On the determination of the earthquake slip distribution via linear programming techniques

Vladimir Bushenkov[1], **Bento Caldeira**[2], **Georgi Smirnov**[3]

[1] *University of Évora, Évora, Portugal;* bushen@uevora.pt
[2] *University of Évora, Évora, Portugal;* bafcc@uevora.pt
[3] *University of Minho, Braga, Portugal;* smirnov@math.uminho.pt

The description that one can have of the seismic source is the manifestation of an imagined model, obviously outlined from Physic Theories and supported by mathematical methods. In that context, the modelling of earthquake rupture consists in finding values of the parameters of the selected physics-mathematical model, through which it becomes possible to reproduce numerically the records of earthquake effects on the Earths surface. Actually, these effects are the elastic records at near field source and at far field source, and inelastic deformations recorded by geodetic techniques. The detail and accuracy level, with which the characteristic parameters for large earthquakes are computed, depends on the combination of two factors - the applied methods and the used data.

Under the hypothesis of constant slip direction and constant rise time of individual source time function, the problem of complete seismic slip time history and distribution reconstruction reduces to the solution of a system of linear equations. It is well-known that this inverse problem is ill-posed [6]. The usual regularization techniques [8] can hardly be applied in this case because of a very high dimension of this problem (see, e.g., [3]). The problem can be overcome by introducing some additional regularizing constraints. Some additional physical hypotheses, like no-backslip constraint, result in condition of non-negativeness of solutions to the system of linear equations.

The positivity that prohibits negative seismic moment values, is a constraint naturally assumed when used the Non Negative Least Squares algorithm (NNLS) [5] to inverts seismic waveforms to slip distribution (e.g., [7]).

We present and test a Linear Programming (LP) inversion in dual form, for reconstructing the kinematics of the rupture of large earthquakes through space-time seismic slip distribution on finite faults planes. The proposed method can be considered as a continuation of the work started in [2]. The proposed algorithm uses strong ground motion waveforms, but it can also used with other types of data as teleseismic waveforms as well as with geodesic data (static deformation). We test the method with data obtained by application to a synthetic model of rupture. To compare it, we rehearsed reconstructions with same data, but made by other strongly used algorithms. Green functions (see, e.g., [1]) were calculated by a finite differences method applied to a 3D structure model [4].

The hypothesis of constant slip direction in general is not verified and the "real" seismic slip time history and distribution reconstruction becomes an hard nonlinear problem. In this work we suggest an algorithm for seismic slip time history and distribution reconstruction allowing to solve the problem in its general setting. The solution of an auxiliary linear programming problem is an essential part of the developed method. To test the algorithm we use a synthetic displacement function for the fault model and perform the inversion.

The slip determination problem can be formalized in the frame of mathematical programming in the following way

$$\begin{aligned} &\langle c, x\rangle \to \min, \\ &A(\lambda)x = b, \\ &x \geq 0. \end{aligned} \tag{1}$$

Here x is the unknown vector of amplitudes and residuals (see [2]) and the vector λ represents the unknown rakes. Note that the displacement field models can be different but the mathematical formalization is always the same. If we fix the rake vector λ, problem (1) becomes a linear programming problem. This observation is the key to an effective solution of problem (1). It turns out that the gradient of the minimized functional $\langle c, x\rangle$ with respect to λ can be calculated in terms of the solution to the linear programming problem dual to (1).

The following algorithm describes the process.

Algorithm:

Given λ_0, $\Delta > 0$, and $\epsilon > 0$.

for $k = 0, 1, 2, \ldots$

Step 1. Solve linear programming problem (1) with $\lambda = \lambda_k$ and obtain x_k.

Step 2. Obtain search direction $\bar{\lambda}_k$ and a step $\delta_k > 0$.
if $\delta_k \|\lambda_k\| < \epsilon$ **break**
else
Step 3. Set $\lambda_{k+1} = \lambda_k + \delta_k \bar{\lambda}_k$.
end (for)

The second step of the algorithm is not trivial. The derivative is calculated using the dual linear programming problem. The latter has a very specific form:

$$\begin{array}{l} \langle c, x \rangle \to \min, \\ Bx \leq 0, \\ -1 \leq (x)_i \leq 1, \quad i = \overline{1, n}, \end{array} \tag{2}$$

where B is an $(m \times n)$-matrix with $m < n$. This special structure of the dual problem allows one to effectively find an admissible vertex.

This research is supported by the Portuguese Foundation for Science and Technologies (FCT), through the project PTDC/CTE-GIN/82704/2006.

REFERENCES

1. *K. Aki, P. Richards.* Quantitative Seismology, 2nd edition, University Science Books, Sausalito, (2002).
2. *S. Das, B.V. Kostrov.* "Inversion for slip rate history and distribution on fault with stabilizing constraints: Application to the 1986 Andreanof Islands earthquake," J. Geophys. Res., **95**, 6899-6913, (1990).
3. *S. Das, P. Suhadolc, B. V. Kostrov.* "Realistic inversions to obtain gross properties of the earthquake faulting process," Tectonophysics, **261**, 165-177, (1996).
4. *S.C. Larsen, C.A. Schultz.* "ELAS3D, 2D/3D Elastic Finite-Difference Wave Propagation Code," Lawrence Livermore National Laboratory, UCRLMA-121792, (1995).
5. *C. Lawson, R. Hanson.* Solving Least Squares Problems, Prentice-Hall (series in automatic computation), New Jersey, (1974).
6. *B.V. Kostrov, S. Das.* Principles of Earthquake Source Mechanics, Cambridge University Press, Cambridge, (1988).
7. *W. Suzuki, S. Aoi, H. Sekiguchi.* "Rupture process of the 2008 Northern Iwate Intraslab earthquake derived from strong-motion records," Bull. Seism. Soc. of America, **99**, 2825-2835, (2009).
8. *A.N. Tikhonov, V. Ya. Arsenin.* Solutions of Ill-Posed Problems, Winston & Sons, New York, (1977).