MARTIN GARDNER and MARILYN vos SAVANT – a Not Always Easy Collaboration

By Jeremiah Farrell, Karen Farrell and Thomas Rodgers

In his book *Gardner's Workout* (2001, AK Peters) Martin Gardner gives a brief history of a now very famous probability problem.

> Jones puts three cards face down in a row. Only one is an ace, but you don't know which. You place a finger on a card. The probability it is on the ace clearly is 1/3. Jones now secretly peeks at each card, then turns face up a card that is *not* the ace. What is the probability your finger is on the ace? Most everyone thinks it has risen from 1/3 to 1/2. After all, only two cards are face down, and one must be the ace. Actually, the probability remains 1/3. If you now switch your finger to the other face-down card, the probability it is on the ace jumps to 2/3.
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> I introduced this problem in my *Scientific American* column (October 1959) in the form of a warden and three prisoners. In 1990 Marilyn vos Savant, in her popular *Parade* column, gave it with three doors and a car behind one of them. Her answer was correct, but she received thousands of angry letters, many from Ph.D. mathematicians, berating her for her ignorance of elementary probability! The fracas generated a front-page story in the *New York Times*.

Now called the Monty Hall problem after the popular "Let's Make a Deal" program seen on TV from 1963-1978 her problem was superimposed on that show's final segment of three doors. In fact every episode we had seen used instead five cards on hooks behind one of which the keys to a new car were hidden. Hall, the host, asked the contestant to choose a card but before revealing what was behind it, exposed a card that did not hide the keys. He offered the contestant a sum of money to give up his choice but the sum was always too low to be tempting. This scenario was repeated until the board was down to exactly two cards; the contestant's and the host's. It was clear to us that Hall knew where the keys were and was therefore always able to show three cards that concealed no keys. Therefore the contestant's card hid the keys with a probability that remained 1/5 and the host's card had probability 4/5 of hiding the keys. Presumably Ms. vos Savant changed from five objects to just three for space reasons in her column.

On November 17, 1996 the three of us visited the Gardner's in Hendersonville, N.C. and were told that Martin had given vos Savant permission to use any of his material in her column (ten years later, the generous Gardner gave our journal "Word Ways" a similar grant,) We recall that Charlotte Gardner expressed some regret that vos Savant didn't always give Martin enough credit in her columns.

Vos Savant's description of the Monty Hall problem was absolutely correct and Martin Gardner was one of her most vocal defenders. However, a later episode became more controversial.

In June 21-23, 1993 Andrew Wiles announced and presented his proof of Fermat's Last Theorem (FLT) in a talk in Cambridge, England. FLT states that no three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ if n is an integer greater than two. Fermat, in the 17th century, had written in a book that he found a proof of this theorem but the margins of the book were too small to contain it. For nearly 400 years many, many mathematicians tried unsuccessfully to prove FLT. Wiles's proof was finally published in 1995 after a few corrections had been made.

Shortly after Wiles's initial declaration in 1993, St. Martin's Press asked vos Savant to write a book about the proof but asked her to write it in only three weeks. And the 80 page paperback *Is it solved? The World's Most Famous Math Problem* was issued in November 1993. On the back cover vos Savant quotes Martin Gardner:

> How Marilyn vos Savant managed to write such a delightful, informative, and accurate book about the probable proof of Fermat's Last Theorem beats me! [This book is] highly recommended even to readers who think they hate math.

In her acknowledgements she claims further "I want to thank the incomparable Martin Gardner for reading my manuscript, for being my dear long-distance friend, and for brightening the world for all thinking people."

We discussed the book with Gardner and we report on several letters he sent to us.

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5 Dec 93

Dear Jerry:

Many thanks for the E-mail and Ian Stewart's good article. I have been struggling with a letter to send to Marilyn that will be diplomatic and not damage our friendship by mail. We've never met, but are friends by correspondence.

I enclose a first draft. Let me know if you think I should change anything in it.

Best,

Martin

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Dear Marilyn:

Thanks for yours of Nov. 29. I've already picked up a copy of your book. You answered that escalator teaser neatly.

By now I'm sure you know that your Parade article is going to bring you more mail than that 3-door problem! Several mathematicians have telephoned me about it, including Dan Asimov, now a Ph.D. in mathematics and working in the area of chaos theory. A friend in Atlanta sent me a batch of e-mail now hitting the networks from math grad students.

