i}^i \\ -Y_{D_jD_i}^j Y_{D_i\alpha_i}^i \end{bmatrix}$$

The signs of these derivatives, taking into account that $Y_{D_jD_j}^j < 0$ and $Y_{D_iD_i}^i Y_{D_jD_j}^j - Y_{D_iD_j}^i Y_{D_jD_i}^j > 0$ is given by:

$$\begin{aligned} \text{sign}\left(\frac{\partial D_i^{**}}{\partial \alpha_i}\right) &= \text{sign}\left(-\frac{Y_{D_jD_j}^j Y_{D_i\alpha_i}^i}{Y_{D_iD_i}^i Y_{D_jD_j}^j - Y_{D_iD_j}^i Y_{D_jD_i}^j}\right) = \text{sign}\left(Y_{D_i\alpha_i}^i\right) \\ \text{sign}\left(\frac{\partial D_j^{**}}{\partial \alpha_i}\right) &= \text{sign}\left(\frac{Y_{D_jD_i}^j Y_{D_i\alpha_i}^i}{Y_{D_iD_i}^i Y_{D_jD_j}^j - Y_{D_iD_j}^i Y_{D_jD_i}^j}\right) = \text{sign}\left(-Y_{D_i\alpha_i}^i\right) \end{aligned}$$

Where $Y^i_{D_i\alpha_i}$ is given by:

$$\left[\int_{-\overline{z}}^{\overline{z}} R^i_{ii}(q^*_i, q^*_j, \cdot)\frac{\partial q^*_i}{\partial \alpha_i} f(z_i)dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{ij}(q^*_i, q^*_j, \cdot)\frac{\partial q^*_j}{\partial \alpha_i} f(z_i)dz_i\right]\frac{\partial q^*_i}{\partial D_i} +$$
$$\left[\int_{-\overline{z}}^{\overline{z}} R^i_{ji}(q^*_i, q^*_j, \cdot)\frac{\partial q^*_i}{\partial \alpha_i} f(z_i)dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{jj}(q^*_i, q^*_j, \cdot)\frac{\partial q^*_j}{\partial \alpha_i} f(z_i)dz_i\right]\frac{\partial q^*_j}{\partial D_i} +$$
$$\left[\int_{-\overline{z}}^{\overline{z}} R^i_{i}(q^*_i, q^*_j, \cdot) f(z_i)dz_i\right]\frac{\partial^2 q^*_i}{\partial D_i \partial \alpha_i} + \left[\int_{-\overline{z}}^{\overline{z}} R^i_{j}(q^*_i, q^*_j, \cdot) f(z_i)dz_i\right]\frac{\partial^2 q^*_j}{\partial D_i \partial \alpha_i} +$$
$$\int_{-\overline{z}}^{\overline{z}} R^i_{i\alpha_i}(q^*_i\left(D_i, D_j, \cdot\right), q^*_j\left(D_i, D_j, \cdot\right), \cdot) f(z_i)dz_i + \int_{-\overline{z}}^{\overline{z}} R^i_{j\alpha_i}(q^*_i\left(D_i, D_j, \cdot\right), q^*_j\left(D_i, D_j, \cdot\right), \cdot) f(z_i)dz_i$$

As proved in lemma 2.4, $\frac{\partial q^*_i}{\partial \alpha_i}$ has the same sign of $V^i_{i\alpha_i}$ and $\frac{\partial q^*_j}{\partial \alpha_i}$ has the opposite sign of $V^i_{i\alpha_i}$. Moreover the sign of $V^i_{i\alpha_i}$ is ambiguous if $R^i_{\alpha_i}$ and $R^i_{i\alpha_i}$ have the same sign, otherwise $V^i_{i\alpha_i}$ has the same sign as $R^i_{i\alpha_i}$. When $R^i_{\alpha_i}$ and $R^i_{i\alpha_i}$ have the same sign, the sign of $Y^i_{D_i\alpha_i}$ is ambiguous as the effect of the parameter α_i on firm i and firm j equilibrium quantities is ambiguous (note that the first and second line have opposite signs). If $R^i_{\alpha_i}$ and $R^i_{i\alpha_i}$ have opposite sign, the sign of $\frac{\partial q^*_i}{\partial \alpha_i}$ is the same than the sign of $R^i_{i\alpha_i}$. The two terms of the expression inside the first parentheses have opposite signs, but the sign of the expression is the opposite sign of $R^i_{i\alpha_i}$ as $\left|\frac{\partial q^*_i}{\partial \alpha_i}\right| > \left|\frac{\partial q^*_j}{\partial \alpha_i}\right|$. However the sign of the second line and also the sign of the penultimate term is the same sign as $R^i_{i\alpha_i}$. Thus the sign of $Y^i_{D_i\alpha_i}$ is ambiguous.■

Proof of Preposition 2.5. The impact of changes in the level of uncertainty on the SPNE default probability is:

$$\frac{\partial \theta^{**}_i}{\partial \overline{z}} = f(\widehat{z}^{**}_i)\frac{\partial \widehat{z}^{**}_i}{\partial \overline{z}}$$

where $\frac{\partial \widehat{z}^{**}_i}{\partial \overline{z}}$ is equivalent to (applying the chain rule to (2.8)):

$$\left(\frac{\partial q^{**}_i}{\partial D^{**}_i}\frac{\partial D^{**}_i}{\partial \overline{z}} + \frac{\partial q^{**}_i}{\partial D^{**}_j}\frac{\partial D^{**}_j}{\partial \overline{z}} + \frac{\partial q^{**}_i}{\partial \overline{z}}\right)\frac{\partial \widehat{z}_i}{\partial q^{**}_i} + \left(\frac{\partial q^{**}_j}{\partial D^{**}_i}\frac{\partial D^{**}_i}{\partial \overline{z}} + \frac{\partial q^{**}_j}{\partial D^{**}_j}\frac{\partial D^{**}_j}{\partial \overline{z}} + \frac{\partial q^{**}_j}{\partial \overline{z}}\right)\frac{\partial \widehat{z}_i}{\partial q^{**}_j} + \frac{\partial \widehat{z}_i}{\partial D^{**}_i}\frac{\partial D^{**}_i}{\partial \overline{z}}$$

The previous expression indicates that increasing the uncertainty has several effects on the default probability. On the one hand, increasing the uncertainty has a direct impact on the second period equilibrium quantities, which affects $\widehat{z}_i$:

$$\frac{\partial q_i^{**}}{\partial \overline{z}}\frac{\partial \widehat{z}_i}{\partial q_i^{**}}+\frac{\partial q_j^{**}}{\partial \overline{z}}\frac{\partial \widehat{z}_i}{\partial q_j^{**}}$$

By proposition 2.8 we know that this direct effect leads to an increase in the probability of default. On the other hand, an increase in the uncertainty level affects the equilibrium debt levels, which in turn affect the second period equilibrium quantities and the equilibrium critical state:

$$\left(\frac{\partial q_i^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \overline{z}}+\frac{\partial q_i^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \overline{z}}\right)\frac{\partial \widehat{z}_i}{\partial q_i^{**}}+\left(\frac{\partial q_j^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \overline{z}}+\frac{\partial q_j^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \overline{z}}\right)\frac{\partial \widehat{z}_i}{\partial q_j^{**}}+\frac{\partial \widehat{z}_i}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \overline{z}}$$

By lemma 2.9 the signs of $\frac{\partial D_i^{**}}{\partial \overline{z}}$ and $\frac{\partial D_j^{**}}{\partial \overline{z}}$ are ambiguous. Thus $\overline{z}$ also has an ambiguous effect on θ_i^{**}.■

Proof of Preposition 2.6. The impact of γ and α_i on the equilibrium default probability is given by:

$$\begin{aligned}\frac{\partial \theta_i^{**}}{\partial \gamma} &= f(\widehat{z}_i^{**})\frac{\partial \widehat{z}_i^{**}}{\partial \gamma}\\ \frac{\partial \theta_i^{**}}{\partial \alpha_i} &= f(\widehat{z}_i^{**})\frac{\partial \widehat{z}_i^{**}}{\partial \alpha_i}\end{aligned}$$

where $\frac{\partial \widehat{z}_i^{**}}{\partial \gamma}$ and $\frac{\partial \widehat{z}_i^{**}}{\partial \alpha_i}$are given by (applying the chain rule to (2.8)):

$$\begin{aligned}
\frac{\partial \widehat{z}_i^{**}}{\partial \gamma} &= \left(\frac{\partial q_i^{**}}{\partial \gamma} + \frac{\partial q_i^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \gamma} + \frac{\partial q_i^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \gamma}\right)\frac{\partial \widehat{z}_i}{\partial q_i^{**}} + \\
&\left(\frac{\partial q_j^{**}}{\partial \gamma} + \frac{\partial q_j^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \gamma} + \frac{\partial q_j^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \gamma}\right)\frac{\partial \widehat{z}_i}{\partial q_j^{**}} + \frac{\partial \widehat{z}_i}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \gamma} + \frac{\partial \widehat{z}_i}{\gamma} \\
\frac{\partial \widehat{z}_i^{**}}{\partial \alpha_i} &= \left(\frac{\partial q_i^{**}}{\partial \alpha_i} + \frac{\partial q_i^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \alpha_i} + \frac{\partial q_i^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \alpha_i}\right)\frac{\partial \widehat{z}_i}{\partial q_i^{**}} + \\
&\left(\frac{\partial q_j^{**}}{\partial \alpha_i} + \frac{\partial q_j^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \alpha_i} + \frac{\partial q_j^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \alpha_i}\right)\frac{\partial \widehat{z}_i}{\partial q_j^{**}} + \frac{\partial \widehat{z}_i}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \alpha_i} + \frac{\partial \widehat{z}_i}{\alpha_i}
\end{aligned}$$

Each of the previous expressions can be rewritten as so as to separate the direct impact of the parameter on the second period equilibrium quantities and default probability, and the impact through the equilibrium debt levels, which in turn influence the second period equilibrium. For instance, $\frac{\partial \widehat{z}_i^{**}}{\partial \gamma}$ can be written as:

$$\left(\frac{\partial q_i^{**}}{\partial \gamma}\frac{\partial \widehat{z}_i}{\partial q_i^{**}} + \frac{\partial q_j^{**}}{\partial \gamma}\frac{\partial \widehat{z}_i}{\partial q_j^{**}} + \frac{\partial \widehat{z}_i}{\gamma}\right) + \left(\frac{\partial q_i^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \gamma} + \frac{\partial q_i^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \gamma}\right)\frac{\partial \widehat{z}_i}{\partial q_i^{**}} +$$
$$\left(\frac{\partial q_j^{**}}{\partial D_j^{**}}\frac{\partial D_j^{**}}{\partial \gamma} + \frac{\partial q_j^{**}}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \gamma}\right)\frac{\partial \widehat{z}_i}{\partial q_j^{**}} + \frac{\partial \widehat{z}_i}{\partial D_i^{**}}\frac{\partial D_i^{**}}{\partial \gamma}.$$

The analysis of the expressions presented above, allows us to conclude that the effects of the parameters γ and α_i on the equilibrium default probability are ambiguous, both because the direct impact on the equilibrium quantities is ambiguous and because the impact on the equilibrium debt levels is also ambiguous.■

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Chapter 3

Default Costs, Financial and Product Market Decisions and Default Risk

3.1 Introduction

In the last decades, the financial literature has addressed the issue of default and default risk. Considering its negative social and economic impact on the economy, it is not surprising that the existing literature has focused mainly on the best form to predict default and default risk (for a survey of the empirical literature see Balcaen and Ooghe, 2006). However, despite of the vast empirical literature, there is a lack of theoretical models aimed at understanding the factors that influence the default probability. The main objective of this essay is to provide a contribution in this direction.

The essay examines analytically and numerically, how the market structure influences financial decisions and product market decisions and, consequently, the default risk, considering the existence of default costs. Our objective is to study the impact of changes in the level of demand uncertainty, in the degree of product substitutability, in the asymmetry between the two firms' marginal production costs and in the direct (ex-post) and indirect (ex-ante) default costs' parameters on the equilibrium default risk of the two firms. Furthermore we also aim to analyze if the impact of the various parameters is the same when firms have equal marginal production costs (symmetric duopoly) or when they have different marginal costs (asymmetric duopoly).

The link between financing and output market decisions began to emerge with the pioneering work of Modigliani and Miller (1958).[1] In their framework there is no relationship between financial structure and output market decisions. Theories of capital structure that followed (the trade-off theory and the pecking order theory) support the existence of an optimal capital structure, but they do not incorporate in their analysis the interdependence of financing decisions and output market decisions.

Riordan's (2003) critical survey summarizes the existing literature on the interaction between capital structure and output market. He argues that the existence of a link between the financial structure and output market decisions has been highlighted both on the Corporate Finance literature and the Industrial Organization literature and begins to emerge in the 80's. Brander and Lewis (1986) were the first to examine this relationship. They consider a two stage Cournot duopoly model with an uncertain environment. In the first stage, each firm decides the capital structure. In the second stage, taking into account their previously chosen financial structure, firms take their decisions in the output market. The model focuses on the effects of the limited liability in debt financing. They assume that the investment decision is taken before the capital structure decision. If this assumption was not made, the debt-equity mix choice would influence the investment which would have further effects on the output market.[2] As pointed out by Brander and Lewis (1986) one possible interpretation of the capital structure choice is that the firm is initially equity financed, when the loan is taken the borrowed money is fully distributed among shareholders. The authors conclude that debt tends to encourage a more aggressive behavior by the indebted firm in the output market, while the competitor

[1] The authors argue (see their propositions I and II, where they consider an economy without taxes) that in a perfect capital market, the capital structure is irrelevant in determining the firm value, the important thing is the value created by the assets. In 1963 Modigliani and Miller, restated propositions I and II considering an economy with taxes, they claim that a firm reaches its maximum value when fully indebted as it is when it gets the maximum tax benefit.

[2] This happens in Clayton (2009) where the investment is made to reduce the marginal cost of production.

tends to produce less. Thus, firms have an incentive to use their financial structure for strategic purposes. Maksimovic (1988) confirms the findings of Brander and Lewis (1986) regarding the aggressiveness of indebted firms in the output market and argues that this effect is due to the existence of limited liability. Nevertheless the author considered a model with multiple periods of interaction and shows that debt is a barrier for firms to be able to maintain collusive outcomes.[3]

One of the criticisms directed at the previously mentioned work, is the fact that it does not consider the agency problems arising between creditors and shareholders. This issue is considered in Clayton (2009) who shows that when firms have an investment option, leverage leads to a less aggressive output competition behavior and this is due to the existence of agency problems. Similarly, Grimaud (2000) follows the formalization of Brander and Lewis (1986), but incorporates the choice of the financial contract as a strategic variable. According to the author, the existence of asymmetric information, between borrowers and lenders, has an important role in the relationship between financial decisions and the output market decisions. Grimaud (2000) shows that the increase in debt leads to a more aggressive behavior but this is offset by the financial costs.

Despite the vast literature on default probability, the existing literature is constituted essentially by prediction models. In other words, there is a lack of theoretical models to explain default probability. The default probability depends not only on the level of debt, but also on operational factors that allows a firm to meet its obligations. The relationship between the financial structure decisions, the output market decisions and the default probability has been analyzed theoretically by a small number of authors. Franck and Le Pape (2008) and Haan and Toolsema (2008) are among these few authors. While Brander and Lewis (1986) present a general model, without specifying whether products are homogenous or differentiated and whether uncertainty affects demand or costs, Franck and Le Pape (2008) and Haan and Toolsema (2008) have explored more specific models and used numerical simulations to analyze the impact of demand uncertainty and the de-

[3]Some empirical papers, namely Campos (2000), Erol (2003) and Lyandres (2006) confirm that debt incites a more aggressive behavior. Others, like Chevalier (1995b), Khanna and Tice (2000) and Zingales (1998) argue that indebted firms tend to adopt a more conservative behavior.

gree of product substitutability on the probability of default. The authors come to similar conclusions: the probability of default is decreasing with the degree of product complementarity (when goods are complements) and it is increasing with the degree of product substitutability (when the goods are substitutes). Moreover, the default probability is decreasing with the level of uncertainty. Although these results are quite interesting, they ignore the existence of default costs and do not consider the possibility of firms having different degrees of efficiency. In this essay, our aim is to extend their results by incorporating these two very important aspects of reality.

The existing literature that relates financial and output market decisions usually ignores the existence of default costs. However these costs can have a substantial effect on the financial structure and output market decisions and they may constrain the firms' behavior (making it more or less conservative). The literature divides default costs into two types: direct or ex-post costs (legal, accounting and administrative costs) and indirect or ex-ante costs (reduced profits resulting from lower sales, in particular, due to the reputation effect).

The relevance of default costs was first addressed by Kraus and Litzenburger's (1973), who analyze the trade-off between default costs and tax benefits associated with debt. Elkami, Ericsson and Parsons (2012) argue that the financial distress costs can offset the debt tax benefits. Altman (1984) concludes that default costs may represent about 20% of the value of assets and the indirect costs can be high (other authors argue that indirect costs are irrelevant). Kwansa and Cho (1995) report the importance of considering indirect default cost since they affect the capital structure decision. Among the studies that analyze the link between financial and output market decisions, Brander and Lewis (1988) and Parsons (1997) take into account the default costs. Brander and Lewis (1988) show that the default costs affect financial and output market decisions. The authors present two alternative ways of modelling default costs: (i) fixed default costs and (ii) proportional default costs. They conclude that firms tend to be more aggressive when more vulnerable. Parsons (1997) introduces a set of specifications that are not considered by Brander and Lewis (1988). He shows that firms tend to adopt a more conservative behavior as debt increases.

There are empirical studies that confirm that there is a relationship between debt and default probability (Chacharat *et al*, 2010) and between debt, quantity or price, market conditions (higher or lower concentration, industry performance) and default probability (Borenstein and Rose, 1995, Evrensel, 2008 and Opler and Titman, 1994). The impact of debt and market conditions on default probability is not so clear-cut.

The essay aims, using a two-stage duopoly model to analyze numerically and analytically, how the default risk changes with the demand uncertainty, the degree of product substitutability and the asymmetry in the marginal costs of the two firms, considering the existence of default costs. This is done in two stages: in the first stage, each firm decides the capital structure and in the second stage, taking into account their previously chosen financial structure, firms take their decisions in the output market. The main contribution of this work for the literature that explores theoretically the equilibrium default risk is the analysis of the case where the two firms have different levels of efficiency and the introduction of default costs in the analysis.

The remainder of the chapter is organized as follows. In the next section we present the model. Section 3.3 analyzes the second stage of the game and the subgame perfect equilibrium. Section 3.4 presents the results in the benchmark case of no default costs whereas section 3.5 presents the results when there are default costs. Finally, section 3.6 summarizes the main conclusions of the study.

3.2 Model

This study considers a particular case of Brander and Lewis (1986) model, where the duopolists produce differentiated products, demand is linear, marginal costs are constant and the uncertainty in the model is on the demand side.[4] In the first stage each firm (firm i and firm j) decides the financial structure, i.e., the level of debt and equity in the

[4] A similar model has been considered by other authors, such as Haan and Toolsema (2008) and Wanzenried (2003). The differences are that we do not assume that firms have the same marginal cost and we introduce default costs in the analysis.

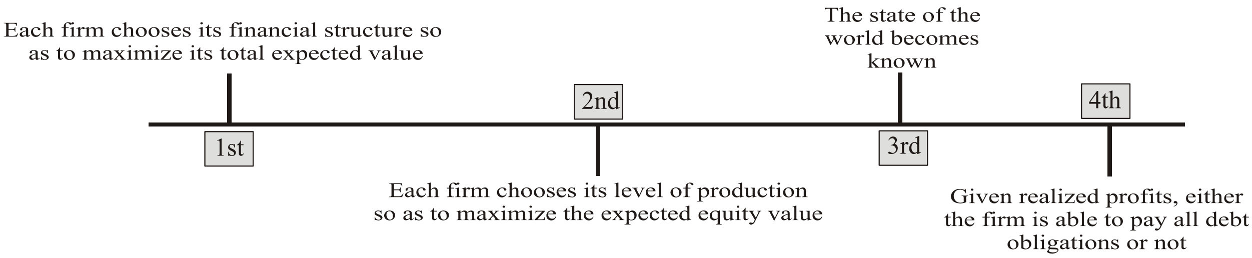


Figure 3.1: Timing of the game: first financial decisions are taken, then output decisions are taken. Output decisions are taken before the uncertainty is resolved.

capital structure. In the second stage each firm chooses the quantity to produce. Figure 3.1 shows the timing of the game.

The demand functions are derived from the solution of the consumer's problem. Following Dixit (1979) and Singh and Vives (1984), we assume that the consumer utility function is quadratic:

$$U(q_i, q_j, q_0) = q_0 + (\alpha_i + z_i) q_i + (\alpha_j + z_j) q_j - \frac{1}{2}\left[\beta_i q_i^2 + 2\gamma q_i q_j + \beta_j q_j^2\right] \tag{3.1}$$

where q_i and q_j are the quantities consumed of firm i and firm j products, respectively, and q_0 represents the quantity consumed of all the other products (with a price normalized to unit). The parameter γ, with $\gamma \in [0, 1]$, corresponds to the degree of substitutability between the two firms products. When $\gamma = 0$, products are completely differentiated, thus each firm can behave as a monopolist. When $\gamma = 1$, the two products are perfect substitutes. Parameter α_i and α_j represent the expected size of the market and α_i , α_j , β_i and β_j are positive constants. The observed size of the market depends on the random variable z_i that represents the effect of an exogenous demand shock, in other words, there is uncertainty regarding the size of the market. It is assumed that this variable is distributed in the interval $[-\overline{z}, \overline{z}]$ according to the uniform density function, i.e., $f(z_i) = \frac{1}{2\overline{z}}$. We assume that z_i and z_j are independent and identically distributed. The random variable z_i represents the uncertainty in the output market, i.e., the deviation from the average market demand (this deviation can be positive or negative). To simplify notation we assume that $\beta_i = \beta_j = 1$.

Let p_i and p_j be the prices of firm i and firm j products, respectively and let M be the consumer's income. The consumer chooses q_i, q_j and q_0 so as to maximize U subject to $p_i q_i + p_j q_j + q_0 = M$. The budget constraint can be written as $q_0 = M - p_i q_i - p_j q_j$. Substituting this expression in the utility function, the consumer's problem can be rewritten as:

$$\max_{q_i, q_j} \; M - p_i q_i - p_j q_j + (\alpha_i + z_i) q_i + (\alpha_j + z_j) q_j - \frac{1}{2}\left[q_i^2 + 2\gamma q_i q_j + q_j^2\right]$$

The first order conditions of this problem are:

$$\left\{\begin{array}{l} -p_i + \alpha_i + z_i - q_i - \gamma q_j = 0 \\ -p_j + \alpha_j + z_j - q_j - \gamma q_i = 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} p_i = \alpha_i + z_i - q_i - \gamma q_j \\ p_j = \alpha_j + z_j - q_j - \gamma q_i \end{array}\right.$$

The last system of equations, gives us the inverse demand functions of the two firms. We assume that:

$$\alpha_i = (1 - a_i \theta_i^2)\alpha$$

where θ_i represents the expected default probability and a_i is a parameter that measures the demand sensitivity to increases in the default probability. This assumption is intended to capture the ex-ante default costs. Everything else constant, if consumers expect firm i to have a higher default probability, their demand of firm i product decreases. In other words, when default is likely and consumers are aware of that, there is a negative effect on the image that customers have about the firm, therefore causing a loss of reputation and a decrease in demand. In our formulation, the reputational demand reduction is increasing with the probability of default. Altman (1984) and Kim (1978) argue that the indirect default costs occur when a potential buyer perceives that default is likely, we believe that our way of modelling the ex-ante default cost is consistent with their definition.

Under the previous assumption, the inverse demand is given by:

$$p_i = (1 - a_i \theta_i^2)\alpha - q_i - \gamma q_j + z_i \tag{3.2}$$

Hence, the operating profits (revenue less costs) are given by:

$$R_i = \left((1 - a_i\theta_i^2)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i \tag{3.3}$$

where c_i is the constant marginal cost of firm i.

In the first stage, firms simultaneously choose their debt levels so as to maximize the value of the firm. We represent the debt obligation of firm i by D_i. Note that D_i is the amount that firm i pays at the end of the game to bondholders, if operating profits are high enough to do so. If the realized operating profits are lower than D_i, all the operating profits obtained will be used to pay bondholders, who become the residual claimants. Let $\widehat{z}_i$ be the critical state of the world such that the operating profit of the firm is just enough for the firm to meet its debt obligations. This critical state of the world is implicitly defined by:

$$\left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i - D_i = 0 \tag{3.4}$$

The default probability of firm i is given by:

$$\theta_i = \Pr(R_i < D_i) = \Pr\left(z_i < \widehat{z}_i\right) = \int_{-\overline{z}}^{\widehat{z}_i} f(z_i)dz_i = \int_{-\overline{z}}^{\widehat{z}_i} \frac{1}{2\overline{z}}dz_i$$

which is equivalent to:

$$\theta_i = \frac{\widehat{z}_i - (-\overline{z})}{2\overline{z}} = \frac{\widehat{z}_i + \overline{z}}{2\overline{z}}$$

The value of indirect default cost, IDC_i, is given by:

$$IDC_i = a_i\theta_i^2\alpha q_i$$

In the second stage of the game the manager maximizes the expected equity value, which is given by:

$$V^i = \int_{\widehat{z}_i}^{\overline{z}} \left(\left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i - D_i\right) \frac{1}{2\overline{z}} dz_i \tag{3.5}$$

$$V^i = \int_{\widehat{z}_i}^{\overline{z}} \left(\left(\left(1 - a_i\left(\frac{\widehat{z}_i + \overline{z}}{2\overline{z}}\right)^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i - D_i\right) \frac{1}{2\overline{z}} dz_i$$

It should be noted that the critical state of the world is influenced by the quantity choices of the two firms and by the firm's debt level. This implies that $\widehat{z}_i(q_{i,}q_{j,}D_i)$ is determined endogenously in the second stage of the game.

When the firm cannot meet its debt obligations in all the states of the world, it will have a positive probability of default and thus positive default costs (direct and indirect). If there are default costs, these are deducted from the expected value for creditors. Considering the default probability and the default costs, the expected value of debt is given by:

$$W_i = \Pr(z_i > \widehat{z}_i)D_i + \int_{-\overline{z}}^{\widehat{z}_i} \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i \frac{1}{2\overline{z}} dz_i - E(DDC) \tag{3.6}$$

Note that D_i is different from W_i. D_i is the amount that firm i promised to pay at the end of the game to bondholders, which includes capital amortization and interest. W_i is the expected value of debt, which takes into account the probability of the firm not paying in full D_i, i.e., if this probability is positive $W_i < D_i$. $E(DDC)$ represents the expected direct default costs.[5] We assume that direct default costs are proportional to the amount that the bondholders fail to receive.[6] Let k be the direct costs proportionality

[5] Direct costs are costs that occur ex-post, which increase with default probability and they are supported primarily by debtholders because in bad states of nature, the remaining profit (after deducting default costs) will be used to pay them. This is supported by Kraus and Litzenberger (1973), Kim (1978), Brander and Lewis(1988), Parsons (1997) and Bris, Schwartz and Welch (2005).

[6] We follow the same formalization as Brander and Lewis (1988) and Parsons (1997).

parameter.

$$E(DDC) = k\int_{-\overline{z}}^{\widehat{z}_i^*} \left(D_i - \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i\right) \frac{1}{2\overline{z}} dz_i$$

Therefore, the expected value of debt is given by:

$$W_i = (1-\theta_i)D_i + (1+k)\int_{-\overline{z}}^{\widehat{z}_i} \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i \frac{1}{2\overline{z}} dz_i - kD_i\theta_i \quad (3.7)$$

Note that the interest rate r is defined implicitly by $W_i(1+r) = D_i$.

As mentioned before, in the first stage of the game the firms choose their debt levels so as to maximize the value of the firm. The value of the firm is equal to the sum of the expected equity value and the expected value of debt:

$$\begin{aligned} Y^i(q_i, q_j, D_{i,}\overline{z}) &= V^i(q_i, q_j, D_{i,}\overline{z}) + W^i(q_i, q_j, D_{i,}\overline{z}) \quad (3.8) \\ &= \int_{\widehat{z}_i}^{\overline{z}} \left(\left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i - D_i\right) \frac{1}{2\overline{z}} dz_i + \\ &\quad (1-\theta_i)D_i + (1+k)\int_{-\overline{z}}^{\widehat{z}_i} \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i \frac{1}{2\overline{z}} dz_i - kD_i\theta_i \end{aligned}$$

The above expression can be written as:

$$\begin{aligned} Y^i(q_i, q_j, D_i, \overline{z}) &= \int_{-\overline{z}}^{\overline{z}} \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i \frac{1}{2\overline{z}} dz_i - \quad (3.9) \\ &\quad k\int_{-\overline{z}}^{\widehat{z}_i} \left(D_i - \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i\right) \frac{1}{2\overline{z}} dz_i \end{aligned}$$

Thus the value of the firm is equal to the expected profits (which incorporate the indirect default costs) minus the expected direct default costs.

The welfare is given by:

$$
\begin{aligned}
Wel \; = \; & \int_{-\overline{z}}^{\overline{z}}\int_{-\overline{z}}^{\overline{z}} \left(\begin{array}{c} M + \left(\left(1 - a_i\theta_i^2\right)\alpha + z_i\right) q_i + \left(\left(1 - a_j\theta_j^2\right)\alpha + z_j\right) q_j \\ -\frac{1}{2}\left[q_i^2 + 2\gamma q_i q_j + q_j^2\right] - c_i q_i - c_j q_j \end{array} \right) \frac{1}{4\overline{z}^2} dz_i dz_j + \\
& -k \int_{-\overline{z}}^{\widehat{z}_i} \left(D_i - \left(\left(1 - a_i\theta_i^2\right)\alpha - q_i - \gamma q_j + z_i - c_i\right) q_i\right) \frac{1}{2\overline{z}} dz_i \\
& -k \int_{-\overline{z}}^{\widehat{z}_j} \left(D_j - \left(\left(1 - a_j\theta_j^2\right)\alpha - q_j - \gamma q_i + z_j - c_j\right) q_j\right) \frac{1}{2\overline{z}} dz_j
\end{aligned} \tag{3.10}
$$

To better follow the model resolution, table 3.1 summarizes the variables used.

Table 3.1: Variables of the model

Variables	Meaning
q_i, q_j	Output of firms i and j
D_i, D_j	Debt obligation of firms i and j
R^i	Operating profits of firm i
z_i	Random variable that represents the uncertainty
$\widehat{z}_i$	Critical value of z_i
γ	Degree of substitutability
M	Consumer's income
α	Expected size of the market
a_i, a_j	Demand sensitivity (ex-ante default cost parameter) of firms i and j
c_i, c_j	Marginal cost of production of firms i and j
θ_i, θ_j	Default probability of firms i and j
k	Direct default costs proportionality parameter (ex-post default cost parameter)
DDC_i, DDC_j	Value of direct default cost of firms i and j
IDC_i, IDC_j	Value of indirect default cost of firms i and j
Y^i, Y^j	Firm value of firms i and j
V^i, V^j	Expected equity value of firms i and j
W^i, W^j	Expected value of debt of firms i and j
Wel	Welfare

3.3 Solving the model

Our game is a dynamic game with two stages, thus to determine the equilibrium financial and output decisions we need to solve the game using the concept of subgame perfect Nash equilibrium (SPNE). The game is solved backwards, that is, one starts by determining the Nash equilibrium in the second stage of the game as a function of the debt levels chosen by the firms in the first stage. Then we solve the first stage game. In this stage firms make their financial decisions, taking into account their impact on the output market equilibrium, so as to maximize the value of the firm, thus determining, the SPNE.

3.3.1 Nash equilibrium in the second stage of the game

In the second stage of the game, firm i chooses its quantity, q_i, so as to maximize the equity value (3.5). Using the Leibniz rule, the first-order condition of this maximization problem is:

$$\int_{\widehat{z}_i}^{\overline{z}} \left[\left(1-a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}}\right)^2\right)\alpha-2q_i-2a_i\alpha\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}}\right)q_i\frac{\partial\theta_i}{\partial q_i}-\gamma q_j+z_i-c_i\right]\frac{1}{2\overline{z}}dz_i -$$
$$\left(\left(1-a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}}\right)^2\right)\alpha-q_i-\gamma q_j+\widehat{z}_i-c_i)q_i-D_i\right)\frac{1}{2\overline{z}}\frac{\partial\widehat{z}_i}{\partial q_i}dz=0$$

However, taking into account the definition of $\widehat{z}_i$, the second term is equal to zero. Thus, the first-order condition is given by:

$$\int_{\widehat{z}_i}^{\overline{z}} \left[\left(1-a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}}\right)^2\right)\alpha-2q_i-2a_i\alpha\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}}\right)q_i\frac{\partial\theta_i}{\partial q_i}-\gamma q_j+z_i-c_i\right]\frac{1}{2\overline{z}}dz_i=0$$

where

$$\frac{\partial\theta_i}{\partial q_i}=\frac{1}{2\overline{z}}\frac{\partial\widehat{z}_i}{\partial q_i}=\frac{1}{2\overline{z}}\frac{\left(1-a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}}\right)^2\right)\alpha-2q_i-\gamma q_j+\widehat{z}_i-c_i}{\left(a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}^2}\right)\alpha-1\right)q_i}$$

After integrating, the first order condition can be written as:

$$2\left(\left(1-a_i\theta_i^2\right)\alpha-2q_i-2a_i\alpha\left(\frac{\widehat{z}_i+\overline{z}}{4\overline{z}^2}\right)\frac{\left(1-a_i\theta_i^2\right)\alpha-2q_i-\gamma q_j+\widehat{z}_i-c_i}{a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}^2}\right)\alpha-1}-\gamma q_j-c_i\right)+\overline{z}+\widehat{z}_i=0$$

The first order condition for firm j is derived in a similar manner. Note that the first order conditions depend on the critical states of the world, $\widehat{z}_i$ and $\widehat{z}_j$, which in turn depend on q_i and q_j. This implies that, in order to get the Nash equilibrium of the second stage game, we need to simultaneously solve the system of the two first order conditions and the two conditions that define the critical states of the world. In other words, for an interior solution (i.e., for $-\overline{z}<\widehat{z}_i<\overline{z}$), the Nash equilibrium is given by the solution of the following system:

$$\begin{cases} 2\left(\left(1-a_i\theta_i^2\right)\alpha-2q_i-2a_i\alpha\left(\frac{\widehat{z}_i+\overline{z}}{4\overline{z}^2}\right)\frac{\left(1-a_i\theta_i^2\right)\alpha-2q_i-\gamma q_j+\widehat{z}_i-c_i}{a_i\left(\frac{\widehat{z}_i+\overline{z}}{2\overline{z}^2}\right)\alpha-1}-\gamma q_j-c_i\right)+\overline{z}+\widehat{z}_i=0 \\ 2\left(\left(1-a_j\theta_j^2\right)\alpha-2q_j-2a_j\alpha\left(\frac{\widehat{z}_j+\overline{z}}{4\overline{z}^2}\right)\frac{\left(1-a_j\theta_j^2\right)\alpha-2q_j-\gamma q_i+\widehat{z}_j-c_j}{a_i\left(\frac{\widehat{z}_j+\overline{z}}{2\overline{z}^2}\right)\alpha-1}-\gamma q_j-c_i\right)+\overline{z}+\widehat{z}_j=0 \\ \left(1-a_i\theta_i^2\right)\alpha-q_i-\gamma q_j+\widehat{z}_i-c_i)q_i-D_i=0 \\ \left(1-a_i\theta_j^2\right)\alpha-q_j-\gamma q_i+\widehat{z}_j-c_i)q_j-D_j=0 \end{cases}$$

This 4 equations system is equivalent to solving a polynomial equation of the fourth order, which does not have a simple analytical solution. In fact, through substitution, it can be shown that solving this system is equivalent to solving a polynomial equation of the fourth order, which does not have a simple analytical solution.

It should be noted that, for some values of (D_i, D_j) one or both of the critical states of the world may be equal to $-\overline{z}$ or equal to $\overline{z}$. In these cases the third and/or fourth conditions need to be substituted by $\widehat{z}_i=-\overline{z}$ or $\widehat{z}_i=\overline{z}$. A complete analysis of all the possible Nash equilibria involves computing these corner solutions.

We developed a GAUSS code (presented in the Appendix) to solve the model numerically. Considering the various types of possible equilibria, we ran simulations for many values of the parameters γ and $\overline{z}$ for the symmetric and asymmetric duopoly cases, so as to analyze how the equilibrium changes with the parameter values. After that we analyze the equilibria for many values of the parameter k and $a_i = a_j$ and for fixed k values for many values of a_i and a_j, considering fixed values of γ and $\overline{z}$. For each set of parameter values, we determine the Nash Equilibrium of the second stage game, for many possible combinations of the debt levels (D_i, D_j). Let $q_i^*(D_i, D_j)$, $q_j^*(D_i, D_j)$, $\widehat{z}_i^*(D_i, D_j)$ and $\widehat{z}_j^*(D_i, D_j)$ be the Nash equilibrium quantities and critical states of the world for given debt levels (D_i, D_j).

3.3.2 Subgame Perfect Nash equilibrium

After computing the Nash equilibrium of the second stage game as a function of the debt levels chosen by the firms in the first stage we solved the first stage game using backwards induction. In this stage, firms take their financial decisions, taking into account their impact on the output market equilibrium, so as to maximize the value of the firm, thus determining the Subgame Perfect Nash Equilibrium (SPNE).

As mentioned above, we developed a GAUSS code to solve the model numerically, considering the various types of possible equilibria, for many values of the parameters γ, $\overline{z}$, k, a_i and a_j (so as to analyze how the equilibrium changes with the parameter values). The program first determines the Nash Equilibrium of the second stage game, for given debt levels (D_i, D_j), and then for each (D_i, D_j) the equilibrium value of each firm (Y_i, Y_j), is computed. This is repeated for many (D_i, D_j) and the equilibrium values of Y_i and Y_j are saved in two matrices. The equilibrium of the first stage game is then determined. We identify, for a given debt level of the other firm, the firm's level of debt that maximizes its value, thus determining the firm's best response. The Nash equilibrium of the debt game occurs when we find a vector (D_i^{**}, D_j^{**}) in such a way that the two firms are simultaneously in their best responses. Thus (D_i^{**}, D_j^{**}) denotes the SPNE levels of debt. Finally, considering (D_i^{**}, D_j^{**}) the corresponding SPNE quantities (q_i^{**}, q_j^{**}) of the second stage game are computed as well as other equilibrium variables like the default probabilities $\left(\theta_i^{**}, \theta_j^{**}\right)$,the equilibrium interest rate (r_i^{**}, r_j^{**}) and so on.

