PUZZLES AND GAMES ON WORD CONFIGURATIONS

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In their article "Numerical Patterns and Geometrical Configurations" on pages 82-92 of Mathematics Magazine 57 (1984), Harold Dorwart and Warren Page note that the venerable David Hilbert once remarked "...there was a time when the study of configurations was considered the most important branch of all geometry." "Today," added Dorwart and Page, "...most students have limited knowledge about configurations—except perhaps the Pappus (9,3), the Desargue (10,3), and the Petersen graph (the logo on the cover of the Journal of Graph Theory)."

Recently, however, there has been a noticeable revival of interest in configurations, due mostly to the extensive research by Germany's Harald Gropp. This topic is summarized in his article "Configurations" in *The CRC Handbook of Combinatorial Designs* (CRC Press, Boca Raton, 1996). Our own contribution to the theory has been to treat points as letters and lines as words and to use configurations to play two-person games (see "Games on Word Configurations" in the November 1994 Word Ways). In this self-contained note we add some puzzles to our previous work and complete the listing of all order-10 configurations.

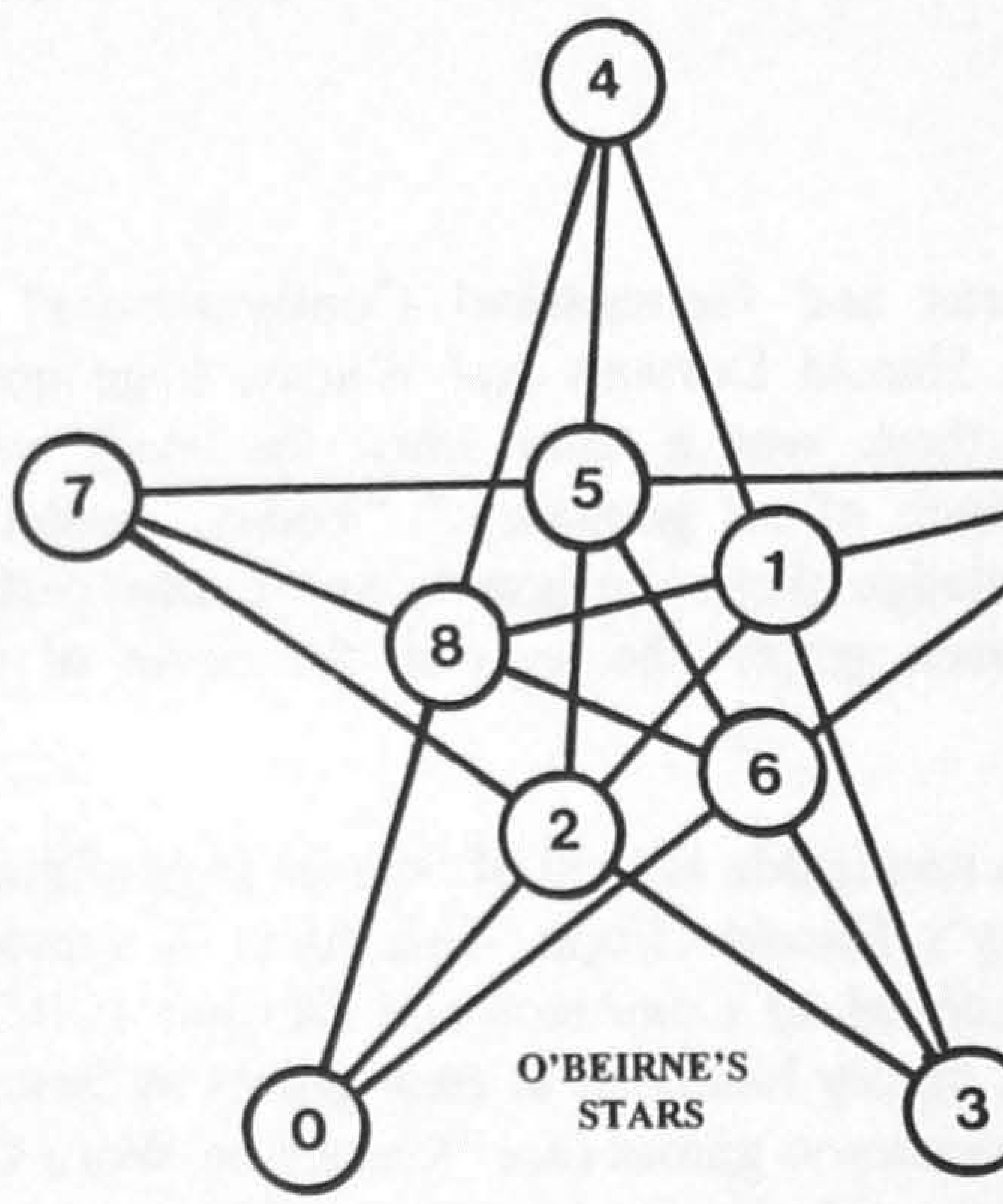
By a word configuration of class (n,r) we mean a collection of distinct letters and a collection of distinct words formed from them subject to the following requirements:

R1 Any two letters are in at most one word and any two words have at most one letter in common
R2 Each word has r letters and each letter is in r words
R3 There are the same number, n, of words as there are letters

We restrict our attention to class (n,3) configurations for n = 7,8,9 and 10. Gropp lists the number of distinct solutions (up to isomorphism) for each type as: one for n = 7 or 8, three for n = 9, and ten for n = 10. In the figure marked Solutions are word lists (or, dually, letter lists) that achieve the configurations. They are based on the letters PYRAMID for Fano's 7-point projective plane, ASTEROID for n = 8, MOUSETRAP for each of the 9's, and ELUCIDATOR for each of the 10's. Two additional figures contain actual geometric realizations of each of the 15 configurations. For the two smallest orders one ideal "line" must be represented in the graph by a circle.

The puzzles are best played by using any one of the 15 graphs as a playing board and printing the words from the Solutions sheet for that puzzle on small tiles. One is then to arrange the tiles on the board so that tiles lying on a line have a letter in common. These puzzles range in difficulty from the very easy (Fano's PYRAMID is automatic) to the extremely difficult (OVERSEA'S CAP has only two solutions).

Another puzzle, based on a diagram used by the Glasgow puzzlist and mathematician Thomas H. O'Beirne, is shown at the left below (see page 109 in O'Beirne's *Puzzles & Paradoxes,* Oxford, 1965). One is to take the ten words from the solution set and arrange these words on the graph so that abutting words have a letter in common. Amazingly, once any given word is initially placed on any node, there will be exactly two solutions to the puzzle, one with lines with a common letter and one with common letters on a triangle. The reader will note that the same ten words are used in ROBIN HOOD'S CAP. We say because of this, that the two puzzles are isoagonic even though they are not graph isomorphic.



Word configuration games are played in normal form by two players alternately selecting letter tiles from the game word (ELUCIDATOR, MOUSETRAP, etc.) until one of them wins by being first to complete, using only his own selected tiles, one of the words from that game's word list. Since it is clearly an advantage to start in the game, we usually play that a draw is also a win for Second.

For fairness we always insist that both players have the same number of moves. Hence for n = 7 or 9, First does not get an extra play. With this proviso, FANO'S PYRAMID and MOUSETRAP are Second player wins, while all others are First player wins. The misère forms of these games are meaningful only for even n, but the analysis must be left to the reader (most seem to be second person wins). Misère means that the first player forced to form a word loses.

