EXPERIMENTAL APPLICATION TO A WATER DELIVERY CANAL OF A DISTRIBUTED MPC WITH STABILITY CONSTRAINTS

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In this work, a novel distributed MPC algorithm, denoted D-SIORHC, is applied to upstream local control of a pilot water delivery canal. The D-SIORHC algorithm is based on MPC control agents that incorporate stability constraints and communicate only with their adjacent neighbors in order to achieve a coordinated action. Experimental results that show the effect of the parameters configuring the local controllers are presented.

INTRODUCTION

Model predictive control (MPC) is a tool for controller design that is generally recognized as a powerful mean to control various types of processes [1] and, in particular, water delivery canals. Indeed, since MPC is an optimization based design technique, it has features that are suitable for this type of application. In the first place, through an appropriate choice of the functional to optimize, MPC directly links the controller design with the objective of saving water. Furthermore, there are several ways to obtain decentralized versions of MPC, an important issue since water delivery canals are large, space distributed, systems. Some examples are provided in [2, 3, 4, 5].

In this article, a novel distributed MPC algorithm, denoted hereafter as D-SIORHC, is applied to local upstream water level control of a pilot water delivery canal. The D-SIORHC algorithm is based on local MPC control agents that communicate only with their neighbors in order to achieve a coordinated action. Each local control agent manipulates the canal gate to which it is associated such as to drive the corresponding local upstream level to the desired target value, but taking into account the control moves of their neighbors.

Local control agent MPC algorithms minimize a quadratic cost function using linear models, under the constraint that the plant state vanishes at a number of coincidence points locate in time close to the end of the prediction horizon [6]. Under appropriate conditions on the number of coincidence points, these constraints ensure stability of the local control loops, in case they are isolated [7].

When several local control agents are simultaneously connected to different plant inputs, they may yield competitive actions that destabilize the overall system. In order to prevent this undesirable effect, the actions of the different local control agents must be

coordinated. For this sake, at the beginning of each sampling interval, a coordination procedure is run during which all control agents compute the value of the respective manipulated variables, using as accessible disturbances the previously computed value of the manipulated variable of their neighbors.

The contribution of this paper consists in the experimental illustration of the effect of selecting different values for the parameters that configure the controller.

The paper is organized as follows. After this initial section that motivates the problem to consider and explains the approach followed in general terms, the water delivery canal used for experiments is described in section 2. The distributed MPC algorithm is described in section 3. The main results are presented in section 4, where the effect of different choices for the controller parameters is shown experimentally. Finally, section 5 draws conclusions.

WATER DELIVERY CANAL DESCRIPTION

The water delivery canal used in the experiments is located in Évora, Portugal, and belongs to *Núcleo de hidráulica e Controlo de Canais*, a research unit of the University of Évora, Portugal [8]. Figure 1 shows some views of it.

Figure 1. Two of the gates of the pilot canal used in the experiments.

The structure of this experimental canal is shown schematically in Figure 2. The canal has four pools, each with a length of 35 m, separated by three undershoot gates (Figure 1 left), with the last pool ended by an overshot gate (Figure 1, right). The water is fed by gravity to the canal from a reservoir, with the inflow Q_0 imposed by a valve equipped with a flow controller of proportional type (denoted MONOVAR in Figure 2). The maximum value for Q_0 is 0.09 m^3s^{-1}. At the downstream section of each canal pool there is an offtake of the type orifice, used to simulate water consumption by farmers.
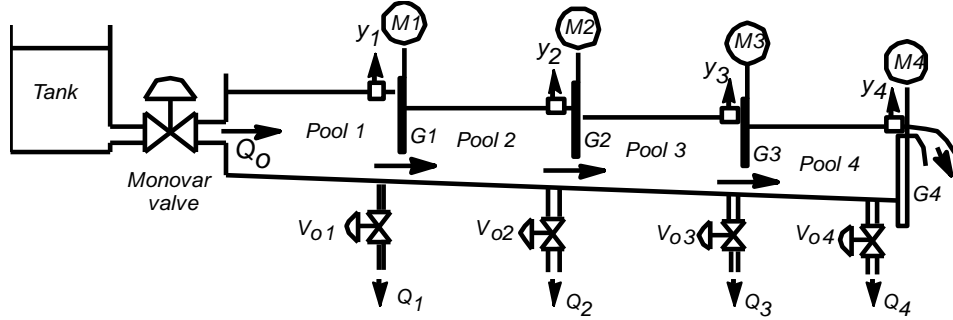
Figure 2. A schematic view of the pilot canal used in the experiments.