Leonard Gillman, a noted mathematician at the U of Texas, asked me to forward the enclosed letter to you. You may recall that he wrote a paper on the 3-door problem defending your solution. (I think I sent you a copy).

I can't pose as an expert on the philosophy of mathematics, but based on phone calls and what I've learned in the past week I think the crux of the matter is this. Hyperbolic geometry does indeed rest on an axiom not in Euclid's system, but it has been shown by Poincaré and others that if Euclidian geometry is true and consistent, so is hyperbolic geometry. Escher's "Circle Limit" is based on Poincaré's model of the hyperbolic plane which proved a one-to-one correspondence between Euclidian and hyperbolic geometry.

Wiles showed that if FLT has a solution, a certain structure in hyperbolic geometry must exist. He proved that the structure did not exist, therefore.... In brief, he went from the integers to hyperbolic geometry, then back to the integers. Assuming his steps are all correct, the result is as certain as any proof in mathematics. Mathematicians constanly go from one axiom system to another then back again. There are elegant proofs in geometry and algebra, for example, that are by way of topology.

By the way, I'm told that there is a gap in Wiles's proof, recently discovered, but most mathematicians assume it can be plugged. In any case, the gap is unrelated to your objection.

When the dust settles, you may want to devote a Parade column to the controversy. It certainly won't damage the book's

When the dust settles, you may want to devote a *Parade* column to the controversy. It certainly won't damage the book's sales, which I gather are brisk. You have plunged into deep, dark waters!

All best,

Martin

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15 Dec 93

Dear Jerry:

Thanks for your comments on my letter to Marilyn, the e-mail, and your good way of solving that 12-coin problem. I'll enclose what I had to say about it in an old *SA* column on the ternary system. Also enclosed is a *NY Times* piece on the gap in Wiles's proof. Personally, as I said on the phone, I hope the proof is invalid!

Will let you know if Marilyn answers my letter.

Happy holiday,

Martin

It seems clear to us that Martin had not read the manuscript before publication and in addition there are many inconsistencies in the work with which he certainly would not agree. Four examples.

(1) Wiles's proof involves a "proof by contradiction". In her book on page 34 vos Savant claims such a proof "is fraught with dangers. For example, let's say the human animal can see only black and white. (To make this point, let's also assume that nothing is gray.) We want to prove that an object is white, so we manage to prove that it isn't black. By double negation, the object is proved to be white. But considering all the colors of the rainbow, this shouldn't be a valid proof. Maybe the object is red. But how would we know it?"

Gardner however refutes this with his Chapter 2 in *Gardner's Workout* (p. 17) where every proof in that chapter uses indirect proofs. (which date back to the ancient Greeks). He adds:

> Note that all the proofs in this article are *reductio ad absurdum* proofs. They illustrate how powerful this type of proof is. As G. H. Hardy put it in his famous *Mathematician's Apology:*
>
> > It is a far finer gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers *the game*.

(2) On pages 44-45 vos Savant mentions Kurt Gödel's proof that any "consistent system of formal logic powerful enough to formulate statements in the theory of numbers must include true statements that cannot be proved." "One wonders whether Gödel's logic could be applied to his own argument, like a dog chasing its tail, thus "proving" itself invalid."

Chapter 10 of Gardner's *Are Universes Thicker than Blackberries?* (2003, W. W. Norton) is an essay on the correctness of Gödel's proof. There Gardner said:

> Because laws of physics are expressed mathematically, the possibility arises that the universe may also have its undecidable laws. "Some of our colleagues in particle physics," writes physicist Freeman Dyson in *Infinite in All Directions,* "think that they are coming to a complete understanding of the basic laws of nature. . . . But I hope [this] . . . will prove as illusory as the notion of a formal decision process for all of mathematics. If it should turn out that the whole of physical reality can be described by a finite set of equations, I would be disappointed. I would feel that the Creator had been uncharacteristically lacking in imagination."

In his 1985 book Dyson succinctly put his view of Gödel's result (page 52):

> Fifty years ago, Kurt Gödel, who afterwards became one of Einstein's closest friends, proved that the world of pure mathematics is inexhaustible. No finite set of axioms and rules of inference can ever encompass the whole of mathematics. Given any finite set of axioms, we can find meaningful mathematical questions which the axioms leave unanswered. This discovery of Gödel came at first as an unwelcome shock to many mathematicians. It destroyed once and for all the hope that they could solve the problem of deciding by a systematic procedure the truth or falsehood of any mathematical statement. After the initial shock was over, the mathematicians realized that Gödel's theorem, in denying them the possibility of a universal algorithm to settle all questions, gave them instead a guarantee that mathematics can never die. No matter

how far mathematics progresses and no matter how many problems are solved, there will always be, thanks to Gödel, fresh questions to ask and fresh ideas to discover.