To help in the play of these games, one can utilize the misgraph for that game. The misgraph is the graph of words that have no common letter, or dually the graph of letters that are not used together in any of the words. That is, two nodes are joined in case the nodes "miss" each other. The figure with the eleven graphs labeled a through k gives the misgraphs for all the games. For the ten "hat" (10,3)'s, the misgraph column also includes the order of the graph automorphism group (which will give the number of solutions to that game's puzzle) and the number of lines (L), angles (A) and triangles (T) for the graph. For example, in OVERSEA'S CAP, the one line indicated in the 1-3-6 listing is the line 793 that "misses" node 1. On that line, node 3 misses angle 018 while the other two nodes miss triangles. This gives a hint on why this puzzle has exactly two solutions.

Our strategy to win at the games as First player is to force Second to waste a move in his misgraph in order to gain time to form a double threat. When Second can win, he will be able to make First take too many moves to accomplish the win. We shall give hints for the three MOUSETRAP games and leave the other games for the reader.

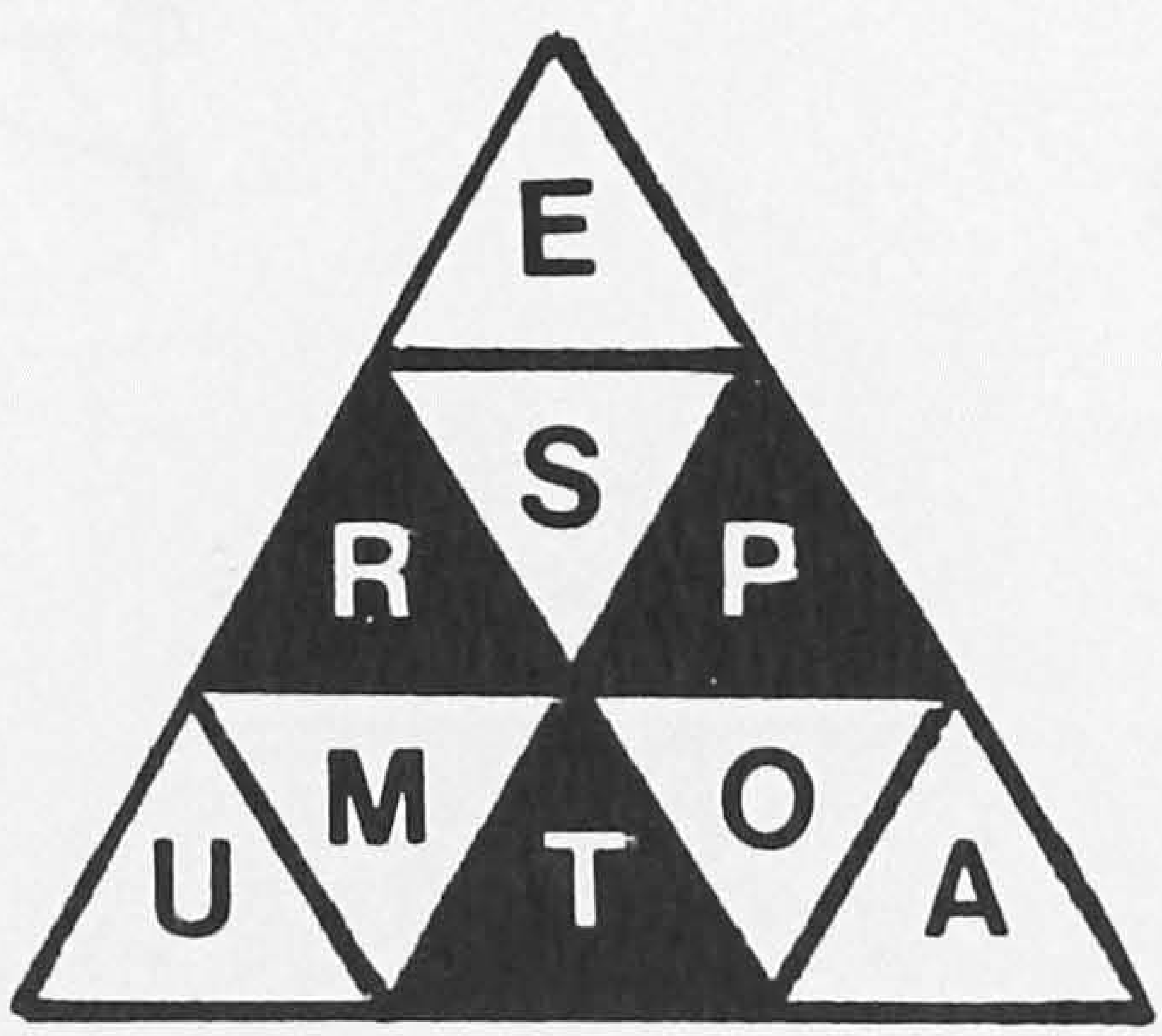
On January 4 1962, in his column “Puzzles & Paradoxes” in the British magazine New Scientist (pages 98-9), O’Beirne experimented with balanced patterns of (9,3) configurations to see if any were suitable for ticktacktoe play. He regarded only our O’BEIRNE’S MOUSETRAP as exceptional, and it was commercially produced by him under the name Tri-Hex. Martin Gardner later reported on this game in his Scientific American column, and reprinted it on page 69 of *Mathematical Magic Show* (Knopf, 1977).

First wins O’Beirne’s MOUSETRAP employing the magic rune above on the right that is derived from misgraph c. He initially must play one of E, A or U. If Second chooses another from this trio, First chooses the remaining letter and plays rationally thereafter to force a four-move win. If Second’s initial play comes from the interior hexagon, it will be one of two letters opposite First’s play, or not. If so, First wins by choosing one of the two adjacent letters to Second’s play, and otherwise the opposite letter (in the hexagon) to Second’s play. For example, the order of choices could go U, P, S (threatens SOU), O (forced), T (threatens RUT or SET). Or, U, O, R (threatens RUT), T (forced), P (threatens PER or UMP).

Second wins at MOUSETRAP by replying one step away from First’s opening in misgraph b. For convenience, this cycle is listed in the interior of the enneagon.

First wins at PAPPUS’ MOUSETRAP by initially selecting any letter. If Second takes in First’s misgraph triangle (graph d), First wins by selecting the remaining letter. When Second starts by choosing a letter from a different trio, First forces Second to waste his next move by forcing him to block with another letter in Second’s misgraph. First will have a double threat on his third play by choosing the remaining letter in Second’s misgraph. For example, play could go M, P, E (threatens MOE), O (forced), T (threatens SET or MAT).

