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Cube Mentalism

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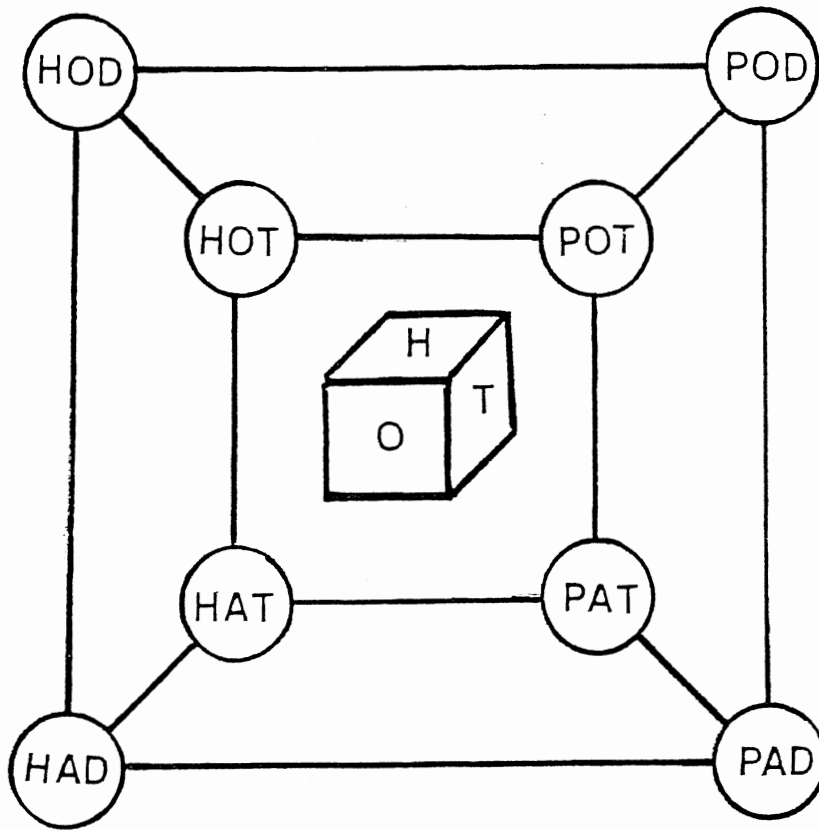
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Presented by Ivan Moscovich
to honor Martin Gardner at G4G8

CUBE MENTALISM

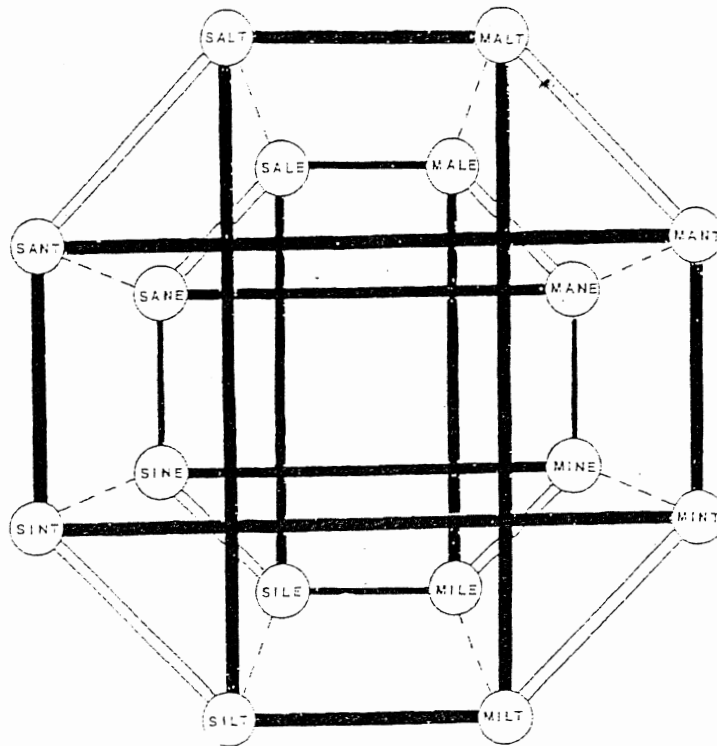
by Jeremiah Farrell and Ivan Moscovich

Our tour of multidimensional cubes begins with the marking of the eight corners of a 3-cube with the eight words HOT, POT, POD, HOD, HAD, HAT, PAT, and PAD. The figure below shows how these eight inherit the labels of the HOT-PAD die where the letter H is opposite P, the letter O is opposite T and the letter T is opposite the letter D.



One of Martin Gardner's passions is a study of all of the works of Lewis Carroll. In 1878, Carroll began writing about "Word Links" or what he would later call "Doublets" (see Reference 1). We now often refer to these as word ladders. A word ladder example from Carroll is COLD, CORD, CARD, WARD, WARM, i.e. COLD is changed to WARM in four steps by changing one letter at a time. All the words in the chain should be legitimate words and Carroll's example is called perfect since it has a number of steps equal to the number of letters in each word. The HOT-PAD list can be perfect for any pair of the eight words. It is only slightly harder to arrange all eight words in an octagon where the abutting words have only one letter different.

