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# Working Toward a Third Space in the Teaching of Elementary Mathematics 

## Ryan Flessner


#### Abstract

Building on work in the area of third space theory, this study documents one teacher's efforts to create third spaces in an elementary mathematics classroom. In an attempt to link the worlds of theory and practice, I examine how the work of other theorists and researchers - inside and outside the field of education - can create new lenses for classroom practitioners. In addition, the article provides evidence that third spaces may be more difficult to realize than others have described. Rather than forcing a third space to emerge, what this study finds more important is creating an environment that will allow third spaces to surface more organically as students and teachers engage in the everyday life of the classroom.


Keywords: third space theory, teacher research, elementary mathematics, critical incidents

## Introduction

Q: Will school math help you later in life?
A: I don't need math. I'm going to be an [sic] doctor. (Tierra, ${ }^{1}$ survey, 5 July 2006)
Q: Three reasons math is important are:
A: for my future
for helping people [other students in class]
good job. (Washington, Survey, 5 July 2006)
Q: Do you see math in the world outside school?
A: No y [I] see brisd [birds] hoses [houses] scool [school] park tree grass cars. (Josef, survey, 5 July 2006)

The survey responses listed above told me much about my fifth graders. Like Tierra, many of my students saw mathematics as a school subject isolated from the rest of their lives. Others, like Washington, saw mathematics as useful to their futures but failed to make connections to their current existence. Most surprising, Josef seemed quite oblivious to the ways in which mathematics was used outside school. At first glance, it seemed that Josef was making light of the question above (Do you see math in the world outside school?); however, when I asked him to tell me more, he rationalized his answer by saying 'You do math, you don't see it.' I asked him if he does mathematics outside school, and he responded 'No, I play.' The fact that Josef and his peers were unable to see the connections between 'school math' and their lives outside school was worrisome, but not surprising.

This paper explores the ways in which teachers can create classroom environments that facilitate the development of third spaces in which, '... local practice, knowledge, and beliefs of both the local community and of the classroom and school community [are] brought to bear in everyday classroom practice’ (Gutiérrez, Baquedano-López, and Tejeda, 1999, 291). As a teacher-researcher and a graduate student examining the preparation of elementary mathematics teachers in the United States, I spent the summer of 2006 in a fifth-grade summer school classroom searching for answers to the question:

- How do I create culturally sensitive third spaces in which students can become mathematically literate?

As an action research study, this paper examines the ways in which I joined my students in constructing a classroom environment that enabled a social culture to develop in which third spaces could be fostered. By attending to notions of mathematical literacy and third-space theories, I attempted to analyze my teaching in hopes of connecting the worlds of educational theory and practice.

This study allowed me to systematically analyze interactions that took place in the classroom, to consciously reflect on the classroom community that evolved throughout the summer, to critically examine the ways in which I constructed activities for my students, and to carefully assess the results of my actions. It is my hope that this study brings the realities of classroom life to the page. Through this study, I hope to uncover ways in which theory can impact classroom life. Yet, more importantly, I hope to offer a genuine portrait of the events that occurred in one classroom as my students and I searched for answers to the question above.

The study that follows examines the context in which I taught, documents the data sources I employed as a teacher-researcher, summarizes and analyzes data I collected during my employment as a summer school teacher, and concludes with my reflections on what it means to create a classroom environment that fosters the development of third spaces in which students can work toward mathematical literacy.

## Context

After teaching elementary school for seven years, I decided to leave the classroom to pursue a career in teacher education. At the university, I have taken classes, I have read books, and I have written papers about a plethora of ideas surrounding teacher education; I have supervised studentteachers ${ }^{2}$ in local elementary and middle schools; and I have taught a mathematics methods course for pre-service teachers. After two years of studying and teaching at the university level, I began to feel the magnetic energies of the classroom pulling me back. I missed the vitality surrounding, the spontaneity of, and the ah-ha moments found within daily classroom life. Furthermore, through my coursework and teaching, I had changed many of my beliefs about instruction in the area of mathematics, I had taken the time to reflect on issues such as classroom culture and students' backgrounds, and I wanted to test my theories in an elementary classroom. For that reason, I applied to teach a fifth-grade summer school mathematics course in the Midwestern city in which the university I attend is located.

After interacting with and reading the work of several classroom educators (Madison Metropolitan School District 2005), and having read publications by several classroom-based and teacher education scholars (for example, Compton-Lilly 2003; Fecho and Allen 2003; Gallas 2003; Gutstein 2006; Vasquez 2004), I was deeply aware of the opportunities teachers could have to influence educational theories and policies. Whereas many educational researchers complete research on those within schools and present their findings for educators to digest and utilize, there are other ways in which research within schools can be envisioned (Evans et al. 1987; Falk and Blumenreich 2005; Zeichner 1995). As Hiebert, Gallimore, and Stigler observe, 'A significant alternative view claims that the knowledge of teachers is of a very different kind than usually produced by educational researchers' (2002, 3). Because of this, and because I believe that research conducted by classroom teachers provides alternative discourses ${ }^{3}$ within the educational research community, I decided to conduct a teacher-research project during the summer I was employed as a fifth-grade mathematics teacher.