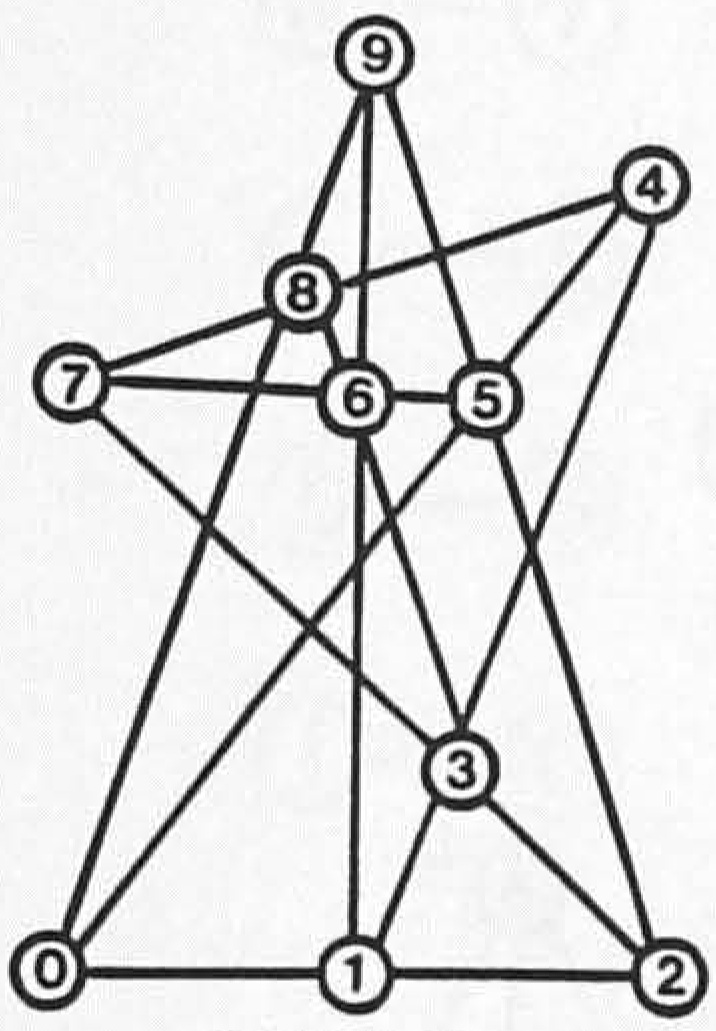
Harald Gropp reports that there are 31 (11,3), 229 (12,3), 2036 (13,3) and 21399 (14,3) configurations. There must be hundreds among these that would make challenging puzzles or word games, and it should be easy to achieve them using words and letters. A good method of constructing such configurations is given in the Dorwart and Page article.



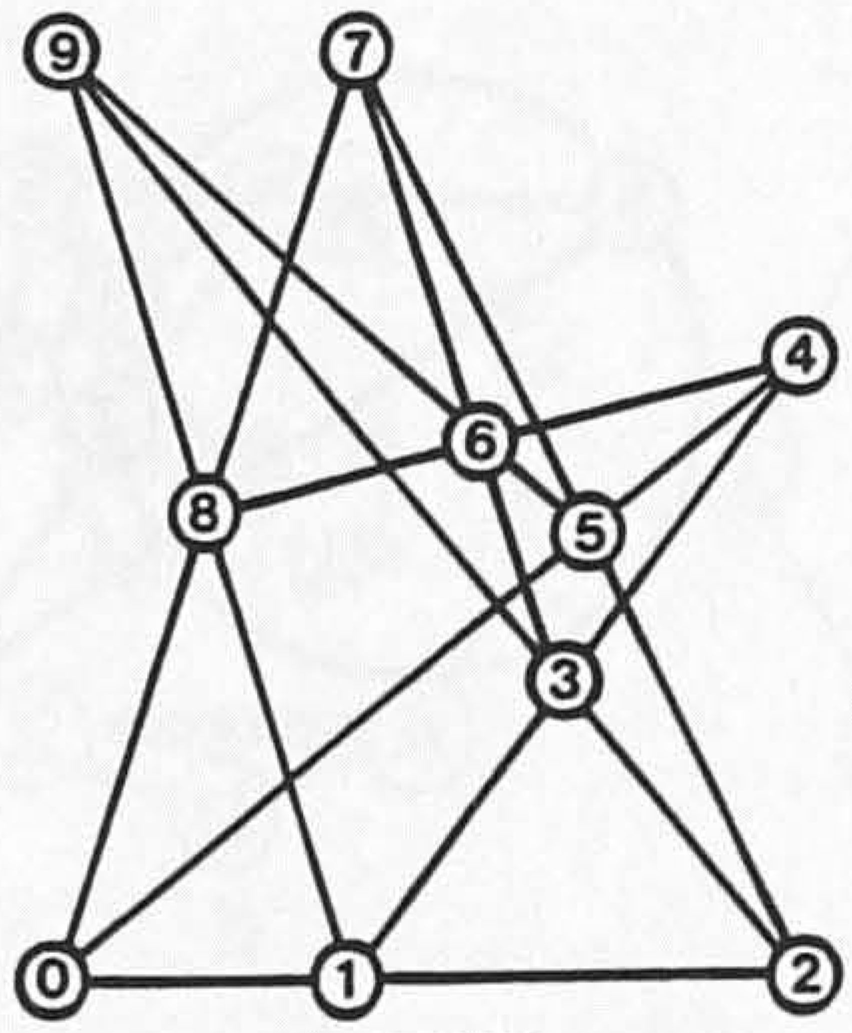
SOLUTIONS

NAME	0	1	2	3	4	5	6	7	8	9	MISGRAPH L – A - T
DESARGUES' MITRE	CUR L	DUE O	TAU I	LET R	ICE A	COT D	OIL U	LAD C	OAR E	RID T	PETERSEN 10-0-0 120e
FOOL'S CAP	AIL U	EAR C	OAT D	OUR I	RID T	TIC E	CUD A	COL R	LED O	UTE L	FRISCO 4-6-0 12f
NIGHTCAP	LAO E	RAD U	TAU D	LET L	ICE O	COT I	CUR R	DUE A	LID T	RIO C	PETERSEN 6-0-4 120e
HEADDRESS	RED U	LID C	CUD D	COL I	ALE T	UTE E	OAT A	OUR R	AIR O	TIC L	FRISCO 2-6-2 12f
FEZ	OAT E	CUT I	ICE T	LIT C	IDO U	DUE O	LAD R	CAR L	OUR D	REL A	SOCCER 1-9-0 6g
BASINET	ICE R	CAT L	OAR I	LAD T	DUE U	IDO O	OUT D	CUR E	REL C	LIT A	SOCCER 0-9-1 6g
PETER PAN'S HAT	LET U	ROT O	TAU L	AIL A	CAD I	DOE R	RID D	CUR E	ICE C	LOU T	STUBE 0-3-7 6h
OVERSEA'S CAP	LID L	ICE O	AIR U	OAT C	CAD T	CUT A	LET D	LOU I	RUE R	ROD E	TRI 1-3-6 2i
CLOWN'S HAT	RUT A	CUD C	LOU L	CAL U	TIC D	TOE E	ADE T	LIE O	AIR R	ROD I	BITRI 0-6-4 8j
ROBIN HOOD'S CAP	ICE D	RED C	CUR O	ROT U	COD R	LOU T	LIT A	TAU L	AID E	ALE I	CYLINDER 0-0-10 20k
O'BEIRNE'S STARS	ALE I	LIT E	LOU C	ROT R	TAU D	CUR O	RED T	COD U	AID A	ICE L	(=ROBIN HOOD'S CAP) 0-0-10
O'BEIRNE'S TRIANGLES	ALE I	AID A	RED T	COD U	ICE L	CUR O	LOU C	ROT R	LIT E	TAU D	0-0-10