Downstream of each pool there are water level sensors that provide measures y_i, $i = 1, \ldots, 4$, ranging from 0 mm to 900 mm (canal bank). The opening of gate i is denoted u_i and varies between 0 mm (gate closed) and 800 mm (gate fully open). In this work only the first three pools are used, and gate 4 is kept at a fixed position.

THE D-SIORHC ALGORITHM

The first step in designing a distributed controller consists in decomposing the plant in a number of subsystems and defining its interactions. The canal is decomposed in three subsystems, each one associated to a pool and a gate. To each subsystem, a local control agent (denoted controller 1, 2 and 3) computes the gate position using local information (pool water level measurement y_i for controller i) and information from their neighbor agents. So, the controller of pool 1 exchanges information about pool water level and gate position with pool 2, pool 2 does so with polls 1 and 3 and pool 3 with pool 2. It is remarked that the decomposition of the canal is purely conceptual and involves an approximation, while the decomposition of the control agents reflects the software structure that actually implements the controller.

The plant decomposition is implemented in practice by describing the overall system, when working around a fixed equilibrium point, by a state-space model written as

$$x(k+1) = ax(k) + Bu(k) \tag{1}$$

$$y(k) = Cx(k) \tag{2}$$

In (1, 2), k is an integer number that denotes the discrete time, x is the state of the overall plant, $u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$ is the vector of manipulated variables, $y = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$ is the vector of observations and A, B and C are matrices of parameters. To reflect the partitioning, matrices A and B are estimated imposing *a priori* the block structure

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \tag{3}$$

where A_{ij} is a matrix that reflects the influence of the state x_j on x_i.

Each of the local controller agents computes its manipulated variable by minimizing a quadratic cost that takes into consideration not only the tracking error of the water level in the pool that it is controlling, but also the tracking error in adjacent pools. Hence, for pools 1, 2 and 3 the cost to be minimized are

$$J_1 = \sum_{i=1}^{N} \left[e_1^2(k+i) + e_2^2(k+i) + \rho_1 \left(\Delta u_1(k+i-1) \right)^2 \right] \tag{4}$$

$$J_2 = \sum_{i=1}^{N} \left[e_1^2(k+i) + e_2^2(k+i) + e_3^2(k+i) + \rho_2 \left(\Delta u_2(k+i-1) \right)^2 \right] \tag{5}$$

$$J_2 = \sum_{i=1}^{N} \left[e_2^2(k+i) + e_3^2(k+i) + \rho_1 \left(\Delta u_3(k+i-1) \right)^2 \right] \tag{6}$$

where $e_i(j) = r_i(j) - y_i(j)$ is the tracking error for the level in pool i at time j, r_i being the reference to be tracked by pool level 1, the ρ_i are positive parameters that weight the penalty on the manipulated variable increments Δu_i of u_i and N is the prediction horizon. This minimization is done in a receding horizon sense under the stability constraint [6]

$$y_i(k+N+j) = r_i(k+N+j) \quad \text{for} \quad j = 1, \ldots, P \tag{7}$$

In a distributed framework, at the beginning of each sampling interval, a coordination recursive procedure is run a number of times that ensures convergence. In each cycle of the recursion, each J_i is optimized with respect to u_i, assuming knowledge of u_{i-1} and u_{i+1} (whenever defined) as computed in the previous iteration.

