CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Repositório Científico da Universidade de Évora

Computers and Mathematics with Applications 59 (2010) 115–125

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Random differential operational calculus: Theory and applications☆

L. Villafuerte [a], C.A. Braumann [b], J.-C. Cortés [c,*], L. Jódar [c]

[a] Centro de Estudios en Física y Matemáticas Básicas y Aplicadas, Universidad Autónoma de Chiapas, Calle 4ª Ote. Nte. 1428, Tuxtla Gutiérrez, Chiapas, Mexico
[b] Centro de Investigação em Matemática e Aplicações, Universidade de Évora, Rua Romão Ramalho 59, 7000-671, Évora, Portugal
[c] Instituto de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Edificio 8G, 46022, Valencia, Spain

ARTICLE INFO

Article history:
Received 5 July 2009
Accepted 26 August 2009

Keywords:
Random differential equation
Mean square calculus
Mean fourth calculus
Random chain rule

ABSTRACT

In this article, we obtain a product rule and a chain rule for mean square derivatives. An application of the chain rule to the mean square solution of random differential equations is shown. However, to achieve such mean square differentiation rules, fourth order properties were needed and, therefore, we first studied a mean fourth order differential and integral calculus. Results are applied to solve random linear variable coefficient differential problems.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

The combination of complexity, uncertainty and ignorance that are present in real problems, not only due to natural phenomena, but also due to human behaviour, requires the consideration of randomness in the mathematical models. Individual behaviour may be erratic, but aggregate behaviour is often quite predictable. Differential equations are powerful tools for representing reality up to a certain point. Statistics is only second to differential equations in the power to model the world around us [1]. The quantification of uncertainty requires a model specifying the mechanism by which randomness is generated.

Random differential equations have been used in the last few decades to deal with errors and uncertainty. For example, see [2] for the case of general randomness and [3,4] for white noise uncertainty. Theoretical approaches of random differential equations probably started with Strand in [5,2]. As regards applications using explicit analytical solutions or numerical methods, a few results may be found in [6–9].

General treatment of the mean square solutions of random differential equations requires some basic operational tools such as, the algebra of the mean square limits, the derivative of the product of two stochastic processes or the chain rule for the composition of stochastic process. Apart from special cases where the independence of the stochastic processes factors is assumed ([7, p. 96], [8]), to our knowledge, general results for the derivative of a product or the chain rule are not available.

Difficulties with the mean square operational calculus of the product result from the fact that the mean square norm is not submultiplicative. This fact together with the Schwarz inequality, links the mean square calculus with the mean fourth order calculus. We, therefore, recognize the need of a mean fourth calculus to get the mean square results of interest for important applications like solving linear random differential equations in the mean square sense.

☆ This work has been partially supported by Mexican Promep, the Centro de Investigação em Matemática e Aplicações da Universidade de Évora (research center financed by the Fundação para a Ciência e a Tecnologia, Portugal), the Generalitat Valenciana and the Spanish M.E.C. Grant TRA2007-68006-C02-02 and MTM2009-08587.

* Corresponding author. Tel.: +34 963879144.
E-mail addresses: laura.villafuerte@unach.mx (L. Villafuerte), braumann@uevora.pt (C.A. Braumann), jccortes@imm.upv.es, jccortes@mat.upv.es (J.-C. Cortés), ljodar@imm.upv.es (L. Jódar).

0898-1221/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2009.08.061