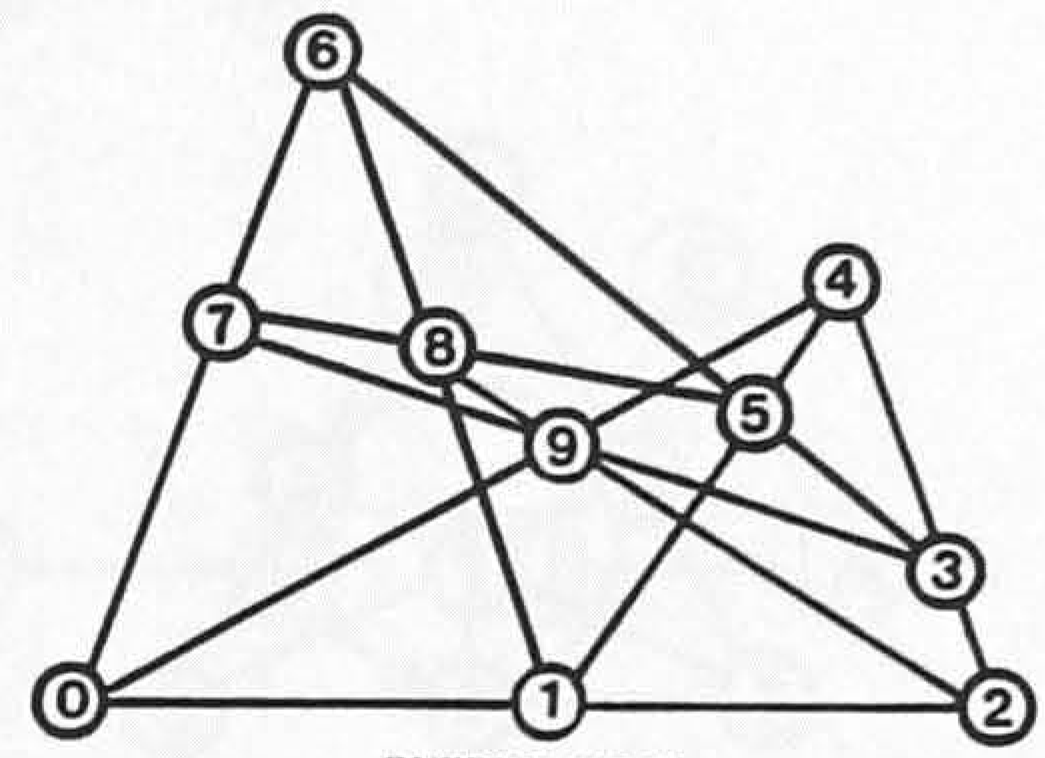
NAME	1	2	3	4	5	6	7	8	9	MISGRAPH
FANO'S PYRAMID	DRY D	AIR R	RPM Y	YIP I	PAD P	DIM A	MAY M			EMPTY
ASTEROID	AID R	OAR S	SAT I	DOT A	TIE D	SIR E	RED O	EOS T		2-2-2-2 a
MOUSETRAP	UMP M	MOE U	SAM P	EAR T	PAT A	PRO S	SOU O	SET R	RUT E	ENNEAGON b
PAPPUS' MOUSETRAP	MAT S	RUT E	SET T	SOU R	UMP P	SAP U	PER M	OAR O	MOE S	3-3-3 CYCLES d
O'BEIRNE'S MOUSETRAP	SAM M	SET E	SOU O	OAR U	MOE S	UMP A	PER T	PAT P	RUT R	3-6 CYCLES c