The default probability is given by:

$$\theta_i^{**} = \frac{\widehat{z}_i^{**} + \overline{z}}{2\overline{z}} \tag{3.11}$$

Calculating the integral in expression (3.7), the SPNE expected value of debt is given by:

$$\begin{aligned} W_i^{**} &= (1-\theta_i^{**})\, D_i^{**} - k\theta_i^{**} D_i^{**} + \\ & (1+k)\frac{1}{2\overline{z}}\left(\widehat{z}_i^{**} + \overline{z}\right)\left(\left(\left(1 - a_i\theta_i^{**2}\right)\alpha - q_i^{**} - \gamma q_j^{**} - c_i\right) q_i^{**} + \frac{q_i^{**}}{2}\left(\widehat{z}_i^{**} - \overline{z}\right)\right) \end{aligned} \tag{3.12}$$

The interest rate r_i is defined implicitly by $W_i(1+r_i) = D_i$ so, in the SPNE:

$$r_i^{**} = \frac{D_i^{**}}{W_i^{**}} - 1 \tag{3.13}$$

Calculating the integral in expression (3.5), the SPNE expected value of equity is given by:

$$V_i^{**} = \frac{1}{2\overline{z}}\left(\overline{z} - \widehat{z}_i^{**}\right)\left(\left(\left(1 - a_i\theta_i^{**2}\right)\alpha - q_i^{**} - \gamma q_j^{**} - c_i\right) q_i^{**} - D_i^{**} + \frac{q_{i^{**}}}{2}\left(\overline{z} + \widehat{z}_i^{**}\right)\right) \tag{3.14}$$

Similarly, calculating the integral in expression (3.10), the SPNE expected welfare level is given by:

$$\begin{aligned} Wel^{**} &= M + \left(\left(1 - a_i\theta_i^{**2}\right)\alpha\right) q_i^{**} + \left(\left(1 - a_j\theta_j^{**2}\right)\alpha\right) q_j^{**} - \frac{1}{2}\left[q_i^{**2} + 2\gamma q_i^{**} q_j^{**} + q_j^{**2}\right] \\ & -c_i q_i^{**} - c_j q_j^{**} - k\theta_i^{**}\left(D_i^{**} - \left(\left(1 - a_i\theta_i^{**2}\right)\alpha - q_i^{**} - \gamma q_j^{**} - c_i\right) q_i^{**} + \frac{q_i^{**}}{2}\left(\overline{z} - \widehat{z}_i^{**}\right)\right) \\ & -k\theta_j^{**}\left(D_j^{**} - \left(\left(1 - a_j\theta_j^{**2}\right)\alpha - q_j^{**} - \gamma q_i^{**} - c_j\right) q_j^{**} + \frac{q_j^{**}}{2}\left(\overline{z} - \widehat{z}_j^{**}\right)\right) \end{aligned} \tag{3.15}$$

The value of direct and indirect default costs are given by:

$$IDC_i^{**} = a_i\theta_i^{**2}\alpha q_i^{**} \tag{3.16}$$

$$E(DDC_i^{**}) = k\theta_i^{**}\left(D_i^{**} - \left(\left(1 - a_i\theta_i^{**2}\right)\alpha - q_i^{**2} - \gamma q_j^{**} - c_i\right)q_i^{**}\right) + \frac{q_i^{**}}{2}\left(\overline{z} - \widehat{z}_i^{**}\right) \quad (3.17)$$

There are certain combinations of the parameters γ and $\overline{z}$ that originate multiple equilibria. Such situation occurs mainly for γ close to 0 and $\overline{z} < 0.8$, so we do not consider these parameter values. In the analytical resolution we need to set values of some variables (α, M, $\overline{z}_{\max}$). Since we are interested in isolating the impact of debt and output market decisions on the default probability we chose parameter values that imply positive operating profits even in the worst state of the world. This assumption implies that a firm would never go bankrupt if it was fully equity financed. We chose to use $\alpha = 5$, $M = 10$ and $\overline{z}_{\max} = 2$.

The next two sections present the results of the numerical simulations. Our study is focused on the equilibrium of the whole game (the subgame perfect equilibrium).[7] The next section presents the results assuming there is no default costs and compares the results in a symmetric duopoly with the results when the two firms have different marginal costs. The results in this section can be interpreted as a benchmark case which can be used for comparison with the case where default costs are considered. Section 3.5 presents the results of the model with default costs and once again both the symmetric duopoly and asymmetric duopoly cases are analyzed. In both sections, we study the impact of changes in the level of demand uncertainty ($\overline{z}$) and in the degree of product differentiation (γ) on the SPNE values of the endogenous variables.

3.4 Results without default costs

In this section we analyze the SPNE of the game considering that the default cost parameters are all nil; i.e., $k = a_i = a_j = 0$. In this setup we first analyze how the equilibrium

[7] It should be noted that one could also analyze the Nash equilibrium of the second stage, which is contingent on the debt levels chosen in the first stage of the game, and study how it changes with the level of debt.

values of the variables change with the uncertainty level, $\overline{z}$, and with the degree of product substitutability, γ, in a duopoly model where the two firms are equally efficient (symmetric duopoly). Next we analyze how the equilibrium values of the variables change when firms differ in their marginal production costs i.e., when we have an asymmetric duopoly model. In addition, we explore how the equilibrium values change with a unilateral increase in the marginal production cost of firm j. In other words we analyze what happens as the asymmetry between the two firms efficiency levels increases.

3.4.1 Symmetric duopoly

In this subsection we consider a duopoly model where the two firms are equally efficient and assume that the marginal costs are equal to zero, i.e. $c_i = c_j = 0$.[8] The objective in this subsection is to study how the equilibrium values of the variables change with the uncertainty level, $\overline{z}$, and with the degree of product substitutability, γ. The analysis was performed for values of $\overline{z} \in [0.8; 2]$ and $\gamma \in]0; 1]$. The figures presented in this section show the impacts in a three dimensions graph (on the left) and in a two dimensions graph (on the right). In the two dimensional graph, γ is represented in the x-axis and $\overline{z}$ is represented by four different curves, where the lower demand uncertainty level is represented with dots and the higher demand uncertainty level with a continuous line. It should be noted that the model studied in this subsection is the same than the one analyzed by Franck's and Le Pape's (2008) and Haan and Toolsema (2008). Therefore, it is not surprising that we obtain the same qualitative results.

Figure 3.2 shows the equilibrium levels of debt as a function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. This figure allow us to conclude the following:

Result 3.1 *The SPNE level of debt obligations, D^{**}, is strictly positive and decreasing with the level of demand uncertainty, $\overline{z}$. On the other hand, the degree of product substitutability does not have a monotonic impact on D^{**}. For small values of demand*

[8] The use of null or positive constant marginal costs does not influence the qualitative results obtained.

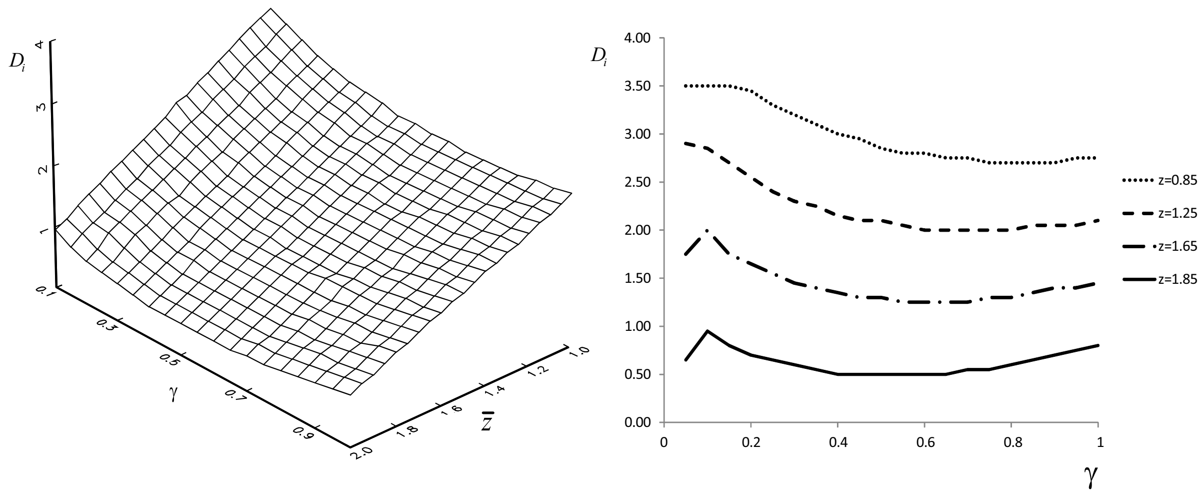


Figure 3.2: SPNE debt obligation as a function of the degree of product substitubility and the level of demand uncertainty without considering the default costs

uncertainty, D^{**} is decreasing with product substitutability. However, for higher values of demand uncertainty, D^{**} initially increases with γ but after a certain point follows a U relationship with γ.

Figure 3.3 shows the equilibrium output level as a function of the degree of product substitutability, γ and as a function of the level of demand uncertainty, $\overline{z}$. The figure allows us to conclude the following:

Result 3.2 *The SPNE level output, q^{**}, is decreasing with the degree of product substitutability, γ and increasing with the level of demand uncertainty $\overline{z}$. However, the impact of demand uncertainty is relatively small.*

To explain the effect of increasing the demand uncertainty level, $\overline{z}$, on the output levels, one needs to consider two effects. The first one is the direct impact that increasing uncertainty has on the equilibrium quantities, for a given debt level. The second one, is the indirect impact through the changes in the equilibrium debt level, which in turn influences the equilibrium quantity levels. The direct impact of increasing the level of uncertainty is positive. In other words, for fixed debt level, firms have a more aggressive behavior in the output market when uncertainty is higher. Intuitively, the increase in

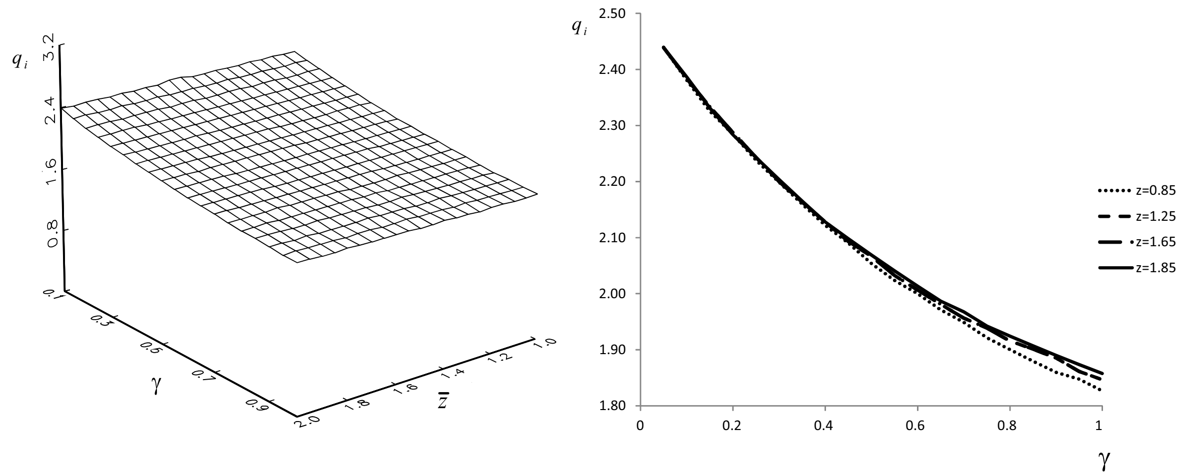


Figure 3.3: SPNE output level as a function of the degree of product substitutability and the level of demand uncertainty without considering the default costs

the uncertainty level implies that there are more good states of the world with positive marginal profits. Thus the expected marginal profits conditional on $z_i > \widehat{z}_i$ increases, hence it is optimal to produce a higher quantity. It is noteworthy that increasing $\overline{z}$ also means that there are more states of the world with more negative marginal profits, but equity holders do not care about these states of the world, unless the firm is all equity financed.

However, the previous effect also implies that firms can get the same strategic effect in the product market with a lower level of debt. Therefore, firms act in a more conservative manner in the debt market when uncertainty increases. This explains why result 3.1 holds. However the fact that higher uncertainty leads to lower equilibrium debt levels has an indirect impact in the equilibrium quantities, which also decrease. It turns out that the direct positive impact of increasing uncertainty on the equilibrium quantities is higher than the negative indirect impact, which implies that the equilibrium quantities are increasing with the uncertainty level. The fact that the impact is very small, is related to the fact that the direct and indirect effects have opposite signs and they almost cancel each other.

Figure 3.4 shows the equilibrium default probability as a function of the degree of

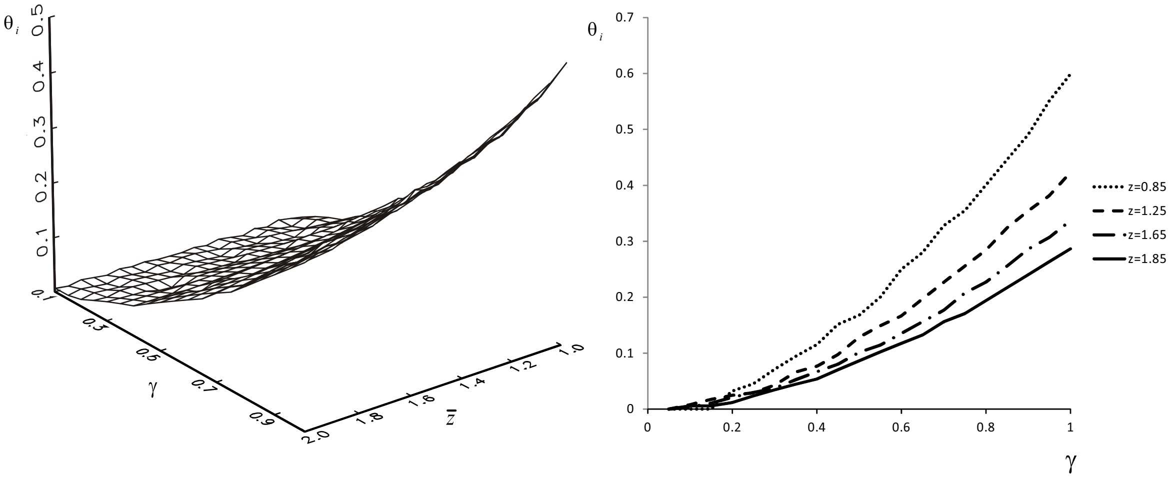


Figure 3.4: SPNE default probability as a function of the degree of product substitutability for various levels of demand uncertainty without considering the default costs.

product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. The figure allows us to conclude the following:

Result 3.3 *The SPNE default probability, θ^{**}, is increasing with the degree of product substitutability, γ, and it is decreasing with the level of demand uncertainty, $\overline{z}$.*

At the first sight, the result that an increase in the level of uncertainty leads to lower equilibrium default probabilities is very surprising. To interpret this result we need to take into account three effects. The first effect is the direct impact of increasing uncertainty on the default probability, for given debt and quantity levels. This effect is positive, since an increase in the uncertainty level increases the default probability. However we also need to consider two indirect effects. The fact that there is larger uncertainty leads firms to behave in a more aggressive manner in the output market. This effect tends to increase the default probability. However, the greater uncertainty also leads firms to be more conservative in the debt market, thus issuing less debt. A lower debt decreases the default probability, directly and indirectly, through its influence on the second period equilibrium quantities. Result 3.3 means that the last effect dominates the first two effects. Although uncertainty has a positive direct impact on the default probability, the fact that

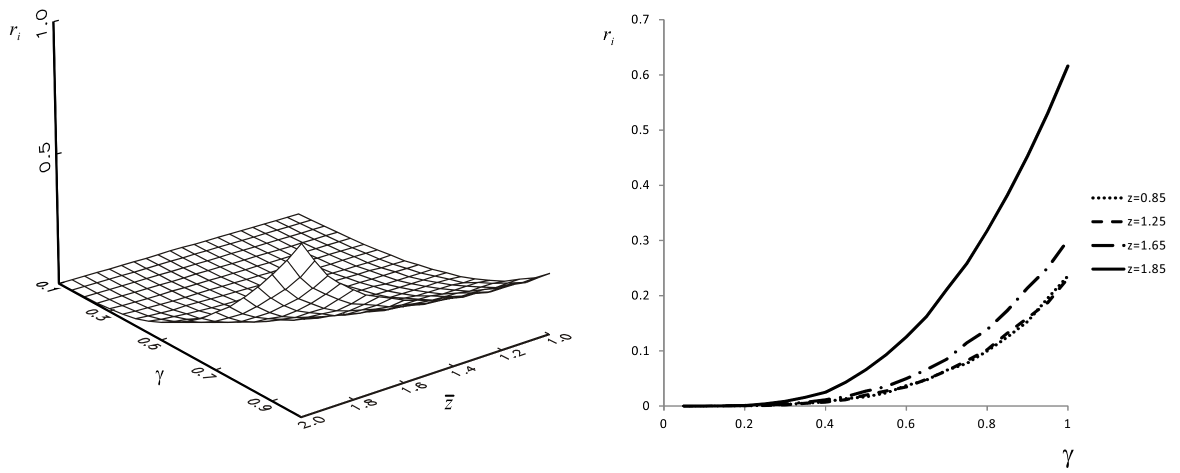


Figure 3.5: SPNE interest rate as a function of the degree of product substitutability and the level of demand uncertainty without considering the default costs

firms behave less aggressively in the debt market when uncertainty is higher outweights that impact and explains why the default probability is decreasing with uncertainty.

Figure 3.5 shows that the equilibrium interest rate depends of the degree of product substitutability, γ, and on the level of demand uncertainty, $\overline{z}$, as follows:

Result 3.4 *The SPNE interest rate, r^{**}, is increasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$. The impact is more significant for higher values of γ and for higher values of $\overline{z}$.*

The interest rate depends on how the promised payment to debtholders, D_i, compares with the expected value of debt, W_i which depends on the default probability associated with D_i. When uncertainty increases, D_i and W_i both decrease, but W_i decreases proportionally more. This is related with the fact that, for a given debt level, increasing uncertainty leads to an increase in the default probability. Bondholders know that under higher uncertainty, for a given D_i, it is more likely that the firm is not able to fulfil its obligations, thus they require an higher interest rate.

Figure 3.6 shows that the equilibrium expected equity value depends on the degree of product substitutability, γ, and it depends on the level of demand uncertainty, $\overline{z}$. The figure allows us to conclude the following:

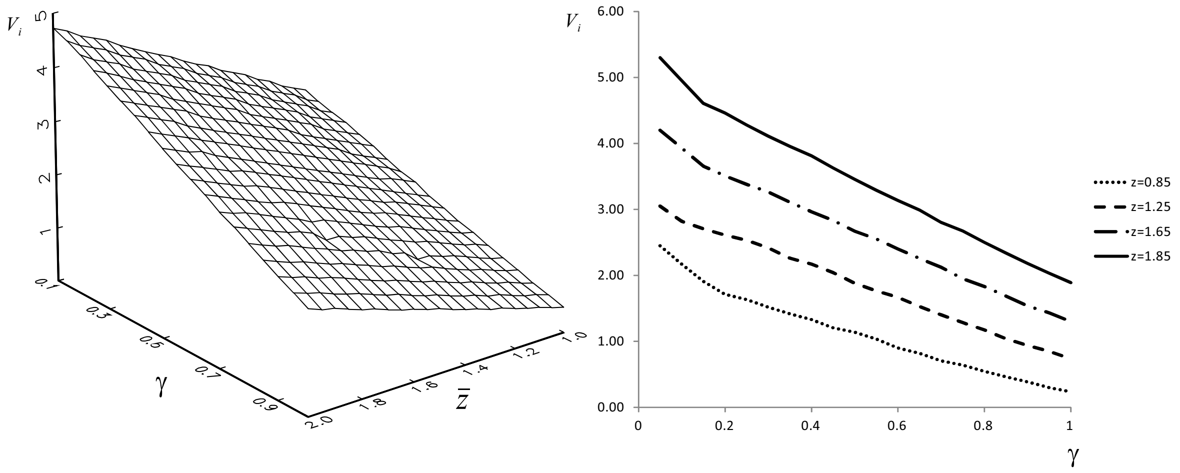


Figure 3.6: SPNE expected equity value as a function of the degree of product substitutability for various levels of demand uncertainty without considering the default costs.

Result 3.5 *The SPNE expected equity value, V^{**}, is decreasing with the degree of product substitutability, γ, and it is increasing with the level of demand uncertainty, $\overline{z}$.*

The first part of the result would be true if the firm was all equity financed. As the degree of product substitutability increases, the competition between the two firms becomes tougher and the equilibrium profits decrease. The second part of the result is due to the fact that equityholders only care about the good states of the world ($z_i > \widehat{z}_i$) and an increase in the uncertainty parameter, increases the good states of the world and hence increases the expected value of equity (which is equal to expected profit net of debt payments, conditional on $z_i > \widehat{z}_i$).

Figure 3.7 shows the equilibrium expected debt value as a function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. The figure allows us to conclude the following:

Result 3.6 *The SPNE expected debt value, W^{**}, is decreasing with the level of demand uncertainty, $\overline{z}$. On the other hand, the degree of product substitutability does not have a monotonic impact on W^{**}. For small values of demand uncertainty, W^{**} is decreasing with product substitutability. However, for higher values of demand uncertainty, W^{**} initially increases with γ but after a certain point follows a U relationship with γ.*

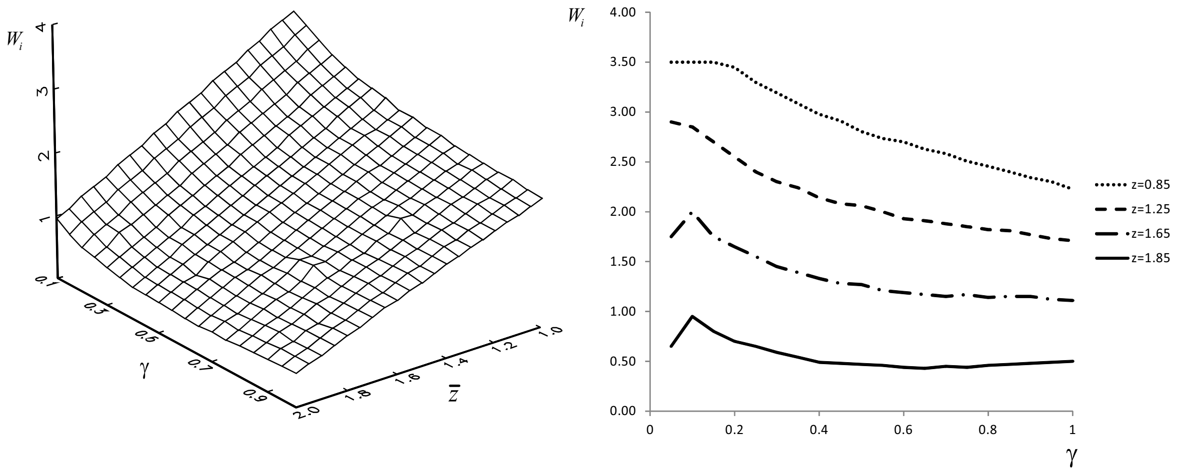


Figure 3.7: SPNE expected debt value as a function of the degree of product substitutability for various levels of demand uncertainty without considering the default costs.

The previous result combines the results regarding the impact of γ and $\overline{z}$ on the equilibrium debt obligations and on the equilibrium interest rate. The shape of the expected debt value is very similar to the shape of the debt obligations, but W^{**} is lower and, for lower levels of uncertainty, it decreases at a higher rate with the increases on the degree of product substitutability, due to the increase in the interest rate.

Figure 3.8 shows the equilibrium firm value as a function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. This figure allows us to conclude the following:

Result 3.7 *The SPNE firm value, Y^{**}, is decreasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$. However, the impact of the level of demand uncertainty, $\overline{z}$, is relatively small.*

The expected value of the firm is the sum of the expected equity value and the expected debt value. The fact that the expected value of the firm is decreasing with the degree of product substitutability is quite obvious considering that the expected equity value decreases very intensely with γ and that, the expected debt level is also decreasing with γ for many parameter values and when it is not decreasing, the increases have a

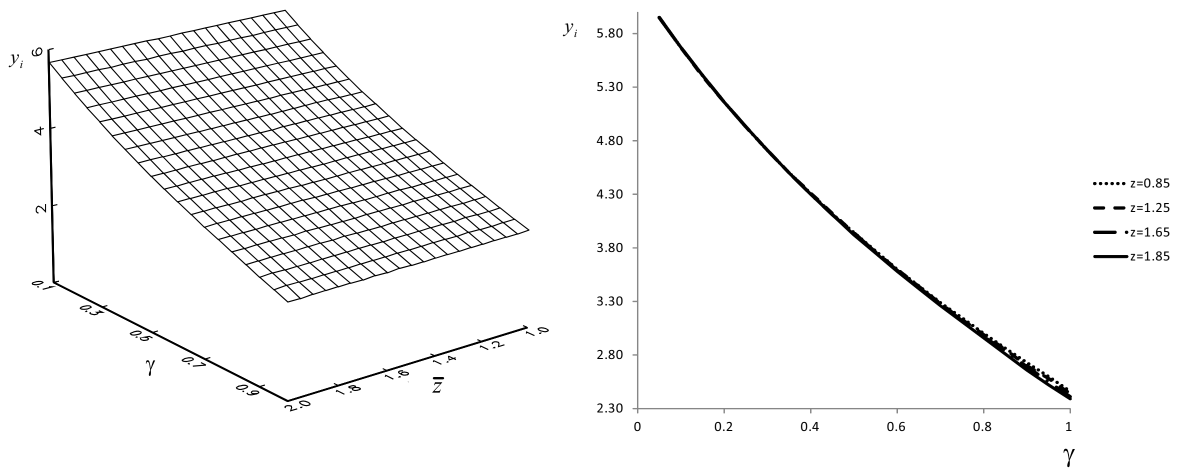


Figure 3.8: SPNE expected firm value as a function of the degree of product substitutability and the level of demand uncertainty without considering the default costs.

much smaller magnitude than the decrease in the expected equity value. Another way of interpreting this result is using the fact that the expected value of the firm is equal to the expected profit of the firm. The higher is the degree of product substitutability, the tougher is the competition among the two firms and hence the lower are the expected profits. The uncertainty degree has a small impact on the firm expected value because it has contradictory impacts on the expected equity value (which increases with $\overline{z}$) and the expected debt level (which decreases with $\overline{z}$).

Figure 3.9 shows that the equilibrium welfare depends on the degree of product substitutability, γ and it depends on the level of demand uncertainty, $\overline{z}$. The figure allows us to conclude the following:

Result 3.8 *The SPNE welfare level, Wel^{**}, is decreasing with the degree of product substitutability, γ, and decreasing with the level of demand uncertainty, $\overline{z}$. However the impact of demand uncertainty is relatively small.*

The social welfare increases with the degree of product differentiation (which is higher, the lower is γ). A high product differentiation increases consumer welfare because the utility function is such that consumers value quality. Moreover, the higher is product differentiation (the closer to 0 is γ) the higher are the two firms profit.

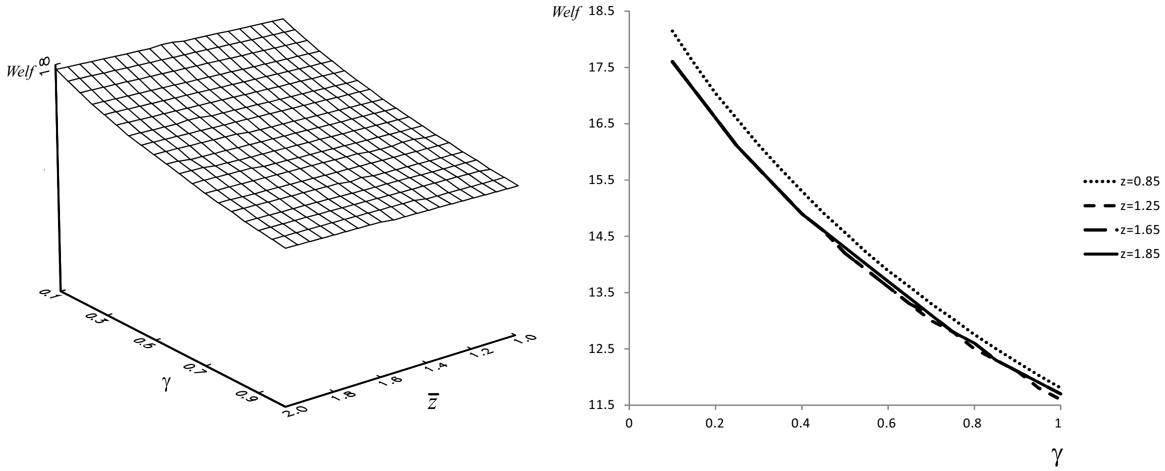


Figure 3.9: SPNE welfare as a function of the degree of product substitutability for various levels of demand uncertainty without considering the default costs.

After analyzing the results we conclude that in the symmetric case we confirmed the results obtained by Toolsema and Haan (2008) and Franck and Le Pape (2008).

3.4.2 Asymmetric duopoly

In this subsection we consider that firms differ in their marginal production cost. Firm i has a null production cost, $c_i = 0$, while firm j has marginal cost c_j. We study what happens as firm j becomes less efficient by analyzing the SPNE as the marginal cost of firm j, c_j, varies between 0 (the symmetric case) and 0.5. It should be noted that the upper limit on c_j was chosen so that the more inefficient firm has positive operating profits even in the worst state of the world. Thus default can only happen as a consequence of having too much debt obligations.

We examine how the variables' equilibrium levels (debt, output, implicit interest rates, default probabilities, equity value, value of the firm and welfare) vary as the marginal cost of firm j increases (c_j is represented in the x-axis), considering three possible values for the degree of product substitutability, γ ($\gamma = 0.2, \gamma = 0.6$ and $\gamma = 1$). Three graphs are presented for each variable (the first corresponds to $\overline{z} = 0.85$, the second to $\overline{z} = 1.25$ and the third one to $\overline{z} = 1.85$). This allows us to check if the behavior is stable with the level

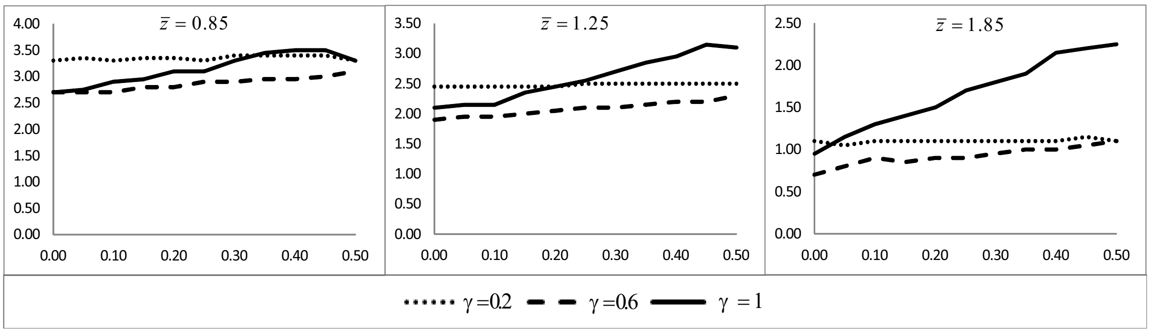


Figure 3.10: SPNE debt level of the more efficient firm as a function of the marginal costs of the rival without considering the default costs.

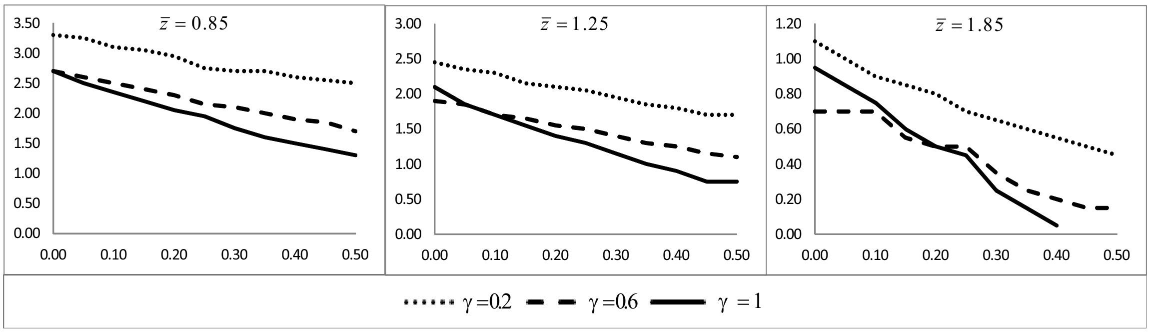


Figure 3.11: SPNE debt level of the more inefficient firm as a function of its marginal costs without considering the default costs.

of demand uncertainty, $\overline{z}$.

Figures 3.10 and 3.11 show the debt obligations of firm i and of firm j, respectively, as a function of the marginal cost of production of the firm j, c_j. These figures allow us to conclude the following:

Result 3.9 *The SPNE level of debt obligation of firm i, D_i^{**}, is increasing with the marginal costs of firm j, c_j. The increase is more pronounced for high levels of demand uncertainty, $\overline{z}$. On the contrary, the SPNE level of debt obligation of firm j, D_j^{**}, is decreasing with the marginal cost of firm j, $c_{j.}$ The decrease is more pronounced for high levels of demand uncertainty, $\overline{z}$.*

Therefore we can conclude that the less efficient firm behaves more cautiously in the debt market whereas the more efficient firm behaves more agressively.

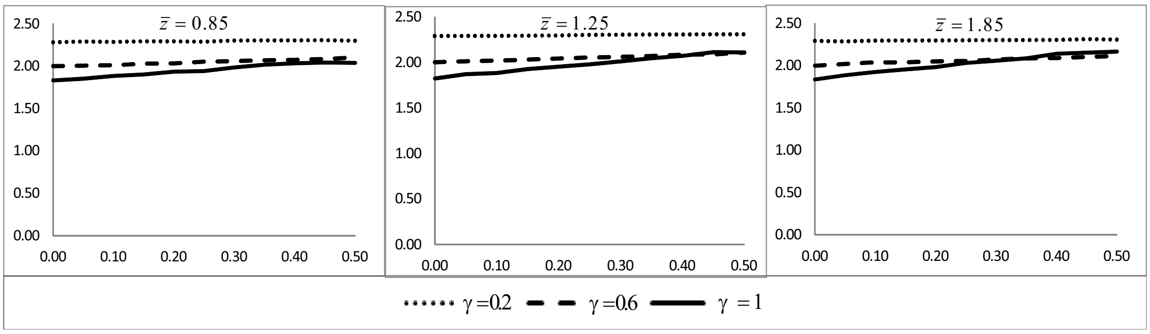


Figure 3.12: SPNE output level of the more efficient firm as a function of the rival's marginal costs without considering the default costs.

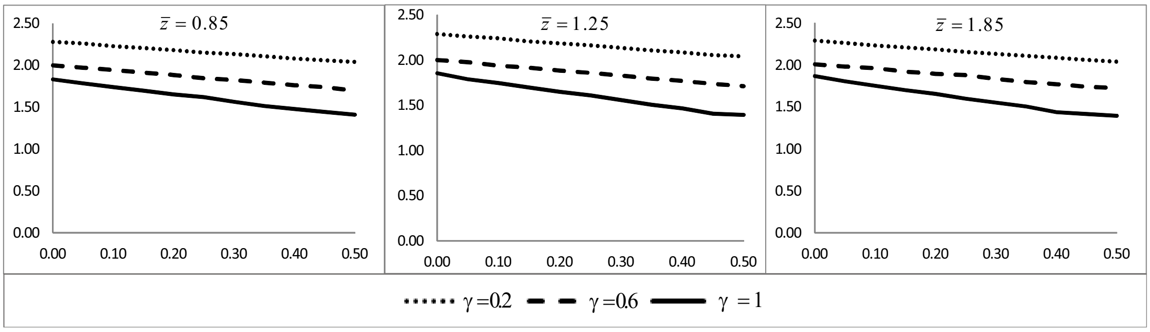


Figure 3.13: SPNE output level of the more inefficient firm as a function of its marginal costs without considering the default costs.

Figures 3.12 and 3.13 show the output level of the firm i and the output level of the firm j as a function of the marginal cost of the firm j, c_j. These figures allow us to conclude the following:

Result 3.10 *The SPNE level of output firm i, q_i^{**}, is increasing with the rival's marginal cost, c_j. On the contrary, the SPNE level of output of firm j, q_j^{**}, is decreasing with the firm's marginal cost.*

Regarding the effect of the marginal production cost of firm j, c_j on the debt obligation and on the output level of the two firms, the results presented above show that as firm j becomes less efficient (i.e., its marginal production costs increases), the firm adopts a more conservative approach in the debt market and in the output market. The intuition for this result is that, an increase in the marginal production cost leads to a decrease in the marginal profit which implies a decrease in the debt and output levels.

The more efficient firm has the opposite behavior, i.e. it becomes more aggressive in the debt market and in the output market. These effects are more pronounced for high levels of uncertainty, which increases the volatility of marginal profit.

Figures 3.14 and 3.15 show the default probability of firm i and the default probability of firm j as a function of the marginal cost of the firm j, c_j. These figures allow us to conclude the following:

Result 3.11 *The SPNE default probability of firm i, θ_i^{**}, is increasing with the marginal cost of firm j, c_j. On the contrary, the SPNE default probability of firm j, θ_j^{**}, is decreasing with the marginal cost of firm j, c_j. The default probabilities are more sensitive to changes in c_j when the degree of product substitutability, γ, is high.*

Note that the increase in the marginal cost of firm j has opposite effects on the default probability of the two firms. The default probability of the inefficient firms decreases while the default probability of the efficient firm increases with c_j.

In order to understand the impact of changes in the marginal cost of firm j on its own default probability, one needs to consider both direct and indirect effects. The direct effect is positive. For given debt and quantity levels, an increase of the marginal cost of the firm j, increases its default probability. A first indirect effects results from the fact that the increase in the marginal cost of firm j leads to a more conservative behavior in the debt and output markets which implies a decrease in the default probability. A second indirect effect is related to the fact that the more efficient increases its quantity, which hurts the inefficient firm's profits and thus increases its default probability (this indirect effect is smaller in magnitude). Thus the total effect of increasing c_j on the default probability of firm j may be positive or negative, depending on which of the effects dominates. We observe that the first indirect effect dominates the other two effects, leading to the counterintuitive result that as the inefficient firm becomes less efficient, its default probability decreases.