We want to extend this idea to higher dimensions before we perform our mental magic on the cubes. For four dimensions we choose the graph of the SALT-MINE die depicted below.

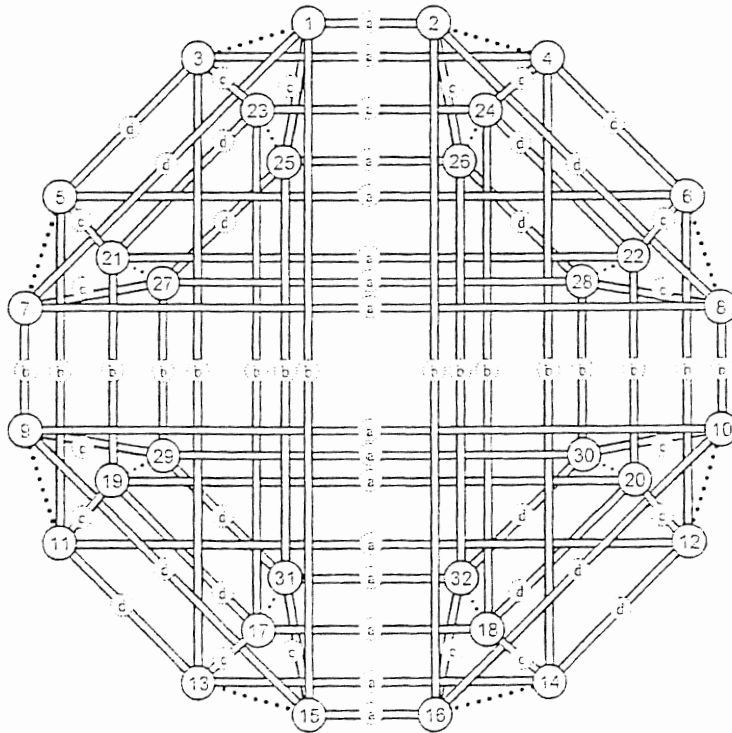


The eight 3-cubes of the graph are marked with a common letter, one of the letters of SALT-MINE. Each of the 16 nodes of the 4-cube inherits the labels of the 3-cubes it bounds. The reader may trace the eight 3-cubes (in perspective) by following the eight words with a common letter, say S for example. Notice that the M 3-cube is "opposite" the S 3-cube. Similarly, the A-cube is "opposite" the I-cube, the L-cube is "opposite" the N-cube and the T-cube is "opposite" the E-cube.

Each of the 16 words on the nodes are bona fide (SINT and SANT are "Saint" in different European languages and are often used in place names). Carroll's word game can be played with the 16 words here also. No pair of words need be more than four steps from another. More challenging is to ask someone to form a 16-gon of the 16 words with adjoining words having exactly three letters in common. There are many solutions.

A step to five dimensions can be had using the ten letters of POLAR-MINES. A 32 node 5-cube is shown below where each node is a bona fide word found in the Merriam-Webster Dictionary 3rd ed. unless noted. Each of the ten letters in its 16 appearances marks the boundaries of a 4-cube. Similarly to the two earlier cubes one can play word ladders. No two of the 32 words are separated by more than five steps and it is a difficult problem to arrange the 32 in a circle with one-letter changes.

- | | |
|---------------------------------------|----------------------------------|
| 1 POLAR | 2 MOLAR |
| 3 POLAS (Beers, Sp.) | 4 MOLAS |
| 5 POLES | 6 MOLES |
| 7 POLER | 8 MOLER |
| 9 PILER | 10 MLER |
| 11 PILES | 12 MILES |
| 13 PILAS (Batteries, Sp.) | 14 MILAS (Mines, Sp.) |
| 15 PILAR | 16 MLAR (Vintage Recording Tape) |
| 17 PINAS | 18 MINAS |
| 19 PINES | 20 MINES |
| 21 PONES | 22 MONES (Nicole, novelist) |
| 23 PONAS (Lith. Slave owner) | 24 MONAS |
| 25 PONAR (Device to scour river beds) | 26 MONAR (Polish help group) |
| 27 PONER (To put, Sp.) | 28 MONER |
| 29 PINER | 30 MINER |
| 31 PINAR | 32 MINAR |