EXPERIMENTAL RESULTS

Several tests have been conducted in the pilot canal in order to study the effect of the parameters that configure the D-SIORHC controller. The results are reported in Figures 4 to 6. In all these experiments, after an initial period in which the setpoint of the level for reference 1 is made to vary, this setpoint is made constant and the valve off-take of pool 1 is open (at the instant corresponding to the vertical linea marked A) and the closed (at the
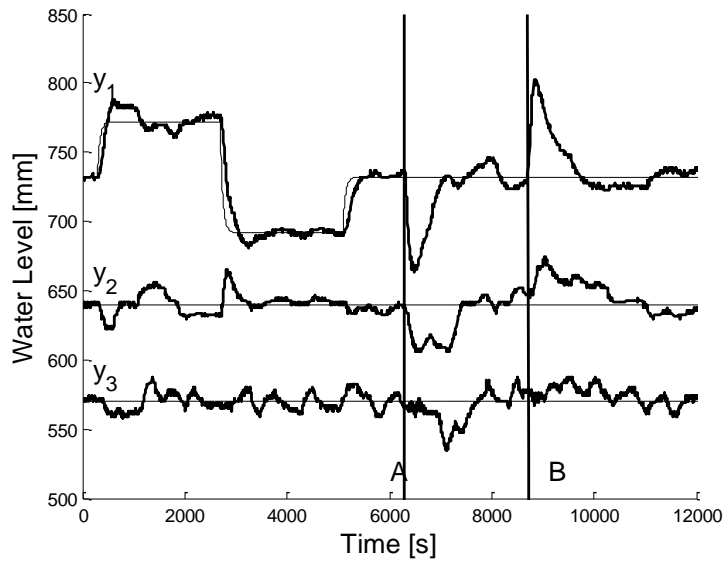
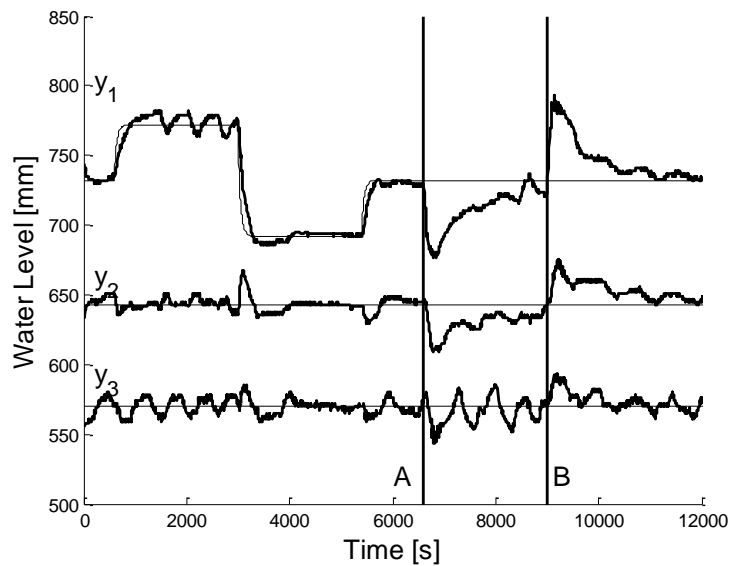Figure 3. Experimental results with D-SIORHC. N=20, P=1, $\rho_1 = \rho_2 = 100$ and $\rho_3 = 1000$.

Figure 4. Experimental results with D-SIORHC. N=25, P=1, $\rho_1 = \rho_2 = 10$ and $\rho_3 = 1000$.
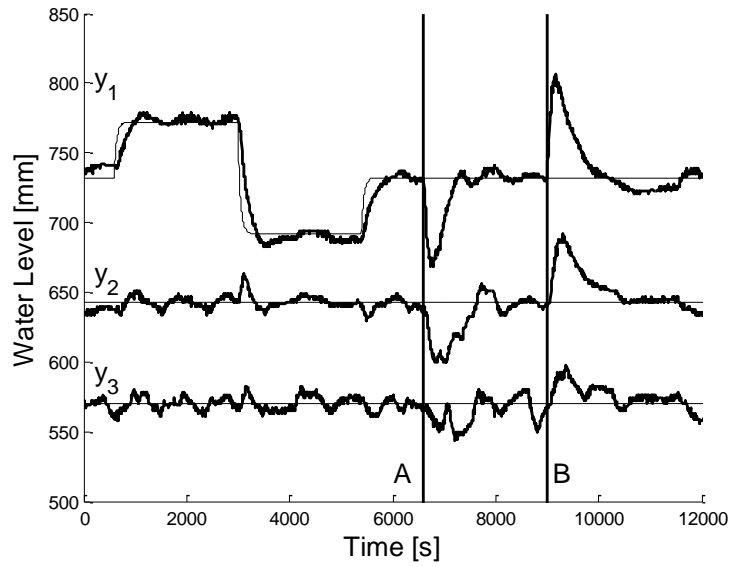
Figure 5. Experimental results with D-SIORHC. N=25, P=1, $\rho_1 = \rho_2 = 100$ and $\rho_3 = 1000$.
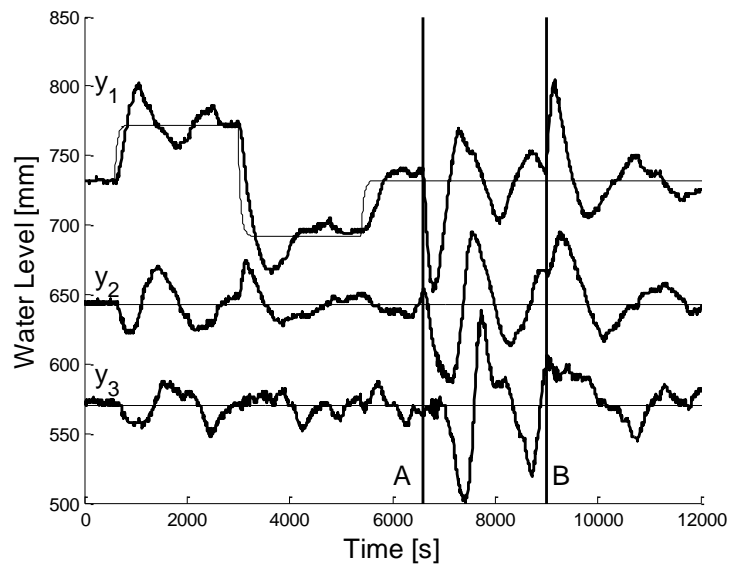