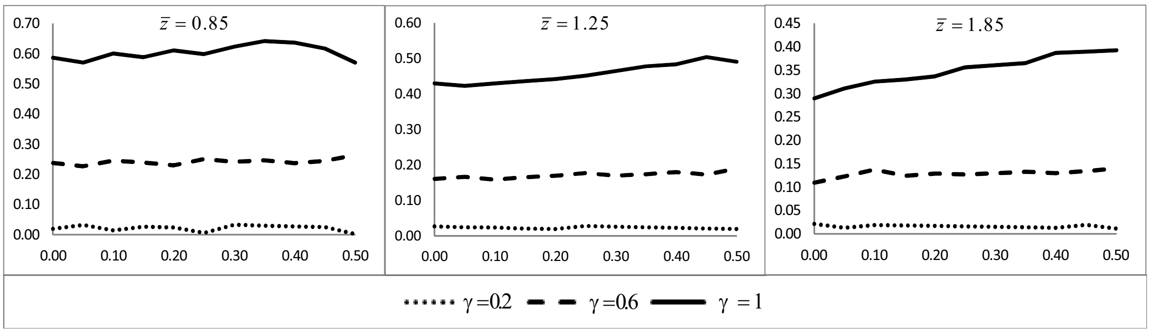


Figure 3.14: SPNE default probability of the more efficient firm as a function of the rival's marginal costs without considering the default costs.

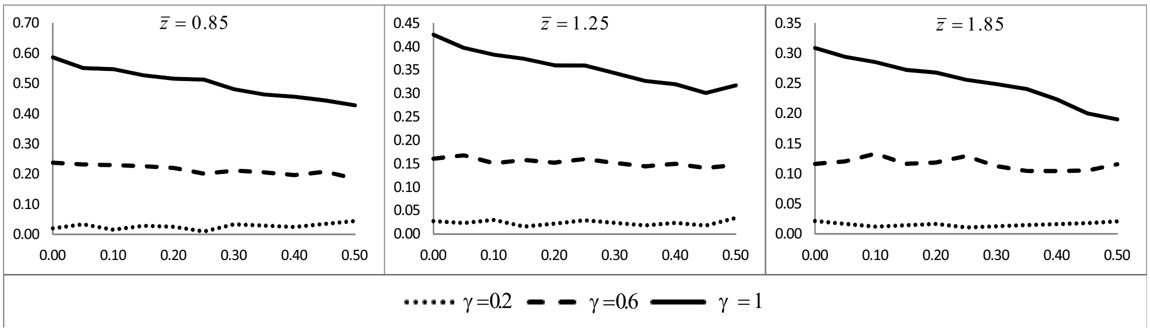


Figure 3.15: SPNE default probability of the less efficient firm as a function of its marginal costs without considering the default costs.

The explanation of the impact on the more efficient firm default probability is similar, although in this case there is no direct impact and thus the ambiguity in the total impact is due to the fact that marginal costs has opposite effects on the quantities and debt of the two firms. It should be highlighted that, under our assumptions, we are not considering the default risk related to operational inefficiency as we are considering levels of c_j that imply positive operating profits for the more inefficient firm even in the worst state of the world, $-\bar{z}$. If this assumption was not made, one would expect that further increases in c_j would imply higher default risk, even if the firm does not issue debt. Such default risk would be due exclusively to operational reasons, and the more inefficient the firm becomes, the higher would this risk be.

Figures 3.16 and 3.17 show the interest rate of the firm i and the interest rate of the

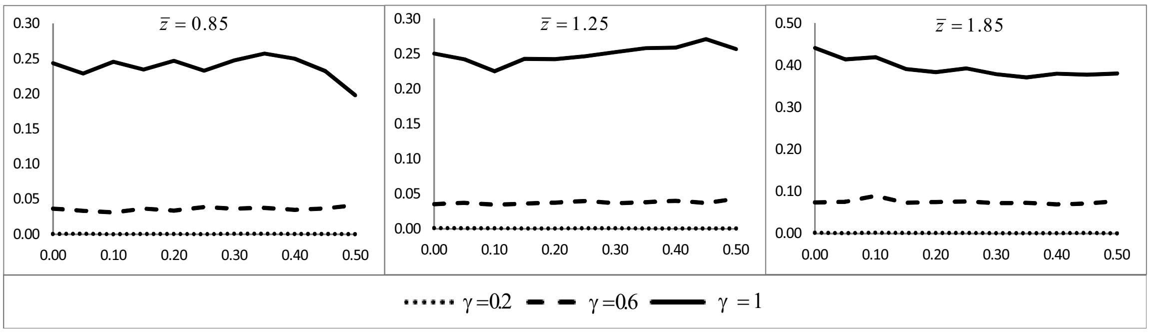


Figure 3.16: SPNE interest rate level of the more efficient firm as a function of the rival's marginal costs without considering the default costs.

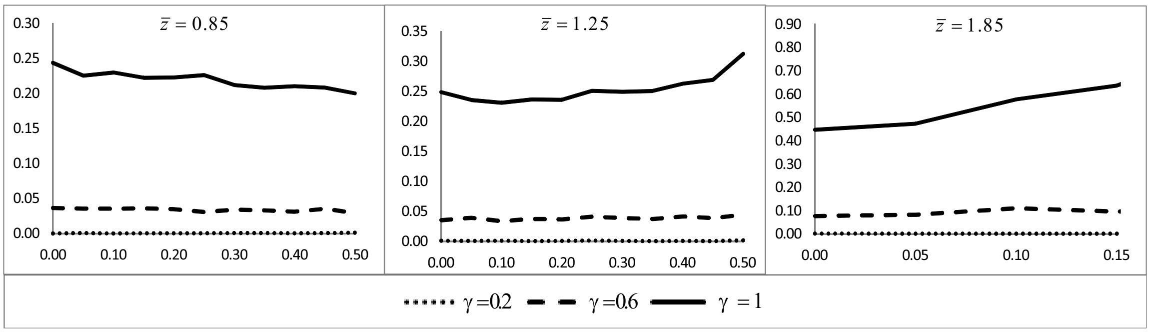


Figure 3.17: SPNE interest rate level of the less efficient firm as a function of its marginal costs without considering the default costs.

firm j as a function of the marginal cost of firm j, c_j. These figures allow us to conclude the following:

Result 3.12 *For high levels of demand uncertainty, an increase in marginal cost of the firm j, c_j, leads to a decrease in the SPNE interest rate of firm i, r_i^{**}, and to increases in the SPNE interest rate of firm j, r_j^{**}. The change (decrease or increase) is more pronounced for high levels of the degree of product substitutability, γ. For intermediate or low values of demand uncertainty, the SPNE interest rate of firm i, r_i^{**} follows an inverted U relation with c_j whereas the SPNE interest rate of firm j, r_j^{**} follows a U relationship.*

Figures 3.18 and 3.19 show the expected equity values of firm i and firm j, respectively, as a function of the marginal cost of production of the firm j, c_j. These figures allow us to conclude the following:

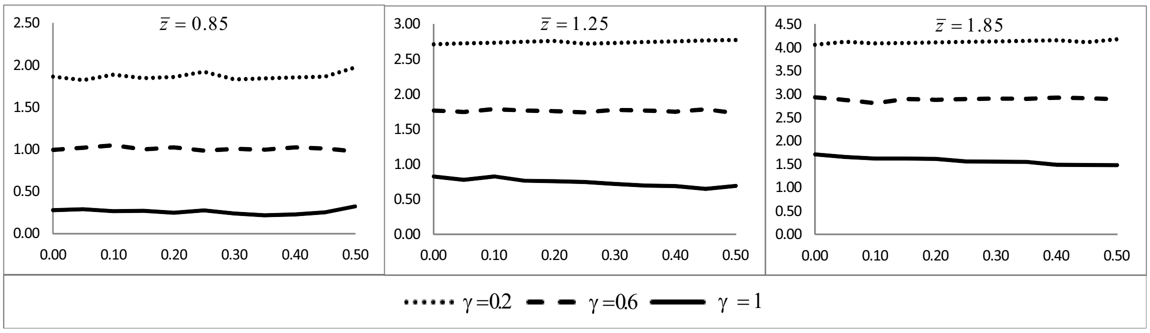


Figure 3.18: SPNE expected equity value of the more efficient firm as a function of the rival's marginal costs without considering the default costs.

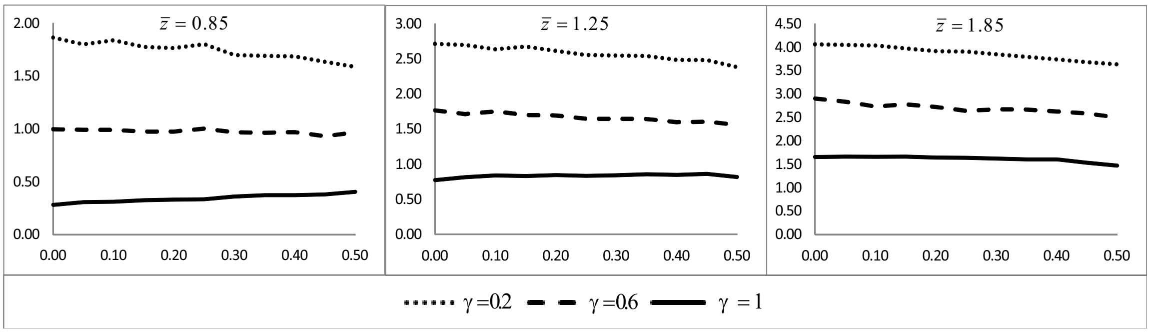


Figure 3.19: SPNE expected equity value of the less efficient firm as a function of its marginal costs without considering the default costs.

Result 3.13 *The SPNE expected equity value of the firm i, V_i^{**}, is increasing with the rivals' marginal cost of production, c_j. On the contrary, the SPNE expected equity value of firm j, V_j^{**}, is decreasing with the firm's marginal cost of production of the firm j, c_j.*

Figures 3.20 and 3.21 show the expected debt values of firm i and firm j as a function of the marginal cost of production of firm j, c_j. The figures allow us to conclude the following:

Result 3.14 *The SPNE expected debt value of firm i, W_i^{**}, is increasing with the marginal cost of firm j, c_j. On the contrary, the SPNE expected debt value of firm j, W_j^{**}, is decreasing with the marginal cost of firm j, c_j.*

Results 3.22 and 3.23 allow us to conclude the following:

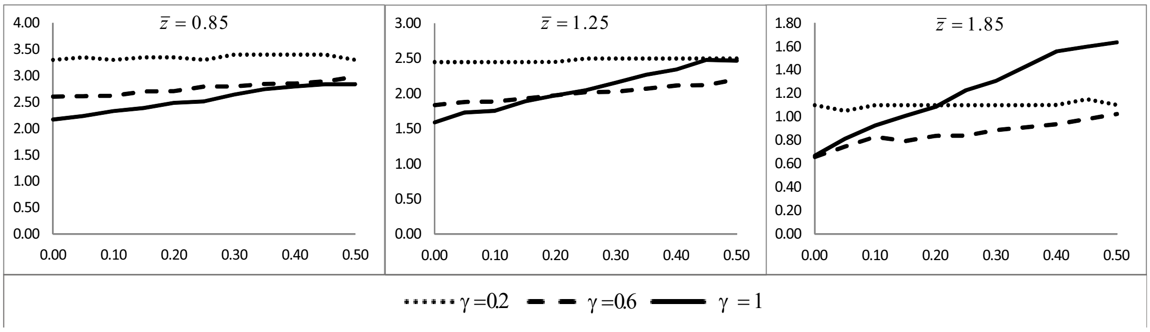


Figure 3.20: SPNE expected debt value of the more efficient firm as a function of the rival's marginal costs without considering the default costs.

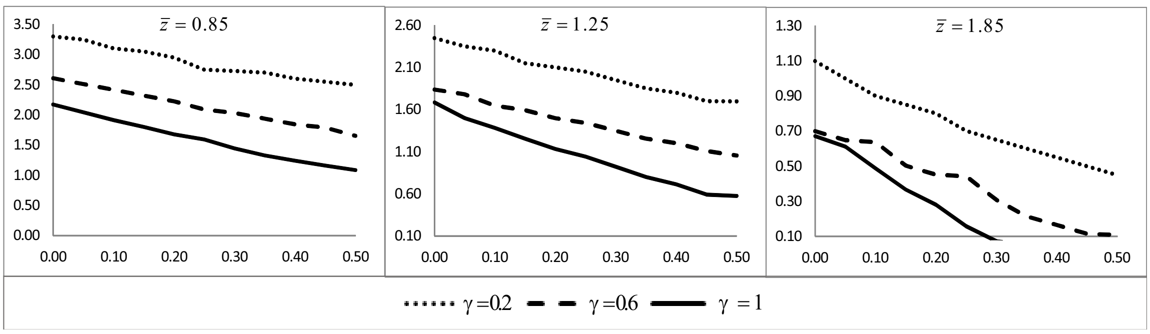


Figure 3.21: SPNE expected debt value of the less efficient firm as a function of its marginal costs without considering the default costs.

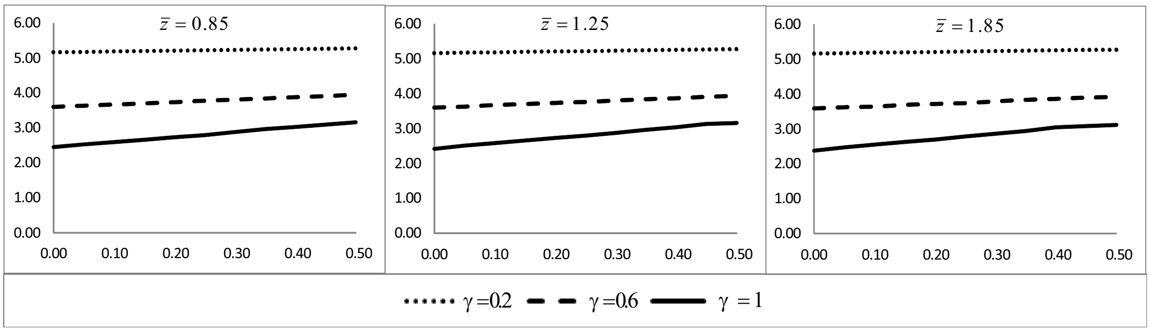


Figure 3.22: SPNE expected firm value of the more efficient firm as a function of the rival's marginal costs without considering the default costs.

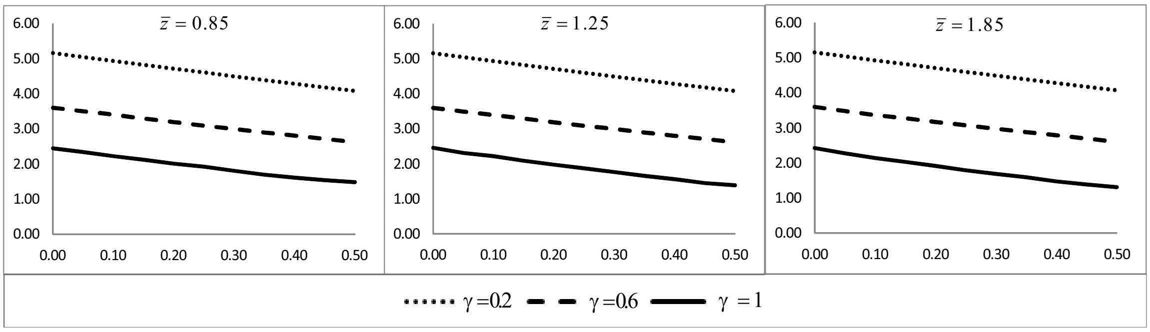


Figure 3.23: SPNE expected firm value of the less efficient firm as a function of its marginal costs without considering the default costs.

Result 3.15 *The SPNE expected value of firm i, Y_i^{**}, is increasing with marginal cost of the firm j, c_j. On the contrary, the SPNE expected value of firm j, Y_j^{**}, is decreasing with the marginal cost of firm j, c_j.*

The previous result is a direct consequence of the expected profit of a firm being negatively related with its marginal costs and positively related with the rival's marginal cost.

Figure 3.24 shows the welfare level as a function of the marginal cost of production of firm j, c_j. The figure allows us to conclude the following:

Result 3.16 *The SPNE of the welfare, Wel^{**}, is decreasing with the marginal cost of the firm j, c_j.*

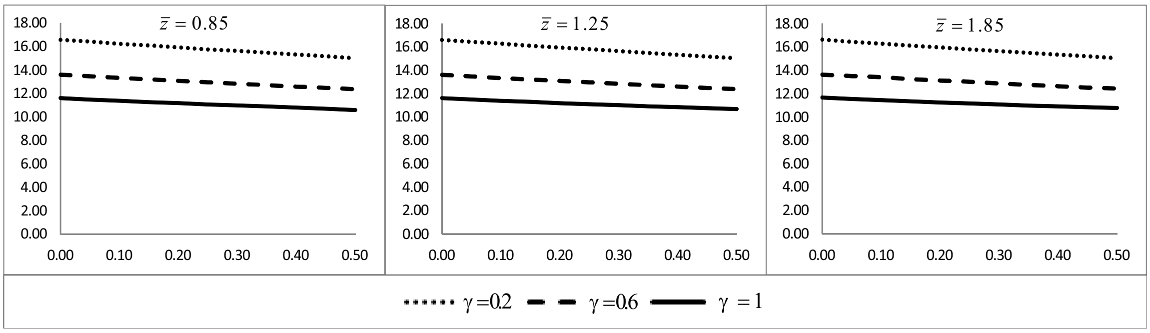


Figure 3.24: SPNE expected social welfare as a function of less efficient firm's marginal costs without considering the default costs.

This result is expected since everything else constant, the society as a all is worse the higher the marginal cost of production.

The result of the asymmetric duopoly reveal that the SPNE output decreases with the degree of product substitutability and with the firm's marginal cost of production and, on the contrary, it is increasing with the rival firm marginal cost. These results are similar to the ones obtained in traditional oligopoly models. Moreover, the equilibrium debt level of the less efficient firm is decreasing with its marginal cost while the most efficient firm has the opposite behavior. This is a quite interesting result as it tells us that the less efficient firm is more cautious and finances less with debt while the more efficient firm becomes «more aggressive» in the debt market. Another interesting result is that the default probability of the inefficient firm decreases as the firm becomes less efficient. This result is due to the existence of direct and indirect effects. On the one hand, for the same debt level, increasing the marginal cost of the firm is expected to lead to an increase on the default probability. On the other hand, since a decrease in efficiency leads to lower levels of debt obligations and output, this leads to a decrease on the default probability. Our results reveal that the last effect dominates, showing that a less efficient firm may have a lower probability of default because it finances less with debt.

3.5 Results with default costs

In this section we analyze the SPNE of the game considering that the default cost parameters are positive; i.e., k, a_i and a_j are positive. In the next subsection we analyze how the equilibrium values of the variables change with the uncertainty level, $\overline{z}$, and with the degree of product substitutability, γ, in a symmetric duopoly model. Comparing the results with the ones in subsection 3.4.1 we can see how the existence of default costs influences the equilibrium. The next two subsections explore the impact of changes in the default costs parameters. We start by analyzing the impact of higher direct and indirect costs, increasing k and a (assuming a symmetric increase for both firms) then we examine the impact of asymmetric changes in the indirect costs parameter.The analysis of the SPNE of the game considering default costs and asymmetry in production costs is presented in the Appendix (the results are similar to the results obtained from the analysis of the impact of cost asymmetry and the existence of default costs). The main contribution of this section is the introduction of the default costs in the analysis, which allows us to verify if the previous results continue to be valid. This part of the study provides a major contribution to the existent literature as default costs were not included in the previous numerical studies.

3.5.1 Symmetric duopoly with default costs

This section studies a model with default cost and two symmetric firms ($c_i = c_j = 0$). In addition, we assume $\alpha = 5$, $\overline{z}_{\max} = 2$, $M = 10$; $k = 0.10$, $a_i = a_j = 0.05$.[9] The aim of this section is to analyze if the impact of changes in the level of demand uncertainty, $\overline{z}$, and in the degree of product substitutability, γ, is different when default costs are considered.

Figure 3.25 shows the equilibrium levels of debt obligation as a function of the degree

[9]The assumed values of k, a_i and a_j are consistent with the literature. In fact, these values allow us to obtain total indirect costs (IDC_i) higher than direct costs (DDC_i). The empirical literature in the area concludes that bankruptcy costs can vary between 0% and 20% of the total value of the firm (see, for instance, Altman (1964), Kwasa and Cho (1995) and Bris, Welch and Zhu (2006)). The percentages assumed also ensure that the total value of bankruptcy costs are less than 20% of the firm value. We opted for not considering very high percentages, because with higher values we would go into areas with multiple equilibria.

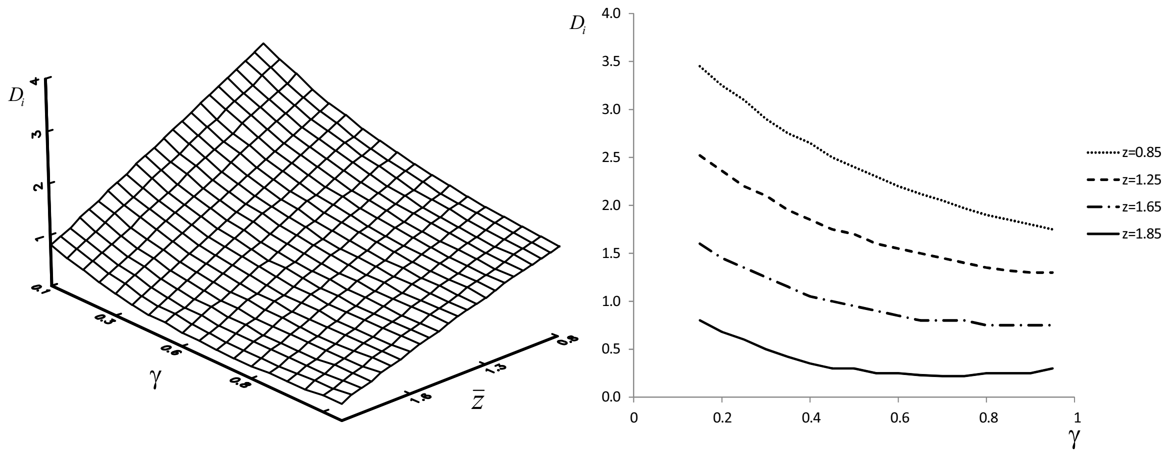


Figure 3.25: SPNE debt obligation as a function of the degree of product substitubility and the level of demand uncertainty considering default costs

of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$, considering default costs. This figure allows us to conclude the following:

Result 3.17 *The SPNE level of debt obligation, D^{**}, is strictly positive and decreasing with the level of demand uncertainty, $\overline{z}$. On the other hand, the degree of product substitutability does not have a monotonic impact on D^{**}. For small values of demand uncertainty, D^{**} is decreasing with product substitutability. However, for higher values of demand uncertainty, D^{**} follows a U relationship with γ.*

The most striking feature of the previous result is that it is qualitatively very similar to the result obtained under the assumption of no default costs. In other words, firms are less aggressive in the debt market when uncertainty increases and the relationship of debt and product substitutability is not monotonic for higher levels of uncertainty. However one also observes that, the optimal level of debt obligations is lower when there are default costs and that, for lower levels of uncertainty, the equilibrium debt obligation is more sensitive to changes in the degree of product substitutability when default costs are considered.

Figures 3.26 shows the equilibrium output level as a function of the degree of product

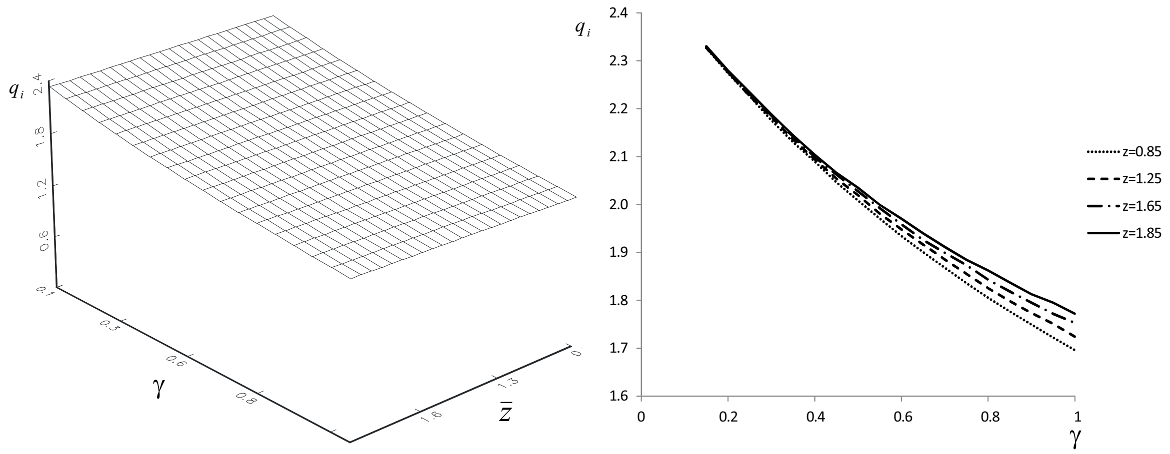


Figure 3.26: SPNE output level as a function of the degree of product substitutability and the level of demand uncertainty considering default costs

substitutability, γ and as a function of the level of demand uncertainty, $\overline{z}$, when default costs are considered. The figure allows us to conclude the following:

Result 3.18 *The SPNE level output, q^{**}, is decreasing with the degree of product substitutability, γ and increasing with the level of demand uncertainty $\overline{z}$. However, the impact of demand uncertainty is relatively small. Nevertheless, when considering default costs, the impact is more pronounced than when they are not considered.*

Like in the case of the debt obligation, the equilibrium output behaviour as a function of γ and $\overline{z}$ is very similar, in qualitative terms, to the one verified without default costs. The main difference is again the higher sensitivity to changes in the degree of product substitutability and also to changes in the level of uncertainty. Like in the case of no default costs case, when interpreting the impact of the uncertainty level on q_i^*, one has to consider both direct and indirect effects. For a fixed level of debt obligations, increasing uncertainty increases the optimal quantity because there are more good states of the world and hence the expected marginal equity is higher, which increases the optimal quantity. However there is also the indirect effect since increasing $\overline{z}$ implies a lower equilibrium debt level, which then leads to a lower optimal quantity. These two effects have opposite

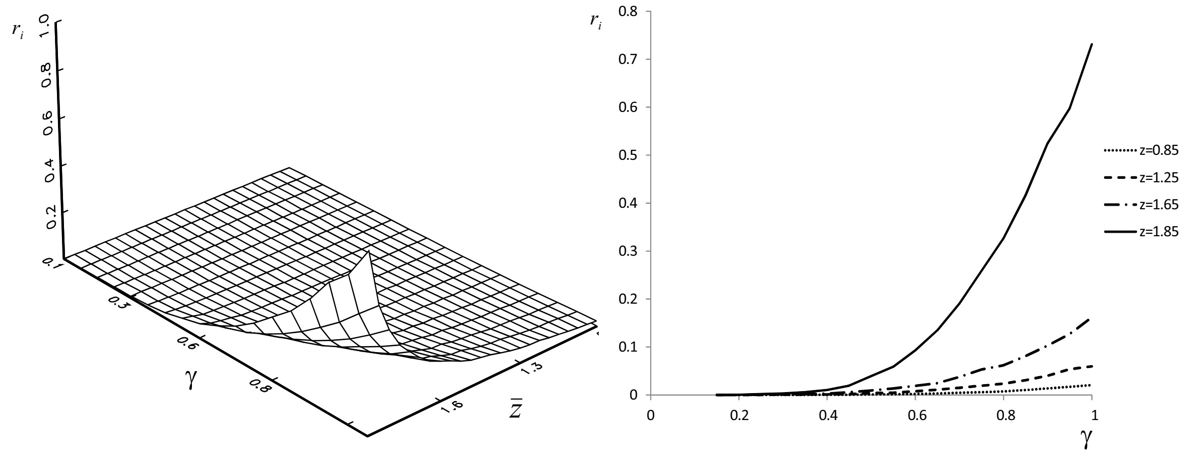


Figure 3.27: SPNE interest rate as a function of the degree of product substitutability and the level of demand uncertainty considering default costs

signs but the first dominates, which explains why the equilibrium quantity is increasing with $\overline{z}$.

Figure 3.27 shows that the equilibrium interest rate depends on the degree of product substitutability, γ, and on the level of demand uncertainty, $\overline{z}$, considering default costs. These figures allow us to conclude the following:

Result 3.19 *The SPNE interest rate, r^{**}, is increasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$.The impact is more significant for high values of γ and $\overline{z}$.*

Once again we can see that the way the interest rate varies with changes in the degree of product substitutability and with $\overline{z}$ is qualitatively very similar to the one observed under no default costs. However, with default costs, the interest rate is higher and more sensitive to changes in the uncertainty level. This result is expected because, for a given debt level, the existence of default costs, decreases the expected profit net of debt obligations and it enlarges the set of states of the world where default occurs, explaining why the debtholder require an higher interest rate.

Figure 3.28 shows the equilibrium default probability as a function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$, when

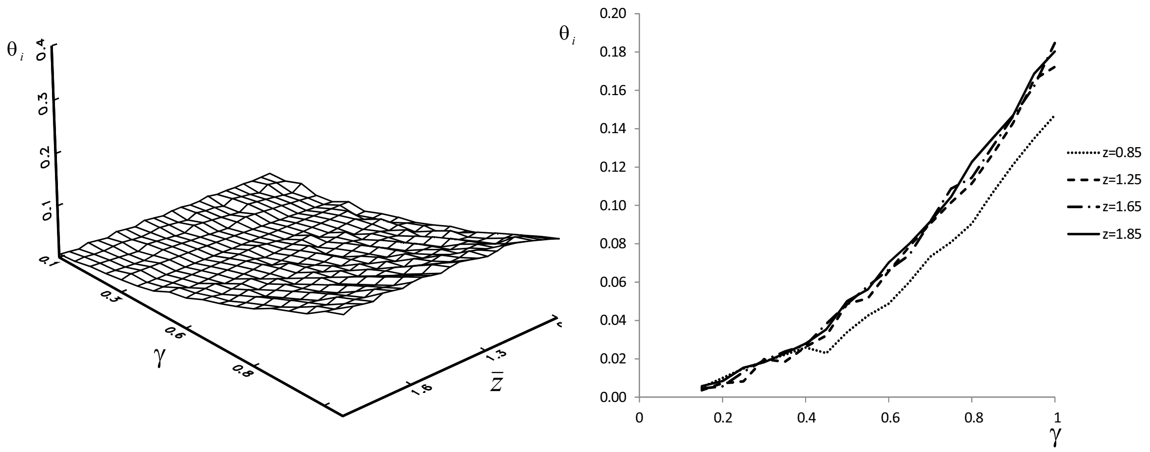


Figure 3.28: SPNE default probability as a function of the degree of product substitutability and the level of demand uncertainty considering default costs.

default costs are considered. The figure allows us to conclude the following:

Result 3.20 *The SPNE default probability, θ^{**}, is increasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$.*

Comparing the default probability when default costs are considered with default probability when there are no default costs, one concludes that with default costs and taking into account the values considered for the parameters, the probability of default is much lower. Moreover, with default costs and increase in the level of uncertainty, $\overline{z}$, has the opposite effect on the default probability than when there are no default costs. With default costs, an increase in the level of uncertainty, leads to an increase in the default probability.

The fact that the default probability is lower with default cost is a consequence of the firm adopting a much more conservative attitude in the debt market. This effect overwhelms the negative direct impact of the default costs on the default probability.

Regarding the effect of increasing the demand uncertainty level, $\overline{z}$, on the default probability, we have to take into account three effects. The direct effect is that, for given debt and quantity levels, the increase in the uncertainty level increases the default probability. Regarding the indirect effects, the fact that there is larger uncertainty leads firms to behave in a more aggressive manner in the output market. This effect also tends to increase the default probability. However, the greater uncertainty leads firms to be much more conservative in the debt market, thus issuing less debt. A lower debt, implies a lower default probability, directly and indirectly, through its influence on the second period equilibrium quantities. We conclude that when default costs are considered the first and the second effects dominate, i.e., the direct effect of the increasing uncertainty and the effect of the more aggressive behavior in the output market dominate the negative impact of having a reduced equilibrium level, which leads to higher equilibrium default probabilities.

Figures 3.29 and 3.30 show that the equilibrium expected equity value and the equilibrium expected debt value depend on the degree of product substitutability, γ, and on the level of demand uncertainty, $\overline{z}$, considering default costs. The figures allow us to conclude the following:

Result 3.21 *The SPNE expected equity value, V^{**}, and the SPNE expected debt value, W^{**}, are both decreasing with the degree of product substitutability, γ. On the other hand, V^{**} is increasing with the level of demand uncertainty, $\overline{z}$ whereas W^{**} is decreasing with $\overline{z}$. When default cost are considered γ no longer has an ambiguous effect on the expected debt value.*

It should be noted that, when default cost are considered, γ no longer has an ambiguous effect on the expected debt value. But the remaining results are qualitatively very similar to the ones obtained without default costs.

Figure 3.31 shows the equilibrium firm value as a function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$, considering default cost. This figure allows us to conclude the following:

Result 3.22 *The SPNE firm value, Y^{**}, is decreasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$. However, the impact of the level of demand uncertainty, $\overline{z}$, is relatively small.*

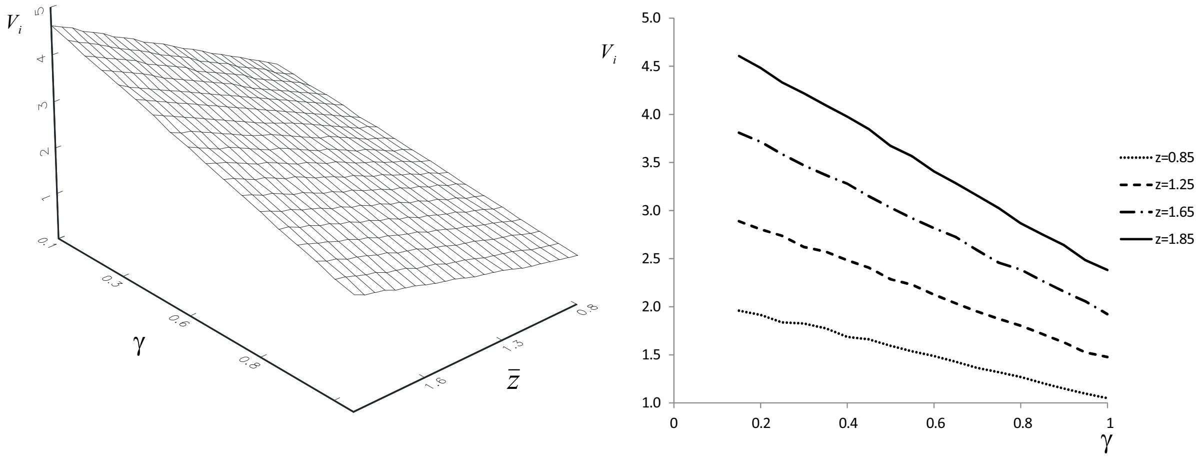


Figure 3.29: SPNE expected equity value as a function of the degree of product substitutability for various levels of demand uncertainty considering default costs.

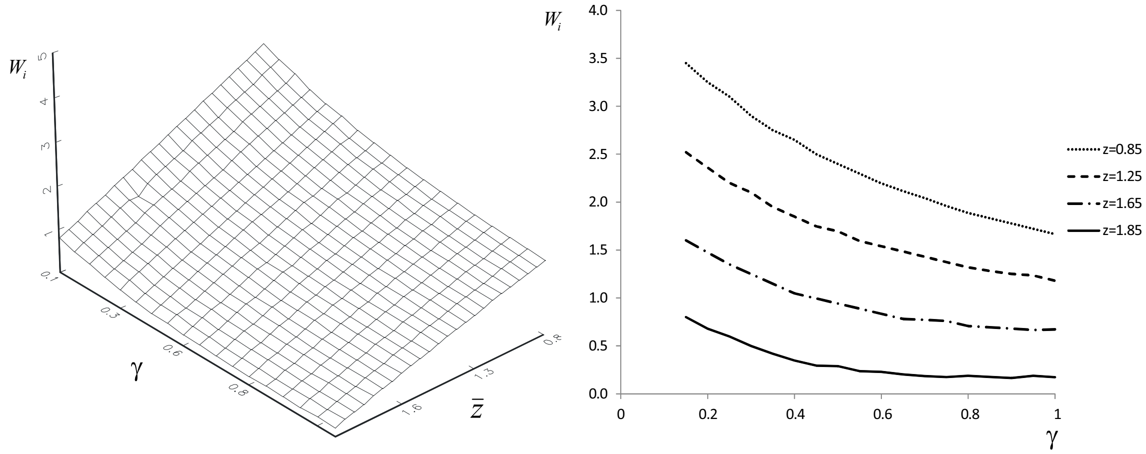


Figure 3.30: SPNE expected debt value as a function of the degree of product substitutability for various levels of demand uncertainty considering default costs.