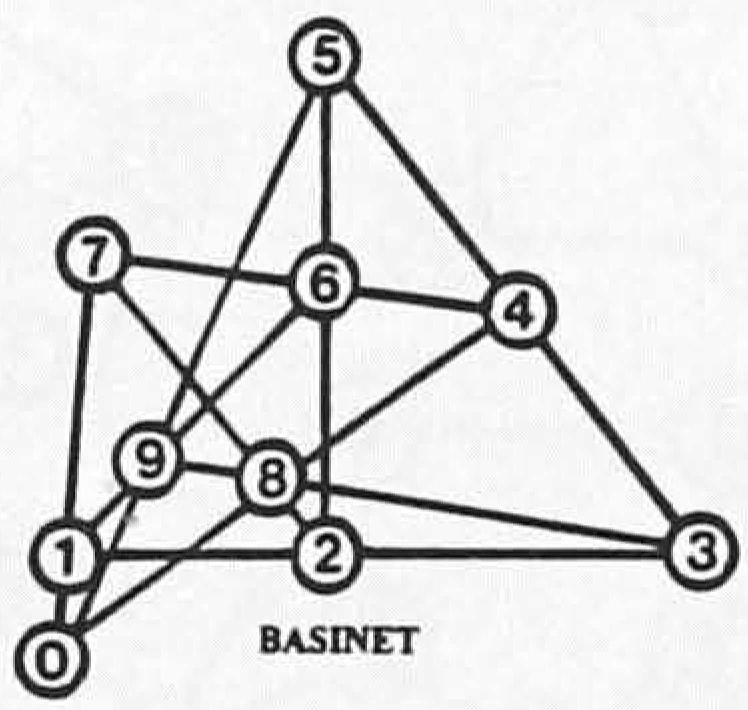
CLOWN'S HAT



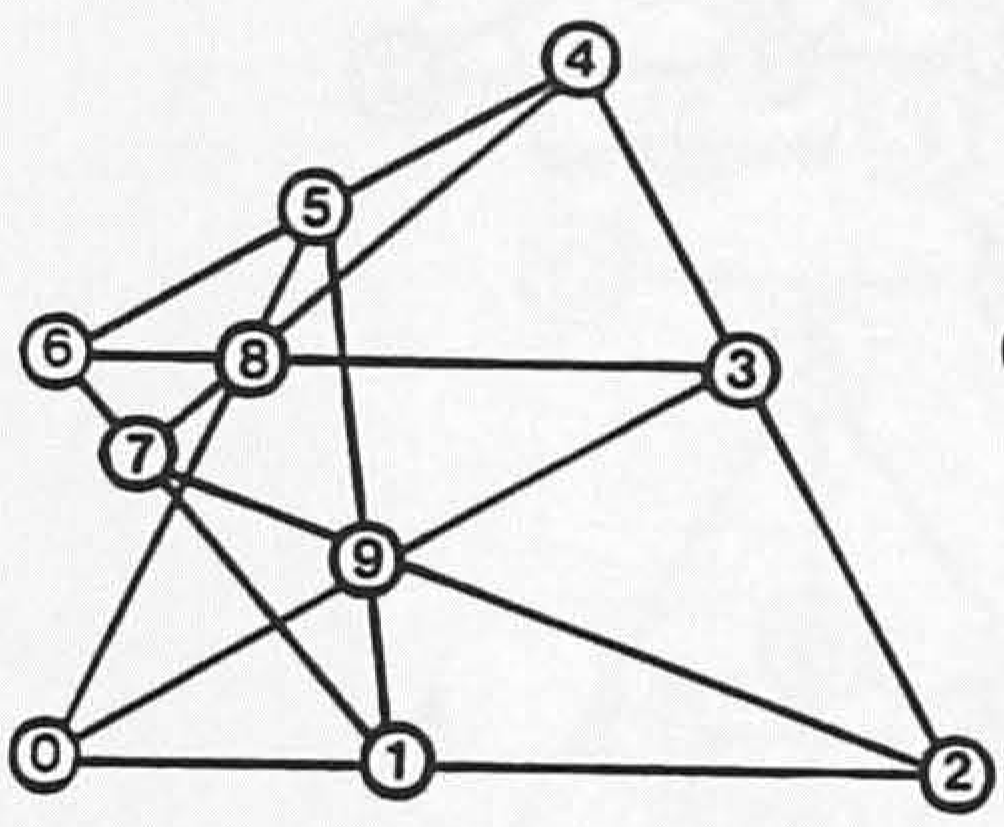
HEADDRESS



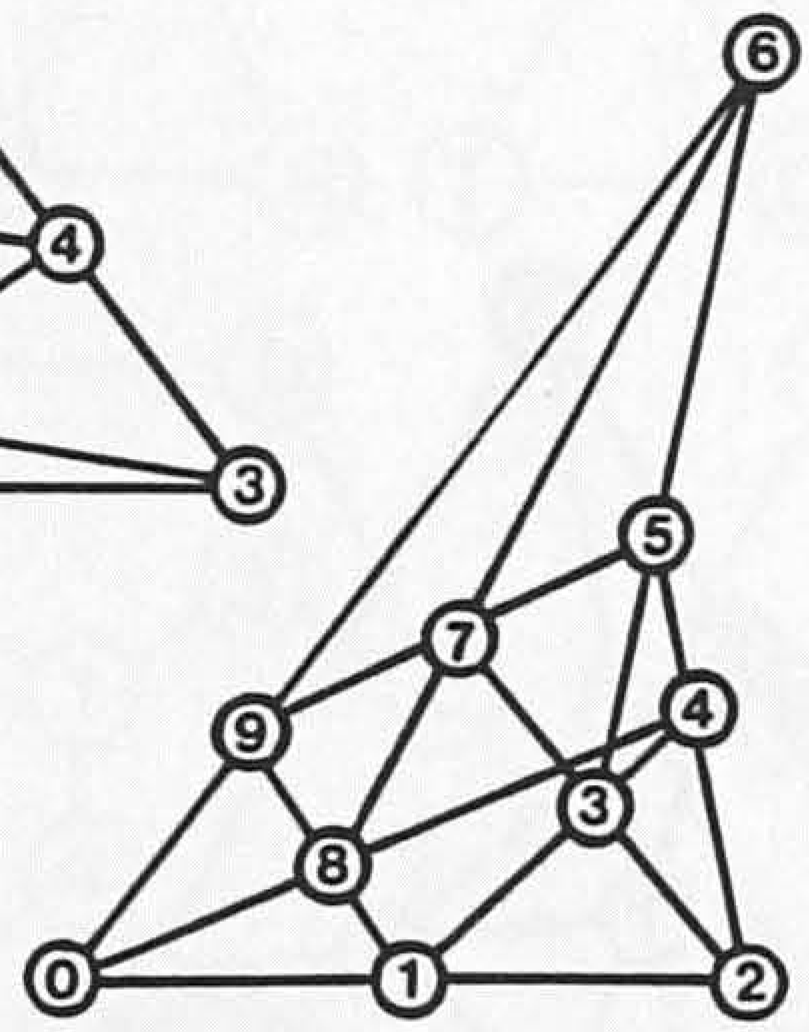
OVERSEA'S CAP



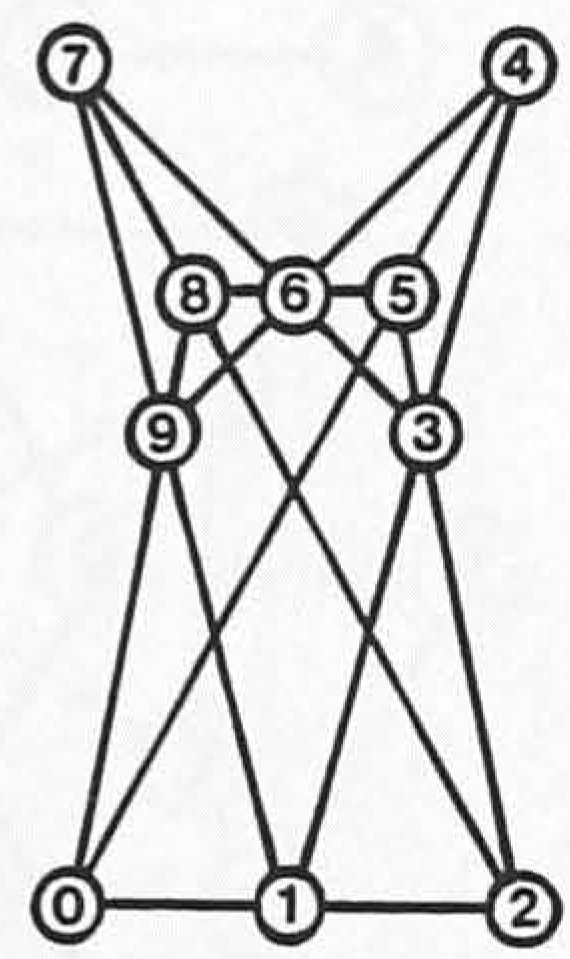
