On the distribution of proportions and ratios as indicators of ungulate body condition

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In the wildlife literature various studies have shown that the amount of fat around the kidneys is often a good indicator of body condition and health of ungulate species ([1],[2]). Wildlife biologies often measure the weights of fat around the kidneys and the kidneys themselves (without fat), comparing these two quantities and transforming them into a ratio or a proportion as surrogates of body condition of dead animals. Later, they use these indices as response variables to model the effect of covariates or treatments, such as age group and season of the year, on body condition. The more commonly used models are ANOVA-type. Most debate has been concentrated on whether or not to use a simple ANOVA model of ratios or proportions with fixed effects, or an ANCOVA model using fat weight as response variable, and the kidneys weight as a covariate ([3]). Results have taken by surprise some biologists because using one or another model with both response variables could lead to very different results for what it considered the most important effects. In this manuscript we attempt to highlight the main differences and the distributional properties of these response variables, ratio and proportion.

The relationship between ratio and proportion

Assume that we have a continuous, positive random vector $(X, Y) \in \Re^2$, and let X be the weight of kidney fat, and Y the weight of the kidneys without fat, so that $Z = X + Y$ is the total weight of the kidneys including fat. Define $R = X/Y$ and $P = X/(X + Y)$. We are interested in the distributional properties of R and P as surrogate variables to assess ungulates body condition. Note that R and P have support on $(0, \infty)$ and $(0, 1)$ respectively.

Note also that R and P are related, and each of them can be expressed as a monotone function of the other, $R = P/(1-P)$ and $P = R/(1+R)$. Because these are monotonic one-to-one transformations, we could find the distribution of one of the variables given that the distribution of other variable is known ([4]). Empirical evidence has shown that it might be reasonable to assume that P could have a symmetric distribution, whereas, R has a more likely skewed distribution. We are interested in identifying in which sampling and parametric spaces one could expect the distributions of R and P would have similar behavior, and thus, they could generate similar results when performing data analyzes.