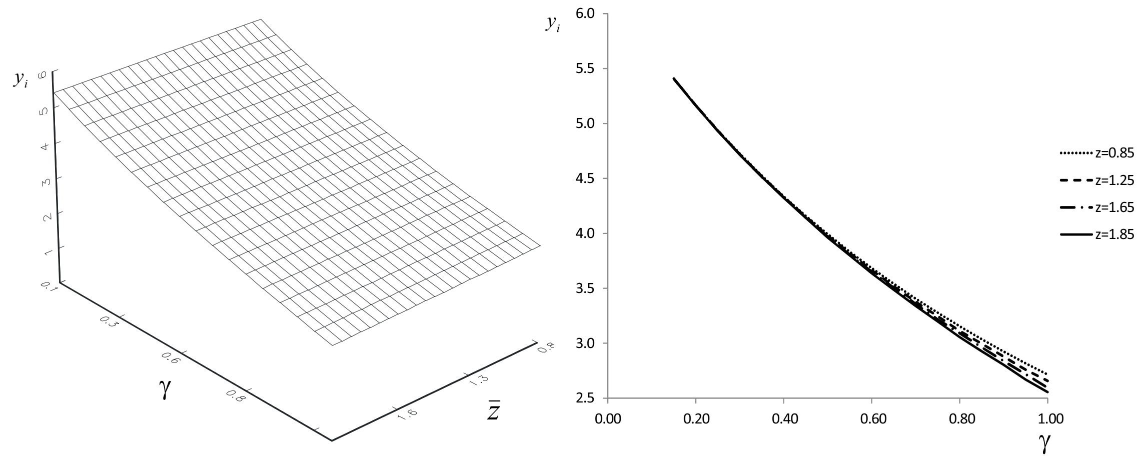


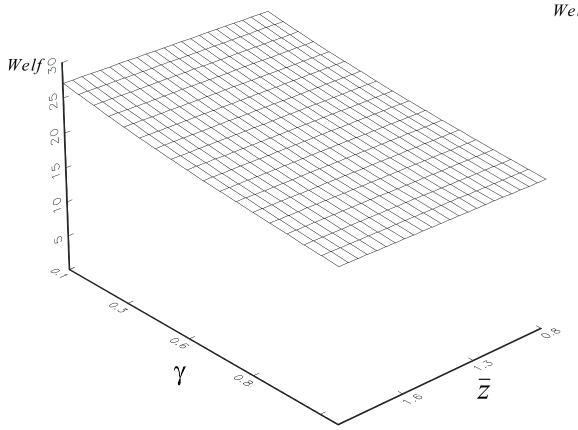
Figure 3.31: SPNE expected firm value as a function of the degree of product substitutability and the level of demand uncertainty **c**onsidering default costs.

In qualitative terms, the previous result is very similar to the one when there are no default costs. However it should be noted that the value of the firm is higher when there is default costs. This counter-intuitive result is due to the fact that having default costs leads the firms to reduce the equilibrium debt levels which ends up having a positive effect on the value of the firm, that outweighs the direct negative impact of the default costs. Thus default cost may be beneficial for the firms. However this result may depend on the value of the default cost parameters. For higher values of these parameter the negative direct impact of the default costs may be higher and dominate the strategic effect through the debt reduction.

Figure 3.32 shows that the equilibrium welfare depends on the degree of product substitutability, γ and it depends on the level of demand uncertainty, $\overline{z}$, when default costs are considered. The figure allows us to conclude the following:

Result 3.23 *The SPNE welfare level, Wel^{**}, is decreasing with the degree of product substitutability, γ, and increasing with the level of demand uncertainty, $\overline{z}$. However the impact of demand uncertainty is relatively small. When default costs are considered, the SPNE welfare level is higher than when they are not considered.*

The previous result is interesting because it tells us that the social welfare is higher



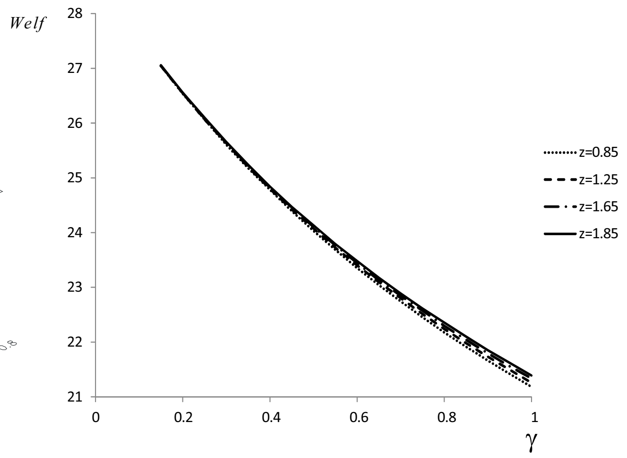


Figure 3.32: SPNE welfare as a function of the degree of product substitutability for various levels of demand uncertainty considering default costs.

when there are default costs. This result happens because the default cost imply a more cautious behaviour by the firms, which is welfare improving.

Figure 3.33 and 3.34 show that equilibrium of direct and indirect default costs depend on the degree of product substitutability, γ, and on the level of demand uncertainty, $\overline{z}$, when default costs are considered. The figure allows us to conclude the following:

Result 3.24 *The SPNE of direct and indirect default costs, DDC_i^{**} and IDC_i^{**}, are increasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$.*

For given values of the default cost parameter, the shape of the total direct default costs and total indirect default cost depends very much on the shape of the default probability. Therefore it is not surprising that they are increasing with the degree of product substitutability and with the level of demand uncertainty.

In this section we analyzed the equilibrium when there are direct and indirect default costs and firms are symmetric. It is interesting to note that most of the qualitative results that were obtained with no default costs also hold when default costs are considered. For instance, in both cases the equilibrium level of debt is decreasing with the uncertainty

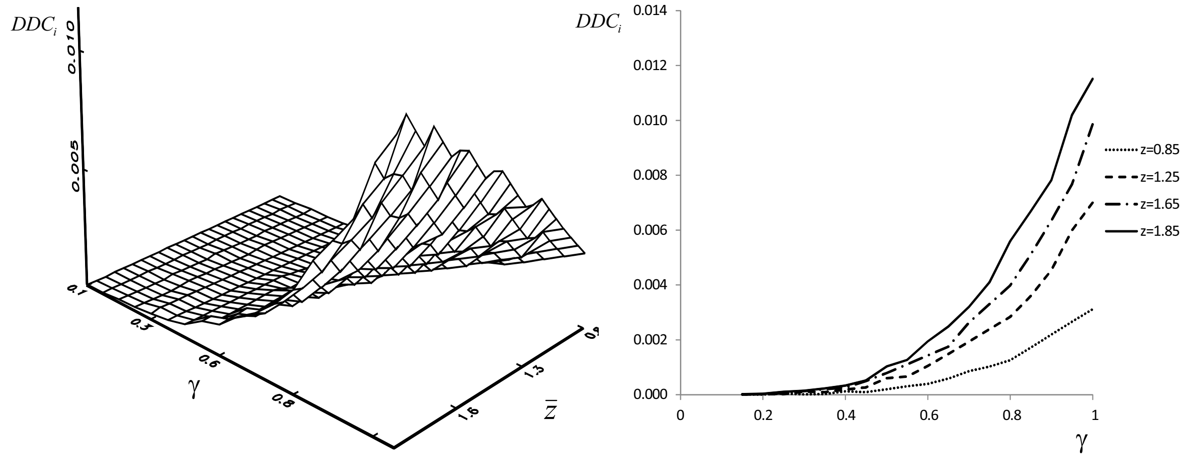


Figure 3.33: SPNE ex-post default costs as a function of the degree of product substitutability and the level of demand uncertainty.

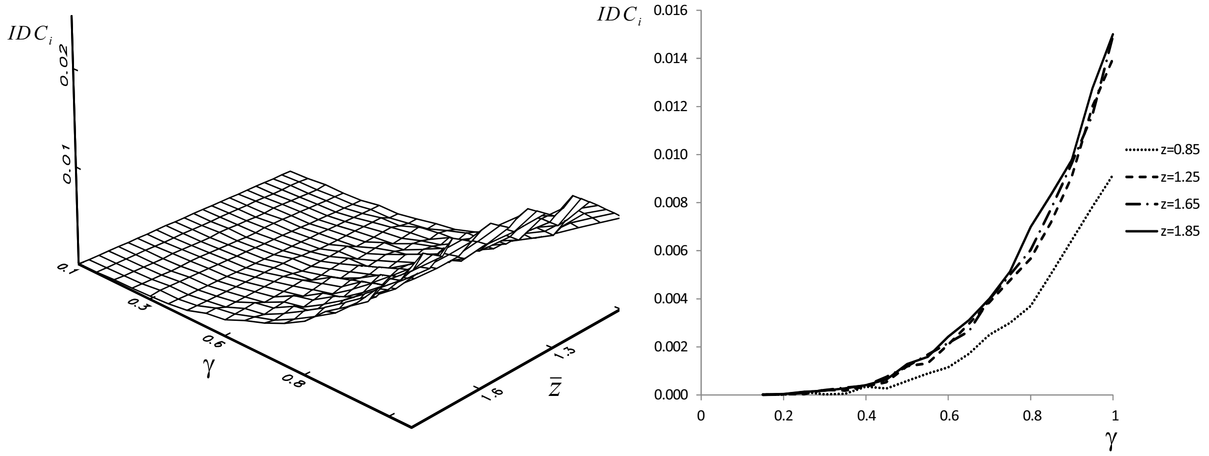


Figure 3.34: SPNE ex-ante default costs as a function of the degree of product substitutability and the level of demand uncertainty.

level whereas the equilibrium quantity level is increasing with uncertainty. However with default cost firms behave more cautiously in the debt market, by issuing less debt. This strategic effect explains why the existence of default cost may lead to higher firms' value and to higher social welfare. The negative impact of the default cost is outweighed by the impact of a more cautious behavior by the firms.

One important difference in the results is that, with default costs and taking into account the values considered for the parameters, a higher uncertainty level leads to a higher default probability, a result which does not hold without default costs. This result is due to the fact that the direct impact of increasing uncertainty has a larger negative direct impact when there are default costs.

The total direct and indirect default costs depend positively on the degree of product substitutability and the level of demand uncertainty.

3.5.2 Impact of symmetric changes in the default cost parameters

In this section we analyze the impact of increasing the default costs parameters in a symmetric two stage duopoly model. In other words, we analyze how the equilibrium values of the variables change with the parameter that captures the decrease in demand due to the loss of reputation when a firm is likely to go bankrupt and the parameter that measures the proportion of profits that bondholders lose when default occur because they have to pay legal, accounting and administrative expenses related with the default process.

In order to analyze the impact of changes at the level of direct (k) and indirect (a_i) default costs parameters on the equilibrium of the variables, we considered two symmetric firms ($c_i = c_j = 0$). We also considered $\alpha = 5$, $\overline{z}_{\max} = 0.9, \gamma = 0.6$, $M = 10$ and $a_i = a_j$. We analyzed the SPNE as a function of the indirect default cost, a_i (represented in the x-axis and we considered values between 0.05 and 0.15) and as a function of direct default cost, k (we represent three k levels: $k = 0.08; k = 0.12; k = 0.18$).

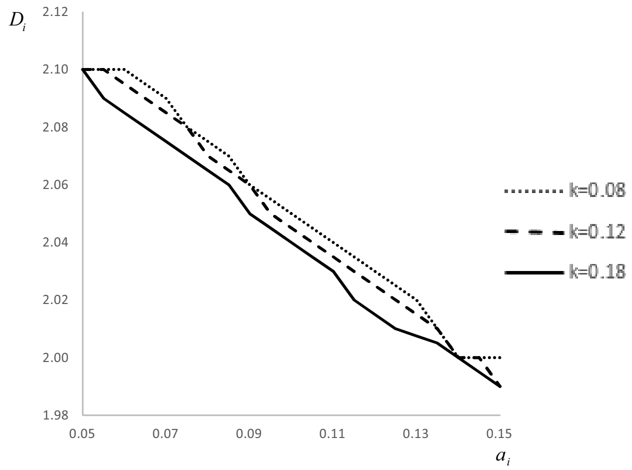


Figure 3.35: SPNE debt obligations as a function of direct and indirect default costs parameters

Figures 3.35, 3.36, 3.37, 3.38 show the equilibrium levels of debt obligations, equilibrium output level, equilibrium interest rate, equilibrium default probability as a function of the indirect default cost parameter, a_i, and as a function of direct default costs parameter, k. These figures allow us to conclude the following:

Result 3.25 *The SPNE level of debt obligation, D^{**}, of output level, q^{**}, of interest rate, r^{**} and of default probability, θ^{**}, are strictly positive and decreasing with the indirect default cost parameter, $a_i = a_j$, and with the direct default cost parameter, k.*

Therefore, the firms tend to adopt a more conservative behavior in the debt market and in the product market as the impact of reputation losses on demand increases and as the losses incurred by bondholders when default occurs increase. The fact that firms behave more cautiously is a quite natural result. If having a high expected default probability has very negative consequence because demand is much lower due to a loss of reputation, firm will try to avoid this reputational demand loss by having lower debt obligation and being less aggressive in the output market. In the case of the direct default cost parameter, when it increases and for given default probability, the interest rate goes up as bondholders will only accept to finance the firm if they receive a compensation high enough to face the

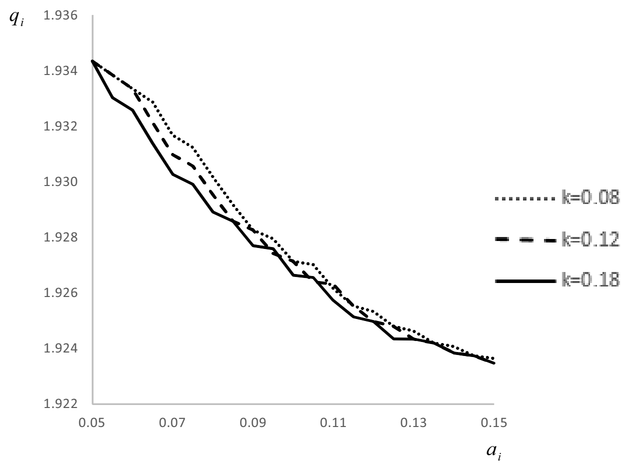


Figure 3.36: SPNE output level as a function of direct and indirect default cost parameters

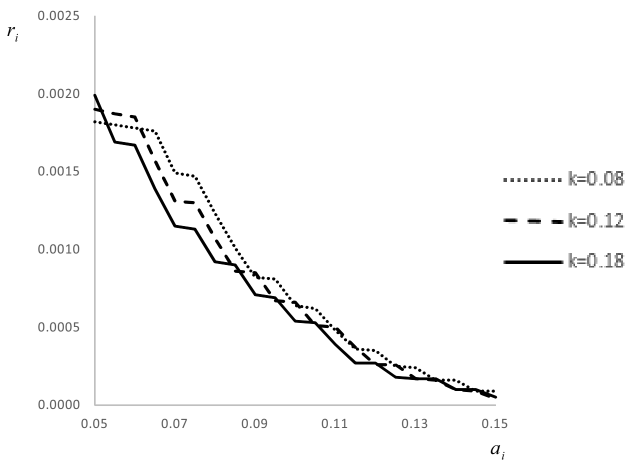


Figure 3.37: SPNE interest rate as a function of direct and indirect default costs parameters

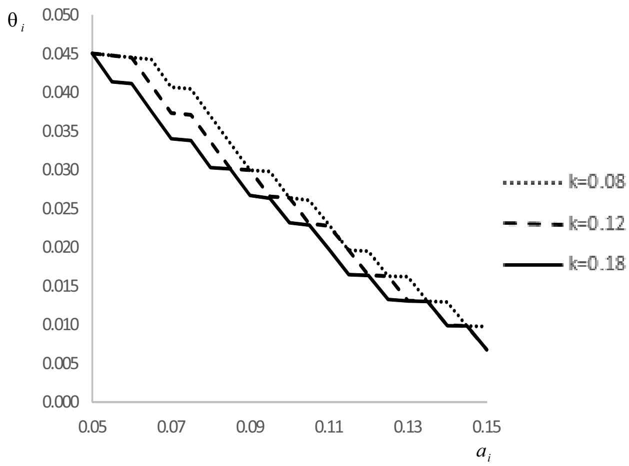


Figure 3.38: SPNE default probability as a function of direct and indirect default costs parameters

higher direct default costs in the event of a default. The best response of the firm to the increase in k is a reduction in the debt obligations. This has a negative indirect impact on the default probability which surpasses the positive direct effect of increasing k. As a result, the equilibrium interest rate also goes down due to the more cautious behavior of the firms.

Figure 3.39 shows the equilibrium expected equity value depends of the indirect default cost parameter, a_i, and it depends on direct default cost parameter, k. The figure allows us to conclude the following:

Result 3.26 *The SPNE expected equity value, V^{**}, is increasing with the indirect default cost parameter, a_i, and with the direct default cost parameter, k.*

Figure 3.40 shows the equilibrium expected debt value as a function of the indirect default cost parameter, a_i, and as a function of the direct default cost parameter, k. The figure allows us to conclude the following:

Result 3.27 *The SPNE expected debt value, W^{**}, is decreasing the indirect default cost parameter, a_i, and with the direct default cost parameter, k.*

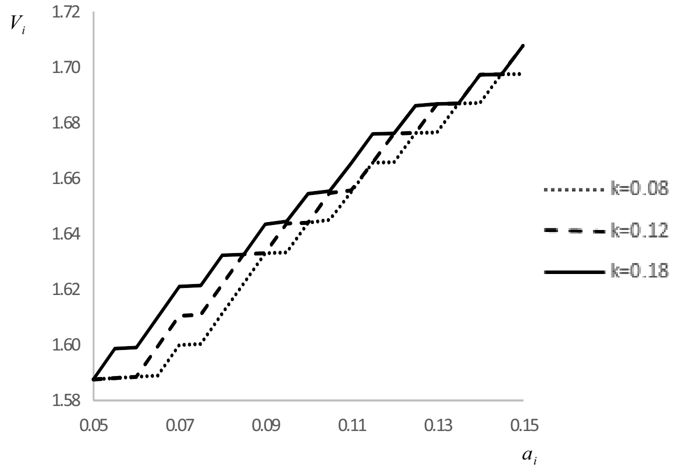


Figure 3.39: SPNE expected equity value as a function of direct and indirect default costs parameters

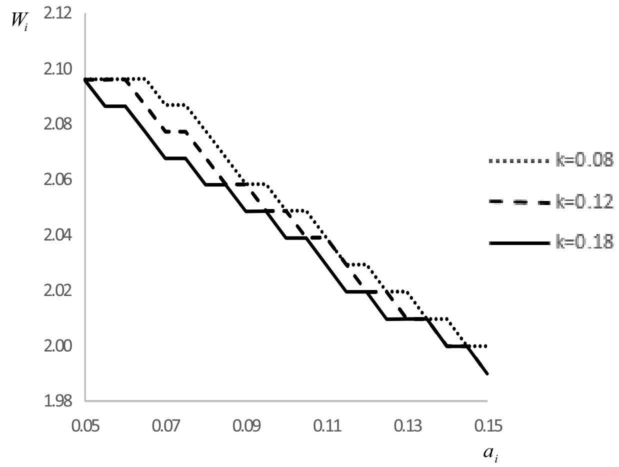


Figure 3.40: SPNE expected debt value as a function of direct and indirect default costs parameters

The behavior of the expected debt value is very similar to the behavior of the debt obligations. In other words, to a great extent the fact that the expected debt value is decreasing with the direct and indirect cost parameters is due to to the fact that debt obligations decrease with these parameter. The fact that the interest rate is also decreasing with default cost parameters implies that W_i should have a smaller decrease than D_i. However, we cannot forget that the default cost parameters have a direct negative impact of the expected value of debt. In fact, the higher are the default cost, the lower is the amount received by bondholders if default happens[10], which for a given interest rate and given debt obligations, reduces the expected value of debt. It seems that the interest rate effect and the direct impact of the default costs tend to cancel each other, explaining why W_i and D_i have a so similar shape.

Figure 3.41 shows the equilibrium firm value as a function of the indirect default cost parameter, a_i, and as a function of the direct default cost parameter, k. This figure allows us to conclude the following:

Result 3.28 *The SPNE firm value, Y^{**}, is increasing with the indirect default cost parameter, a_i, and with the direct default cost parameter, k.*

When demand is more sensitive to the expected default probability (the reputational effect is higher) and the direct losses incurred by bondholders when default occurs increase, we know that the expected value of equity increases whereas the expected value of debt decreases. In other words, increases in the default costs change the capital structure of the firm since the firm becomes more equity financed. Furthermore, the additional value received by the equity holders is higher than the reduction in the expected debt value, which causes an increase in the firm value. Consequently, the increase in the default cost parameters is beneficial for the firm. This counter-intuitive result is explained by the fact that, as a response to the higher default costs, in equilibrium firms behave more cautiously in the debt market and in the product market.

[10] In the case of the direct default cost, the amount received by bondholders is lower because of the default expenses that have to be paid. In the case of the indirect costs, the amount received by bondholders is lower because operating profits are lower when demand is more sensitive to the firm's reputation.

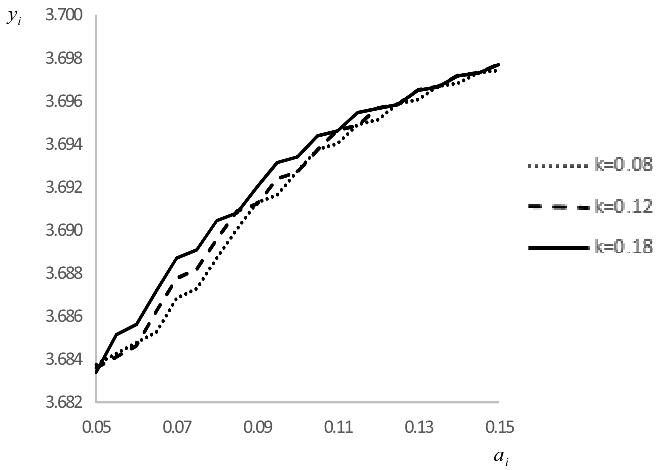


Figure 3.41: SPNE expected firm value as a function of direct and indirect default costs parameters

Figure 3.42 shows the equilibrium welfare as a function of the indirect default cost parameter, a_i, and as a function of the direct default cost parameter, k. The figure allows us to conclude the following:

Result 3.29 *The SPNE welfare level, Wel^{**}, is decreasing with the indirect default cost parameter, a_i, and increasing with the direct default cost parameter, k.*

The previous result shows that the impact of increasing the direct default cost and the indirect default cost parameters on social welfare is very different. While increases in the direct default costs are welfare improving, increases in the indirect cost parameters are welfare reducing. The explanation for the opposite impacts is related with the impact on the consumer surplus. In our model consumers are clearly worse off when a firm is expected to fail. An increase in the indirect default cost parameter, has a direct negative impact on the consumer surplus whereas an increase in direct default costs only influences the consumer surplus indirectly.

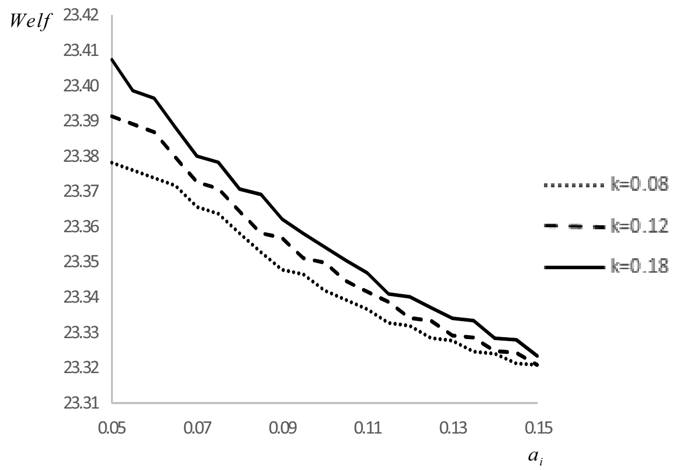


Figure 3.42: SPNE welfare as a function of direct and indirect default costs parameters

3.5.3 Impact of asymmetric changes in the indirect default cost parameters

In this section, the objective is to study the impact of asymmetric changes in the indirect default cost parameters, a_i and a_j. One interpretation of this exercise is that the reputational demand loss may not be the same for the two firms. In other words, we consider that firms can differ in their reputation effect parameter, i.e. we can have $a_i \neq a_j$. We consider two symmetric firms $(c_i = c_j = 0)$ and $\alpha = 5$, $\overline{z}_{\max} = 0.9, \gamma = 0.6$ and $M = 10$. We analyze the SPNE as a function of the indirect default cost parameter of firm i, a_i (represented in the x-axis) and as a function of the indirect default cost parameter of firm j, a_j (we represent three a_j levels: $a_j = 0.05; a_j = 0.08$; $a_j = 0.10$).

Figures 3.43, 3.44, 3.45, 3.46 show the equilibrium levels of debt, equilibrium output level, equilibrium interest rate, equilibrium default probability as a function of the indirect default cost parameter of firm i, a_i and as a function of the indirect default cost parameter of firm j, a_j.

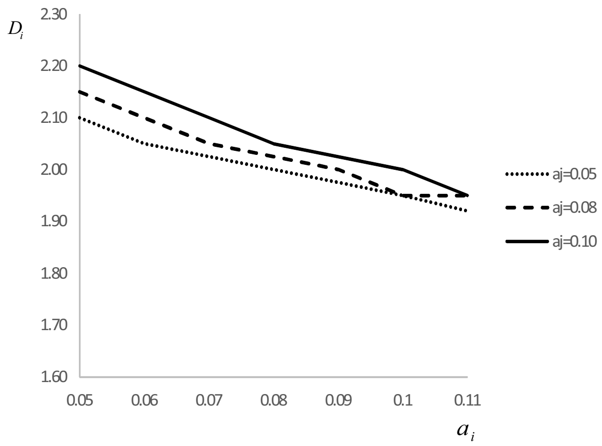


Figure 3.43: SPNE debt obligation of firm i as a function of indirect default cost parameters of both firms

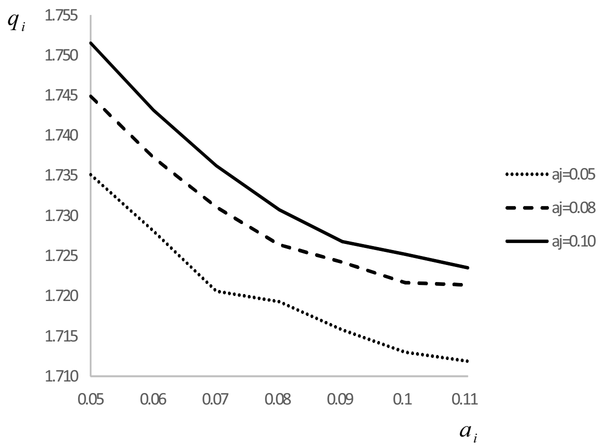


Figure 3.44: SPNE output level as a function of indirect default cost parameters of both firms

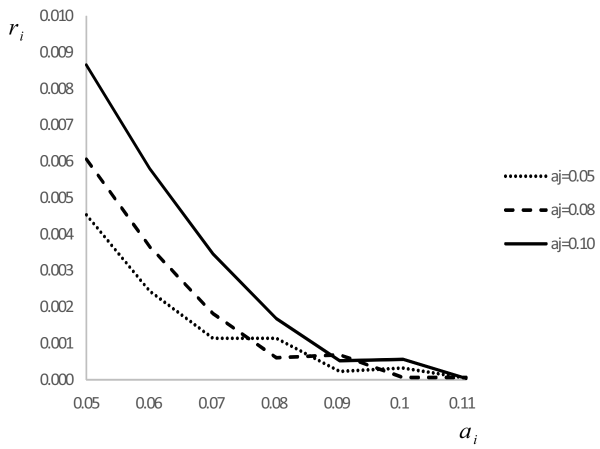


Figure 3.45: SPNE expected interest rate of firm i as a function of indirect default cost parameters of both firms

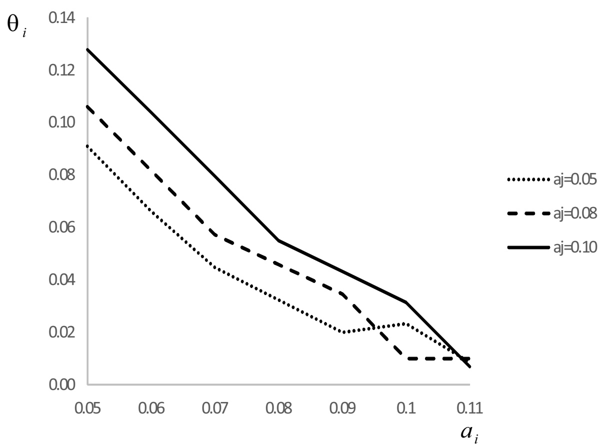


Figure 3.46: SPNE default probability of firm i as a function of indirect default cost parameters of both firms

These figures allow us to conclude the following:

Result 3.30 *For firm i, the SPNE debt obligation, D_i^{**}, the SPNE level of output, q_i^{**}, the SPNE level of interest rate, r_i^{**}, and the SPNE level of default probability, θ_i^{**}, are strictly positive and decreasing with its indirect default cost parameter, a_i, and increasing with the indirect default cost parameter of the rival firm, a_j.*

Regarding the effect of the indirect default cost of the firm i and indirect default cost of the firm j, the previous result shows that the effect of the increase in the reputation effect of the firm i, a_i, leads to a more conservative behavior of the firm in the debt and in the output market, which implies a decrease in the interest rate and in the default probability. Therefore the strategic effect outweighs the direct negative impact of increasing a_i on the default probability.

On the contrary, an increase in the demand sensitivity to the reputation of firm j, a_j, implies a more aggressive behavior in the debt market and in output market by firm i, which leads to an increase in the interest rate and in the default probability of firm i.

Figure 3.47 shows the equilibrium expected equity value as a function of the indirect default cost parameter of firm i, a_i and as a function of the indirect default cost parameter of firm j, a_j. The figure allows us to conclude the following:

Result 3.31 *The SPNE expected equity value of firm i, V^{**}, is increasing with the indirect default cost parameter of the firm, a_i, and decreasing with the indirect default cost parameter of the rival firm, a_j.*

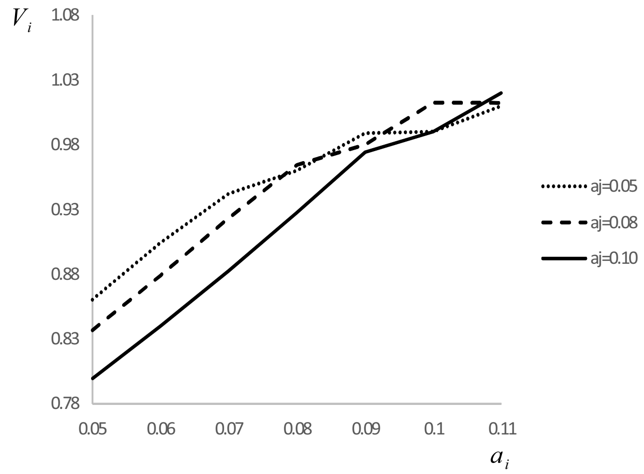


Figure 3.47: SPNE expected equity value as a function of indirect default cost parameters of both firms

The figures 3.48 and 3.49 allow us to conclude the following:

Result 3.32 *The SPNE expected debt value of firm i, W_i^{**}, and the SPNE value of firm i, Y_i^{**}, are decreasing with the indirect default cost parameter of the firm, a_i, and increasing with the indirect default cost parameter of rival, a_j.*

As a consequence, a unilateral increase in the indirect default cost parameter of firm i, a_i, is detrimental for the firm and beneficial for the rival firm. As a_i increases, the firm finances less with debt and more with equity (the equity values increases while the debt value decreases). Since the decrease in the debt value has a larger magnitude than the increase in the equity value, the value of the firm decreases. Despite the more cautious behaviour of firm i when a_i increases, which leads to a reduction in the default probability, the negative direct impact of the increase in the indirect default costs implies that the total effect on on the firm's value is negative. On the contrary, unilateral increases in the indirect default cost parameter of the rival firm, a_j are beneficial for firm i. The expected value of firm i increases because the decrease in the demand of the rival firm, leads the rival to produce less, which increases the expected profit of firm i.

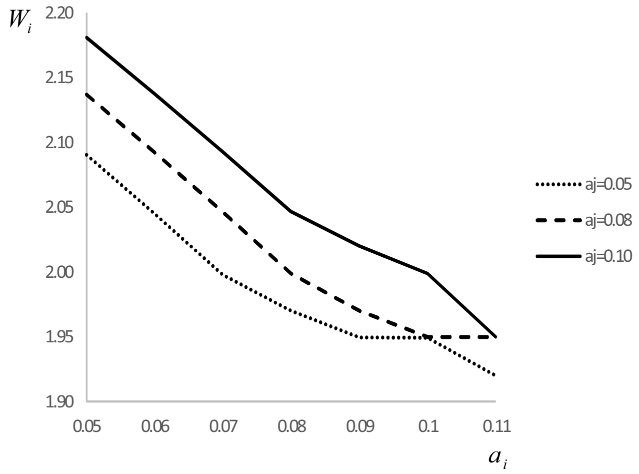


Figure 3.48: SPNE expected debt value as a function of indirect default cost parameters of both firms

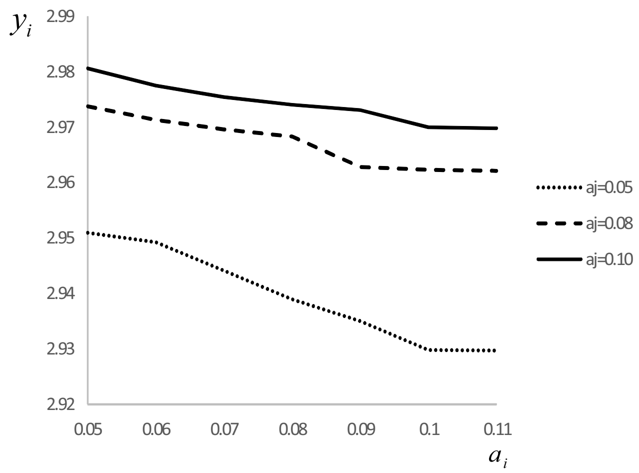


Figure 3.49: SPNE expected firm value as a function of indirect default cost parameters of both firms

Figure 3.50 shows that the equilibrium welfare depends on the indirect default cost parameter of firm i, a_i and on the indirect default cost parameter of firm j, a_j. The figure allows us to conclude the following:

Result 3.33 *The SPNE welfare level, Wel^{**} is decreasing with the indirect default cost parameter of the firm i, a_i, and with the indirect default cost parameter of the firm j , a_j.*

When a_i increases, firm i is worse off while firm j is better off. But besides these two contradictory effects, an increase in a_j also implies lower expected consumer surplus. Therefore it is not surprising that social welfare decreases when the indirect default cost parameters increase. The fact that consumers are uncertain regarding whether the firm is going to survive or not and that because of that uncertainty buy less in this market represents a real loss for society as consumer shift part of their expenditures to goods with smaller utility (but without uncertainty).

Figures 3.51 and 3.52 show the equilibrium direct and indirect total default costs as a function of the indirect default cost parameter of the firm i, a_i, and as a function of the indirect default cost parameter of the firm j, a_j. The figure allows us to conclude the following:

Result 3.34 *The SPNE direct and indirect default costs of firm i, DDC_i^{**} and IDC_i^{**}, are decreasing with its indirect default cost parameter, a_i, and increasing with the indirect default cost parameter of the rival firm, a_j.*

Regarding the effect of the indirect default cost parameter of firm i, a_i, on its own total direct and indirect default costs, DDC_i^{**} and IDC_i^{**}, we need to consider three effects. The first effect is the direct effect (for given, output and debt levels). This direct effect is clearly positive, that is, higher a_i implies higher default costs because a higher a_i leads to a lower demand, which decreases the operating profits and therefore increases the default probability. Regarding the indirect effects, the fact that there is a larger indirect default cost parameter, a_i, leads firm i to behave in a more conservative manner in the debt market and in output market. This effect tends to decrease the default probability

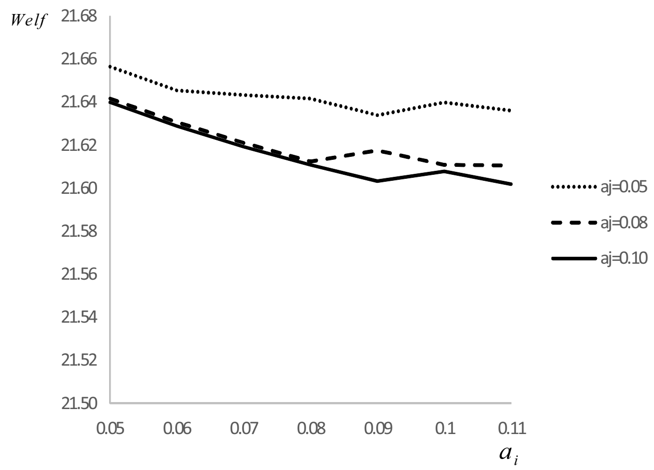


Figure 3.50: SPNE welfare as a function of indirect default cost parameters of both firms

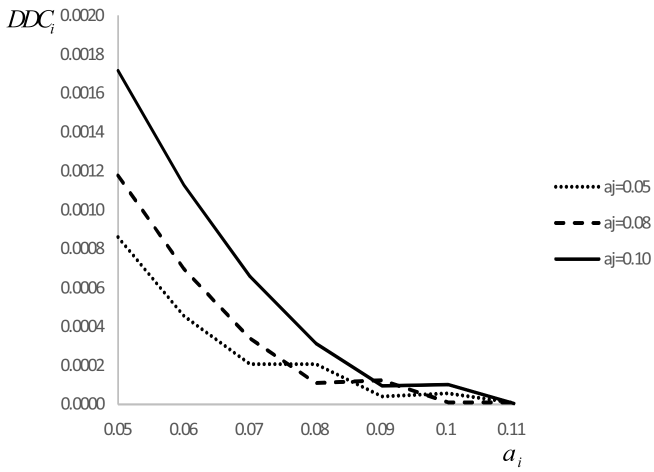


Figure 3.51: SPNE direct default cost as a function of indirect default cost parameters of both firms

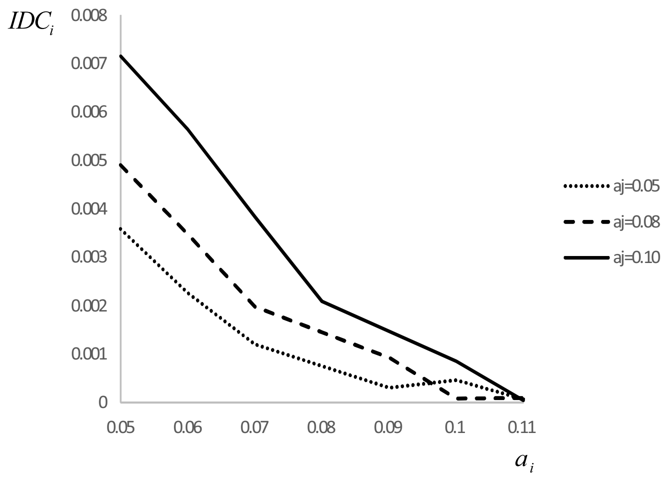


Figure 3.52: SPNE indirect default cost as a function of indirect default cost parameters of both firms

and therefore decrease the direct and indirect default cost of the firm i. We conclude that the indirect effects dominate, i.e. firms behave less aggressively in the debt and output market when the indirect default cost parameter of the firm i is higher, which leads to lower total direct and indirect default costs. The opposite behavior is observed when we analyze the increase in an indirect default cost parameter of the rival firm, a_j.

