# SOLVABILITY FOR A THIRD ORDER DISCONTINUOUS FULLY EQUATION WITH NONLINEAR FUNCTIONAL BOUNDARY CONDITIONS

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Abstract. We prove an existence and location result for the third order functional nonlinear boundary value problem

$$\begin{aligned} u'''(t) &= f(t,u,u'(t),u''(t)), \text{ for } t\in[a,b],\\ 0 &= L_0(u,u',u(t_0)),\\ 0 &= L_1(u,u',u'(a),u''(a)),\\ 0 &= L_2(u,u',u'(b),u''(b)), \end{aligned}$$

with $t_0\in[a,b]$ given, $f: I\times C(I)\times\mathbb{R}^2\to\mathbb{R}$ is a $L^1$ – Carathéodory function allowing some discontinuities on $t$ and $L_0, L_1$, $L_2$ are continuous functions depending functionally on $u$ and $u'$.

The arguments make use of an *a priori* estimate on $u''$, lower and upper solutions method and degree theory.

Applications to a multipoint problem and to a beam equation will be presented.

## 1. Introduction

In this paper it is studied the third order nonlinear functional equation

$$u'''(t)=f(t,u,u'(t),u''(t)), \text{ for } t\in I, \tag{1.1}$$

where $I=[a,b]$, $f: I\times C(I)\times\mathbb{R}^2\to\mathbb{R}$ is a $L^1$ – Carathéodory function together with the nonlinear functional boundary conditions

$$\begin{aligned} L_0(u,u',u(t_0)) &= 0,\\ L_1(u,u',u'(a),u''(a)) &= 0,\\ L_2(u,u',u'(b),u''(b)) &= 0, \end{aligned} \tag{1.2}$$

where $t_0\in I$ is given and $L_0$, $L_1$, $L_2$ are continuous functions satisfying some monotonicity assumptions to be defined in Section 2.

We remark that functional dependence on the solution is allowed in $f$, moreover functions $L_0$, $L_1$ and $L_2$ depend functionally on the solution of the equation and on the first derivative. Such dependence allows us to consider, amongst others, integro - differential equations, delay equations or equations with maxima coupled with Sturm - Liouville or multipoint boundary value conditions under the same formulation.

This type of fully third order differential equation has been studied by several authors, considering nonlinear boundary conditions (see [5, 7, 14]) or two functional

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