BASINET



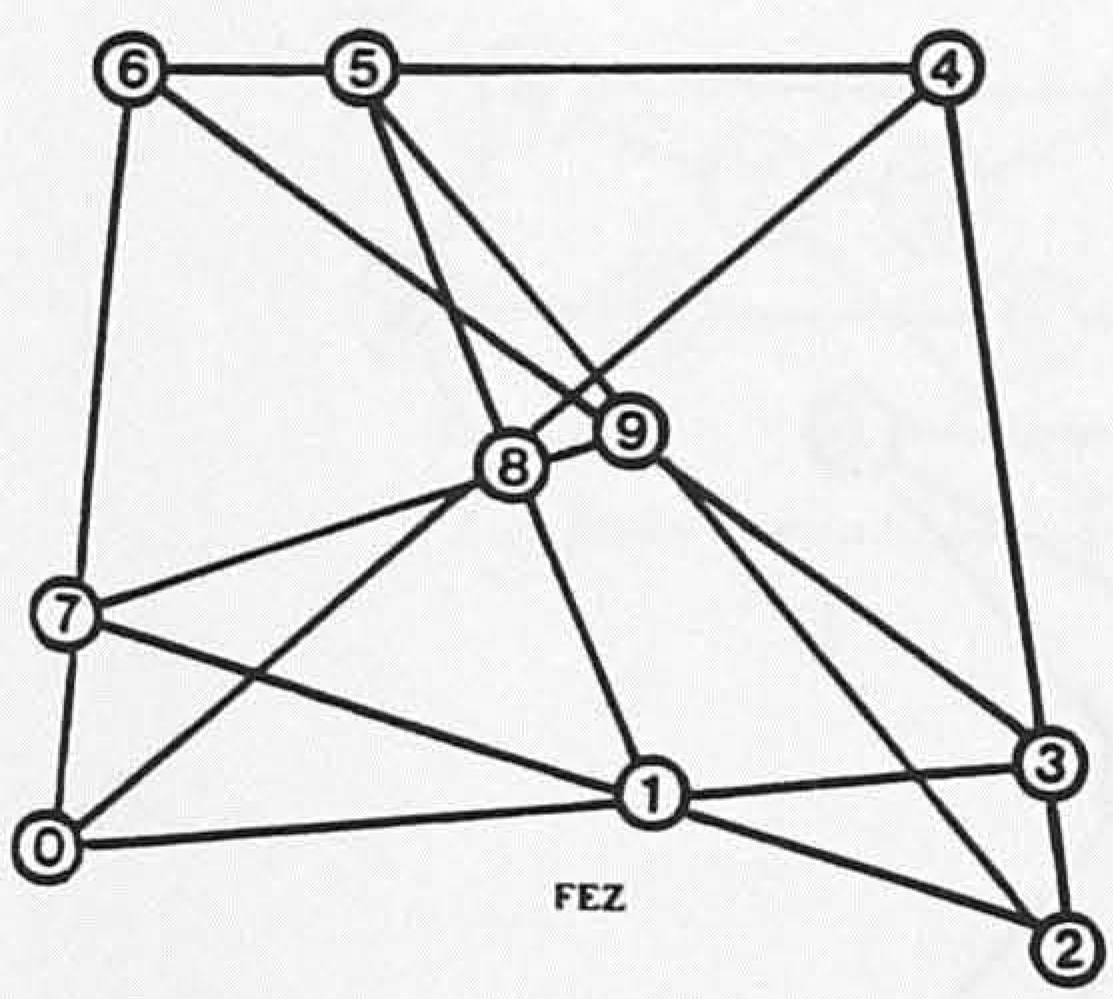
PETER PAN'S CAP



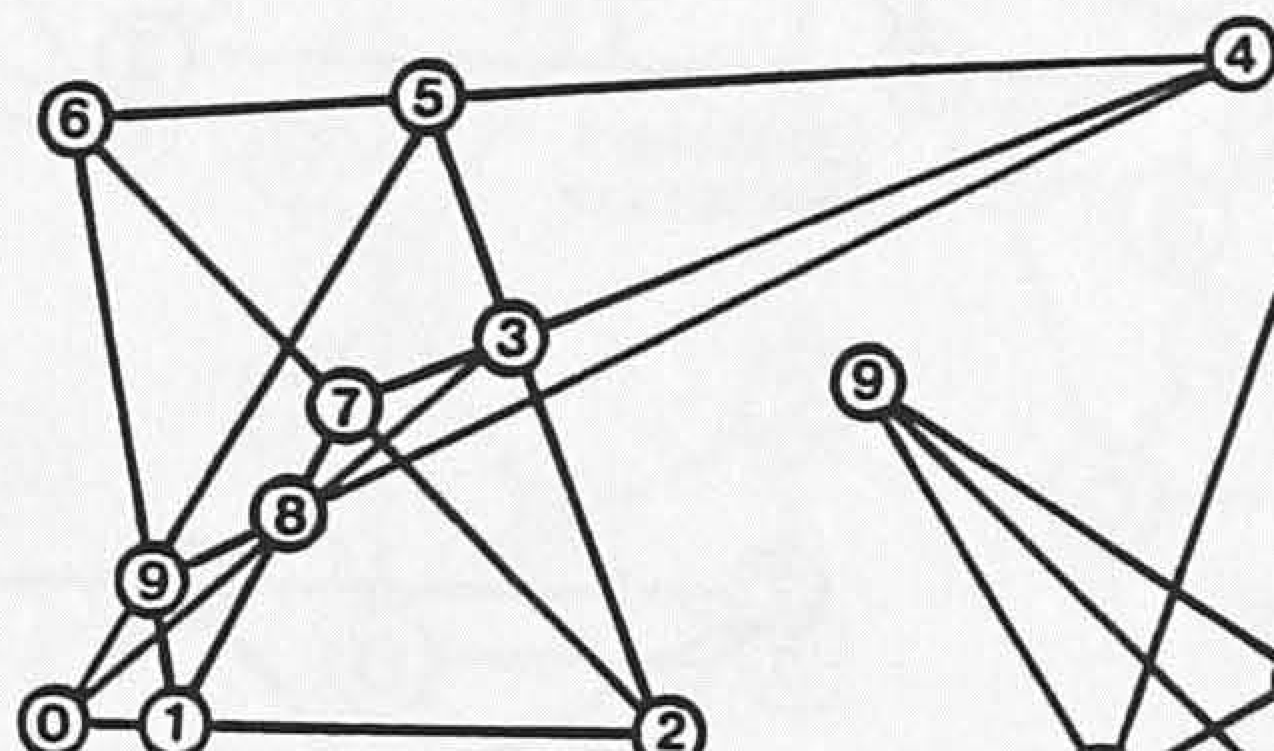
ROBIN HOOD'S CAP



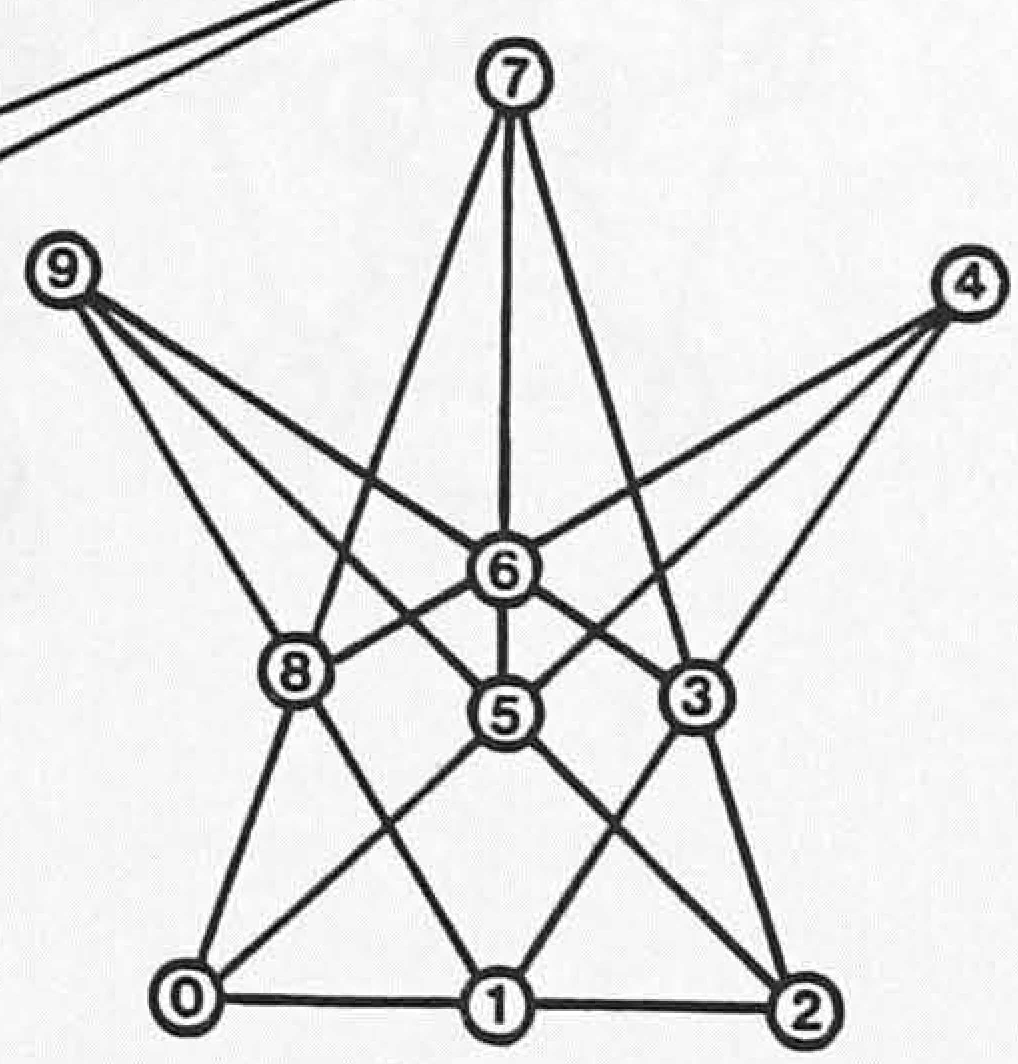
DESARGUES'
MITRE