3.6 Conclusion

The present work examined, analytically and numerically, how the market structure and the default costs influence financial and product market decisions and, consequently, how they affect the default risk. We considered a two stage duopoly model. In the first stage, firms simultaneously decide the level of debt that maximizes the firm value and, in the second stage of the game, firms simultaneously decide the quantity that maximizes the equity value. To find the subgame perfect Nash equilibrium, the model was solved backwards. We first determined the Nash equilibrium of the quantity competition game and then determined the equilibrium levels of debt. Due to the complexity of the problem, we had to solve the model analytically using GAUSS. We determined the equilibrium values of several variables: debt obligation, output, expected equity value, expected debt value, expected value of the firm, implicit interest rates, default probabilities and social welfare. The numerical model was run for many values of the parameter of the model in order to allow us to study the impact of changes in the level of demand uncertainty, in the degree of product substitutability, in the default costs and in the level of asymmetry in marginal production costs on the equilibrium values of the previous variables and, in particular, on the equilibrium default probability.

We studied two scenarios. In the first one, we assume that there were no default costs. In the second scenario we incorporated default cost into our analysis. We considered two types of default costs: (i) Direct default costs, stemming from the legal, accounting and administrative expenditures that have to be supported if default occurs. This direct default costs are supported primarily by bondholders as they keep the firm's profit when default occurs and; (ii) Indirect default costs, related to the reputation effect, i.e., to the reduction in demand resulting from the fear that customers may have when the probability of default is positive. In each scenario, we analyzed the case where the two firms have the same marginal costs (symmetric case) and the case where firms differ in their marginal cost of production (asymmetric case).

When default costs are not considered, we concluded that, in the symmetric model, debt obligations decreases with uncertainty whereas the degree of product substitutability does not have a monotonic impact on the equilibrium debt obligation. Moreover the default probability is increasing with the degree of product substitutability and decreasing with the level of uncertainty. This last result seems strange, but it is easy to explain. There are direct and indirect effect of the level of uncertainty on the default probability. It is true that, for given debt level and quantity levels, increasing uncertainty leads to an increase in the default probability. However, the increase in the uncertainty level also decreases the equilibrium debt obligations and this leads to a decrease in the default probability (there are also indirect effects through the equilibrium quantity levels). It turns out that the impact of the firm reducing its debt obligations outweights the direct impact of increasing uncertainty, which explains the result.

In our second setup we analyzed the impact marginal costs asymmetry. Regarding the equilibrium output, the qualitative results are very similar to ones obtained in a traditional oligopoly (without limited responsability): the equilibrium output decreases whith the firm's marginal cost and increases with the rival firm's marginal cost. However we also obtain the interesting result that as the firm becomes less efficient, in equilibrium it issues less debt whereas the more efficient firm becomes «more aggressive» in the debt market. As a consequence of this more cautious behavior, the default probability of the inefficient firm decreases as the firm becomes less efficient. This is counter-intuitive result happens because the impact of the more cautious behavior in the debt market outweight the direct impact of an increase in the firm's marginal costs, which obvioulsy increases the default probability.

The analysis of the model with symmetric default costs shows that the impact of changes in the level of uncertainty and changes in the degree of product substitutability are, in general, quite similar to the ones verified when default costs were not considered, if we just look at the qualitative. For instance, in both cases the equilibrium level of debt is decreasing with the uncertainty level whereas the equilibrium quantity level is increasing with uncertainty. There are however some differences that should be highlighted. The first one is that with default cost firms behave more cautiously in the debt market, by issuing less debt. This strategic effect explains why the existence of default cost may lead to higher firms' value and to higher social welfare. This counter-intuitive results is due to the fact that the negative direct impact of the default cost is outweighed by the positive impact of a more cautious behavior by the firms. Another important difference in the results is that, with default costs and taking into account the values considered for the parameters, a higher uncertainty level leads to a higher default probability, a result which does not hold without default costs. With default costs, increasing uncertainty has a much larger negative direct impact on the default probability and this effect is not totally compensated by the more cautious behaviour of the firm. In other words, with the default costs, the direct effect of increasing uncertainty dominates the indirect impact through changes in the equilibrium debt levels. Finally, this section showed that the total direct and indirect default costs depend positively on the degree of product substitutability and the level of demand uncertainty.

In the two last sections we analyzed the impact of changing the direct and indirect default cost parameter. We considered first symmetric changes in these parameter (i.e., the parameters change equally for the two firms) and next we explored what happens when the reputation effect parameter of firm increases unilaterally. When we considered symmetric changes in the default cost parameters we concluded that firms behave in a more conservative way both in debt market and in the output market. As a consequence the default probabilities as well as the interest rates go down when default cost parameters increase. Furthermore, the expected debt value decreases but the expected equity value increase when the default costs parameters increasing. The additional expected equity value is higher than the reduction in the expected debt value, which means that the expected value of the firm goes up. This happens because the reduction in the default probability has a positive impact on the expected profits that more than compensates the decrease in the expected profits due to the increase in the default cost parameters. In other words, symmetric increases in the default cost parameter lead to an increase in the expected profits due to the more cautious behavior of the two firms. A similar reasoning explains why, despite the increase in the default cost parameters, the total direct and total indirect default costs go down. Finally, changes in the direct and indirect default cost parameter have different effects on the welfare. While increases in the direct default cost parameter are welfare improving, increases in indirect default cost parameter reduce welfare, which is due to the fact that the indirect default cost parameter have a direct negative impact on the consumer surplus.

The last section reveals that unilateral changes in the indirect default cost parameter have quite different impacts. A firm behaves less aggressively in the debt and in output market when its indirect default cost increases, while the rival firm behaves more aggressively. Moreover, as expected, a unilateral increase of a firm indirect default costs is detrimental for the firm, but beneficial for the rival firm.

Overall, our results suggest that the equilibrium default probability is greatly influenced by the financial and product market decisions of the firms, who optimally adjust their behavior to structural changes in the industry. Therefore a less favorable environment does not necessarily imply higher default probability, as the firm may respond by financing less with debt.

3.7 Appendix

3.7.1 A- Asymmetric duopoly with default costs

In this appendix we study what happens when firm j becomes less efficient by analyzing the SPNE as the marginal cost of firm j, c_j, increases between 0 and 0.5. The aim is to analyze if the impact of a unilateral increase in marginal production cost is different when default costs are considered.

Figures 3.53 and 3.54 show the debt level of the firm i and the debt level of the firm j as a function of the marginal cost of the firm j, cj, considering the default costs. These figures allow us to conclude the following:

Result 3.35 *The SPNE debt obligation of the firm i, D_i^{**}, is increasing with the marginal cost of production of the firm j, c_j. On the contrary, the SPNE debt obligation of firm j, D_j^{**}, is decreasing with the marginal cost of production of the firm j, c_j.*

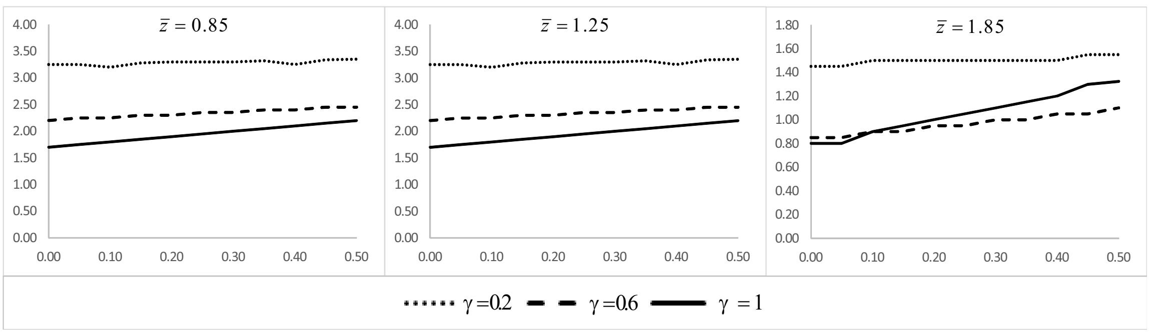


Figure 3.53: SPNE debt obligation of the more efficient firm as a function of the rival's marginal costs considering default costs.

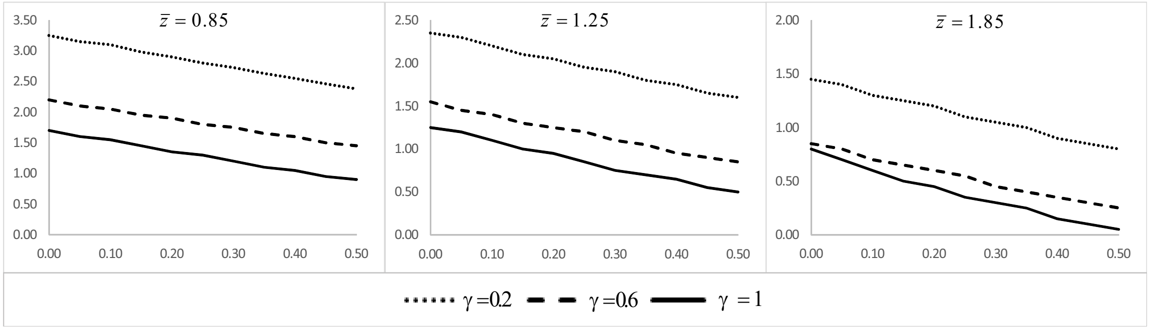


Figure 3.54: SPNE debt obligation of the more inefficient firm as a function of its marginal costs considering default costs.

Therefore the impact of increasing the marginal costs of firm j is qualitatively very similar to what happens without default costs: the less efficient firm becomes more cautious in the debt market by issuing less debt while the opposite happens for the more efficient firm.

Figures 3.55 and 3.56 show the output level of firm i and the output level of firm j as a function of the marginal cost of firm j, c_j, when the default costs are considered. These figures allow us to conclude the following:

Result 3.36 *The SPNE level of output firm i, q_i^{**}, is increasing with the rival's marginal cost of production, c_j. On the contrary, the SPNE level of output of firm j, q_j^{**}, is decreasing with the firm's marginal cost of production.*

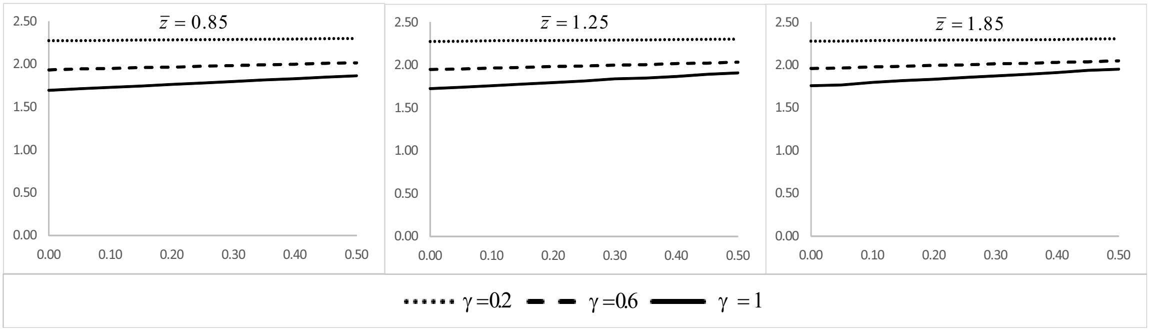


Figure 3.55: SPNE output level of the more efficient firm as a function of the rival's marginal costs considering default costs

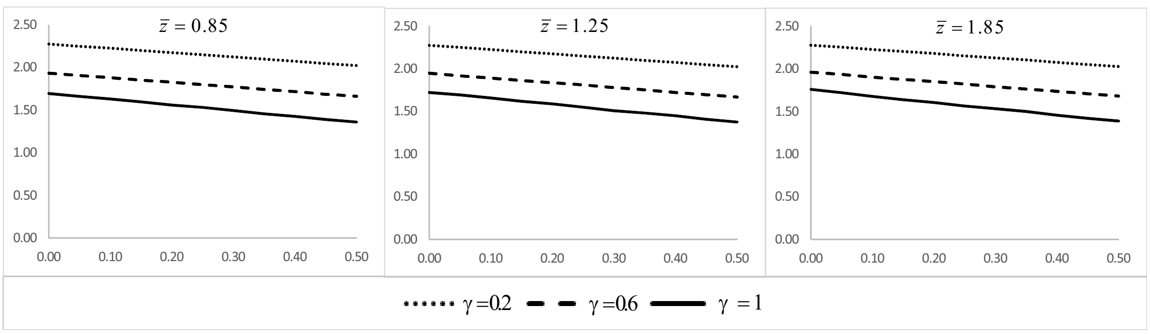


Figure 3.56: SPNE output level of the more inefficient firm as a function of its marginal costs considering default costs.

Therefore, the impact of a unilateral increase in marginal cost of firm j has the same sign than when default costs are not considered and the same sign that would occur in a traditional duopoloy model (without limited liability).

Figures 3.57 and 3.58 show the interest rate of firm i and the interest rate of firm j as a function of the marginal cost of firm j, c_j, considering the default costs. These figures allow us to conclude the following:

Result 3.37 *The SPNE interest rate of firm i, r_i^{**}, is decreasing with the marginal cost of firm j, c_j. On the contrary, the SPNE interest rate of firm j, r_j^{**}, is increasing with its marginal cost, c_j.*

Comparing the previous figures to the ones when there are no default cost, the most remarkable difference is that interest rates are now much lower, which is a result of the more cautious behavior of the two firms when default costs are present.

Figures 3.59 and 3.60 show the default probability of firm i and the default probability of firm j as a function of the marginal cost of production of the firm j, c_j, when default costs are considered. These figures allow us to conclude the following:

Result 3.38 *The SPNE default probability of firm i, θ_i^{**}, increases slightly with the marginal cost firm j, c_j and the SPNE default probability of firm j, θ_j^{**}, is decreasing with its marginal cost, c_j. The default probabilities are more sensitive to changes in c_j when the degree of product substitutability is high.*

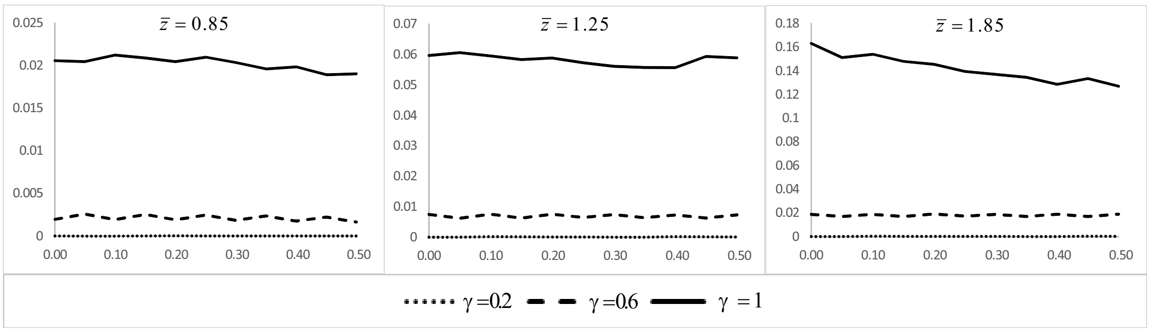


Figure 3.57: SPNE interest rate level of the more efficient firm as a function of the rival's marginal costs considering default costs.

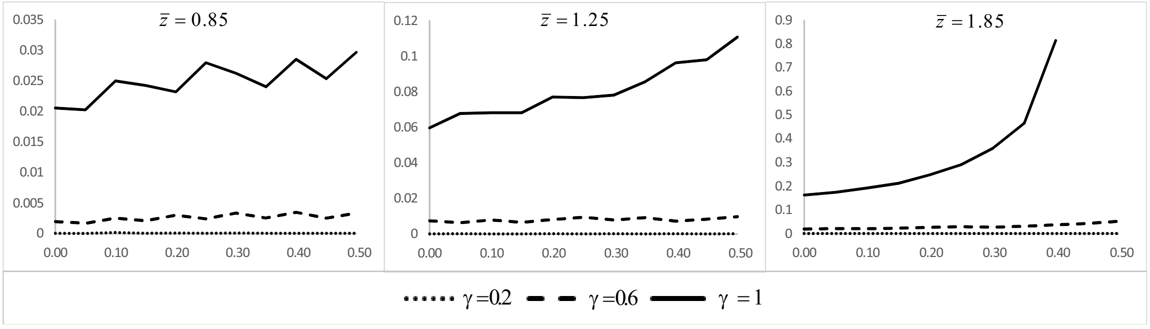


Figure 3.58: SPNE interest rate level of the less efficient firm as a function of its marginal costs considering default costs.

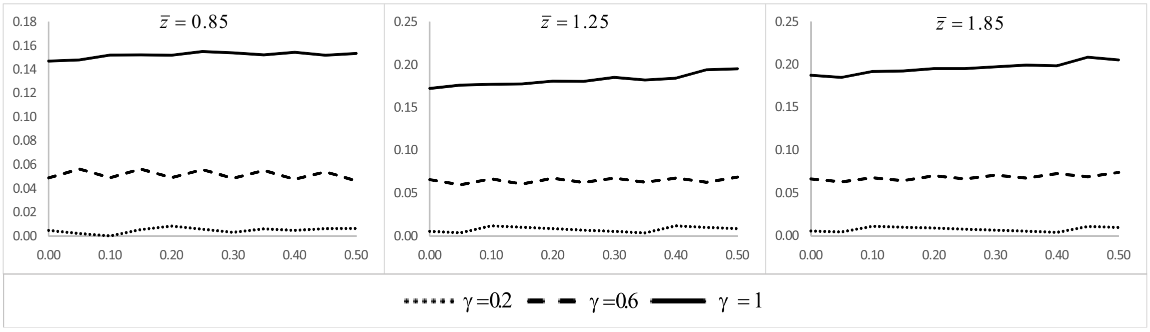


Figure 3.59: SPNE default probability of the more efficient firm as a function of the rival's marginal costs considering default costs.

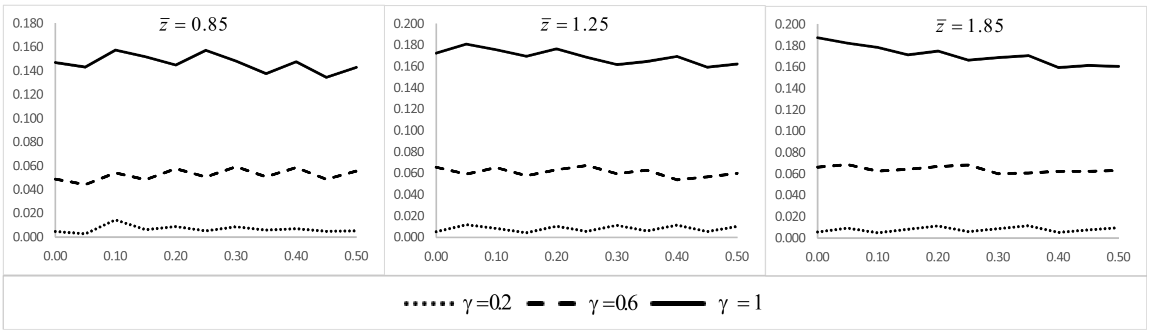


Figure 3.60: SPNE default probability of the less efficient firm as a function of its marginal costs considering default costs.

This result shows that, in qualitative terms, the effect of the increase in c_j on the default probabilities is very similar to the one without default costs. In particular, an increase in c_j has opposite effects on the default probability of firm i and firm j. However it should be emphasized that the default probabilities are much lower with default costs. This is explained by the fact that, in equilibrium, both firms have lower debt obligations as a consequence of the default costs.

Figures 3.61, 3.62, 3.63, 3.64, 3.65 and 3.66 show the expected equity values, expected debt values and expected firm values of firm i and firm j, respectively, as a function of the marginal cost of production of the firm j, c_j, considering the default cost. These figures allow us to conclude the following:

Result 3.39 *The SPNE expected equity value of the firm i, V_i^{**}, the SPNE expected debt value of the firm i, W_i^{**}, and the expected value of the firm i, Y_i^{**}, are increasing with the rivals' marginal cost of production. On the contrary, the SPNE expected equity value of firm j, V_j^{**}, the SPNE expected debt value of the firm j, W_j^{**}, and the expected value of the firm j, Y_j^{**}, are decreasing with the firm's marginal cost of production of the firm j, c_j .*

The previous result reveals that the effect of a unilateral increase in the marginal cost of firm j on the two firms' value (total value, value for shareholders and value for bondholders) has the same sign than when no default costs were considered.

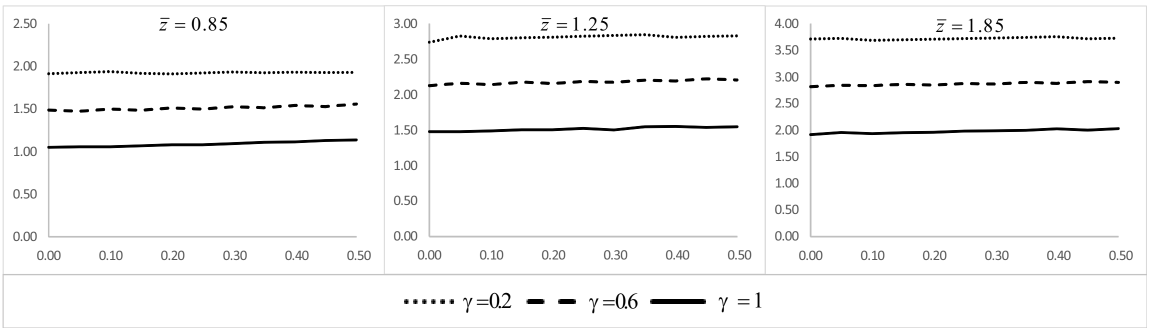


Figure 3.61: SPNE expected equity value of the more efficient firm as a function of the rival's marginal costs considering default costs.

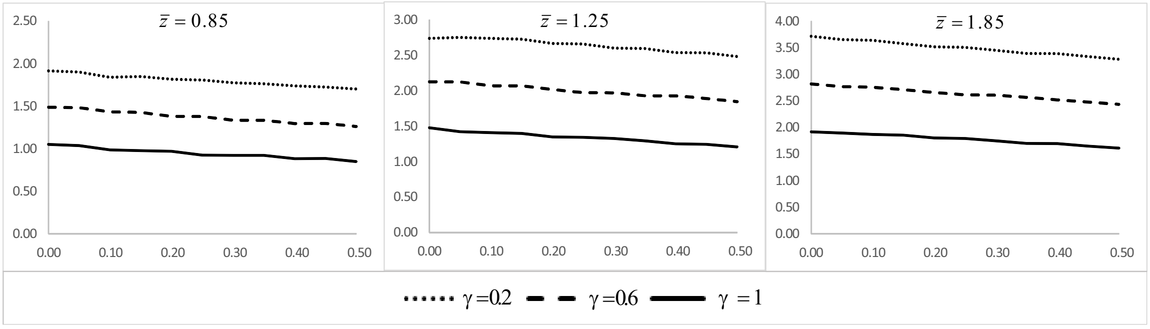


Figure 3.62: SPNE expected equity value of the less efficient firm as a function of its marginal costs considering default costs.

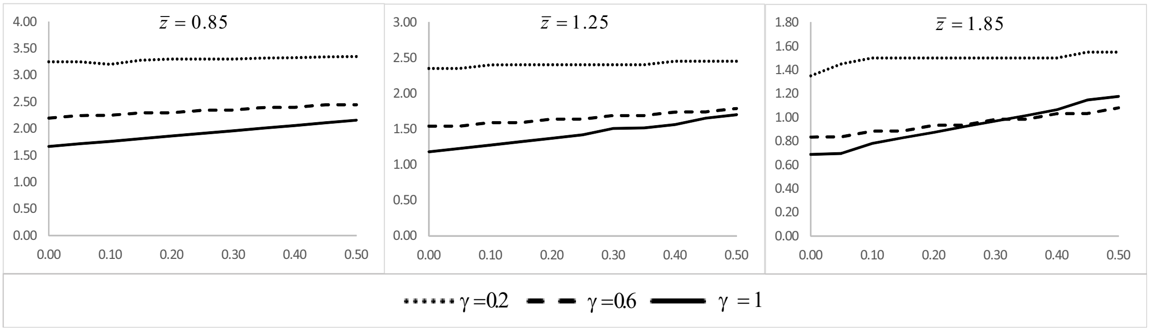


Figure 3.63: SPNE expected debt value of the more efficient firm as a function of the rival's marginal costs considering default costs.

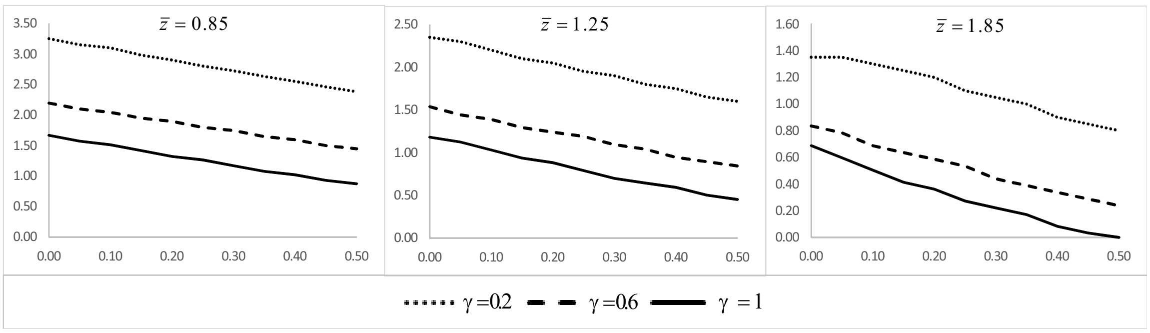


Figure 3.64: SPNE expected debt value of the less efficient firm as a function of its marginal costs considering default costs.

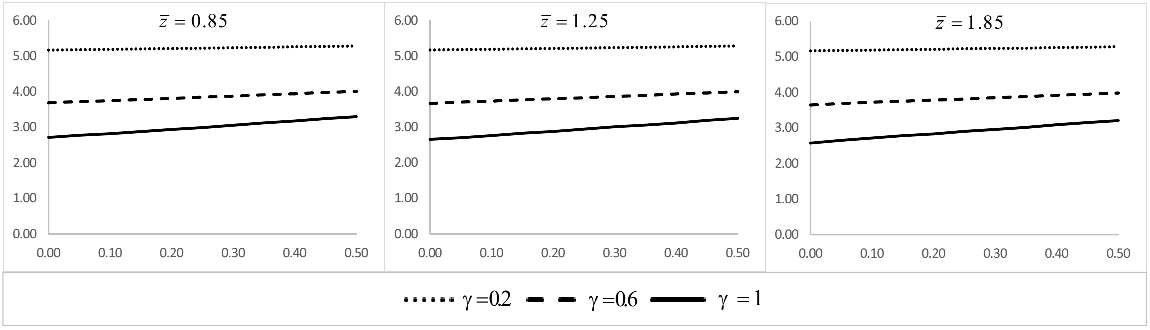


Figure 3.65: SPNE expected firm value of the more efficient firm as a function of the rival's marginal costs considering default costs.

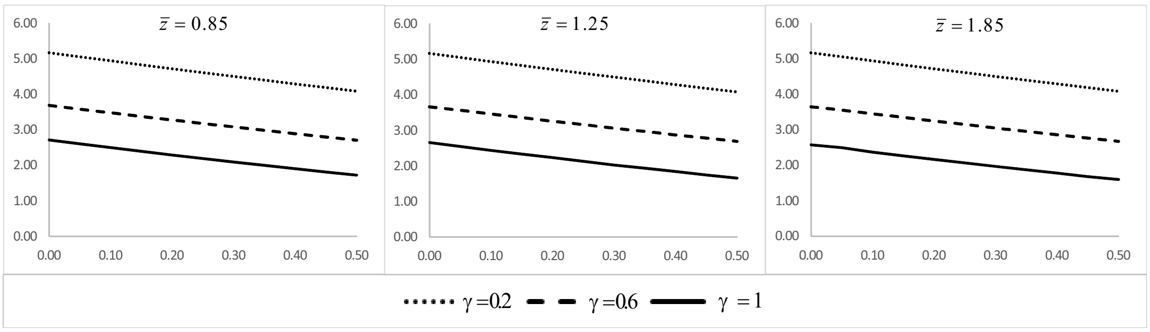


Figure 3.66: SPNE expected firm value of the less efficient firm as a function of its marginal costs considering default costs.

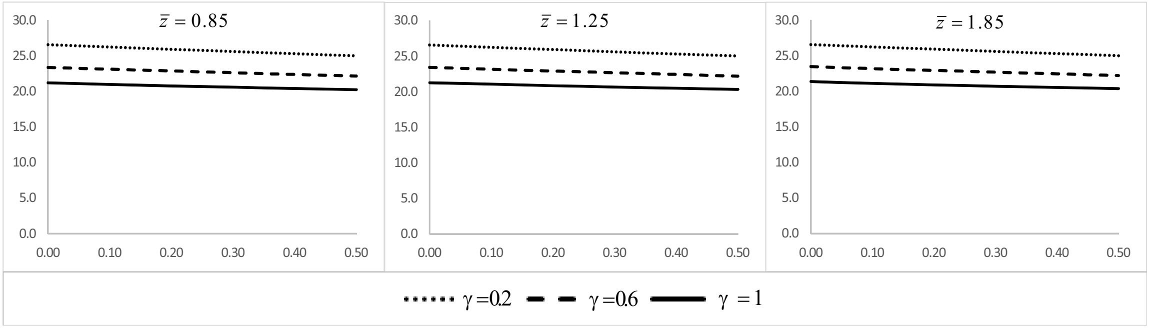


Figure 3.67: SPNE expected social welfare as a function of the less efficient firm's marginal costs considering default costs.

Figure 3.67 shows the welfare level as a function of the marginal cost of production of firm j, c_j, when default costs are considered. The figure allows us to conclude the following:

Result 3.40 *The SPNE of the welfare, Wel^{**}, is decreasing with the marginal cost of production of the firm j, c_j. When default costs are considered the SPNE welfare level is higher than when they are not considered.*

Figures 3.68, 3.69, 3.70 and 3.71 show the equilibrium of direct and indirect default costs as a function of the marginal cost of production of firm j, c_j, considering the default costs. The figures allow us to conclude the following:

Result 3.41 *The SPNE total direct and indirect default costs of the firm i, DDC_i^{**} and IDC_i^{**}, is increasing with the marginal cost of firm j, c_j. On the contrary, the SPNE of direct and indirect default costs of the firm j, DDC_j^{**} and IDC_j^{**} follows a U relationship with the marginal cost of production of the firm j, c_j. The change of behavior occurs for intermediate levels of marginal cost of production of the firm j, c_j.*

After analyzing the results we conclude that when default costs are considered, the impact of a unilateral increase in the marginal production cost is less pronounced in the more efficient firm. For the less efficient firm we can conclude that when default cost are considered, there are some effects that are less pronounced and others that are

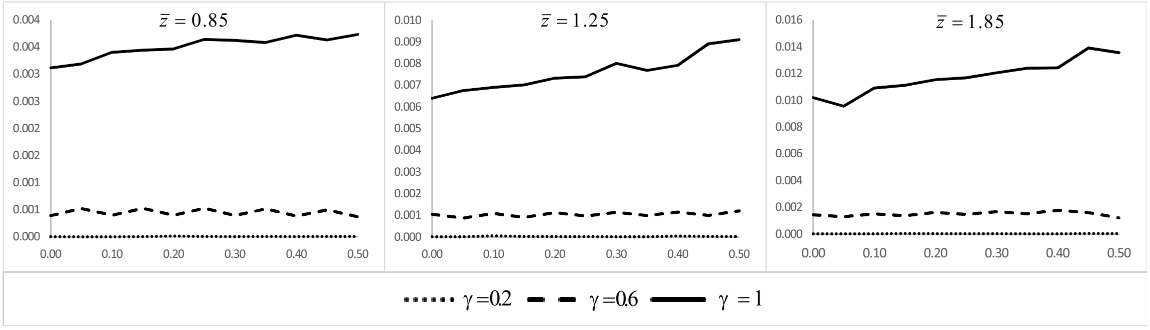


Figure 3.68: SPNE ex-post default costs of the more efficient firm as a function of the rival's marginal costs

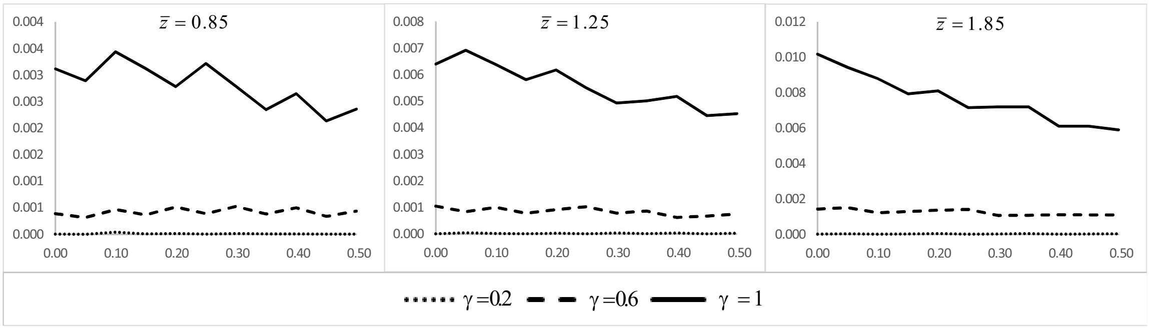


Figure 3.69: SPNE ex-post default costs of the less efficient firm as a function of its marginal costs

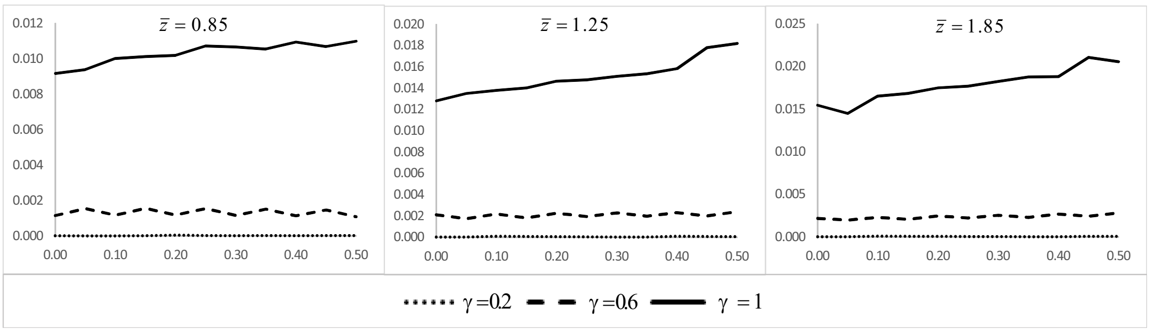


Figure 3.70: SPNE ex-ante default costs of the more efficient firm as a function of the rival's marginal costs

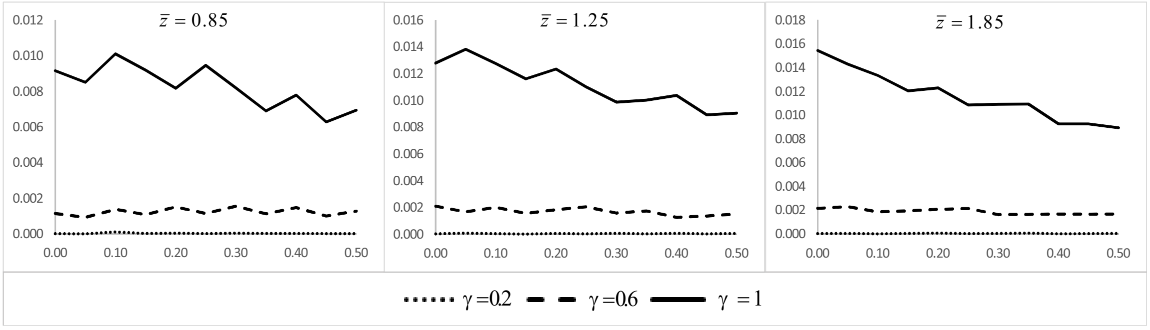


Figure 3.71: SPNE ex-ante default costs of the less efficient firm as a function of its marginal costs

more evident. As direct and indirect default costs depend directly on the probability of default, it is natural to present a similar relationship to a unilateral increase in marginal production costs.