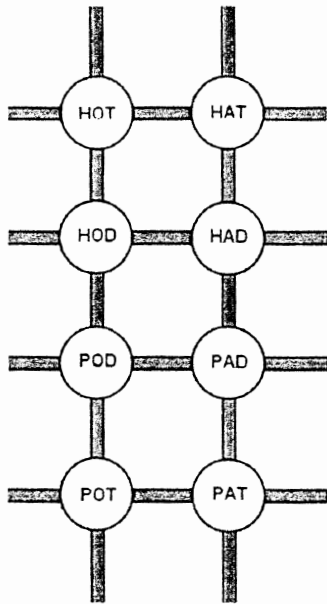


Figure 1

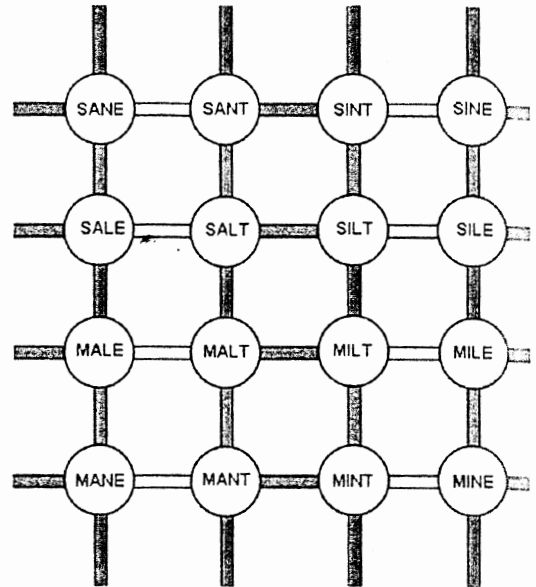


Figure 2

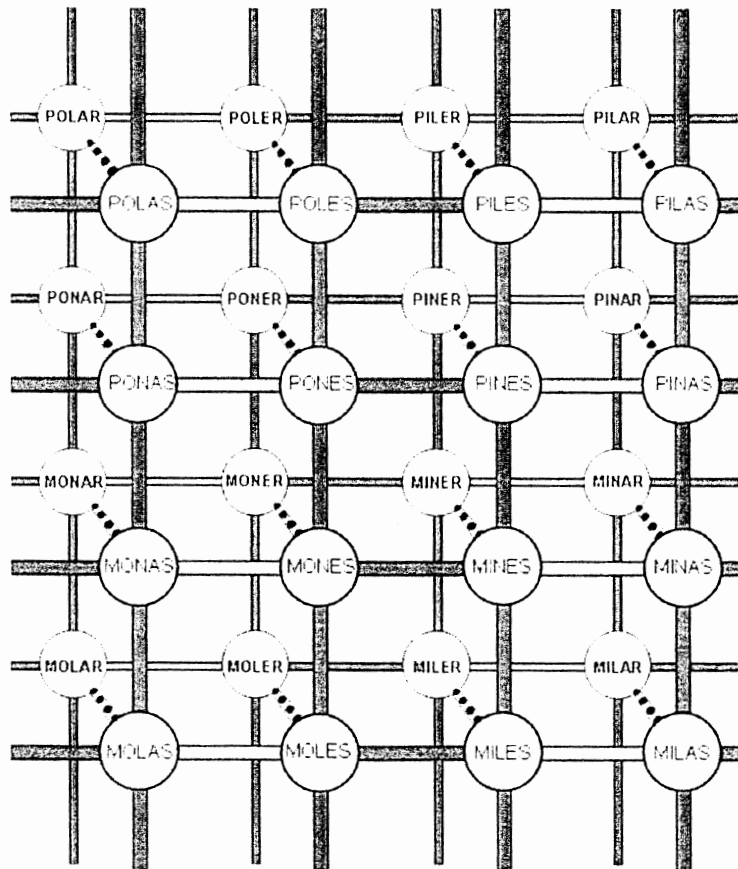


Figure 3

Of course all three of our cubes are drawn in the two dimensional plane and this gets to be very "busy" for higher dimensions. To perform our mental magic on them we use instead the equivalent toroidal (doughnut-like) graphs shown in color as Figures 1, 2, and 3. Each of the three is to be thought of as bent into a torus (Figure 3 is a double torus). That is, the top side bends around to join the bottom and then the right side bends around to join the left side completing the torus.

The magic is performed on any one of the three figures. While the magician's back is turned the subject places a token on any node (and remembers his start). Then the subject tours the board, calling out only the colors of his routes as he traverses them. After a time the subject stops and tells the magician either his start or stop node, but without mentioning whether it is the start or stop. The magician immediately names the other node of the pair.

THE METHOD: We have colored the toruses so that RED is a change in the first letter, BLUE is a change in the second letter, GREEN is a change in the third letter, YELLOW is a change in the fourth letter and DASHED is a change in the fifth letter. Notice that an even number of calls of a color "cancels" that change and an odd number of calls of a color "makes" the change. So the magician simply keeps track of the parity of the colors and the start and end word differ only in the odd places.

For example, suppose the subject calls out YELLOW, GREEN, BLUE, YELLOW, RED, DASHED, and BLUE, i.e., the 3rd, 5th and 1st places since YELLOW and BLUE cancel. So if one terminal was POLAR the other was MONAS.

Mathematicians will recognize this as using direct sums of the group Z_2 3, 4 or 5 times. Magicians may ignore the mathematics claiming only a prodigious memory that can operate in multiple dimensions. For more information, please consult the references.

REFERENCES

- (1) Gardner, Martin. "Doublets", *The Universe in a Handkerchief*, 1996, Copernicus, NY.
- (2) Farrell, Jeremiah. "Magic Word Dice", *Word Ways*, Vol. 40, No. 2, May 2007.
- (3) Dillon, David S. and J. Farrell. "A 5-Dimensional Magic Die", *Word Ways*, Vol. 40, No. 4, Nov. 2007.