



# A note on a class of problems for a higher-order fully nonlinear equation under one-sided Nagumo-type condition

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## ABSTRACT

The purpose of this work is to establish existence and location results for the higher-order fully nonlinear differential equation

$$u^{(n)}(t) = f(t, u(t), u'(t), \dots, u^{(n-1)}(t)), \quad n \geq 2,$$

with the boundary conditions

$$u^{(i)}(a) = A_i, \quad \text{for } i = 0, \dots, n-3,$$

$$u^{(n-1)}(a) = B, \quad u^{(n-1)}(b) = C$$

or

$$u^{(i)}(a) = A_i, \quad \text{for } i = 0, \dots, n-3,$$

$$c_1 u^{(n-2)}(a) - c_2 u^{(n-1)}(a) = B, \quad c_3 u^{(n-2)}(b) + c_4 u^{(n-1)}(b) = C,$$

with  $A_i, B, C \in \mathbb{R}$ , for  $i = 0, \dots, n-3$ , and  $c_1, c_2, c_3, c_4$  real positive constants.

It is assumed that  $f : [a, b] \times \mathbb{R}^{n-1} \rightarrow \mathbb{R}$  is a continuous function satisfying one-sided Nagumo-type conditions which allows an asymmetric unbounded behaviour on the nonlinearity. The arguments are based on the Leray–Schauder topological degree and lower and upper solutions method.

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## 1. Introduction

Let us consider the  $n$ th-order differential equation

$$u^{(n)}(t) = f(t, u(t), \dots, u^{(n-1)}(t)), \tag{1}$$

for  $n \geq 2$ , with the following boundary conditions

$$u^{(i)}(a) = A_i, \tag{2}$$

$$u^{(n-1)}(a) = B, \quad u^{(n-1)}(b) = C,$$

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