3.7.2 B- Gauss Program

```
/*This program computes the SPNE of a model, considering linear demands with differentiated*/
/* products, constant marginal costs which may be asymmetric, demand uncertainty with a */
/*uniform distribution of the uncertainty parameter and Cournot competition. We consider in */
/* our analysis the default costs. Subdivided into two types: a1 and a2 that represent the */
/* reputation effect, i.e. a decrease in demand caused by consumers fear (indirect costs); the k */
/*suffered by bondholders when the probability of default is greater than zero (direct costs). */
/*Alpha is the effect, represents the losses dimension of  the market, gama is the differentiation*/
/*parameter zbarra is the uncertainty parameter and c1 and c2 are marginal costs. The program */
/*determines first the NE of the second stage game, for given debt levels (D1,D2) and for each */
/*(D1,D2) the equilibrium value of each firm, (Y1,Y2), is computed.This is repeated for many*/
/* (D1,D2) and the equilibrium values of Y1 and Y2 are saved in two matrices. Next the NE of */
/*the first stage game is determined  (D1eq,D1eq) and the corresponding NE of the second stage*/
/* game and equilibrium default probabilities are determined. This procedure is repeated for many*/
/* values of the parameter values so as to analyze how the equilibrium changes with changes in */
/* the parameter values. */
library co;
#include co.ext;
coset;
/****************************** Inicial parameters of the model *****************************/
alpha=5;/* expected size of the market*/
k=0.10; /* direct default cost: loss of pay suffered by creditors */
M=10; /* budget constraint*/
zbarmax=1.85; /* maximum value of the uncertainty degree */ ;
zbarmin=1.25; /* minimum value of the uncertainty degree */
c1=0; /* marginal cost of firm 1 */
c2=0.45; /* marginal cost of firm 2 */
a1=0.05; /* indirect marginal default costs of firm 1: reputation effect*/
a2=0.05; /* indirect marginal default costs of firm 2: reputation effect*/
gamamax=1; /* maximum the differentiation parameter */
gamamin=0.2; /* minimum value of the differentiation parameter */
zbarra=zbarmin; /* start value of uncertainty degree */
saltozbar=0.40; /* step size for the iterations on the uncertainty degree */
saltgama=0.40; /* step size for the iterations on the uncertainty degree */
niterzbar=int((zbarmax-zbarmin)/saltozbar)+1; /* number of iterations for uncertainty degree */
nitergam=int((gamamax-gamamin)/saltgama)+1; /*number of iterations for differentiation */
parameter*/
/***Create matrices to keep the SPNE values of quantities, debt and default probabilities ***/
D1eqmat=zeros(niterzbar,nitergam);
D2eqmat=zeros(niterzbar,nitergam);
teta1eqmat=zeros(niterzbar,nitergam);
```

```
teta2eqmat=zeros(niterzbar,nitergam);
q1eqmat=zeros(niterzbar,nitergam);
q2eqmat=zeros(niterzbar,nitergam);
zbarmat=zeros(niterzbar,1);
gamamat=zeros(1, nitergam);
w1eqmat=zeros(niterzbar,nitergam);
w2eqmat=zeros(niterzbar,nitergam);
v1eqmat=zeros(niterzbar,nitergam);
v2eqmat=zeros(niterzbar,nitergam);
y1eqpmat=zeros(niterzbar,nitergam);
y2eqpmat=zeros(niterzbar,nitergam);
r1eqmat=zeros(niterzbar,nitergam);
r2eqmat=zeros(niterzbar,nitergam);
welfeqmat=zeros(niterzbar,nitergam);
idc1eqmat=zeros(niterzbar,nitergam);
idc2eqmat=zeros(niterzbar,nitergam);
ddc1eqmat=zeros(niterzbar,nitergam);
ddc2eqmat=zeros(niterzbar,nitergam);
numberENmat=zeros(niterzbar,nitergam);
/******* Start iterations of level of uncertainty (zbarra) and differentiation level (gama) ********/
iterzb=1;
do while zbarra <=zbarmax;
 gama=gamamin;
 iterga=1;
do while gama <=gamamax;
/*******************************************************************************
/*********** Finding the second stage NE for various levels of (D1,D2) and saving *************/
/********** the NE value of each firm in a matrix which will be use to find SPNE **************/
/*******************************************************************************/
/************** This is to obtain the lower and upper bounds for D1 and D2 ***************/
D1min=0;
D2min=0;
D1max=3; /* Debt cannot be higher than expected monopoly profits. Here we are using an */
D2max=3; /* weighted average of monopoly and duopoly profits as the upper bound of debt */
saltob=0.05; /* step size for the iterations on the debt levels */
niterD1=int((D1max-D1min)/saltob)+1; /* number of iterations for debt level of firm 1 */
niterD2=int((D2max-D2min)/saltob)+1; /* number of iterations for debt level of firm 2 */
y1mat=ones(niterD1,niterD2); /* create matrix to save the NE total value of firm 1 */
y1mat=y1mat*(-500);
y2mat=ones(niterD1,niterD2); /* create matrix to save the NE total value of firm 2 */
y2mat=y2mat*(-500);
D2mat=zeros(1,niterD2);
D1mat=zeros(niterD1,1);
```

```
D1=D1min;
/************************Start iterations of levels of D1and D2) *******************/
iterD1=1;
do while D1<= D1max;
 D2=D2min;
 iterD2=1;
 do while D2<= D2max;
/************************** Check if NE is interior for all 4 variables ****************/
 x0 = {3, 3,0,0}; /* starting values of variables to be used in the constrained optimization routine*/
_co_IneqProc=&ineqlim; /* ineqlim is the procedure where the inequality constraints are defined*/
 _co_MaxIters=100; /* maximum number of iterations in the constrained optimization */
 {x,f,g,ret}=co(&fob,x0); /* this calls the routine to solve constrained optimization problem.
Objective function defined in procedure fob */
call coprt(x,f,g,ret);
if f<0.00001; /* if the optimal value of the objective function is very close to zero it means that we
found an interior NE */
goto nefound; /* if previous condition true can «jump» to the end of if cycles, since NE was already
found.  Jump to line with level «nefound» */
else;
/************* Check if NE is z^1 = -zbar and z^2 = -zbar, q1 and q2 interior **********/
 _co_IneqProc=&ineqlim1;
 _co_MaxIters=100;
 {x,f,g,ret}=co(&foD1,x0);
call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound;
else;
/************Check if NE is z^1 = -zbar and z2 interior, q1 and q2 interior ***********/
 _co_MaxIters=100;
 _co_IneqProc=&ineqlim2;
 {x,f,g,ret}=co(&foD2,x0);
 call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound;
else;
/************* Check if NE is z^1 interior and z2= -zbar , q1 and q2 interior **********/
 _co_MaxIters=100;
 _co_IneqProc=&ineqlim3;
 {x,f,g,ret}=co(&fob3,x0);
 call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound;
else;
```

```
 goto nefound2; /* If we arrive here it means that no NE was found in the feasible region
(z^1 and z^2 cannot be in the upper limit) */
endif;
endif;
endif;
endif;
nefound:
/********* Compute the NE total value of each firm and save it in the matrix **********/
y1=-k*((x[3]+zbarra)/(2*zbarra))*(D1-(((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-
gama*x[2]-c1+(x[3]-zbarra)/2)*x[1]))+((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-
gama*x[2]-c1)*x[1]; /* The NE value of firm 1 is equal to the equilibrium expected profit
(considering the NE values of q1 and q2) */
y2=-k*((x[4]+zbarra)/(2*zbarra))*(D2-(((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-
gama*x[1]-c2+(x[4]-zbarra)/2)*x[2]))+((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-
gama*x[1]-c2)*x[2]; /* The NE value of firm 2 is equal to the equilibrium expected profit
(considering the NE values of q1 and q2) */
 y1mat[iterD1,iterD2]=y1; /* save the NE value of firm 1 in a matrix, where each row
corresponds to a value of D1, and each columnn to the value of D2 */
y2mat[iterD1,iterD2]=y2; /* save the NE value of firm 1 in a matrix, where each row
corresponds to a value of D1, and each columnn to the value of D2 */
nefound2:
 D2mat[1,iterD2]=D2;
 D2=D2+saltob;
 iterD2=iterD2+1;
 endo;
 D1mat[iterD1,1]=D1;
 D1=D1+saltob;
 iterD1=iterD1+1;
 endo;
/*********************************************************************/
/******** The iterations for the NE of the second stage game end here *************/
/***************** Find the SPNE levels of D1 and D2 **********************/
/*******************************************************************/
iterD1=1;
numberEN=0;
do while iterD1 <= niterD1;
iterD2=1;
do while iterD2 <= niterD2;
y1col=y1mat[.,iterD2];
y2row=y2mat[iterD1,.];
y2col=y2row';
if y1mat[iterD1,iterD2]==maxc(y1col) and y2mat[iterD1,iterD2]==maxc(y2col); /* this
checks if a given (D1,D2) is a NE */
```

```
if y1mat[iterD1,iterD2]==(-500) or y2mat[iterD1,iterD2]==(-500); /* if we are in the
region where no NE of  2nd stage game was found, jump to line with level notane */
goto notane;
else;
D1eq=D1min+saltob*(iterD1-1); /* if NE is in feasible region, this gives us SPNE value of D1 */
D2eq=D2min+saltob*(iterD2-1); /* if NE is in feasible region, this gives us SPNE value of D2 */
print "SPNE is equal to" D1eq D2eq;
numberEN=numberEN+1;
else;
endif;
endif;
notane:
iterD2=iterD2+1;
endo;
iterD1=iterD1+1;
endo;
numberENmat[iterzb,iterga]=numberEN;
D1eqmat[ iterzb,iterga]=D1eq; /* save the SPNE of D1 in a matrix */
D2eqmat[ iterzb,iterga]=D2eq; /* save the SPNE of D2 in a matrix */
D1=D1eq;
D2=D2eq;
/*********************************************************************************/
/******Compute the SPNE levels of q1, q2, theta1, theta2. W1, W2, V1, V2, r1, r2 e welfare *****/
/*****This is done by compute NE of the 2nd stage game, for the SPNE value of (D1,D2) ******/
/*********************************************************************************/
 x0 = {3, 3,0,0}; /* starting values */
 _co_IneqProc=&ineqlim;
 _co_MaxIters=100;
 {x,f,g,ret}=co(&fob,x0);
call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound1;
 else;
 _co_IneqProc=&ineqlim1;
 _co_MaxIters=100;
 {x,f,g,ret}=co(&foD1,x0);
call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound1;
else;
 _co_MaxIters=100;
 _co_IneqProc=&ineqlim2;
 {x,f,g,ret}=co(&foD2,x0);
```

```
 call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound1;
else;
 _co_MaxIters=100;
 _co_IneqProc=&ineqlim3;
 {x,f,g,ret}=co(&fob3,x0);
 call coprt(x,f,g,ret);
 if f<0.00001;
 goto nefound1;
else;
 goto nefound3;
endif;
endif;
endif;
endif;
nefound1:
 teta1eq=(x[3]+zbarra)/(2*zbarra); /* compute the SPNE of theta1 */
 teta2eq=(x[4]+zbarra)/(2*zbarra); /* compute the SPNE of theta2 */
 teta1eqmat[ iterzb,iterga]=teta1eq; /* save the SPNE of theta1 in a matrix */
 teta2eqmat[ iterzb,iterga]=teta2eq; /* save the SPNE of theta2 in a matrix */
w1eq=(1-(1+k)*((x[3]+zbarra)/(2*zbarra)))*D1+(1+k)*(1/(2*zbarra))*(x[3]+zbarra)*(((1-a1*((x[3]+zbarra)/
(2*zbarra))^2)*alpha-x[1]-gama*x[2]-c1)*x[1]+(x[1]/2)*(x[3]-zbarra)); /* compute the SPNE of w1*/
w2eq=(1-(1+k)*((x[4]+zbarra)/(2*zbarra)))*D2+(1+k)*(1/(2*zbarra))*(x[4]+zbarra)*(((1-a2*((x[4]+zbarra)/
(2*zbarra))^2)*alpha-x[2]-gama*x[1]-c2)*x[2]+(x[2]/2)*(x[4]-zbarra)); /* compute the SPNE of w2*/
 w1eqmat[ iterzb,iterga]=w1eq; /* save the SPNE of w1 in a matrix */
 w2eqmat[ iterzb,iterga]=w2eq; /* save the SPNE of w2 in a matrix */
v1eq=(1/(2*zbarra))*(zbarra-x[3])*(((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]-c1)*x[1]-

D1+(x[1]/2)*(zbarra+x[3])); /* compute the SPNE of v1 */
v2eq=(1/(2*zbarra))*(zbarra-x[4])*(((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]-c2)*x[2]-

D2+(x[2]/2)*(zbarra+x[4])); /* compute the SPNE of v2 */
v1eqmat[ iterzb,iterga]=v1eq; /* save the SPNE of v1 in a matrix */
v2eqmat[ iterzb,iterga]=v2eq; /* save the SPNE of v2 in a matrix */
y1eqp=-k*((x[3]+zbarra)/(2*zbarra))*(D1-(((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]-

c1+(x[3]-zbarra)/2)*x[1]))+((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]-c1)*x[1];
/*computethe SPNE of y1 */
y2eqp=-k*((x[4]+zbarra)/(2*zbarra))*(D2-(((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]-

c2+(x[4]-zbarra)/2)*x[2]))+((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]-c2)*x[2];
/*compute the SPNE of y2 */
```

```
y1eqpmat[ iterzb,iterga]=y1eqp; /* save the SPNE of y1 in a matrix */
y2eqpmat[ iterzb,iterga]=y2eqp; /* save the SPNE of y2 in a matrix */
r1eq=(D1/w1eq)-1; /* compute the SPNE of r1 */
 r2eq=(D2/w2eq)-1; /* compute the SPNE of r2 */
 r1eqmat[ iterzb,iterga]=r1eq; /* save the SPNE of r1 in a matrix */
 r2eqmat[ iterzb,iterga]=r2eq; /* save the SPNE of r2 in a matrix */
welfeq=M+(1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha*x[1]+(1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha*x[2]-

(1/2)*((x[1])^2+2*gama*x[1]*x[2]+(x[2])^2)-c1*x[1]-c2*x[2]-k*((x[3]+zbarra)/(2*zbarra))*(D1-((1-a1*((x[3]+
zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]-c1)*x[1]+(x[1]/2)*(zbarra-x[3]))-k*((x[4]+zbarra)/(2*zbarra))*
(D2-((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]-c2)*x[2]+(x[2]/2)*(zbarra-x[4]));
/* compute the SPNE of welfare */
welfeqmat[ iterzb,iterga]=welfeq; /* save the SPNE of welfare in a matrix */
idc1eq=a1*((x[3]+zbarra)/(2*zbarra))^2*alpha*x[1]; /* compute the SPNE of lds1 */
idc1eqmat[ iterzb,iterga]=idc1eq; /* save the SPNE of lds1 in a matrix */
idc2eq=a2*((x[4]+zbarra)/(2*zbarra))^2*alpha*x[2]; /* compute the SPNE of lds2 */
 idc2eqmat[ iterzb,iterga]=idc2eq; /* save the SPNE of lds2 in a matrix */
ddc1eq=k*((x[3]+zbarra)/(2*zbarra))*(D1-((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]-

c1)*x[1]+(x[1]/2)*(zbarra-x[3]));/* compute the SPNE of ddc1*/
 ddc1eqmat[ iterzb,iterga]=ddc1eq; /* save the SPNE of ddc1 in a matrix */
ddc2eq=k*((x[4]+zbarra)/(2*zbarra))*(D2-((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]-

c2)*x[2]+(x[2]/2)*(zbarra-x[4]));/* compute the SPNE of bcts2*/
 ddc2eqmat[ iterzb,iterga]=ddc2eq; /* save the SPNE of bcts2 in a matrix */
q1eqmat[ iterzb,iterga]=x[1]; /* save the SPNE of q1 in a matrix */
q2eqmat[ iterzb,iterga]=x[2]; /* save the SPNE of q1 in a matrix */
nefound3:
gamamat[1,iterga]=gama;
 gama=gama+saltgama;
 iterga=iterga+1;
 endo;
zbarmat[ iterzb,1]=zbarra;
 zbarra=zbarra+saltozbar;
 iterzb=iterzb+1;
 endo;
/************************************************************************************/
/******************** Here end the iteractions for zbarra and gama *************************/
/************************************* Output Section **********************************/
/***********************************************************************************/
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\matD1eq.out reset; /* output file just the matrix D1eq*/
```

```
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
 print D1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\matD2eq.out reset; /* output file just the matrix D2eq*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print D2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\teta1eq.out reset; /* output file just the matrix teta1eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print teta1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7/* print number formatation */
output file=d:\Doutoramento\gaussres\teta2eq.out reset; /* output file just the matrix teta2eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print teta2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\w1eq.out reset; /* output file just the matrix w1eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
```

```
print w1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\w2eq.out reset; /* output file just the matrix w2eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print w2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\v1eq.out reset; /* output file just the matrix v1eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print v1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\v2eq.out reset; /* output file just the matrix v2eqmat*/
outwidth 150;/* dimension of output print columns*/
iterzb=1;
 do while iterzb <= niterzbar;
print v2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\y1eqp.out reset; /* output file just the matrix y1eqpmat*/
outwidth 150;/* dimension of output print columns*/
iterzb=1;
 do while iterzb <= niterzbar;
print y1eqpmat[iterzb,.];
 iterzb= iterzb+1;
endo;
```

```
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\y2eqp.out reset; /* output file just the matrix y2eqpmat*/
outwidth 150;/* dimension of output print columns*/
iterzb=1;
 do while iterzb <= niterzbar;
print y2eqpmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\r1eq.out reset; /* output file just the matrix r1eqmat*/
outwidth 150;/* dimension of output print columns*/
iterzb=1;
 do while iterzb <= niterzbar;
print r1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\r2eq.out reset; /* output file just the matrix r2eqmat*/
outwidth 150;/* dimension of output print columns*/
iterzb=1;
 do while iterzb <= niterzbar;
print r2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\welfeq.out reset; /* output file just the matrix */
welfeqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print welfeqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
```

```
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\q1eq.out reset; /* output file just the matrix q1eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print q1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\q2eq.out reset; /* output file just the matrix q2eqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print q2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\idc1eq.out reset; /* output file just the matrix welfeqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print idc1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\idc2eq.out reset; /* output file just the matrix welfeqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print idc2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\ddc1eq.out reset; /* output file just the matrix welfeqmat*/
outwidth 150;/* dimension of output print columns*/
```

```
 iterzb=1;
 do while iterzb <= niterzbar;
print ddc1eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\ddc2eq.out reset; /* output file just the matrix welfeqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print ddc2eqmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,7;/* print number formatation */
output file=d:\Doutoramento\gaussres\numberEN.out reset;/* output file just the matrix
welfeqmat*/
outwidth 150;/* dimension of output print columns*/
 iterzb=1;
 do while iterzb <= niterzbar;
print numberENmat[iterzb,.];
 iterzb= iterzb+1;
endo;
output off;
/************************** Procedures for the constrained optimization *******************/
/****************************** Procedures for an interior NE ************************/
/*** Procedure for objective function *****/
proc fob(x);
 local x1,x2,x3,x4,y1,y2,y3,y4;
 x1 = x[1];
 x2 = x[2];
 x3 = x[3];
 x4 = x[4];
 y1 = 2*((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-2*a1*alpha*((x3+zbarra)/(4*(zbarra^2)))*
(((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-gama*x2+x3-c1)/(a1*((x3+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x2-c1)+zbarra+x3;
 y2 = 2*((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-2*a2*alpha*((x4+zbarra)/(4*(zbarra^2)))*
(((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-gama*x1+x4-c2)/(a2*((x4+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x1-c2)+zbarra+x4;
 y3 = ((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-x1-gama*x2+x3-c1)*x1-D1;
```

```
 y4 = ((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-x2-gama*x1+x4-c2)*x2-D2;
 retp (y1^2+y2^2+y3^2+y4^2);
endp;
/************************ Procedures for inequality constraints ************************/
proc ineqlim(x);
local limits;
limits=zeros(6,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
retp (limits);
endp;
/********* Procedures for an z^1 = -zbar and z^2 = -zbar, q1 and q2 interior *********/
/****************** Procedure for inequality constraints ************************/
proc foD1(x);
 local x1,x2,x3,x4,y1,y2,y3,y4;
 x1 = x[1];
 x2 = x[2];
 x3 = x[3];
 x4 = x[4];
 y1 = 2*((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-2*a1*alpha*((x3+zbarra)/(4*(zbarra^2)))*
(((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-gama*x2+x3-c1)/(a1*((x3+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x2-c1)+zbarra+x3;
 y2 = 2*((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-2*a2*alpha*((x4+zbarra)/(4*(zbarra^2)))*
(((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-gama*x1+x4-c2)/(a2*((x4+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x1-c2)+zbarra+x4;
 y3 = x3+zbarra;
 y4 = x4+zbarra;
 retp (y1^2+y2^2+y3^2+y4^2);
endp;
/****************** Procedure for inequality constraints **************************/
proc ineqlim1(x);
local limits;
limits=zeros(8,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
```

```
limits[7]=((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]+x[3]-c1)*x[1]-D1;
limits[8]=((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]+x[4]-c2)*x[2]-D2;
retp (limits);
endp;
/******** Procedures for an z^1 = -zbar and z2 interior, q1 and q2 interior ***********/
/******************* Procedure for objective function ***************************/
proc foD2(x);
 local x1,x2,x3,x4,y1,y2,y3,y4;
 x1 = x[1];
 x2 = x[2];
 x3 = x[3];
 x4 = x[4];
 y1 = 2*((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-2*a1*alpha*((x3+zbarra)/(4*(zbarra^2)))*
(((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-gama*x2+x3-c1)/(a1*((x3+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x2-c1)+zbarra+x3;
 y2 = 2*((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-2*a2*alpha*((x4+zbarra)/(4*(zbarra^2)))*
(((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-gama*x1+x4-c2)/(a2*((x4+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x1-c2)+zbarra+x4;
 y3 = x3+zbarra;
 y4 = ((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-x2-gama*x1+x4-c2)*x2-D2;
 retp (y1^2+y2^2+y3^2+y4^2);
endp;
/******************* Procedure for inequality constraints *************************/
proc ineqlim2(x);
local limits;
limits=zeros(7,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
limits[7]=((1-a1*((x[3]+zbarra)/(2*zbarra))^2)*alpha-x[1]-gama*x[2]+x[3]-c1)*x[1]-D1;
retp (limits);
endp;
/*********Procedures for an z^1 interior and z2= -zbar , q1 and q2 interior **********/
/****************** Procedure for objective function ***************************/
proc fob3(x);
 local x1,x2,x3,x4,y1,y2,y3,y4;
 x1 = x[1];
 x2 = x[2];
 x3 = x[3];
 x4 = x[4];
```

```
 y1 = 2*((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-2*a1*alpha*((x3+zbarra)/(4*(zbarra^2)))*
(((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-2*x1-gama*x2+x3-c1)/(a1*((x3+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x2-c1)+zbarra+x3;
 y2 = 2*((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-2*a2*alpha*((x4+zbarra)/(4*(zbarra^2)))*
(((1-a2*((x4+zbarra)/(2*zbarra))^2)*alpha-2*x2-gama*x1+x4-c2)/(a2*((x4+zbarra)/(2*zbarra^2))*
alpha-1))-gama*x1-c2)+zbarra+x4;
 y3 = ((1-a1*((x3+zbarra)/(2*zbarra))^2)*alpha-x1-gama*x2+x3-c1)*x1-D1;
 y4 = x4+zbarra;
 retp (y1^2+y2^2+y3^2+y4^2);
endp;
/****************** Procedure for inequality constraints ************************/
proc ineqlim3(x);
local limits;
limits=zeros(7,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
limits[7]=((1-a2*((x[4]+zbarra)/(2*zbarra))^2)*alpha-x[2]-gama*x[1]+x[4]-c2)*x[2]-D2;
retp (limits);
endp;
```

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Chapter 4

Empirical Links Between Market Structure, Capital Structure Decisions and Default Risk

4.1 Introduction

Default and bankruptcy are areas that, in recent years, have undergone systematic analysis by researchers. The growing interest of accountants, financial analysts, managers, creditors, regulators and the community in general has borne fruit to a set of studies aiming to present the best default prediction model. As noted by Gepp and Kumar (2008), timely knowledge enables various entities to avoid default. However, the prediction of default is not the only relevant issue. It is also important to identify the factors that influence the probability of a firm not meeting its debt obligations.

Operational and financial decisions are important pillars in the economic and financial sustainability of a firm. Nevertheless, there is a lack of empirical studies, which simultaneously relate the financial decisions, the operational decisions and the default risk (however, there are studies that connect two of these three areas). The study carried out by Antunes, Mata and Portugal (2011) relates debt and default risk, whereas Opler and

Titman (1994), Borenstein and Rose (1995) and Evrensel (2008) study the relationship between operational decisions and default risk. The aim of this study is to analyze the impact of firm and industry characteristics, including the degree of market concentration, on the capital structure and on the financial distress.

Since financial and operational decisions are likely to depend on the country's characteristics, as concluded, for example, by Demirgüç-Kunt and Maksimovic (1999), Booth el al (2001), Bancel and Mitoo (2004) and Jong, Kabir and Nguyen (2008), our study incorporates a sample of 11 OECD countries[1] which have different economic and financial development levels, which allows us to test the statistical significance of the variable country. Moreover, the study contains data before (2007) and after (2013) the period of financial crisis in the early twenty-first century (with a clear peak in 2008). According to some studies after 2008 operating and financial decisions have changed. Furceri and Mourougane (2012) concluded that after the crisis the output level of the OECD members decreased on average between 1.5% and 2.4%. Erkens Hung and Matos (2012) concluded that after 2008 firms tended to use more equity. After 2008 and the Basel III (which notably aimed stricter supervision and disclosure practices for risk reduction) an increase of banking restrictions in granting credit was observed.

Reviewing the literature on capital structure and financial distress, there are a set of variables that appear in theoretical studies on the subject, but regarding which there is a shortage of empirical testing. In particular, demand uncertainty and the degree of product substitutability (which increases the degree of competition) were referred in a theoretical model by Brander and Lewis (1986) and numerically by Haan and Toolsema (2008) and Franck and Le Pape (2008) as well as in the two previous chapters of this thesis. This essay aims to analyze the empirical significance of the uncertainty and the degree of market concentration (the reverse of the degree of competition) using a sample of 11 OECD countries. The literature considers that default costs have an impact on debt and on distress, however, the empirical testing of default cost relevance is mostly confined to the direct costs (administrative costs and legal costs). The literature refers

[1] OCDE-Organisation for Economic Cooperation and Development.

to the importance of indirect default costs (reputation effect on profit). However, due to the difficulty in their estimation, they are rarely considered. In this study, using a proxy suggested by Altman (1984), we consider the impact of the direct and indirect default costs on debt and on default risk.

The theoretical and numerical models previously mentioned as well as the two previous chapters of this thesis, suggest that debt is endogenously determined (firms choose the equilibrium levels of debt). Therefore, when we wish to analyze the effect of the capital structure on default risk we need to address this potential endogeneity problem (this issue was also noted by Zingales, 1998). In order to solve this problem we use a set of instrumental variables.

The first contribution of this study is to check how debt and default risk vary when the degree of market concentration increases. A second contribution is the study of the impact of uncertainty and default costs (direct and indirect) on debt and on default risk. As a third contribution, we can highlight the wide sample of 11 OECD countries, over a sampling period before and after a period of financial crisis. Finally, the methodology used to explain the default probability, takes into account that debt is an endogenous variable and allows us to evaluate the direct and the total impact of the exogenous explanatory variables on default risk.

The structure of this chapter is as follows. In section 2 a literature review about capital structure, financial distress and the firm and industry effects is elaborated. Section 3 presents the hypotheses to be tested whereas section 4 describes the data. In section 5, the main results are presented. Finally, section 6 summarizes the main conclusions of this essay.

4.2 Related work

4.2.1 Capital Structure

To operate and thrive firms need financial resources and rely essentially on two major sources: equity and debt. According to theory, the choice of the financing form always has two objectives: to maximize the firm's value and to minimize the capital cost. This theme has originated a large number of studies which have attempted to explain the factors/determinants of capital structure and how these factors influence decisions in this area.

Theories of capital structure date back to the works of Modigliani and Miller (1958, 1963). In both papers there are two propositions, one dealing with the capital cost and the other one with the firm value. Modigliani and Miller (1958) established a set of simplifying assumptions, including: the absence of taxes, transaction costs and bankruptcy costs; the equivalence in borrowing costs for firms and investors; and the existence of symmetric market information. The main conclusion of the paper is that the capital structure is irrelevant to the maximization of firm value. This happens because the value of a leveraged firm has to be equal to the value of a non-leveraged firm. If it were not, it would be possible for the investor to obtain risk free gains (arbitrage), by selling overvalued securities and buying undervalued securities. In 1963, Modigliani and Miller refined their original model considering the existence of taxes. The authors argue that the tax benefit that comes from debt implies that the maximum value is obtained when the firm is fully leveraged. Due to the limitations of the previous models, new explanatory theories of capital structure have emerged in recent years, including the trade-off theory, the pecking order theory, the signalling theory, the agency theory and the market timing theory.

According to Kraus and Litzenberger (1973) the trade-off theory argues that there is an optimal capital structure, which is the one that balances the effects of debt on taxes and the bankruptcy costs associated with leverage. Therefore, a firm increases its debt until the point where the marginal tax benefit is offset by the increase in the present value costs of financial distress. As can be seen in the work of Rajan and Zingales (1995), Bancel and

Mittoo (2004) and Franck and Goyal (2007), this theory points to a positive relationship between the tax rate and debt and a negative relationship between bankruptcy costs and debt using the amount of tangible assets and growth (the higher the tangible assets, the lower are the losses in the case of bankruptcy and the higher is growth, the bigger are the losses in the case of bankruptcy) as proxies for bankruptcy costs.

Myers (1977,1984) and Myers and Majluf (1984) argue that the pecking order theory assumes that there is no target capital structure. The firms choose the capital structure according to a preference order: internal finance, debt and equity. The authors claim that the existence of transaction costs and asymmetric information between managers and investors means that investments are preferably carried out with the firm's internal funds. Some studies confirm the predictions of the pecking order theory (Titman and Wessels, 1988; Rajan and Zingales, 1995; Sunder and Myers, 1999; and Franck and Goyal, 2003) while others do not (Brennan and Kraus, 1987; and Leary and Roberts, 2010).

The signalling theory was suggested by Ross (1977) and Leland and Pyle (1977). Firms use internal funds in order to avoid adverse selection problems. The capital structure of a firm can send quality signals to the market. Thus, the signalling theory and pecking order theory are interconnected. The way to test the signalling theory is to find the market reaction when there is an increase or a decrease in debt.

The agency theory (Jensen and Meckling, 1976) analyzes the potential conflicts of interest that may arise among some stakeholders throughout the company's life, such as, managers, shareholders and creditors. The occurrence of these conflicts has costs that are called agency costs. Empirically and according to this theory, the firms holding a higher cash flow and profitability tend to have higher debt.

The market timing theory, originally presented by Baker and Wurgler (2002) appeared more recently. This theory argues that market timing is a determinant of capital structure, which means the firm chooses the form of financing taking into account the financial market value over time.

Table 4.1 presents the main determinants of the capital structure considered by the existent empirical literature, indicating the sign of the impact of the determinants obtained in these studies, the way the variables have been operationalized and representative papers in this literature.

The collateral (also known as tangibility) is expected to have a positive impact on the debt level. This sign is predicted both by the trade-off theory as well as the agency costs theory. The larger the volume of tangible assets of a firm (collateral), the higher the guarantee given to the lender, making it easier to resort to credit. In addition, a higher guarantee reduces the agency problems between creditors and managers, and hence debt increases (Harris and Raviv, 1990). The empirical studies confirm the expected theoretical impact.

The non-debt tax shields, liquidity and income variability are all expected to have a negative impact on debt. Tax deductions that come from depreciation and investment tax credits can be seen as substitute for tax savings stemming from the financial expenses. Thus, according to the trade-off theory the non-debt tax shields have a negative impact on debt (DeAngelo and Masulis,1980). Moreover these tax benefits not related with debt reduce the cash outflows, which leads to an increase of the internal funds. Therefore, the negative impact of the non-tax shields is also predicted by the pecking order theory. Liquidity is also expected to have a negative impact on debt, since, and according to the pecking order theory, a firm prefers to use internal funds rather than external ones (Myers and Majulf, 1984). Greater volatility of income means greater risk, a riskier firm has greater difficulty in obtaining credit and when it does it will have to pay a higher interest rate. Thus it is expected that greater income volatility has a negative impact on debt.

There are determinants that have an ambiguous impact on debt, including profitability, growth opportunities, size and age. According to the pecking order theory, the higher the profitability, the greater the internal resources generated, so the use of debt is lower (Myers ,1984 and Myers and Majluf, 1984). However, according to the trade-off theory, the higher the profitability, the higher the ability of the firm to fulfil its commitments quickly, so the higher the debt ratio (Jensen, 1986).

Table 4.1: Determinants of the capital structure: expected impact, proxies used and representative studies.

Variable	Expect impact	Calculation	Some References
Collateral Tangibility	+	$\frac{\text{Fixed Assets}}{\text{TA}}$ $\frac{\text{Intagible Asset}}{\text{Equity}}$	Rajan & Zingales(1995); Berger, Ofek & Yermack (1997); Lööf (2004); Pindado,Rodrigues & La Torre(2006); Lyandres(2006) Margaritis & Psillaki(2007); Ramalho & Vidigal(2009,2013); Aggarwal & Kyaw (2010); Chakraborty (2010); Serrasqueiro & Nunes (2010); Kayo & Kimura (2011);Guney, Li & Fairchild(2011); Zhang(2012)
Non-debt tax shields	-	$\frac{\text{Depretiation}}{\text{TA}}$	Berger, Ofek & Yermack(1997); Lööf(2004); Pindado, Rodrigues & La Torre(2006) Ramalho & Vidigal (2009,2013); Aggarwal & Kyaw(2010); Chakraborty(2010); Guney, Li & Fairchild(2011)
Liquidity	−	$\frac{\text{Working Capital}}{\text{TA}}$ $\frac{\text{Current Asset}}{\text{Current Liability}}$	Ramalho & Vidigal(2009,2013); Guney, Li & Fairchild(2011)
Income variability	−	Variance of Sales Beta	Lööf(2004); Aggarwal & Kyaw(2010)
Profitability	+/−	$\frac{\text{Operating Income}}{\text{TA}}$ $\frac{\text{Net Income}}{\text{Equity}}$	Rajan e Zingales(1995); Berger, Ofek & Yermack (1997); Lööf(2004); Lyandres(2006); Margaritis & Psillaki(2007); Ramalho & Vidigal(2009,2013) Aggarwal & Kyaw(2010); Chakraborty (2010); Serrasqueiro & Nunes (2010); Kayo & Kimura(2011); Guney Li & Fairchild (2011); Zhang(2012)
Growth opportunities	+/-	$\frac{\text{TA}_t-\text{TA}_{t-1}}{\text{TA}_{t-1}}$ $\frac{\text{OP}_t-\text{OP}_{t-1}}{\text{OP}_{t-1}}$ $\text{Tobin}'\text{s Q}$	Petersen & Rajan (1994), Lööf(2004);Pindado, Rodrigues & La Torre(2006); Ramalho & Vidigal (2009,2013); Aggarwal & Kyaw(2010); Chakraborty(2010); Serrasqueiro & Nunes(2010);Kayo & Kimura(2011); Guney, Li & Fairchild(2011)
Size	+/−	ln(sales) ln(asset) ln(#employees)	Petersen & Rajan(1994), Rajan & Zingales(1995); Berger, Ofek & Yermack(1997); Lööf(2004); Lyandres(2006); Pindado Rodrigues and La Torre(2006); Margaritis & Psillaki(2007); Ramalho & Vidigal (2009,2013) Aggarwal & Kyaw(2010); Chakraborty (2010); Serrasqueiro & Nunes (2010) Kayo & Kimura(2011); Guney et al(2011); Zhang(2012)
Age	+/-	Years since foundation	Berger, Ofek and Yermack (1997);Petersen & Rajan (1994); Ramalho & Vidigal (2009,2013)

Note:T-Total Asset;OP-operating Profit

The expected impact of growth opportunities on debt is also ambiguous and, according to some authors, non-linear. According to the signalling and the pecking order theories, growth opportunities has a positive impact on leverage. The higher the growth opportunities, the higher are the investment needs and thus more likely is that internal funds are not enough, increasing debt financing. On the other hand, according to the trade-off theory, the financial distress has higher costs for firms that have higher growth opportunitties. Thus the firm may be less willing to finance with debt in order not to increase the probability of default. Finally, the agency theory argues that the impact of growth opportunities can be either positive or negative depending on overinvestment and underinvestment costs respectively (Stulz, 1990).

The effect of the firm's size on debt is also ambiguous. According to Rajan and Zingales (1995), larger firms have less information asymmetry so there is greater use of equity. However, according to Titman and Wessels (1988), larger firms are more diversified, which reduces the probability of failure and thus increases the use of debt. The empirical literature confirms the ambiguity of the impact of size on debt and suggests that the sign depends on whether the dependent variable is a short-term debt ratio or a long-term debt ratio.

Finally, according to the literature (Diamond, 1989 and Petersen and Rajan, 1994), age also has an ambiguous impact. On the one hand, age can influence debt positively because the older is the firm the greater is its debt payment history and the corresponding reputation of the firm. On the other hand, an older firm is expected to have more accumulated retained earning, thus reducing the need for debt.

4.2.2 Financial Distress

Over the past few years, the scientific and business community has been researching into the default risk and the best way to prevent situations of financial distress. Several studies have tried to present the best model to predict the default risk, so as to have a preventive model of any default event. An anticipated knowledge of a default event is important in order to avoid the economic and social costs that affect shareholders, creditors, suppliers, customers and the community in general.

The models developed since the 60's can be classified into the classical systems of default risk analysis in the Rating systems[2] and Scoring Systems.[3] In addition to these systems, there are models which are based on the use of the interest rate term structure, mortality rates and migration of credit measures and models that use the real option theory. Sun *et al* (2014) present a survey of the most recently developed methodologies, which are classified as: Artificial Intelligence Single Methods, Hybrid Single Classifier Methods, Ensemble Methods and Dynamics Models.[4]

The literature highlights the use of Scoring Systems namely the univariate and multivariate analysis (discriminant, logit, probit and survival analysis). One of the pioneers in the study of failure was Beaver (1966), who using the univariate analysis, studies thirty financial ratios separately and checks for values that can classify a firm with financial difficulties, based on the industry average, the region or group of firms. Univariate analysis assumes a linear relationship between the variables and the definition of default. This model is criticized for studying the effect of each ratio separately since the predictive power increases when the ratios are considered together.

The use of multiple linear discriminant analysis arose from Altman's (1968) work, a crucial milestone for the study of financial distress. The Z-Score and Zeta models, devel-

[2]The Rating System or Credit Rating is a preventive system and consists of the credit rating classification (assigned by an entity), i.e. the probability of the issuer to pay the debt. This scale or rating aims to transmit the ability to pay the debt. This system is used by rating agencies, which assign a rating to the debt securities of the firm. The banks also use the rating system in loans, through methodologies developed by analysts. One of the methodologies used is the 6 C's system, which is used to assess the person/firm requesting the loan in six areas: capacity, capital, collateral, conditions, and character. The objective is to assist the analyst in the perception of the previously defined risk group that a specific firm belongs to.

[3]The Scoring System or Statistical Single Classifier Method is based mainly on the use of financial ratios. This system uses statistical techniques that allow us to select the ratios and their weighting, estimating the default risk of a firm.

[4]In the 90's appeared new methodologies to measure the default risk based on the application of artificial intelligence, hybrid single classifier, ensemble methods and dynamic models. According to the literature this methodology is considered new and promising but the introduction of ratios is complex, which hinders economic validation. Sun *et al* (2014) claim that this methodology presents, as a main advantage, the fact that it is not subject to statistical assumptions. However, the single classifier's statistical methods can obtain a fixed model structure in different times of training on a certain data set.

oped by Altman (1968) and Altman, Haldeman and Narayanan (1977), respectively, are the most discussed techniques in empirical studies. These techniques allows us to classify firms into bankrupt and non-bankrupt based on accounting and market information. Through a linear combination of normally distributed independent variables, a multivariate model establishes a cutoff point. The Z-score model was developed by Altman (1968) and in this model twenty-two ratios are used, from which the ratios that measure liquidity, profitability, return on investment, market information and management efficiency stand out. The Zeta model was developed by Altman, Haldeman and Narayanan (1997) and it is based on an analysis of seven dimensions: return on investment, risk, interest coverage, liquidity, market information, accumulated profitability, and size. These models are often criticized for assuming that the independent variables have a normal distribution and an equal covariance matrix. Furthermore, it is assumed that the dependent variable (i.e. the probability that the firm does not fulfill its obligations) is continuous.