## Site and participants

Located in a university town, Laketon School District ${ }^{4}$ educates approximately 25,000 students and employs nearly 3000 teachers. As one of the 250 largest school districts in the United States, Laketon faces a plethora of challenges similar to the nation's other large school districts (Anyon 1997, 2005; Cuban and Usdan 2003; Gittell 1998; Oakes and Lipton 1999). An achievement gap between white students and students of color, underfunded or unfunded mandates from the state and federal governments, and facilities in need of updating and repair are only a few of the challenges faced. Because of its commitment to its students, the district has created an extended learning program that it believes will assist students in continuing their educational growth throughout the summer months. A common mantra recited during a week of professional development prior to summer school and at several staff meetings was 'Summer school in Laketon is "highly researched and data driven."" Although the district officials I asked for access to the reports indicated that the research had happened, it was not readily available to teachers.

Nested within this district context, the school at which I worked, Salinger Elementary, educates students in grades Pre-Kindergarten through Grade Five. During the summer of 2006, children from four area elementary schools were bused to Salinger for the six-week program. Classroom teachers identified students in need of extra support at the end of the regular academic year. Students qualified for the mathematics program by receiving a score of one (emerging) or two (progressing) ${ }^{5}$ on their report cards in two of the following areas: solves story problems; solves addition, subtraction, multiplication and division problems; knows division facts and multiples of $2-10$ and $25 .{ }^{6}$

During the summer, I taught two sections of fifth-grade mathematics. Ten students were in my first-period class, and eight students were in my second-period class. ${ }^{7}$ Table 1 summarizes demographic information about individual students in the class. All information is self-reported by each of the students. Because I have been influenced by the writings of Zentella (2005) and González (2005), I am hesitant to classify students as 'Latino,' 'African American,' 'white,' or any other politically charged label. Rather than drawing stereotypical caricatures of my students, I present their self-descriptions in Table 1.

Table 1. Students' demographic information. Name,Race/ethnicity, Language(s) spoken at home First period, ,
Jessica, Ecuadorian, Spanish
Dreon, Senegalese, English/French
Isabella,White, English
Zachary,White, English
Antonio, Mexican, Spanish/English
David,White, English
Washington, Uruguayan, Spanish/English
Lois, Mexican/Asian, Spanish/English
Angel, German/Dutch, English
Alma, Mexican, Spanish
Second period, ,
Alex, Chicano, Spanish/English
Josef, Mexican, Spanish
Kavon, Black, English
Tierra,Black,English
Alejandro, Mexican, Spanish

In order to address ethical considerations, each student's parent/guardian was asked to sign a consent form permitting her/his child to participate in this study. The consent form was available in both English and Spanish, and it asked for permission to use a variety of data collection methods including videotaping the children and compiling classroom artifacts throughout the summer.

## Methodological considerations

As a teacher-researcher, I am sensitive to critiques about the validity of action research conducted in one's classroom. While many scholars, wary of non-traditional forms of research, question the knowledge created by teacher research (Fenstermacher 1994) or criticize the methods used in classroom-based inquiries (Huberman 1996), Cochran-Smith and Lytle argue that '... teachers' work ... represents ... one of the promising avenues that may lead to fundamental educational change over the next decade’ $(1999,22)$.

Because teachers are often seen as beings who 'suffer from knowledge deficiency’ (Eraut 1994, 54), I believe one responsibility of classroom-based professionals is to report events that have occurred in their schools, theories they have developed, and the results of their efforts (positive and/or negative) for the larger educational community. In doing so, teachers can document, share, and build upon the tacit knowledge they possess (Handal and Lauvas 1987).

In order to address the critiques of teacher research, I have done everything within my power as a classroom teacher to accurately report my data. In writing this paper, I heed Clay’s words: ‘... [T]he ethics of action research is really the merger of the ethically defensible actions of the practitioner and the ethically defensible actions of the researcher' (2001, 25). In the following sections, I describe the multiple data sources I used to attend to issues concerning the validity and reliability of this project (Lankshear and Knobel 2004): a teacher journal, videotapes of classroom discussions and lessons, district mandated pre-assessments and post-assessments, and other classroom artifacts.

## Data collection

During my time as a teacher of elementary mathematics, a variety of data were collected. Included in the data-set were entries in a teacher journal, videotapes of classroom interactions, district mandated assessments, surveys, and other classroom artifacts. Each of these sources is explored in the sections below.

## Teacher journal

Throughout my time in the classroom, I kept a journal that highlighted daily events, captured the words of my students (that might not have been documented in any other form), and recorded my reflections about individual students, the success or failure of particular lessons or activities, changes I intended to make in my instruction, and connections I was making between relevant
readings and theories. In writing this paper, my teacher journal was invaluable as I attempted to recreate moments from classroom life several weeks after they occurred.