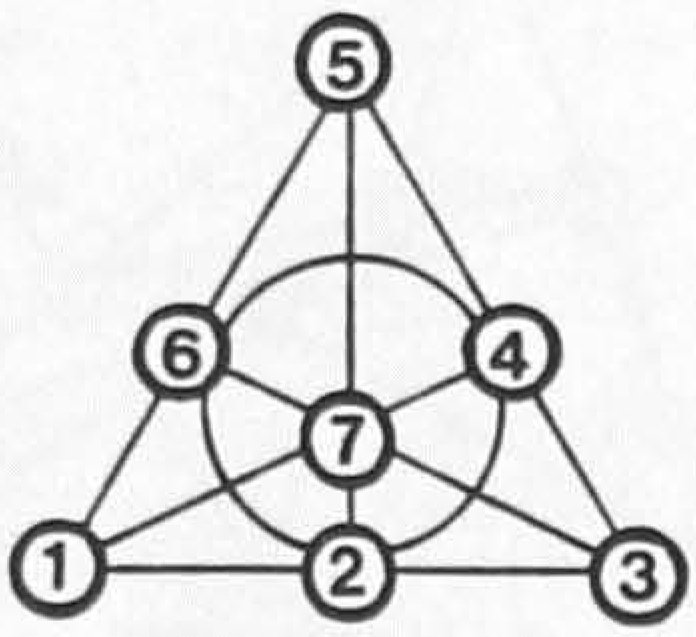
FEZ



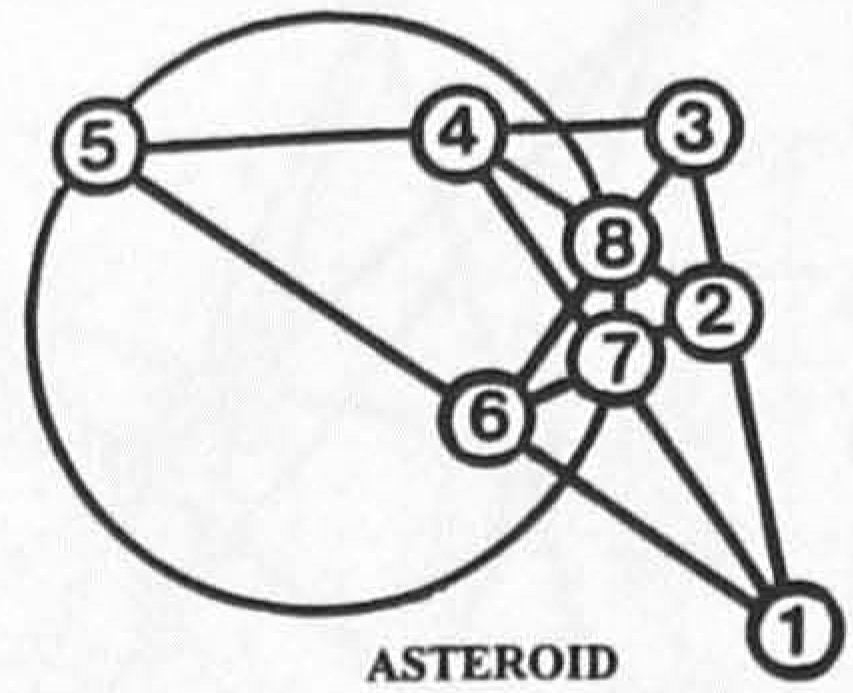
NIGHTCAP



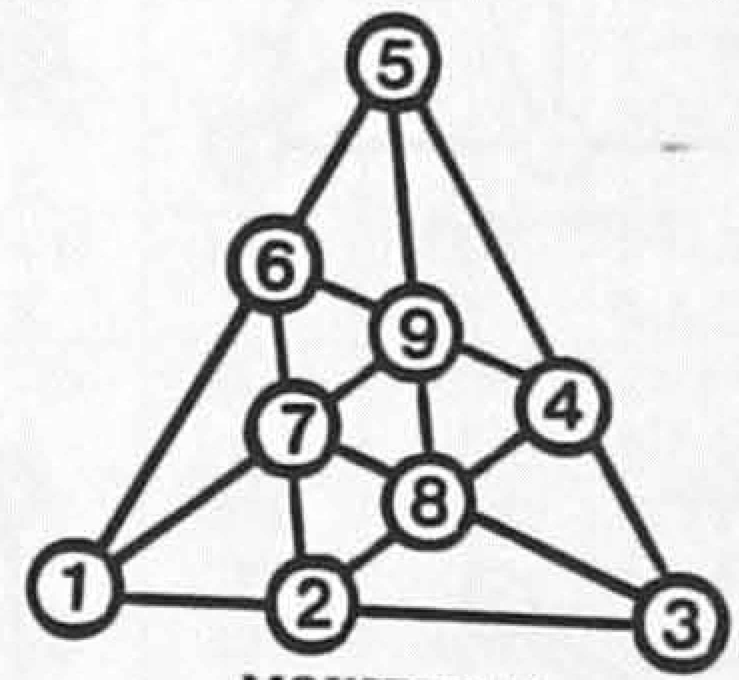
FOOL'S CAP



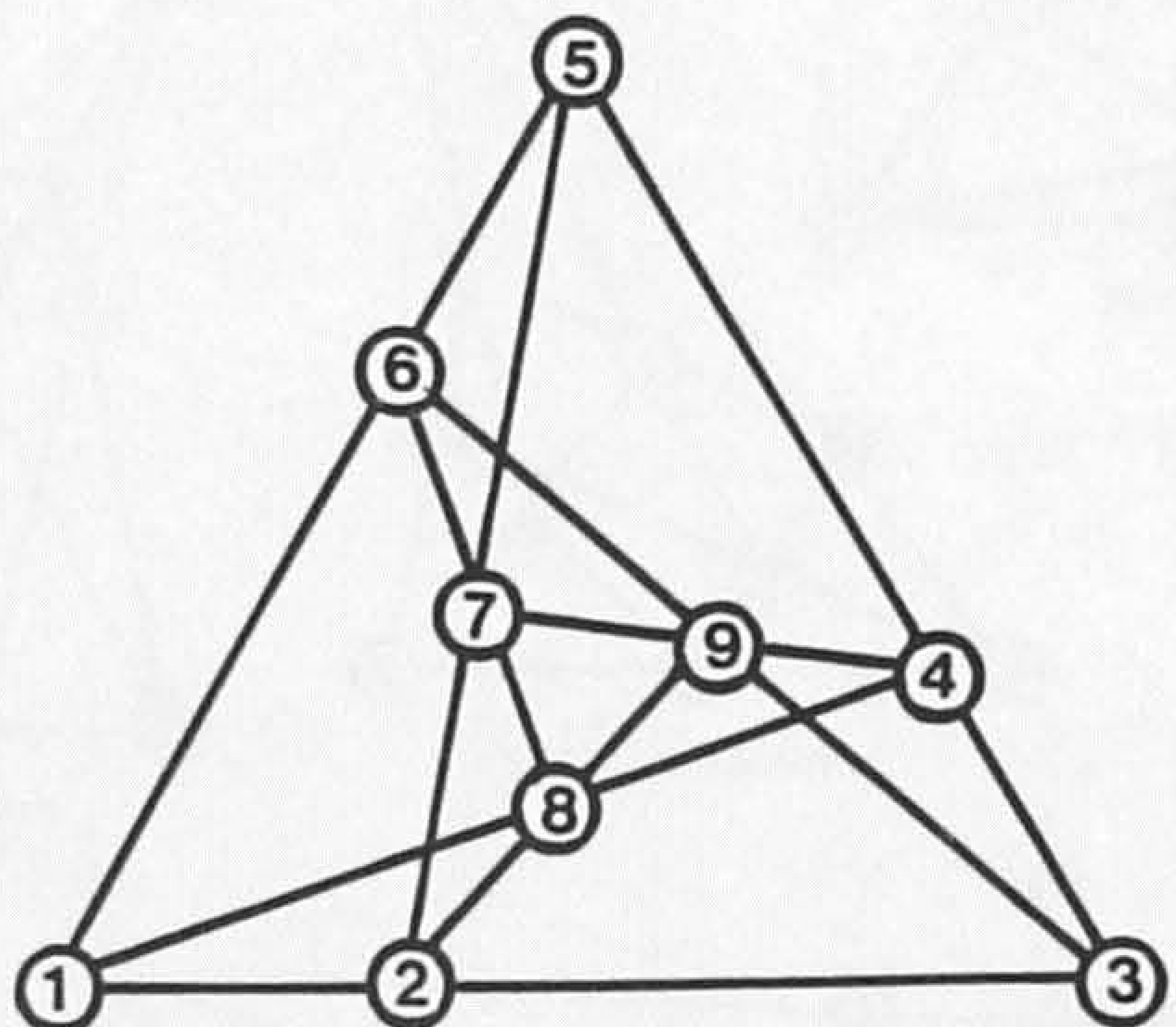
FANO'S PYRAMID