Ohlson (1980) developed a model of bankruptcy prediction based on the logistic analysis (logit),[5] a model also used later by Zavgren (1985). Zmijewski (1984) used the probit model to estimate the probability of financial distress. These models use a set of accounting variables to predict the probability of default, assuming that default may have two possible values, zero or one. These models have the advantage that they can be applied to binary variables and are not based on the assumption that the independent variables follow a normal distribution and have an equal covariance matrix. However, the models require that the independent variables do not have a linear relationship. These model use ratios that assess profitability, leverage, liquidity, activity and size.

Studies have shown that Z-Score, Zeta, logit and probit models have a high accuracy in the classification of firms, and despite their limitations, the percentage of errors committed is low.

Survival analysis is a more refined technique than the traditional discriminant analysis. This technique uses time series and assumes that the default process is stable over time.

[5] Ohlson (1980) uses, among others, ratios that measure: size, financial leverage, liquidity, profitability, return on investment and growth.

Survival analysis was first used by Lane, Looney and Wansley (1986), who applied the model to 130 bankrupt banks and 334 financially healthy banks during the period of 1979 to 1983. According to Gepp and Kumar (2008), survival analysis is not as popular nor as used as discriminant analysis or logit, but it is starting to show some popularity.

Table 4.13, in the appendix, presents a summary of empirical studies that use the aforementioned methodologies.

Table 4.2 presents the determinants of default risk that have been tested in the literature.

The literature points to a negative effect of liquidity; profitability, size and management efficiency on the probability of default.

Liquidity reflects the firms' ability to liquidate its assets and settle the short-term liabilities. When a firm has liquidity this means that it can easily sell its assets.

Profitability indicates the firms' ability to reward investors and it evaluates management performance. A more profitable firm can generate a higher return for investors (own capital and borrowed capital) so the probability of the firm not meeting its obligations decreases with profitability. Therefore the expected impact of this variable is negative.

Management efficiency (measured by the asset turnover ratio) indicates the degree of utilization of total assets. According to Altman (1968), this ratio measures the ability of the firm's assets to generate sales. The negative impact suggests that firms with a greater ability to use their assets are less likely not to meet their obligation.

The impact of the firm size on the probability of default is ambiguous. On one hand, the size variable is expected to have a negative impact on the default probability since, according to the literature, larger firms are more diversified and, as a result, it is easier for them to deal with difficulties in a particular market, therefore they are less likely to fail. On the other hand, other authors (Turetsky and McEwen, 2001; Chancharat *et al*, 2010) found a positive relationship between size and default. This result can be justified by the higher difficulty in managing and monitoring employees in a larger firm.

According to the literature, the variable leverage has an ambiguous expected impact on the default probability. Leverage measures the capital structure of a firm, which

Table 4.2: Determinants of the default risk: expected impact, proxies used and representative studies.

Variable	Expected impact	Calculation	Some References
Liquidity	–	$\frac{\text{Working Capital}}{\text{TA}}$ $\frac{\text{Current Asset}}{\text{Current Liability}}$	Beaver(1966); Altman (1968); Altman,Haldeman & Narayanan(1977); Ohlson(1980); Kahya & Theodossiou(1999) Turetsky & McEwen(2001); Pindado & Rodrigues (2004) Gepp & Kumar (2008); Chancharat et al(2010); Ng, Wong, Zhang(2011) Dionne & Laajimi(2012) Wu, Gaunt & Gray(2010) Sneideire & Bruna (2011) Ho, McCarthy & Yang (2013);Tinoco & Wilson(2013)
Profitability	–	$\frac{\text{Operating Income}}{\text{TA}}$ $\frac{\text{Net Income}}{\text{Equity}}$	Altman(1968); Altman, Haldeman & Narayanan (1977);Ohlson(1980); Kahya & Theodossiou(1999); Turetsky & McEwen(2001); Pindado & Rodrigues (2004); Beaver,Mcnichols & Rhie(2005); Gepp & Kumar(2008); Chancharat et al(2010); Ng, Wong, Zhang (2011); Ho, McCarthy & Yang(2013); Johnstone et al,(2013)
Management Efficiency	-	$\frac{\text{Sales}}{\text{Asset}}$	Beaver(1966);Altman(1968);Turetsky & McEwen (2001); Gepp & Kumar(2008); Chancharat et al (2010);Ng, Wong Zhang (2011);Sneideire & Bruna (2011)
Size	+/-	ln(employees) ln(sales) ln(asset)	Altman, Haldeman & Narayanan(1977); Ohlson (1980); Kahya & Theodossiou(1999);Turetsky & McEwen(2001); Gepp & Kumar(2008) Wu, Gaunt & Gray(2010); Pérez,Llopis and Llopis(2010); Antunes, Mata & Portugal(2011);Ng, Wong, Zhang (2011); Dionne & Laajimi(2012) Johnstone et al,(2013) Tinoco & Wilson(2013)
Age	+/-	Years since foundation	Chancharat et al(2010), Pérez, Llopis & Llopis (2010) Antunes, Mata & Portugal (2011)
Leverage	+/-	$\frac{\text{Long term debt}}{\text{TA}}$ $\frac{\text{Debt}}{\text{TA}}$ $\frac{\text{Short term debt}}{\text{TA}}$	Beaver(1966); Altman (1968); Ohlson(1980); Kahya &Theodossiou(1999); Campos(2000) Turetsky & McEwen(2001);Pindado & Rodrigues (2004) Beaver Mcnichols & Rhie (2005) Gepp & Kumar(2008); Chancharat et al(2010); Dionne & Laajimi(2012) Wu, Gaunt & Gray(2010) Sneideire & Bruna (2011) Ho, McCarthy & Yang(2013);Tinoco & Wilson(2013)

represents the proportion of liabilities and equity capital. Most studies point to a positive correlation between leverage and default probability. A more leveraged firm has higher debt obligations and hence, maintaining constant the firm profitability, the higher is the probability of the firm not being able to meet its obligations. However, a more leveraged firm is subject to greater supervision, leading it to invest in lower risk projects, which may reduce the default probability. Another important issue reported in the literature is the strategic impact that debt can have on decisions in the product market and its indirect impact on the probability of default. According to the literature, the relationship between capital structure and product market decisions can be divided into two types of models: the ones that emphasize the role of limited liability and the ones which are based on predatory behavior. In the first type of models, an increase in debt leads the firm to be more aggressive in the output market, i.e. when firms have limited liability they tend to produce more. This may lead to a positive direct impact on the probability of default. The models of predatory behavior defend the opposite, i.e., the most indebted firms tend to adopt a more conservative approach, while firms without financial constraints tend to be more aggressive in the product market. Thus, if this theory holds, more debt may lead to a negative impact on the probability of default.

With regard to age, some studies suggest a non-linear relationship. Young firms are protected by the resources initially placed and thus for some time are unlikely to fail. The literature suggests that the probability of default start being increasing with age, reaching a maximum after a given period and decreasing thereafter, in other words the impact of age on the default probability follows an inverted U shape (Geroski,1995; Fichman and Levinthal, 1991; and Pérez, Llopis and Llopis, 2010).

4.2.3 Firm and Industry Variables

There are firm specific variables that affect not only the capital structure of a firm, but also the default probability which were not mentioned in the previous subsection because there is a lack of empirical analysis testing their statistical significance. This includes uncertainty and default costs.

The results of the financial and operating decisions depend on the context in which firms operate. Moreover, the events/shocks that influence these decisions also have a direct effect on the firm's survival probability. The analysis of the uncertainty effect on the financial and operational structure of a firm was analyzed theoretically, for example, by Brander and Lewis (1986), Franck and Le Pape (2008), Haan and Toolsema (2008). The authors concluded that an increase in the demand uncertainty reduces the debt ratio. Moreover, regarding the effect on default probability, Haan and Toolsema (2008) conclude that uncertainty has a negative effect on the default risk. Similarly, Franck and Le Pape (2008) claim that the indirect effect through the decrease in the debt outweighs the direct effect of uncertainty on default risk. These results as well as the analytical and numerical results obtained in the two previous chapters of this thesis suggest that it is important to distinguish between the effect of uncertainty on default for a given debt level and the total impact which also incorporates the fact that uncertainty decreases debt, which in turn influences the default probability. Thus in our empirical results it is very likely that the sign of the direct effect of the variable uncertainty may differ from the sign of the total effect. Moreover our numerical results suggest that the total effect is ambiguous. In particular, the sign of the total impact depends on the default costs and their magnitude. The empirical analysis regarding the impact of uncertainty is limited to its effects on financial and operating decisions (Chevalier, 1995b; Showalter, 1999; and Khanna and Tice, 2000). The empirical literature points to a negative relationship between uncertainty and debt. Thus, it is also important to analyze empirically what is the effect of uncertainty on the default risk. Uncertainty has been considered either through the standard deviation of log-changes in sales or by considering a sample period that has a certain shock.

Lastly, the relevance of default costs was first highlighted by Kraus and Litzenburger (1973). They analyze the trade-off between default costs and the debt tax benefits. Elkami, Ericsson and Parsons (2012) argue that the financial distress costs can offset the tax benefits of debt. Literature divides default costs into two types: direct or ex-post costs (legal, accounting and administrative costs) and indirect or ex-ante costs (reduced profits resulting from lower sales, in particular, due to the reputation effect).

In the capital structure empirical literature, when the impact of financial distress costs is analyzed, most often only the direct default costs are considered. To analyze the direct cost, studies incorporate the value of legal, accounting and administrative costs. There are some studies that use direct cost proxies, including a percentage (between 1% and 23%) of the firms' value (Andrade and Kaplan, 1998; Warner, 1977; and Singhal and Zhu, 2013), whereas others consider as a proxy the percentage of intangible assets (Jonh, 1993; Hwang *et al*, 2009; and Dionne and Laajimi, 2012). In the event of liquidation, there is greater difficulty in the adaptation of workers to other functions (Titman and Wessels, 1988) and greater difficulty in selling its assets when a firm has a high proportion of intangible assets. Therefore the bankruptcy costs are higher. The literature that uses this proxy is not clear about which type of costs is being captured by this proxy. But the difficulty mentioned above arises after default occurred (ex-post), and it will be a cost borne by creditors in settlement.

There are very few studies that incorporate indirect costs in their analysis. Pindado, Rodrigues and La Torre (2008) consider that these costs are directly proportional to the default probability and analyze the effect of these costs on investment. Altman (1984), Opler and Titman (1994) and Kwansa and Cho (1995) interpret the indirect costs as lost profits.

Most studies consider default costs as a dependent variable, but by considering them an explanatory factor, it is expected that the default costs (direct and indirect) will have a negative impact on debt and a positive impact on default risk. However, when we look at the total effect, default costs can have a negative impact on the probability of default. According to the literature, the default cost influences negatively the amount of debt (firms behave more cautiously when default costs are higher). This lower level of debt may imply that the total effect of increasing default costs on the default probability may be negative.

Despite the strong role of the firm's specific variables we must not forget that the firm belongs to a certain sector/industry and country. According to Gepp & Kumar (2008) some studies use industry indicators but most of them treat the industry and country effect as a dummy variable. However, Gungoraydinoglu & Öztekin (2011) concluded that there is a large percentage of leverage variation that is explained by specific factors of the industry and the country. Thus, the capital structure and the likelihood of a firm not meeting its obligations is not explained only by factors specific to the firm, but also by industry and country effects.

The industry effects have been incorporated in the empirical Industrial Organization literature. In particular, in the study of the impact on the firms' profitability, explanatory variables such as industry concentration, entry barriers and product differentiation have been introduced. In most studies on capital structure and default risk, industry has been incorporated in the analysis by introducing dummies. However, according to Brander and Lewis (1986), Evrensel (2008), Haan and Toolsema (2008) and Franck and Le Pape (2008), competition in the product market (competition in quantities or prices) influences debt and default risk. Opler and Titman (1994), Zingales (1998), Campos (2000), Erol (2003) and Lyandres (2006) empirically concluded that there is a correlation between debt and product market concentration. The degree of market concentration is measured in literature by the Herfindahl–Hirschman Index, Tobin's Q and the market share of the largest four firms in the industry. With regard to the degree of product differentiation, the literature uses the degree of advertising intensity in the industry and defines advertising expenses as a percentage of sales as proxy.

Some studies have analyzed the impact of the industry/sector on debt. We can highlight the work of Degryse, Goeij, and Kappert (2012) which studies the impact of firm and industry characteristics on small firms' capital structure. The work of Kayo and Kimura (2011) analyzes the influence of time, firm, industry and country-level determinants on capital structure. The works of Lyandres (2006) and Margaritis and Psillaki (2007) are also very relevant. The first studies the connection between the sector competitiveness and indebtedness and the second investigates the relation between firm efficiency and leverage, where firm efficiency is measured by the distance from the industry's 'best practice' production frontier.

In the literature, the impact of the degree of product market concentration on the capital structure, is ambiguous and, eventually, non-linear. In the Brander and Lewis (1986) framework, if there was a monopoly, it would not get financing through debt because, in that framework debt is used because of its strategic impact in the product market, and under monopoly, the strategic effect is non-existent. With a oligopoly, the strategic effect is present and in equilibrium firms have a positive amount of debt. Finally, in a more competitive market, the strategic effect is also low as it each firm has a very small capacity to affect the others. Consequently debt is expected to be higher for intermediate degrees of concentration. To summarize, in the Brander and Lewis (1986) framework we expect debt to have an inverted U relationship with the degree of market concentration. Note, however, that if we exclude the monopoly case, the more concentrated is the market, the higher is the expected level of debt. On the other hand, a younger and/or smaller firm in a concentrated market may opt to use less debt so as to be less vulnerable to predation. Market concentration will have a direct and an indirect impact (through debt) on the probability of default. When the degree of concentration in the product market increases, a decrease in the probability of default is expected since there are fewer competitors. However, according to the limited liability theory, and ignoring the monopoly case, when the level of market concentration increases the strategic effect of debt increases and, as explained before, leverage may have a positive or negative effect on default probability.

Despite the limited empirical testing, table 4.3 summarizes the expected impact of uncertainty, default costs and market concentration. Note that the expected impat on default risk is subdivided into direct impact and total impact (taking into account the impact on debt).

4.3 Hypotheses

Our empirical study is divided into two parts. In the first part we analyze the determinants of the capital structure. In the second part we study the determinants of the

Table 4.3: Expected impact of main explanatory variables.

	Expected impact on debt	Expected impact on default risk	
		Direct impact	Total Impact
Uncertainty	-	+	+/-
Default Costs	-	+	+/-
Market Concentration	+/-	-	+/-

default probability, including the capital structure as an endogenous explanatory variable. Obviously, in our sample, we are interested in analyzing the impact of the variables that have already been tested in the literature. But, we want to give a special emphasis to the variables that have not been studied before, in particular in the default risk literature. In addition, we also want to test if there is a structural difference before and after 2008.

The collected sample allows us to analyze the impact of the global financial crisis which culminated in 2008. Our first set of hypotheses is:

H1a: There is a statistically significant difference of the impact of the various determinants on debt before and after 2008.

H1b: There is a statistically significant difference of the impact of the various determinants on default risk before and after 2008.

The aim is (estimating the models in 2007 and 2013) to analyze if there are differences in the significance and impact of determinants on debt and on default risk. According to some studies there is a change in financial and operational decisions after 2008, which will affect the impact of the debt and default determinants.

As discussed numerically by Haan and Toolsema (2008) and Franck and Le Pape (2008) and as analyzed in the two previous chapters of this thesis, uncertainty affects the decisions regarding capital structure and the default risk. Considering the literature, it is expected that the impact of uncertainty on debt is negative. When firms go through a greater uncertainty period, they tend to adopt a more conservative financial behavior. The previous chapters suggest that the total impact of uncertainty on the default risk is ambiguous. The greater the uncertainty, the more likely firms are not to meet their debt obligations (for a given debt level). However, the total impact can be negative since uncertainty leads to a more conservative behavior. Therefore we intend to empirically test the following two hypotheses:

H2a: Uncertainty has a negative impact on debt.

H2b: Uncertainty has an impact on default risk.

The literature recognizes the importance of the default costs in determining the capital structure. However, the measurement of the indirect default costs and the analysis of its effect on the risk of default is not widespread. The literature in the area (in particular, the trade-off theory) argues that default costs have a negative impact on debt. The higher the default costs, the more expensive it is for the firm to borrow money. However the total effect of the default costs on the default probability is ambiguous. On one hand, there is the direct impact of the costs (the higher the costs, the greater the probability of firm not meeting its obligations). On the other hand, there is also an indirect effect through debt which decreases which in turn decreases the probability of default. An important question to be tested is the significance of these costs for determining the debt level and the default probability, since the literature that studies the impact of these costs is not consensual. Thus, our third set of hypotheses is:

H3a: Default costs (direct and indirect) have a negative impact on debt.

H3b: Default costs (direct and indirect) have an impact on default risk.

According to Opler and Titman (1994), Zingales (1998), Campos (2000), Erol (2003) and Lyandres (2006), the degree of competition in the product market and the capital structure are related. As explained above the impact of the degree of market concentration on the debt level is ambiguous. The Brander and Lewis (1986) framework leads us to predict and inverted U relationship between the degree of market concentration and debt since the strategic effect of debt is stronger for intermediate levels of competition (under monopoly and under perfect competition the strategic effect is nil). However, in a more concentrated market the entrants are more exposed to predation by the incumbents if they are indebted. Thus it is expected that younger firms in concentrated markets finance less with debt. With regard to the default risk, the expected sign of the impact of market concentration is also ambiguous. On the one hand, the higher is the degree of market concentration (i.e., the lower the degree of competition) the higher are the expected profits and therefore the lower is the probability of default. On the other hand, younger firm are more exposed to predation, by established incumbent firms, in concentrated market. Note however that these younger firms are expected to finance less with debt and hence the total impact of market concentration on default risk is ambiguous. Therefore the impact of market concentration on the default risk is not clear cut. Thus our fourth set of hypotheses is:

H4a: Market concentration has impact on debt.

H4b: Market concentration has impact on default risk.

4.4 Methodology

4.4.1 Sample and Variables

To achieve the objectives of the study we used four sources of information: Datastream,Wordscope, OECD and World Bank. The data collected relates to listed firms from 11 OECD countries.[6] The collected data covers the years of 2007 and 2013. However for the calculation of the indirect default costs, which are proxied by the loss of profits relatively to the expected profits (see details in the appendix), it was necessary to collect information from 1997 to 2013. The number of firms is not the same in 2007 and 2013 (the sample contains 9023 firms in 2007 and 7937 in 2013). The sample contains firms belonging to 8 sectors according to the Standard Industrial Classification (SIC). The Financial, Insurance, Real Estate and Public Administration sectors (due to different accounting treatment) and firms which had data for a period of less than two years were excluded from the analysis.

Table 4.4 shows the number of firms in the sample by country and sector, for the two years considered.

[6] The countries included are: Canada, Germany, Greece, Israel, Poland, Portugal, Spain, Switzerland, Sweden, the United Kingdom and the United States.

Table 4.4: Number of firms by country and sector in 2007 and 2013.

Country	Year	Total	Agric.	Mining	Const.	Manuf.	Transp.	Wholesale	Retail	Services
Canada	2007	1857	37	1174	29	277	125	46	29	140
	2013	2082	40	1367	29	285	133	47	33	148
Germany	2007	664	12	7	40	187	174	111	22	111
	2013	514	11	6	33	157	129	89	18	71
Greece	2007	181	20	2	25	45	20	28	8	33
	2013	183	20	2	23	46	26	25	9	32
Israel	2007	277	14	16	23	106	33	41	5	39
	2013	242	12	18	22	91	25	39	5	30
Poland	2007	297	23	5	58	77	40	44	9	41
	2013	199	19	6	44	43	27	29	5	26
Portugal	2007	42	6	1	9	3	6	5	1	11
	2013	37	5	1	8	2	5	4	1	11
Spain	2007	98	13	3	14	24	15	10	3	16
	2013	96	11	2	14	24	15	12	3	15
Switzerland	2007	160	12	1	12	69	20	15	6	25
	2013	144	12	0	12	66	18	10	5	21
Sweden	2007	308	13	20	13	100	45	31	17	69
	2013	203	10	14	12	70	25	23	11	38
UK	2007	980	27	183	38	188	145	82	39	278
	2013	980	26	189	38	188	149	74	37	279
US	2007	4159	137	378	141	1342	754	394	234	779
	2013	3257	112	268	116	1066	593	313	203	586
Total	2007	9023	314	1790	402	2418	1377	807	373	1542
	2013	7937	278	1873	351	2038	1145	665	330	1257

The sample consists of countries with different levels of development, different legal structures and, as we can see in table 4.4, the sectorial structure of firms listed in each country is also different. Canada and UK have a larger number of firms belonging to the Mining sector, Portugal has many firms in the construction sector and, in the remaining countries the manufacturing sector is predominant.

The study is divided into two parts. In the first part, the explained variable is leverage and in the second part the explained variable is default risk. In the first part the long book debt ratio is used. In the second part, default is measured by a dummy variable following the works of Wruck (1990), Asquith, Getner and Scharfstein (1994), Andrade and Kaplan (1998) and Pindado, Rodrigues and La Torre (2008). Table 4.5 shows the dependent variables an how they are calculated in each part of the study.

Table 4.5: Dependent variables.

Part	Dependent variable	Calculation
I	LEV	LEV $=\dfrac{\text{long term debt}}{\text{debt+book equity value}}$
II	DEF	DEF$=\begin{cases} 1 - \text{With Financial distress} \\ 0 - \text{Without Financial distress} \end{cases}$

Default is measured by a binary variable. A firm is considered to be in financial distress if cumulatively it presents the two following conditions for two consecutive years: (1) EBITDA is lower than the financial obligations (2) The market value decreases as defined by Asquith, Getner and Scharfstein (1994); Pindado, Rodrigues and La Torre (2004) and Tinoco and Wilson (2013).

Figure 4.1 shows the average leverage ratio in 2007 and in 2013. As it can be observed, in 2007 and 2013 the most indebted country is Greece. Canada is the lowest indebted country. All countries show an increase in their average debt ratio from 2007 to 2013 except Greece, Israel, Portugal and the UK.

When analyzing the financial structure by sector, we can conclude that the sectors with the greatest long-term debt are agriculture, forestry, fishing and construction. Mining is the sector with the lowest leverage ratio. All sectors increased the average debt ratio from 2007 to 2013.

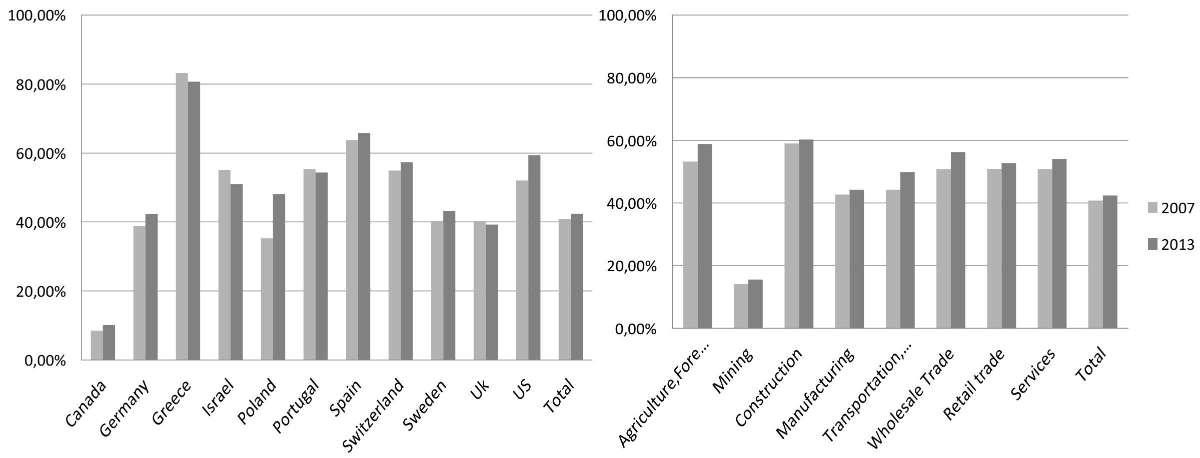

Figure 4.1: Average leverage ratio by country and sector.

Figure 4.2 shows the percentage of firms that are in default by country and sector in 2007 and in 2013. Analyzing the figure, one can concluded that all countries have a higher number of firms in default in 2013 compared to 2007 (except the US). The most significant increase is in Canada. When analyzing the percentage by sector, the sectors that increased the number of firms in default were agriculture, forestry, fishing, mining, construction and retail trade. The sector with the highest percentage of default firms is the mining sector. The data also reveals that there is a substantial number of firms with zero leverage ratio. In 2007 this percentage is around 38% while in 2013 it is 39%. According to table 4.6 and analyzing the average debt, there is a considerable increase in the leverage ratio both when we consider the whole sample as well as when analyzing only firms with a positive debt ratio. One interesting feature is that the average debt is lower for firms that are in financial distress. This suggest that many firm that are in financial distress did not finance with long-term debt and they are in financial distress due to operational reasons and not because of an excessive use of debt. The average indebtedness increased from 2007 to 2013, but decreased for default firms. The percentage of firms in default increased from 2007 to 2013 whereas in the case of the indebted firms the percentage decreased.

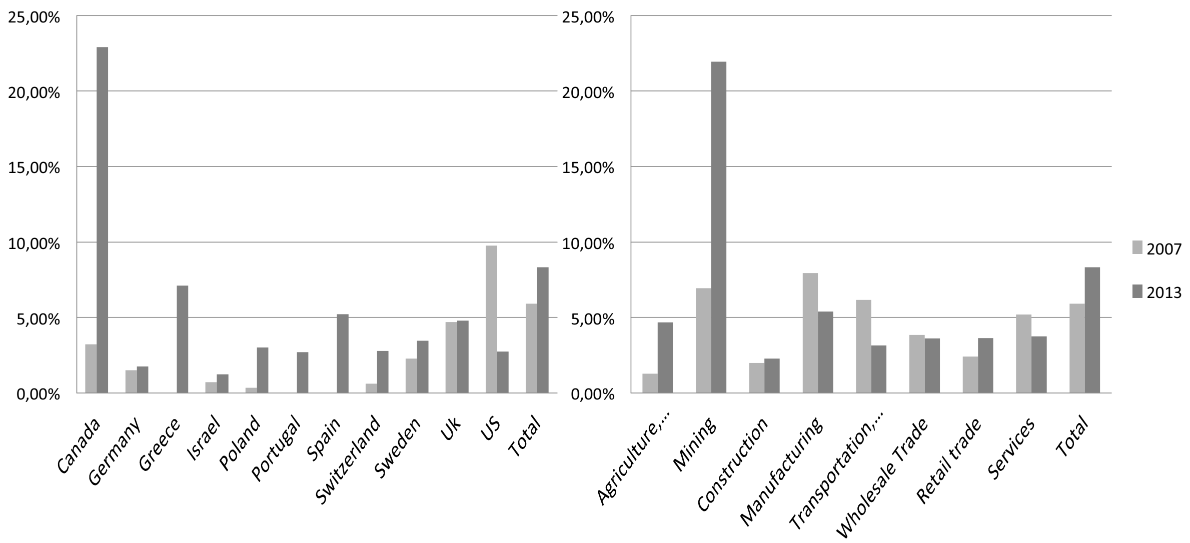

Figure 4.2: Percentage of firms in financial distress by country and by sector.

Table 4.6: Average leverage and proportion of firms in financial distress.

Dependent	All Sample		Lev >0		Default=1		Lev>0 and Default=1	
Variable	2007	2013	2007	2013	2007	2013	2007	2013
Leverage Ratio (%)	40.79	42.40	66.90	68.55	33.37	14.53	67.90	46.62
Default firms (%)	5.92	8.33	2.90	2.60	-	-	-	-

The analysis will be divided into two parts. In the first part the aim is to test hypotheses H1a, H2a, H3a and H4a regarding the impact on debt, therefore debt is the dependent variable. The independent variables used in the first model are: uncertainty, direct default cost, indirect default cost, market concentration and the square of market concentration. The control variables used are: non-tax-shield, profitability, expected growth, square of expected growth, collateral, size, age, and the set of sectorial and country dummies.

In the second part the objective is to test hypotheses H1b, H2b, H3b and H4b regarding the impact on the default probability, and to test the impact of debt on the default risk. Thus, the default probability is used as the dependent variable and the main independent variables used in the second model are: leverage, uncertainty, direct default cost, indirect default cost, market concentration and square of market concentration. The control variables used are: management efficiency, size, age, squared of age, and the set of sectorial and country dummies. The last model considers debt as an endogenous variable, using the following instrumental variables: non-tax-shield, expected growth, square of expected growth and collateral.[7]

Table 4.7 summarizes the variables used (references are presented in the Appendix). Table 4.8 shows some descriptive statistics of the main variables and table 4.9 presents the correlation between the main variables. According to table 4.9 there is a negative correlation between leverage and default. The variables uncertainty and direct default costs do not have the same correlation with debt and default. Indirect default costs and market competition are negatively related to debt and default.

4.4.2 Econometric Models

In order to test the hypotheses mentioned in section 4.3, the econometric analysis will be divided into three models. Each of these models will analyze data from 2007 and 2013. The aim is to understand whether there are differences in the impact of the variables before (2007) and after (2013) the financial crisis.

[7] The square of the variables market concentration, growth and age were introduced due to their possible non linear effect.

Table 4.7: Independent Variables

Variables	Notation	Calculation
Uncertainty	UNC_i	Standard deviation of log-changes in sales
Indirect Default Cost	IDC_i	Difference between profits and estimated profits (see Altman 1984)
Direct Default Cost	DDC_i	R&D expenses to total asset
Market Competition	HHI_i	Sum of the squares of the percentage market shares
Squared Market Competition	$\mathrm{HHI2}_i$	Square of Market Competition
Leverage	LEV_i	Long book debt ratio
Leverage Transformed	LEVT_i	Transformation of fractional variable for continuous variable
Non-Tax Shields	$\mathrm{NTAXSHI}_i$	Depreciation to total asset
Profitabilty	PROF_i	Operating profit to total asset
Growth opportunities	GROWTH_i	Percentage change in total asset from previous to current year
Squared growth opportunities	$\mathrm{GROWTH2}_i$	Square of growth opportunities
Collateral	COLL_i	Fixed assets to total asset
Age	Age_i	Years since foundation
Squared Age	$\mathrm{Age2}_i$	Square of age
Management Effciency	$\mathrm{MANEFFIC}_i$	Sales to asset
Size	SIZE_i	ln (total asset)
Country	$\mathrm{COUNTRY}_{iz}$	Dummy variables, equal to 1 if firm i belongs to country z
Sector	SECTOR_{ij}	Dummy variables, equal to 1 if firm i belongs to sector j

* The Appendix presents references that use these variables and explains the calculation of IDC.

The first model aims to analyze the impact of uncertainty, direct and indirect default costs and market concentration on debt. Most studies that analyze the financial leverage decisions use linear regression models, however the variable debt ratio has two important characteristics: it is a limited variable between 0 and 1 and there may be many firms that do not use debt. To address this reality some authors use the logistic transformation or other non-linear transformation so as that the transformed variable takes values between 0 and infinity or such that the transformed variable can take any real value. However, the two aforementioned characteristics of the leverage ratio can be taken into account using a two-part fractional regression model (as indicated by Papke and Wooldridge, 1996; Ramalho and Silva, 2009; Ramalho and Ramalho, 2010; Ramalho, Ramalho and

Table 4.8: Descriptive statistics for dependent and independent variables.

	All Sample			LEV +			Default			LEV+ and Default		
Variable	Obs	Mean	Std.Dev	Obs	Mean	Std.Dev	Obs	Mean	Std.Dev	Obs	Mean	Std.Dev
LEV	16960	0.416	0.430	10412	0.677	0.351	1194	0.230	0.382	468	0.585	0.406
DEF	12909	0.093	0.290	8283	0.057	0.231	1194	1.000	0.000	468	1.000	0.000
UNC	14746	0.433	0.538	10140	0.361	0.460	798	0.722	0.709	422	0.666	0.685
DDC	12953	0.254	13.30	8018	0.061	0.389	1014	0.174	0.719	398	0.125	0.405
IDC	7155	0.364	13.78	5603	0.387	15.51	245	0.288	1.693	176	0.181	0.718
HHI	16960	218.6	193.4	10412	221.0	203.9	1194	200.9	156.6	468	197.9	184.4
HHI2	16960	85173	175549	10412	90391	184487	1194	64854	148480	468	73103	73104

Table 4.9: Correlation between variables.

	LEV	DEF	UNC	DDC	IDC	HHI	HHI2
LEV	1.0000						
DEF	-0.0241	1.0000					
UNC	-0.1564	0.1230	1.0000				
DDC	-0.1317	0.0839	0.2382	1.0000			
IDC	-0.0274	-0.0030	0.0866	0.0961	1.0000		
HHI	-0.0692	-0.030	0.0421	-0.0233	-0.0073	1.0000	
HHI2	-0.0422	-0.0088	0.0225	-0.0238	-0.0059	0.9374	1.0000

Murteira, 2011; and Ramalho and Silva, 2013).

By using a two-part model for a variable that is between 0 and 1 and with a non-neglible proportion of zeros, in the first part the aim is to analyze the use or not of debt (a binary choice), i.e., the probability of the debt ratio being positive. The use of leverage variable, LEV^*, is defined as follows:

$$LEV^* = \begin{cases} 0 & \text{if } \ LEV = 0 \\ 1 & \text{if } LEV \in (0,1] \end{cases} \tag{4.1}$$

The first part is modelled as follows:

$$\Pr\left(LEV^* = 1|\mathbf{X}\right) = \Pr\left(LEV \in (0,1]\,|\mathbf{X}\right) = F(\mathbf{X}\boldsymbol{\theta}) \tag{4.2}$$

Where $\mathbf{X}$ is a vector of explanatory variables and $\boldsymbol{\theta}$ is the vector of coefficients and $F(\cdot)$ can be the Cauchy (Cauchit), Logistic (Logit), Standard normal (Probit), Extreme maximum (Loglog) or Extreme minimum (Complementary loglog) distribution function and can be estimated by Maximum Likelihood (ML) using the whole sample.

The second part of the model aims to analyze the non-zero leverage, i.e., aims to study the impact of the determinants of the debt ratio. The second part is modelled as follows:

$$E\left(LEV|LEV^{*}=1,\mathbf{X}\right)=G(\mathbf{X}\boldsymbol{\gamma}) \tag{4.3}$$

Where $\boldsymbol{\gamma}$ is the vector of coefficients and $G(\cdot)$ can be the Cauchy (Cauchit), Logistic (Logit), Standard normal (Probit), Extreme maximum (Loglog) or Extreme minimum (Complementary loglog) distribution function and can be estimated by Quasi-Maximum Likelihood (QML) using observations with positive leverage ratio. Note that, the vector of coefficients $\boldsymbol{\theta}$ and $\boldsymbol{\gamma}$ may be different since the factors that influence resorting or not to debt may be different from the factors that influence the amount of debt.

In order to analyze the impact of a variable on the expected leverage ratio, one needs to take into account its effect on the probability of the debt being positive and its effect on the debt ratio given that leverage is positive. In other words, the conditional expected leverage ratio, $E(LEV|\mathbf{X})$, is given by:

$$E\left(LEV|\mathbf{X}\right)=E\left(LEV|LEV^{*}=1,\mathbf{X}\right)\cdot\Pr\left(LEV^{*}=1|\mathbf{X}\right)=F(\mathbf{X}\boldsymbol{\theta})\cdot G(\mathbf{X}\boldsymbol{\gamma}) \tag{4.4}$$

In our case, the $\mathbf{X}$ is a vector composed by the following variables: uncertainty, UNC, direct default cost, DDC, indirect default cost, IDC, the Herfindahl-Hirschman Index, HHI, the square of the Herfindahl-Hirschman Index, HHI^2, non-tax-shield, $NTAXS$, profitability, $PROF$, expected growth, $GROWTH$, square of expected growth, $GROWTH^2$, collateral, $COLL$, size, $SIZE$, age, AGE and the set of sectorial and country dummies. Considering the next model, it is useful to decompose vector $\mathbf{X}$ into two vectors: $\mathbf{X}_1$ which includes the variables that also influence directly the default risk and $\mathbf{Z}$ which is formed by the variables that influence debt but do not directly influence the default probability.

The second model aims to test the impact of debt and the effect of the independent variables on the default probability. Since the dependent variable, DEF, is dichotomous we need to use an estimation method that takes into account the binary nature of the variable. Moreover, considering the potential endogeneity of the leverage variable we need to use a method such as instrumental variables. Therefore we decided to use instrumental variables probit model (ivprobit in STATA). However this model assumes that the endogenous variable is a continuous and non-limited variable. To solve this issue we used a transformation of the leverage ratio that takes values between minus infinitum and plus infinitum. The transformation is the following:[8]

$$LEVT_i = \ln\left(\frac{LEV_i}{1-LEV_i}\right) = \ln\left(\frac{\text{long term debt}}{\text{equity}}\right)$$

The estimated model is:

$$\begin{aligned}\Pr\left(DEF_{1i}=1|LEVT,\mathbf{X}_1,\mathbf{X}_2\right) \quad = \quad & \Phi(\alpha_0+\alpha_1 LEVT_i,+\alpha_2 UNC_i+\alpha_3 DDC_i+ \\ & \alpha_4 IDC_i+\alpha_5 HHI_i+\alpha_6 HHI_i^2+\alpha_7 MANEF_i+ \\ & \alpha_8 SIZE_i+\alpha_9 AGE_i+\alpha_{10} AGE_i^2+ \\ & \sum_{j=1}^{7}\rho_j SECTOR_{ij}+\sum_{k=1}^{10}\lambda_k COUNTRY_{ik}) \qquad (4.5)\end{aligned}$$

where Φ is the cumulative normal distribution and $\mathbf{X}_1$ and $\mathbf{X}_2$ are vectors of exogenous variables that influences the default probability, where $\mathbf{X}_2$ are the variables that do not influence leverage. The instrumental variables used were the set of variables in vector $\mathbf{Z}$, which includes non-tax shields, collateral, growth and growth squared.[9]

Another model that can be estimated is the reduced form of the default probability

[8] The transformed variable can only be calculated for observations with a positive debt ratio. Therefore, the estimation of the default probability is only done for leveraged firms.

