Longitudinal Analysis for Matched Series of Studies

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Abstract. In the first phase of the STATIS methodology the information is contained in a series of studies is first condensed into a symmetric matrix. Models based in the spectral analysis of their mean matrices were developped, see Areia et al.(2008). When their first engenvalue θ is dominant a further condensation of the information into a structure vector $\theta\gamma$, with γ the eigenvector associated to θ, and a sum of square of residues is carried out.

This enables the joint analysis of matched series of studies associated to the treatments of a base design. Then in the longitudinal analysis of such families of series we study the action of the factors in the base design (contrasts) taken for the components of the structure vectors. An application to elections in Portugal is presented.

Keywords: ANOVA, Inference, Matched Series of Studies, STATIS.
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INTRODUTION

Escoufier (1973) developed a geometric representation for the series of studies. For the series of studies of the first type he obtained operators $\mathbf{A}_i = \mathbf{X}_i\mathbf{D}_{pi}\mathbf{X}_i^t\mathbf{D}_n$, and the symmetric matrix $\mathbf{S} = [s_{ij}], i,j = 1,\ldots,k$, with $s_{ij} = tr\left(\mathbf{A}_i\mathbf{A}_j^t\right)$.

A model was developed for the $\mathbf{S}$ matrix, see Areia et al. (2008), with a common structure of degree h, given by

$$\mathbf{S} = \sum_{i=1}^{h} \lambda_i \alpha_i \alpha_i^t + \mathbf{E} = \sum_{i=1}^{h} \beta_i \alpha_i^t + \mathbf{E} \tag{1}$$

with $\beta_i = \lambda_i\alpha_i$, $i = 1,\ldots,k$, and $\mathbf{E}$ an errors matrix and presented the tests of validation of this model. In this paper, based on this model, we apply a longitudinal analysis that allows to study the action of factors on the evolution of the series of studies. This type of analysis is relevant when studies are carried out in series in each successive times, with simultaneous studies that correspond to the different series. We present an application of the results of local elections in mainland Portugal.

LONGITUDINAL ANALYSIS

The longitudinal analysis allows the study of matched series of studies. We admit structure common of degree one $(h = 1)$, that is, a single structure vector represented by $\widetilde{\beta}(l)$ for the l serie, with $l = 1,\ldots,m$ and $V(l)$ the corresponding sum of the squares residuals. This analysis is made by using the contrasts. A contrast between the components of the vector $\mathbf{v} = (v_1, v_2, ..., v_k)^t$ can be defined as a linear combination, $\sum_{i=1}^{k}(c_i v_i)$, where the components have null sum, $\sum_{i=1}^{k} c_i = 0$. We admit

$$\widetilde{\beta}(l) \sim N\left(\beta(l), \sigma^2\mathbf{W}(l)\right), l = 1,\ldots,m, \tag{2}$$

independent of $V(l) \sim \sigma^2\chi^2_{g(l)}, l = 1,\ldots,m$. As we have m series of studies, $V = \sum_{l=1}^{m} V(l) \sim \sigma^2\chi^2_g$ is independent of $\widetilde{\beta}(l), l = 1,\ldots,m$.

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