Figure 6. Experimental results with D-SIORHC. N=25, P=1, $\rho_1 = \rho_2 = 1000$ and $\rho_3 = 1000$.

ρ_1	ρ_2	ρ_3	N	D_1 [mm]	D_2 [mm]	D_3 [mm]
100	100	1000	20	10	8	6
10	10	1000	25	10	7	5
100	100	1000	25	9	7	6
1000	1000	1000	25	13	14	11

Table 1. Comparison of controller performances for different configurations.

instant corresponding to the vertical line marked B). The setpoints for the water levels of pools 2 and 3 are kept constant during the whose experiment. During all the experiments, gate 4 was kept at a constant position. Table 1 shows the mean deviation of the level with respect to the setpoint in pool i, denoted D_i. This figure of merit is expressed in [mm] and is computed using all the data points for each experiment and hence it depends not only on the initial phase, in which the setpoint is varied, but also on the second phase, in which a disturbance is induced by opening the off-take of pool 1. The smaller D_i, the better is the controller performance. Of-course the values of D_i would be different for other sequences of manoeuvers.

In all the cases reported in this paper, the number of coincidence points was made P=1 and $\rho_3 = 1000$. In Figure 1, the prediction horizon is N=20. In the experiments reported in Figures 2, 3 and 4, the prediction horizon was N=25, greater than in Figure 1 and this led to an improvement in performance. From Figure 2 to Figure 4, the values of ρ_1 and ρ_2 are selected with the increasing values of 10, 100 and 1000. As concluded from Table 1 and confirmed by the visual inspection of the figures, the better performance is obtained in this set of experiments for $\rho_1 = \rho_2 = 100$.

The SCADA system that interfaces the D-SIORHC controller to the canal imposes that a gate is moved with a dead band of 5 mm. This actuator nonlinearity causes a limitation in performance and, in particular, induces the oscillations observed in Figure 2, for $\rho_1 = \rho_2 = 10$. By observing Figure 3, it is concluded that the choice $\rho_1 = \rho_2 = 100$ is less sensitive to these undesirable effects.

CONCLUSIONS

The effect of different values of the parameters that configure the distributed MPC D-SIORHC controller has been studied experimentally in a pilot water delivery canal.

With the two values of the prediction horizon considered, the largest yields the best result. This illustrates the advantage of using predictive control. It is remarked that, in other tests not reported in this paper, the performance degrades for N=30. A factor that contributes to this degradation is the loss of precision of predictive models when the predictive horizon grows, a fact observed in general in MPC algorithms. The number of coincidence points P is also an important variable. Increasing P with N=25 does not lead to better results, but other experiments (unreported here) show that the combination P=5 and N=25 leads to a better performance.

When selecting the weight in the penalty of the manipulated variables the best results are obtained with an intermediate value.

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