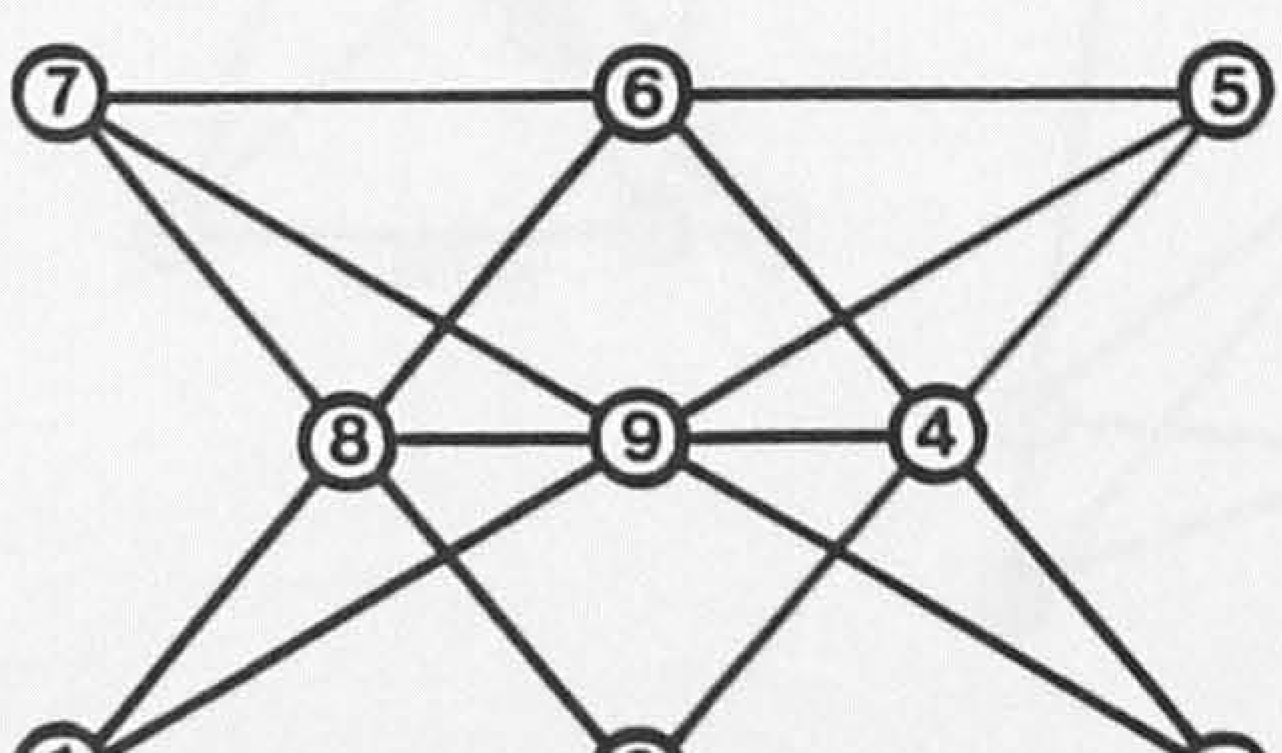
ASTEROID



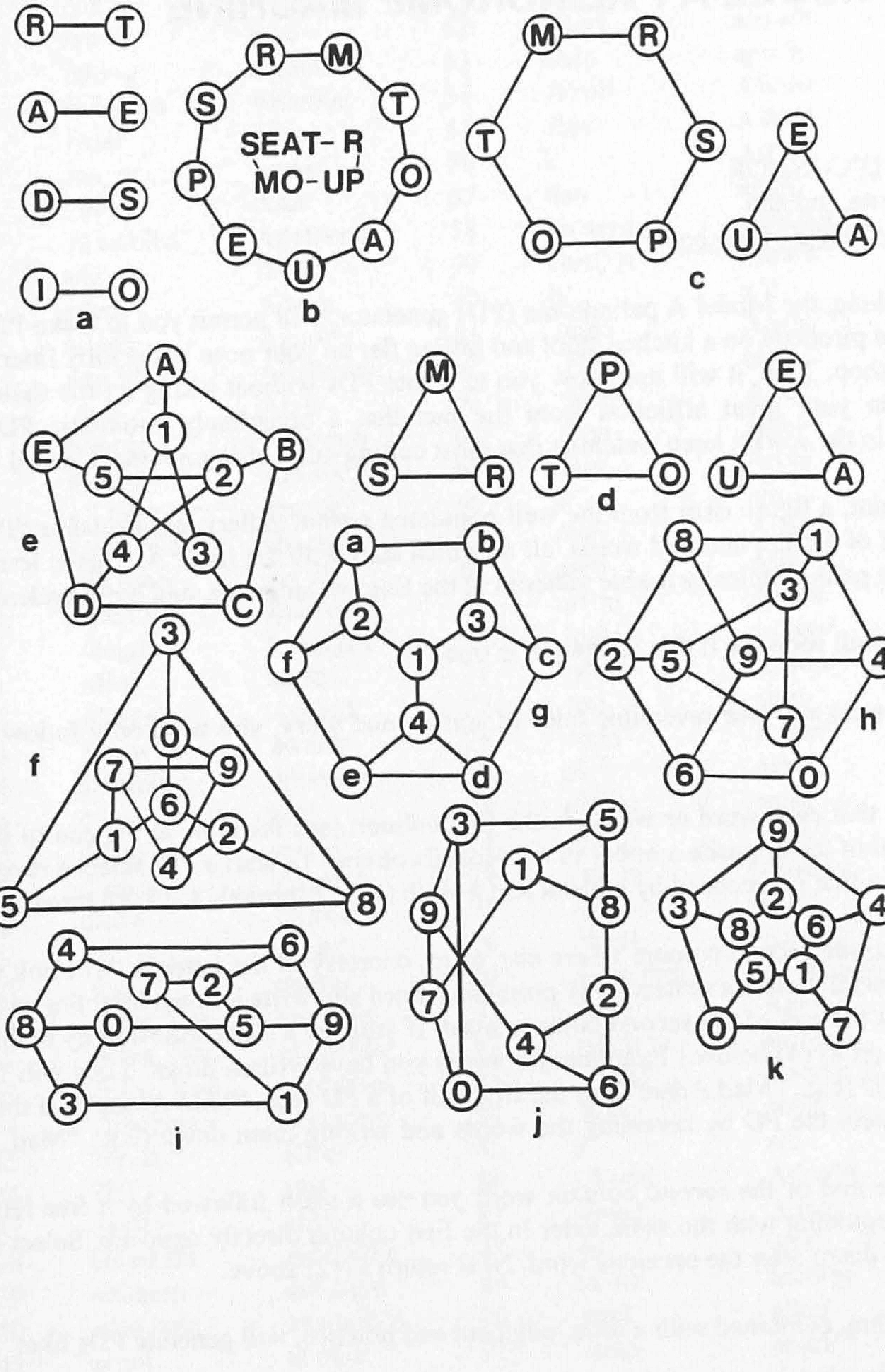
MOUSETRAP



O'BEIRNE'S
MOUSETRAP



PAPPUS'
MOUSETRAP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