[9] Note that the variable profitability was not introduced neither as an explanatory variable of the default risk nor as an instrumental variable. The reason is that profitability is used to identify the firms under financial distress, and thus it is likely to be correlated with the errors in the default regression.

equation, where $LEVT$ is substituted by its expected value conditional on $\mathbf{X}_1$ and $\mathbf{Z}$. In order words, to estimate the probability of the firm being in financial distress as a function of all exogenous variables, $\mathbf{X}_1, \mathbf{X}_2$ and $\mathbf{Z}$:

$$\begin{aligned}
\Pr\left(DEF_{2i} = 1 | \mathbf{X}_1, \mathbf{X}_2, \mathbf{Z}, LEV > 0\right) \;\; = \;\; & \Phi(\gamma_0 + \gamma_1 UNC_i + \gamma_2 DDC_i + \gamma_3 IDC_i + \\
& \gamma_4 HHI_i + \gamma_5 HHI_i^2 + \gamma_6 NTAXS_i + \gamma_7 GROWTH_i + \\
& \gamma_8 GROWTH_i^2 + \gamma_9 COLL_i + \gamma_{10} AGE_i + \\
& \gamma_{11} AGE_i^2 + \gamma_{12} MANEF_i + \gamma_{13} SIZE_i + \\
& \sum_{j=1}^{7} \mu_j SECTOR_{ij} + \sum_{k=1}^{10} \phi_k COUNTRY_{ik}) \qquad (4.6)
\end{aligned}$$

It worthwhile to related the two previous estimations with the models analyzed in the two previous chapters. Equation (4.5) corresponds to estimating the default probability in the second stage of the game (i.e., for a given debt level) and it allows us to study the impact of debt on the default probability as well as the impact of the exogenous variables, considering debt as fixed. Therefore the impacts estimated with this regression do not take into account the indirect impact of the variable on the default probability through the induced changes on debt. However, it should be noted that indirect effects through the more or less aggressive behavior in the product market (for a given debt level) are captured with this regression.

On the other hand, equation (4.6) is related to the solution of the whole game in the previous chapters, i.e. the subgame perfect equilibrium default probability. The partial effects of this equation measure the total impact of the explanatory variable on the default risk because it includes the direct impact of the variable, given by the partial effects of equation (4.5), as well as the indirect impact through the impact of the explanatory variable on debt which in turn influences default probability. Therefore, with the estimation of these 2 equations we are able to distinguish between the direct and total impact of each explanatory variable.

4.5 Empirical Results

4.5.1 Main findings of the leverage model

In Table 4.10 the results of the estimation of the two-parts fractional model are presented.[10] The left part of the table presents the results regarding the decision on whether to finance with debt or not. The right part of the table presents the results regarding the decision of the debt ratio (for the firms that have a positive leverage). We present the results including the observations of the two years, the results with 2007 observations and the results with 2013 observations. To facilitate interpretation, the table also presents the expected impact of each variable.[11]

According to the RESET test there seems to be a correct adequacy of both models to the data. In addition, the Pseudo R^2 values are similar to the values reported in the literature in this topic. The Chow-type statistic (for the test of structural break) was carried out to check if the variables have a different effect on the two years (2007 and 2013). The null hypothesis was rejected for a 1% level of significance. Therefore it can be concluded that there is a structural break since the variables have different effects in 2007 and 2013. Considering this result, it is more adequate to look at the results for each year, than to interpret the regression with observations of the two years.

An overall look at the signs and statistical significance of the coefficients in the two regressions (observations of 2007, observations of 2013) reveals that the results tend to be consistent with each other. There are however cases where a variable only has a statistical significant impact in one regression and where the sign is not the same (but when this happens one of the coefficients is not significant, hence the change in sign is not really relevant).

With regard to the decision of a firm financing with debt (binary model) we can

[10] For reasons of space the results regarding the coefficients of the sectorial and country dummies were omitted. However the results of the tests carried out to test their significance is presented (LR/LM Tests).

[11] The review of the literature presented above is based primarily on determining the amount of the debt, hence it is more related with the second part in the two-part model. Thus the expected impact of variables has been placed on the right side of table 4.10.

conclude that direct default costs (DDC), collateral ($COLL$) and size ($SIZE$) are all statistically significant in the two years under analysis. The signs obtained for these variables are the ones that are expected according to the theory. Higher direct default costs decrease the probability of a firm financing with debt. On the contrary, the higher is the collateral and the larger is the firm, the more likely is that the firm decides to use debt as a financing alternative.

Besides the previous variables, we observe that in 2013 the profitability ($PROF$) has a negative impact on the likelihood of a firm financing with debt. This result is consistent with the pecking order theory. Note that, after the crisis, banks started being more careful in the evaluation of credit applications and thus it started being more difficult for a firm to obtain debt if it desired to do so. This may be a reason for an increasing reliance on internal funds for financing. This is a potential reason why the variable is only significant after the crisis.

On the other hand, the variables age (AGE), uncertainty (UNC), indirect default cost (IDC) and square of HHI (HHI^2) are only significant in 2007. The variable AGE has a positive impact on the probability of the firm financing with debt. This result is consistent with the idea that a older firm already build a reputation and it is better know in the debt market, making it easier to resort to debt financing. The fact that the variable is not significant in 2013, may be related with the fact that the restriction on bank credit were so high that even older firms had difficulties in obtaining credit. In other words, the reputational advantage was not so important as before the crisis. The impacts of UNC and IDC are both positive. These are an unexpected result, which are not easy to explain. In the case of the indirect default costs, it may be related with the way they are measured.

Analyzing the amount of debt (second part of the model) we can see that the only variables that have a significant impact in the two years are $COLL$, $SIZE$ and AGE. The two first variables influence positively the debt ratio and therefore they have the same impact on both parts of the two-parts model. On the contrary the variable AGE has a negative impact on the debt ratio, which can be explained by the fact that older firms are likely to have higher accumulated retained earnings, and therefore need a lower amount of debt. Note that while the decision to use debt seems to be more influenced by the reputation story, the amount of debt seems more related with retained earnings.

In addition to the previous variables, the DDC, HHI, $PROF$ and $GROWTH^2$ are all significant in 2007 whereas UNC and IDC are significant in 2013. Note that the impact of uncertainty, direct default costs and indirect default costs on the debt ratio, is the expected one, as these three variables lead to a lower debt ratio. Moreover a higher market concentration implies higher leverage, which is consistent the larger strategic effect of debt in more concentrated markets (except, the limit case of a monopoly, which does not occur in our dataset). The control variables profitability and growth also have the expected signs.

The coefficients shown in table 4.10 allows us to analyze the sign and the significance of the coefficients but we cannot evaluate the effect of the variables on the debt (due to the non-linearity of the function F and G). Table 4.11 shows the estimated partial effects for each variable (considering the average of the partial effects for all observations).

Taking into account the results mentioned above and the hypotheses postulated in section 4.3, we can conclude that the result support the hypotheses *H1a, H2a, H3a,* and *H4a*.

First, *there is a statistically significant difference in the determinants' impact on debt before and after 2008* as verified in the Chow test, and as observed in tables 4.10 and 4.11 there are some variables which have different statistically effects in 2007 and 2013. We can, therefore, state that the determinants have a different impact before and after the peak of the crisis. This difference is significant both in probability of using debt (binary model) and the amount of debt (fractional model). For instance, the indirect default costs has a negative effect on the debt ratio in 2013 (considering the firms that finance with debt) but does not have a significant impact on the debt ratio before the crisis.

Second, *uncertainty affects negatively the debt ratio* (however the result is only statistically significant in 2013). According to the theoretical literature in the area (Brander and Lewis, 1986; Franck and Le Pape, 2008; Haan and Toolsema, 2008) a negative im-

Table 4.10: Results of the two-parts regression to estimate leverage.

Independent Variable	Part I Binary model			Part II: Fractional regression model			
	All Sample	2007	2013	All Sample	2007	2013	Exp sign
UNC	0.1451**	0.4073***	-0.0040	-0.1138*	-0.0611	-0.1549*	-
	(0.0661)	(0.1316)	(0.0823)	(0.0632)	(0.0934)	(0.0819)	
DDC	-0.7467***	-0.6417*	-1.1327***	0.0140	-0.7875*	0.4824	-
	0.2158	0.3481	0.2797	0.2635	0.4242	0.3233	
IDC	0.0180***	0.0199***	0.0028	-0.0020***	-0.0050	-0.0012***	-
	(0.0066)	(0.0099)	(0.0023)	(0.0006)	(0.0081)	(0.0005)	
HHI	-0.0008**	-0.0012	-0.0007	0.0005	0.0005***	0.0005	+/-
	(0.0004)	(0.0008)	(0.0004)	(0.0003)	(0.0006)	(0.0004)	
HHI^2	0.0000**	0.0000*	0.0000	0.0000*	0.0000	0.0000	+/-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
NTAXS	0.6933*	0.3496	0.7203*	0.1379	1.2116	-0.0604	-
	(0.3914)	(1.1947)	(0.4217)	(0.3947)	(0.9398)	(0.4473)	
PROF	-0.0001	0.0210	-0.2990***	-0.2641**	-0.9042***	-0.0402	+/-
	(0.0031)	(0.0735)	(0.0788)	(0.1277)	(0.2137)	(0.1107)	
GROWTH	0.1005**	0.1494	0.0774	-0.0179	0.1009	0.1239	+/-
	(0.0444)	(0.1024)	(0.0738)	(0.0535)	(0.1029)	(0.1051)	
$GROWTH^2$	-0.0012	-0.0045	-0.0004	0.0007	-0.0132***	-0.0010	+/-
	(0.0011)	(0.0106)	(0.0044)	(0.0009)	(0.0069)	(0.0018)	
COLL	1.1498***	1.4080***	1.0857***	0.6050***	0.7733***	0.5020***	+
	(0.1051)	(0.2314)	(0.1214)	(0.0916)	(0.2133)	(0.1024)	
SIZE	0.2479***	0.2211***	0.2714***	0.2479***	0.2497***	0.2520***	+/-
	(0.0105)	(0.0185)	(0.0134)	(0.010)	(0.0173)	(0.0127)	
AGE	0.0020	0.0103***	-0.0012	-0.0167***	-0.0220***	-0.0157***	+/-
	(0.0019)	(0.0036)	(0.0023)	(0.0019)	(0.0039)	(0.0023)	
CONSTANT	-2.0511***	0.0795	-0.1358***	-1.6976***	-1.7864***	-1.7450***	
	(0.2706)	(63.109)	(67.066)	0.1996	(0.3681)	(0.2357)	
# of obs	5920	1844	4076	4593	1445	3148	
Pseudo R^2	0.2141	0.2122	0.2300	0.3725	0.3213	0.4122	
RESET test	0.672	0.683	0.715	0.625	1.755	0.661	
Chow test		75.804***			25.003***		
LR/LM sector	42.261***	13.385*	29.331***	16.786**	8.4010	10.179	
LR/LM country	62.268***	36.727***	52.402***	423.82***	91.052***	336.48***	

Notes: The statistics reported are obtained through Stata (version 13.1) Below the coefficients we report standard deviation in parentheses; for the test statistics we report p-values; ***, ** and * denote coefficients which are statistically significant at 1%, 5% or 10%, respectively. For the binary model, the joint significance of the sector and country dummies was tested using a standard LR statistic,while the pseudo R2 and the RESET and heteroskedasticity tests were computed as Ramalho, Ramalho and Murteira (2011).

Table 4.11: Partial effects of the two-parts leverage model.

Independent	Probability of use debt			Proportion of debt			Total Effect		
Variable	All Sample	2007	2013	All Sample	2007	2013	All Sample	2007	2013
UNC	0.036	0.098	-0.001	-0.022	-0.012	-0.029	0.008	0.060	-0.023
DDC	-0.184	-0.155	-0.272	0.003	-0.154	0.091	-0.124	-0.230	-0.115
IDC	0.004	0.005	0.001	-0.000	-0.001	-0.000	0.003	0.003	0.000
HHI	-0.000	-0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	0.000
HHI^2	0.000	0.000	0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000
NTAXS	0.170	0.085	0.173	0.027	0.237	-0.011	0.137	0.245	0.109
PROF	-0.000	0.005	-0.072	-0.051	-0.177	-0.008	-0.039	-0.135	-0.0547
GROWTH	0.025	0.036	0.019	-0.003	0.020	0.023	0.014	0.041	0.0306
$GROWTH^2$	-0.000	-0.001	-0.000	0.000	-0.003	-0.000	-0.000	-0.003	-0.0002
COLL	0.283	0.340	0.261	0.116	0.151	0.095	0.284	0.358	0.2503
SIZE	0.061	0.053	0.065	0.048	0.049	0.048	0.079	0.358	0.0809
AGE	0.001	0.003	-0.000	-0.003	-0.004	-0.023	-0.002	-0.002	-0.0025

pact of the uncertainty on the debt level was expected as firms behave more cautiously when uncertainty is higher. Thus our result support the hypothesis *H2a* when we consider leveraged firms. However, as mentioned before, uncertainty positively affects the probability of a firm borrowing (binary model), although the coefficient is only significant in 2007. This indicates that, before the crisis, the probability of a firm opting for debt was higher when uncertainty was higher. One possible explanation, following the logic of the pecking order theory, is that when uncertainty is higher the firm is more likely to face liquidity problems and has a higher probability of having to use debt to face those liquidity problems. Another potential explanation is that greater uncertainty may be correlated with greater information asymmetry. For these two reasons the firm may prefer to resort to borrowing than to issue new equity. However, after the crisis the uncertainty variable ceased to have statistical significance and this may be explained by the increased constraints on borrowing making it more difficult to resort to debt to finance liquidity problems.

The *default costs (direct and indirect) have a negative impact on the debt ratio* (however for the direct default costs the effect is statistically significant only in 2007 whereas for the indirect default costs the impact is statistically significant only in 2013). This result is consistent with the trade-off theory, since the higher are the default costs, the higher are the costs of holding debt and therefore the lower is the optimal debt ratio. Moreover it is also consistent with the model presented in the previous chapter. It should be noted that our results reveal that direct default costs also influence negatively the probability of firm financing with debt, which was expected. More difficult to explain is the fact that in 2007, higher indirect default costs, increase the probability of the firm using debt. As mentioned above this may be related with the way indirect default costs are measured. They are evaluated by the profit loss with respect to the expected profit. However that profit loss may be related with a negative demand shock (in that particular period the firm had a unusually low demand) and not to a reputation loss associated with the probability of the firm meeting its obligations. This negative shock may lead the firm to face liquidity problems, that increase the need for financing.

Market concentration has an impact on the debt ratio (however the impact is significant only in 2007). Regarding the 2007 regression, it can be seen that the degree of market concentration has a positive effect on the debt ratio. Note that if we ignore the monopoly case (which actually does not occur in our data) the more concentrated is the market, the higher will be the strategic effect of debt and, therefore, the higher is the equilibrium debt level (in the Brander and Lewis (1986) type of models).

It should also be highlighted that the significant control variables have the expected signs. However there are some variables that were significant before the crisis but not significant after the crisis and variables for which the reverse is true. This suggests that, after the crisis, some of the debt determinants may have changed and that it may be wise to test the relevance of other variables.

4.5.2 Main findings of the default model

In table 4.12 the results of default estimation are presented.[12] We present the results including the observations of the two years, the results with 2007 observations and the results with 2013 observations. To facilitate interpretation, the table also presents the expected impact (direct and total) of each variable. To estimate the direct impact instrumental variable probit model was used (ivprobit in Stata) while for the estimation of the reduced form of the model we used a Probit model.

The exogeneity tests (Durbin χ^2, Wu- Hausman F and Wald Test) presented in the Appendix were conducted and confirm the correlation of the variable debt with the residuals for the all sample and for 2013, which suggests that it is appropriate to use the instrumental variables method. The Pseudo R^2 values presented are in accordance with the values reported in the literature in the area.[13] In general, the instrumental variables have statistical significance according to the t test for a level of significance of 10%. The Chow-type statistic (for the test of structural break) was carried out to check if the variables have different effects on the two groups (2007 and 2013). The null hypothesis was rejected for a 1% level of significance. Therefore, it can be concluded that there is a structural break since the variables have different effects in 2007 and 2013. Considering this result, it is more appropriate to look at the results for each year, than to interpret the regression with observations of the two years.

Analyzing the signs and the significance of the variables, one observes some differences between 2007 and 2013, with the coefficient of some variables changing their sign and significance. Similarly, there are some differences between the direct and total effects, which is expectable.

The impact of debt on the default probability is not significant in 2007, but it is negative and significant in 2013. One explanation for this results is that more indebted firms tend to behave more cautiously in the product market, with this effect dominating

[12] For reasons of space the results regarding the coefficients of the sectorial and country dummies were omitted. However the results of the tests carried out to test their significance is presented (LR/LM Tests).

[13] The RESET test suggests a change in the functional form. There was no time to explore the functional form of the model.

the fact that more debt increases the probability of default (maintaining constant the product market behavior), as defended by predation theories. However the fact that the variable is only significant in 2013 suggests another explanation. After the crisis there has been an increase in the credit constraints and banks are only willing to give loans to firms which offer a high guarantee of meeting their obligations. Thus only the best firms with less risky projects are able to get financing. This screening mechanism implies that debt and default probability are negatively related.

An increase on the uncertainty leads to an increase in the default probability in the two years (although in 2013 the direct effect is not significant). This result is consistent with the numerical results in the previous chapter, where with default costs, the direct and the total impact of uncertainty are both positive.

Regarding the default costs, in our regressions only the total effect of the indirect default costs in 2013 is significant, which gives only a very mild support to *H3b*. In 2013, the total impact of the indirect default costs on the debt probability is negative, which is consistent with the numerical results of the previous chapter.

Similarly, the impact of the degree of market concentration is only significant in 2013. Since the coefficients of HHI and HHI^2 are both significant, we can conclude that impact of the degree of market concentration is non-linear. In particular, the result suggest that the higher is the market concentration the lower is the default risk (the coefficient associated with HHI is negative), but the decrease in the default risk is progressively smaller (the coefficient associated with HHI^2 is positive, suggesting a relationship in U, but the coefficient is extremely small, hence the impact reaches a minimum for very high concentration values). This result is consistent with what we expected as profits are expected to be higher in more concentrated markets, and although debt may also be higher, the probability of default is lower.

Regarding the control variables, in 2013 management efficiency ($MANEF$) has a direct negative impact on the probability of default, which is consistent with the expected sign. The size variable has a direct positive impact on the default probability but a negative total impact on the default probability (although the total impact is not significant in 2013). In other words, considering the total effect, the larger is the firm, the less likely it is to fail.

In 2007, since the coefficient associated with AGE is positive while the coefficient associated with AGE^2 is negative, one can conclude that the total effect of the variable age on the default probability follows the inverted U shape relationship previously identified in the literature. In other words, the default probability increases till a certain age and decreases thereafter. However, for a given debt level the result show that the default risk is decreasing with age. This can be explained by the presence of learning by doing and acquired experience effects which, for a given debt level, imply a reduction in the default probability as the firm becomes older.

Taking into account the results mentioned above and the hypotheses postulated in section 4.3, we can conclude that the result support hypotheses *H1b, H2b, H3b*, and *H4b* although the support for *H3b* is relatively mild.

First, there is a statistically significant difference in the determinants' impact on default risk before and after 2008 as verified in the Chow test and, as can be seen in table 4.12, where some variables have a different statistical effect in 2007 and 2013. Therefore we can conclude that the determinants have a different impacts before and after the peak of the crisis. This may indicate that the 2008 financial crisis may have altered the determinants of the default probability. For instance, the indirect default costs, the market concentration and the management efficiency are only significant in 2013.

Second, uncertainty has an impact on default risk since the default probability is increasing with uncertainty. The results confirm what is suggested by the numerical analysis performed in the previous chapter.

However, there is only a relative mild support for hypothesis *H3b* because only the indirect default costs in 2013 influence the default risk (considering the total effect). This negative impact of the indirect default costs was expected since higher default costs lead to the adoption of a more conservative behavior in the debt market and also in the output market, hence default probability decrease. However we expected the result to hold in the two years and for both direct and indirect default costs, which did not occur.

Table 4.12: Results of the default risk regressions.

Independent variables	Exp sign Direct	Exp sign Total	All Sample Direct	All Sample Total	2007 Direct	2007 Total	2013 Direct	2013 Total
LEVT	+/-		-0.3749***	-	-0.1909	-	-0.3789***	-
			(0.0162)		(0.0800)		(0.0069)	
UNC	+	+/-	0.1105	0.4454***	0.6189***	0.5879***	-0.0606	0.3172**
			(0.1018)	(0.0982)	(0.2199)	(0.1583)	(0.0808)	(0.1479)
DDC	+	+/-	-0.2189	0.1937	0.1199	0.4406	0.1251	-0.4866
			(0.2399)	(0.3173)	(0.4775)	(0.5869)	(0.2524)	(0.4050)
IDC	+	+/-	-0.0019	-0.0031**	-0.0706	-0.0262	-0.0014	-0.0016***
			(0.0030)	(0.0013)	(0.0462)	(0.0262)	(0.0016)	(0.0006)
HHI	-	+/-	-0.0002	-0.0018**	-0.0004	-0.0009	0.0001	-0.0019**
			(0.0005)	(0.0008)	(0.0011)	(0.0014)	(0.0004)	(0.0009)
HHI^2	+		0.0000	0.0000***	0.0000	0.0000	0.0000	0.0000**
			(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
MANEF	-		-0.0950***	-0.0430	-0.0876	-0.0874	-0.1022***	-0.0009
			(0.0279)	(0.0601)	(0.0844)	(0.1073)	(0.0306)	(0.0696)
SIZE	+/-		0.1620***	-0.0601***	0.0091***	-0.0920***	0.1759***	-0.0262
			(0.0502)	(0.0207)	(0.0340)	(0.0345)	(0.0120)	(0.0266)
AGE	+/-		-0.0208**	0.0173	0.0512***	0.0685*	-0.0262***	0.0093
			(0.0093)	(0.0179)	(0.0314)	(0.0352)	(0.0094)	(0.031)
AGE^2	+/-		-0.0000	-0.0003	-0.0015	-0.0016**	0.0001	-0.0001
			(0.0002)	(0.0004)	(0.0008)	(0.0008)	(0.0002)	(0.0004)
NTAXS			-	0.2938	-	-0.5789	-	0.2150
				(1.0136)		(2.7430)		(1.1080)
GROWTH			-	-0.5598**	-	-0.3920*	-	-1.0339***
				(0.2287)		(0.2244)		(0.3323)
$GROWTH^2$			-	0.0081**	-	0.0191	-	0.0166***
				(0.0034)		(0.0195)		(0.0050)
COLL			-	-0.5474**	-	-0.4993	-	-0.5703**
				(0.2079)		(0.4538)		(0.2513)
CONSTANT			0.5517**	-0.5849***	-6.3168	-4.9103***	-1.2395***	-0.9213*
			(0.2523)	(0.4117)	(944.85)	(0.8088)	(0.2779)	(0.4882)
# of obs			3864	3864	1046	1050	2686	2686
Pseudo-R^2			-	0.1099	-	0.1903	-	0.0860
Wald Test exogeneity			7.14***	-	0.48	-	38.0***	-
RESET test			227.60***	2.9397	849.05***	1.0581	2.967	1.4676
Chow test			745.45***	56.650***	-	-	-	-
LR/LM sector			37.056**	6.3519	19.311***	3.5608	30.517***	6.2754
LR/LM country			1354.2***	30.033**	849.05***	21.158**	1109.3***	17.034*

Notes: Results using Stata (version13.1). Below the coefficients we report standard deviation in parentheses; for the test statistics report p-values; ***, ** and * denote coefficients which are statistically significantly 1%, 5% or 10%, respectively. Direct impacts were estimated using IVProbit whereas total effects were estimated with Probit.

Market concentration has an impact on default risk (however the impact is significant only in 2013). The impact obtained is the expected one.

4.6 Conclusion

The determinants of capital structure and default probability have been research areas attracting lots of attention. However after reviewing the literature we came to the conclusion that there are a set of variables whose importance is admitted but, so far, with limited empirical testing. In particular, the level of uncertainty, the degree of market concentration and the default costs have been little explored, especially in the default risk literature. The indirect default costs, due to their difficult measurement have been even more ignored. The main purpose of this chapter was to test the significance of this set of variables on the leverage ratio and on the default probability. In addition, and considering that the two previous chapters suggest that debt is endogenously determined, our estimation of the default probability used a instrumental variables approach.

Our estimations used data of listed firms from 11 countries belonging to the OECD in 2007 and in 2013. This allowed us to estimate the effects of the variables before (2007) and after (2013) the peak of the financial crisis in 2008. The results showed that the determinants of capital structure and probability of default were different before and after 2008. After 2008, obtaining credit become more difficult and only firms with a sustainable capacity to meet their obligations were able to get credit as banks become more selective in approving credit applications.

To analyze the factors that influence the capital structure we used a two-part fractional model to take into account the specificities of the debt variable. That allowed us to analyze which determinants affect the decision on whether to finance with debt or not and which affect the decision of the debt ratio. To analyze the default risk, an instrumental variable probit model (to estimate the direct impact) and a Probit model (to estimate the total impact) were used. The potential endogeneity of the debt variable, was confirmed in 2013 but not in 2007.

Regarding our main independent variables, we concluded that uncertainty affects negatively the debt ratio and positively the probability of default. In addition, direct and indirect default costs have a negative impact on the debt ratio, confirming the idea that higher default costs lead the firm to behave more cautiously in the debt market. Similarly, the result of the default probability regressions suggest that in order to avoid the loss of reputation, the firm are less aggressive in the product market, reducing the probability of default. Finally, there is also some support to the hypotheses that the degree of market concentration influences debt and probability of default and that the later effect is non-linear.

Time constraints did not allow us to study some additional aspects. One of them would be to explore new ways of measuring the indirect default costs. The results obtained, namely regarding the impact on debt, suggest that the current measure may be capturing idiosyncratic negative shocks, which reflect uncertainty but not necessarily a reputation loss. It would be worthwhile to explore new ways of measuring the indirect default costs, so that the measurement is more aligned with the reputation loss. Another interesting extension would be to include advertising as a measure of the degree of product differentiation (the reverse of the degree of product substitutability in the theoretical framework of the second essay). Finally, it would be interesting to see the effect of the competitors' indebtedness on the firm default probability.

Appendix

Indirect Default Cost (IDC$_i$)

The estimation of indirect costs follows Altman's (1984) methodology and corresponds to the difference between profits and estimated profits (Altman 1984 and Kwansa and Cho 1995)

We first estimated the firms' sales as a function of the industry sales. To this end, firm and industry data were used 10 years before the date of the estimation. For the 2007 estimation, we used data from 1997 to 2006. To estimate the 2013 sales, data from 2003 to 2012 were used The parameters a and b were estimated for each firm.

$$S_{i,j,t} = a + bS_{j,t}$$

where $S_{i,j,t}$ denotes the sales of firm i, in industry j at time t and $S_{j,t}$ denotes the aggregate sales of industry j at time t. In other words, $S_{j,t} = \sum_{i=1}^{n_j} S_{i,j,t}$ where n_j is the number of firms in industry j. and $t = t-10, t-9, .., t-1$ (i.e., for each period we consider the 10 previous years).

The second step was to insert 2007 and 2013 aggregate industry sales so as to estimate the firm sales in ach of these years:

$$\widehat{S}_{i,j,t} = a + bS_{j,t}$$

with $t = 2007$ and 2013

The third step was to calculate the average of historical profit margin of each firm and multiply it by the estimated sales, so as to calculate the expected profit.

$$\widehat{P}_{i,j,t} = \overline{PM}_i \cdot \widehat{S}_{i,j,t}$$

where $\widehat{P}_{i,j,t}$is the estimated profit of firm i, in industry j at time t and $\overline{PM}_i$ is the average historical profit margin of firm i.

The last step allows us to calculate the difference between the actual profit and the estimated profit. When this difference is negative it is interpreted as the lost profit. Thus the lost profit is:

$$LO_{i,j,t} = P_{i,j,t} - \widehat{P}_{i,j,t} \quad \text{if} \;\; P_{i,j,t} < \widehat{P}_{i,j,t}$$

For consistency with the calculation of the direct default costs,we considere the profit loss in relative terms (as a percentage of the total assets). That the indirect default costs are given by:

$$IDC_{i,j,t} = \frac{LO_{i,j,t}}{\text{Total Assets}_{i,t}}$$

Table 4.13: Methods used in default risk studies.

Author(s)	Univariate analysis	Multiple linear discriminant analysis	Logit/ Probit analysis	Survival analysis
Beaver (1966)	X			
Altman (1968)		X		
Deakin (1972)	X	X		
Edmister (1972)		X		
Blum (1974)		X		
Altman, Haldeman & Narayanan(1977)		X		
Ohlson (1980)			X	
Zmijewski (1984)			X	
Zavgren (1985)			X	
Aziz, Emanuel & Lawson(1988)		X	X	
Kaya and Theodossi (1999)				X
Turetsky and Mceween (2001)				X
Pindado & Rodrigues (2004)			X	
Beaver (2005)				X
Gepp and Kumar (2008)		X	X	X
Chacharat et al (2010)				X
Pérez, Llopis & Llopis(2010)				X
Wu, Gaunt & Gray (2010)		X	X	X
Zhang, Altman, Yen(2010)		X		
Ng, Wong & Zhang(2011)		X		
Seneidere and Bruna (2011)		X	X	
Tinoco and Wilson (2013)			X	
Ho,McCarthy & Yang(2013)			X	
Huang and Lee (2013)				X
Singhal and Zhu (2013)			X	

Table 4.14: References of independent variables

Notation	References
UNC_i	Dechow and Dichev (2002); Kothari, Laguerre and Leone (2002); Stock and Watson(2002) Zhang (2006); Banker, Byzalov and Plehn-Dujowich(2014)
IDC_i	Jonh(1993); Dionne and Laajimi(2012)
DDC_i	Altman (1984); Kwansa and Cho(1995)
HHI_i	Spnaos, Zaralis and Lioukas(2004); Guney, Li and Fairchild(2011); Kayo and Kimura(2011); Valta(2012)
$HHI2_i$	Guney, Li and Fairchild(2011)
LEV_i	Beaver(1966); Altman(1968); Ohlson(1980); Zingales(1998); Kahya and Theodossiou(1999); Turetsky and McEwen(2001); Beaver et al (2005); Gepp and Kumar(2008); Chancharat et al(2010); Wu, Gaunt and Gray(2010); Dionne and Laajimi(2012); Ho, McCarthy and Yang(2013); Johnstone et al(2013); Tinoco and Wilson(2013)
$NTAXSHI_i$	Berger, Ofek and Yermack(1997); Lööf (2004); Pindado, Rodrigues and La Torre(2006); Ramalho and Silva (2009); Aggarwal and Kyaw(2010); Chakraborty(2010); Guney, Li and Fairchild (2011)
$PROF_i$	Rajan and Zingales(1995); Berger, Ofek and Yermack (1997); Lyandres (2006); Ramalho and Silva (2009); Aggarwal and Kyaw(2010); Chakraborty(2010); Kayo and Kimura(2011); Guney, Li and Fairchild(2011); Zhang(2012)
$GROWTH_i$	Rajan and Zingales(1995); Lööf(2004); Ramalho and Silva(2009); Chakraborty(2010); Guney, Li and Fairchild(2011)
$GROWTH2_i$	Bhaduri,(2002); Chen(2004); Serrasqueiro and Nunes(2010)
$COLL_i$	Rajan and Zingales(1995); Berger, Ofek and Yermack(1997); Lööf (2004); Pindado, Rodrigues and La Torre(2006); Kayo and Kimura,(2011);Lyandres(2006); Chakraborty(2010); Kayo and Kimura(2011); Margaritis and Psillaki(2007); Ramalho and Silva (2009, 2013); Aggarwal and Kyaw(2010); Guney, Li and Fairchild,(2011)
AGE_i	Berger, Ofek and Yermack (1997); Petersen and Rajan (1994); Ramalho and Silva (2009,2013); Chancharat et al(2010); Pérez, Llopis and Llopis (2010); Antunes, Mata and Portugal (2011)
$AGE2_i$	Geroski 1995 Stinchcombe(1965); Fichman and Levinthal(1991); Pérez, Llopis and Llopis(2010)
$MANEFFIC_i$	Beaver(1966); Turetsky and McEwen (2001); Gepp and Kumar(2008); Chancharat et al(2010); Ng, Wong, and Zhang,(2011); Sneideire & Bruna (2011)
$SIZE_i$	Rajan and Zingales(1995); Berger, Ofek and Yermack (1997); Lööf (2004); Lyandres (2006); Pindado, Rodrigues and La Torre (2006); Margaritis and Psillaki (2007); Ramalho and Silva (2009,2013); Aggarwal and Kyaw (2010); Chakraborty (2010); Kayo and Kimura (2011) Guney,Li and Fairchild (2011); Zhang (2012)

Table 4.15: Endogeneity test

	All Sample		2007		2013	
	Statistic	P-value	Statistic	P-value	Statistic	P-value
Durbin χ^2	3.360	0.0668	0.351	0.5534	12.651	0.0004
Wu- Hausman F	3.338	0.0678	0.343	0.5584	12.514	0.0004

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Chapter 5

Conclusion

This thesis presented three interconnected essays that study how several market structure parameters influence financing and output market decisions and the default risk.

The first essay further developed the ideas of Brander and Lewis (1986) by analyzing the implications of financial structure decisions and output market decisions on the default probability and also by studying the impact of changes in the parameters of the model on the equilibrium. This analysis is done both for the second stage Nash equilibrium (considering the financial structure as given but taking into account the impact on the output market decisions) as well as for the subgame perfect equilibrium (i.e., taking into account the impact on the financial structure decisions as well as on the product market decisions).

The second essay analytically and numerically examined how market structure parameter (such as the level of uncertainty and the degree of product substitutability) and the default costs influence financial and product market decisions and, consequently, how they affect the default risk. We considered a two stage duopoly model. In the first stage, firms simultaneously decide the level of debt that maximizes the firm value and, in the second stage of the game, firms simultaneously decide the quantity that maximizes the equity value. To find the subgame perfect Nash equilibrium, the model was solved backwards. We first determined the Nash equilibrium of the quantity competition game and then determined the equilibrium levels of debt. Due to the complexity of the problem, we had to solve the model analytically using GAUSS. We determined the equilibrium values of several variables: debt obligation, output, expected equity value, expected debt value, expected value of the firm, implicit interest rates, default probabilities and social welfare. The numerical model was run for many values of the parameter of the model in order to allow us to study the impact of changes on the level of demand uncertainty, on the degree of product substitutability, on the default costs and on the level of asymmetry in marginal production costs on the equilibrium values of the previous variables and, in particular, on the equilibrium default probability.

The last essay aimed to test empirically the relevance of uncertainty, market concentration and direct and indirect default costs on the financing decisions and on the default probability. Therefore, we tested whether debt (considering both the probability of a firm financing with debt and, when it does, the amount of the debt) vary with uncertainty, the direct and indirect default costs and the degree of market concentration. To do this, a sample of 11 OCDE countries was used and regressions were run for 2007 and 2013, allowing us to examine whether there is a significant difference before and after the peak of the financial crisis that occurred in 2008.

An important conclusion from the first essay, is that the impact of a change in parameter may be different depending on whether we assume that financing decisions are fixed or not (i.e., depending on whether we are looking at the second stage of the game or looking at the whole game). The results in this essay confirm the importance of considering both direct and indirect impacts (through the changes in the equilibrium financing and output decisions) on the default probability. For instance, an increase in a firm marginal cost, has a positive direct effect on the default probability (if the marginal cost increases, the default probability increases, for fixed debt and quantity levels), however the increase in marginal costs also leads the firm to behave more cautiously in the debt and product markets, which decreases the default probability.

The results in the second essay confirm the importance of considering both direct and indirect impacts (through the changes in the equilibrium financing and output decisions) on the default probability. For example, we confirm numerically that an increase in the marginal costs of a firm, reduce its default probability whereas the reverse happens for the rival firm. This counter-intuitive result can be explained by the fact that as the firm becomes more inefficient, it tends to become more cautious in the debt and in the product market. As consequence, the probability of default decreases because the previous indirect impact through the changes in the behavior of the firm outweigh the direct effect of increasing marginal cost on the default probability. Moreover, the second essay also allowed us to conclude that direct and indirect default costs are extremely relevant in the firm financing decision, as firms decrease their debt level when the default costs increase. In addition the impact of changes in other parameter may be different when default costs are considered. For instance, without considering default cost, an increase in the uncertainty level leads firms to behave more cautiously in the debt market and this implies a decreased probability of default (the indirect effect through debt dominates). However, when the default costs are considered and taking into account the values considered for the parameters, an increase in the uncertainty level leads to an increase in the default probability, i.e. the direct impact of the variable prevails. The empirical analysis is in line with these findings since the impact of uncertainty on debt is negative and on the default probability is positive.

With regard to the changes in degree of product substitutability (analyzed in the second essay), it was concluded that it increases the default probability but that its impact on the capital structure is not monotonic. On the other hand, the third essay shows that the degree of market concentration influences the default probability. Moreover, the default costs are an important determinant of debt, as it leads firms to take more cautious positions and such behavior leads to a decreased probability of default.

Like any other study, this thesis has some limitations, which open opportunities for further extensions of this study. At a theoretical level an obvious extension would be to consider taxes in our model, thus allowing us to incorporate the tax benefits of debt, which would to some extent balance the default costs. In principle, this modification would lead to higher equilibrium debt levels than the ones obtained in our second essay. Other obvious extensions of the theoretical model would be to consider n firms instead of two firms and to study a model where firms compete in prices in the second stage of the game. However, one can also think of less obvious extensions. An important one would be to abandon the assumption that the financing decisions are taken after the investments decisions were already taken. If the financing decisions were taken before the investment decisions, we would need to consider other strategic effects, which would increase the complexity of the model but would increase its realism. We believe this is an important extension that should be explored in the future.

At the empirical level, the estimated model would also benefit with some additional refinements. One of them would be to explore new ways of measuring the indirect default costs of default. In addition, it would be interesting to see the effect of the competitors' indebtedness on the firm default probability. The last extension is particularly interesting, because it is suggested by the theoretical framework proposed in this thesis.

Overall, our results suggest that the default probability is greatly influenced by the firms' financial and product market decisions, which optimally adjust their behavior to structural changes in the industry. Therefore, a less favorable environment does not necessarily imply higher default probability, as the firm may respond by financing less with debt.