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Abstract

We start by reviewing several results that have been obtained by the author concerning a general model of population growth in a randomly varying environment of the form $dN/dt = (g(N)+\sigma(N)\varepsilon(t))N$, where $N=N(t)$ is the population size at time t. In this general model we have assumed that the per capita growth rate $(1/N)dN/dt$ has an "average" value $g(N)$ and is perturbed by a white noise $\sigma(N)\varepsilon(t)$ describing the effect of environmental fluctuations, where $\sigma(N)$ is the noise intensity and $\varepsilon(t)$ is a standard white noise. This model generalizes the specific models that have been proposed in the literature by several authors, which consider specific functional forms for the "average" growth rate $g(N)$ and for the noise intensity $\sigma(N)$. Since little is known about the specific functional forms, by considering general functions $g(N)$ and $\sigma(N)$, subjected only to a few mild assumptions mostly dictated by biological considerations, we aim at obtaining results that are model robust. We review results on "mathematical" extinction and existence of stationary densities. By "mathematical" extinction we mean population size reaching zero or converging to zero as time goes to infinity. Since these models are approximations, it is preferable to talk about "realistic" extinction, by which we mean population size dropping to some low threshold size. We will review previous results by the author and a co-author on the probability of that happening and on the first passage time by the threshold. We also study the time to reach a high threshold size, which may be of importance for the study of pest outbreaks. We give an example of application, including the issues of parameter estimation and prediction. Finally, we briefly consider the extension of these results to harvesting models.

Keywords: Stochastic differential equations; Population growth; Random environments; Extinction; Itô and Stratonovich calculi