Vos Savant states at least two more preposterous claims.

(3) [A Possible Fatal Flaw]
A possible fatal flaw in Wiles's proof is whether the same basic arguments could be constructed to hold true for *all* exponents, instead of just the exponents equal to or greater than 3. If it could, the same proof would "prove" the Pythagorean theorem ($x^2+ y^2 = z^2$) to be false. Page 62.

(4) To demolish Einstein's theories of relativity (in elliptic geometry), you could go back and prove the parallel postulate, bringing down not only Einstein, but all of the non-Euclidean geometries, as well. (Or you could go after Einstein selectively by the route of proving a contradiction in elliptic geometry.) Page 67.

There actually is a second edition of vos Savant's book apparently published in 1995. All of the above examples appear in both editions and a change in the text occurs on the back cover where Gardner's tribute is shortened.

> A delightful, informative, and accurate book about the probable proof of Fermat's Last Theorem! [This book is] highly recommended even to readers who think they hate math.

Also, on page 18 in both volumes vos Savant comments about Bolyai,. . . one of the three founders of hyperbolic geometry:

> Finally, he concluded by solving in non-Euclidean geometry one of the classical problems of geometry: SQUARING THE CIRCLE [capitalization added for emphasis] i.e., constructing a square equal in area to a given circle. At the time, it was not known whether this could be done in Euclidean geometry, the first conclusive proof of impossibility being given in 1882. . .Bolyai indeed pointed out that his proof will not work in the case of Euclidean geometry.

Vos Savant adds:

> So one of the founders of hyperbolic geometry managed to square the circle. Then why is it known as such a famous impossibility? Has the circle been squared, or has it not? An issue of *Scientific American* noted that squaring the circle is "known to be impossible, and so any purported solutions can be rejected out of hand." So has Fermat's last theorem been proved, or has it not? *That is, if we reject a hyperbolic method of squaring the circle, we should also reject a hyperbolic proof of Fermat's last theorem!*

In the second edition after the date "July, 1995" she writes:

> That was what I thought when this book was first published. But I've since read many letters on the subject from mathematicians, who say I was wrong to draw an analogy between Fermat's last theorem and squaring the circle. They say I should have viewed F. L. T. as an unsolved problem, not as an academic exercise the way I did.
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> That is, I viewed F. L. T. as an intellectual challenge--to "find a proof with Fermat's tools", and this modern-day proof doesn't do that. But the mathematicians who wrote asserted that there are no restrictions on the tools we use.
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> If that's the case--and I'm willing to agree that it is--I can't imagine why anyone would bother searching for a proof. (Part Two details how that same sentiment was expressed by eminent mathematicians of the past.) As a theorem, F. L. T. is considered to be both uninteresting and unimportant.
>
> As of this date, gaps in the proof have been found and fixed and found and fixed again. The process of review is virtually complete. When it is, Andrew Wiles is likely to emerge triumphant.

The long time collaboration between Martin and Marilyn continued in spite of this difference of opinion on FLT. In Gardner's 2001 book *Mind-Boggling Word Puzzles* (Sterling) appears the dedication:

For Marilyn vos Savant

A maven who can't fail to write gracefully about
anything from word puzzles to psychology,
and how to spell *hippopotamus* and *paleontology*.
--A clerihew by Armand T. Ringer

On page 52 of that book is the word puzzle "Guess the Pseudonym".

> Armand T. Ringer is an anagram of the name of a writer with whom you are familiar. Who is he or she?

Just who is Armand T. Ringer?

We have long admired Gardner's concise answer to the question "What is it about mathematics that you find so attractive?" His answer was given in "The College Mathematics Journal", vol. 38, No.4, September 2005:

> I suppose it's the fact that in mathematics, unlike in science, which is fallible, you can prove astonishing results with absolute certainty. Of course a proof must always be within a formal system. The Pythagorean theorem, for example, is certain only with the formal system of Euclidean geometry. It doesn't become false when it fails in non-Euclidean geometries because such geometries are different formal systems. Mathematical theorems are timeless truths, analytic in nature like the great truth that there are three feet in a yard.

Martin and Charlotte Gardner with Karen Farrell, Hendersonville, N.C. 1996.