## Videotapes

My most treasured data-set is the collection of videotapes recorded throughout the summer. With the help of a colleague who volunteered several times throughout the summer, I was able to focus my attention on the students as she assisted in the capturing of video documentation. Small group activities, classroom conversations, or my work with individual students were recorded on a regular basis. These recordings have proven extremely useful in that they are permanent documentation of the activities within the classroom. Transcripts from the videos have provided me with the opportunity to sort, code, and identify key conversations throughout the six-week program.

## District mandated pre-assessments and post-assessments

As part of my responsibilities as the teacher of record, I was required to administer pre-assessments and post-assessments to each of my students. Fact interviews, in which students were presented with a list of mathematical expressions (i.e. $5+1=\ldots$ ) were conducted with each student at the beginning and at the end of the summer. Additionally, a problem-solving assessment - drawing on the research of cognitive scientists such as Griffin and Case (1997) and Carpenter et al. (1999) was designed by mathematics resource teachers within the district. In this assessment, students were presented with a variety of word problems and asked to show two strategies that they could use to solve each problem. Every child completed this assessment twice during her/his time in my class. While the mathematics resource teachers lament the fact that the many intricacies of problem-solving have been reduced to numerical values, the assessments - if carefully inspected by the classroom teacher - provide a plethora of knowledge for practitioners. Furthermore, the numerical scores recorded on report cards and in district data files present educators within the district with a general idea of a child's ability to comprehend contextualized problems and to utilize appropriate strategies as he/she solves the problems with which he/she has been presented.

## Surveys

During the first half of the summer, I attempted to gain insight into my students' feelings about, and understandings concerning, mathematics. To do so, I created a survey that asked several true/false questions (with additional room for students to explain their answers) as well as several open-ended statements for students to complete. The survey has been included in Appendix 1.

## Classroom artifacts

In addition to the above data sources, a variety of other artifacts was collected throughout the summer. These artifacts included student worksheets, chart paper tablets documenting students' strategies volunteered during small-group and whole-class sharing sessions, overhead transparencies used during mini-lessons and class conversations, as well as other, miscellaneous items. While many of the items simply took up space in my files, several of them provided unexpected, little-remembered pieces of evidence that support some of the claims I make in this paper.

## Analyzing the data

Throughout the summer, and for several weeks after summer school, I analyzed the data gathered. At various points throughout my time in the classroom, I was completely overwhelmed with the amount of data and considered abandoning the project. These feelings were fueled by my concerns that I was focusing too much on theory, that I was selfishly pursuing my own agenda, and that time could be better spent constructing lessons for individual students. However, the readings on third space helped to justify my research and provided a way in which I could focus the work so that I could use my own personal learning experiences to affect the educational opportunities I created for - and with - the children. Furthermore, I found comfort in the words of Glesne, who encourages her readers to push forward:

Unlike a squirrel hoarding acorns for the winter, you should not keep collecting data for devouring later. Rather, examine your data periodically to insure that your acorns represent the variety or varieties desired, and that they are meaty nuggets, worthy of your effort. (Glesne 1999, 133)

With these words as inspiration, I stepped into my data. I began by scouring my teacher journal for entries related to classroom culture. After identifying several key moments throughout the semester, I turned to the videotapes for verification of these moments. When videotapes were not available, I examined students’ work, my kidwatching notes, and other classroom artifacts. By sifting through these data sources, I was able to identify several critical incidents (Hole and McEntee 1999; Loughran, Mitchell, and Mitchell 2002) that addressed issues of classroom community and moved my instruction and interactions with students toward the creation of third spaces.

These critical incidents allowed me to think about the order in which I would present my findings. By creating a table to document the data I would present, I was able to see where the data interacted with, contradicted, and/or supported each other. By triangulating my data sources (Falk and Blumenreich 2005; Gall, Gall, and Borg 2003), I worked to '... establish converging lines of evidence to make [my] findings as robust as possible’ (Yin 2006, 115). Although action research is admittedly subjective (Anderson, Herr, and Nihlen 1994), I nonetheless drew on a variety of events and data sources to create a mosaic that accurately represented the life of the classroom. By creating themes that grew out of my coding schema, I systematically prepared to present my findings.

While this methodological process has been laid out in linear fashion - identification of critical incidents, systematic analysis of the data, development of themes, and then the presentation of my findings - it is imperative that the reader understands the messiness of the action research process (Cook 1998; Zeichner 1999). In no way do I intend to imply that the analysis of data was a seamless, coherent progression that simply fell into place. Instead, I frequently found myself returning to the data to validate claims, check the accuracy of my interpretations, rethink codes, and add or eliminate a theme.

As a result, this paper is quite incomplete. It represents only those critical incidents that address the question on which I chose to focus. The paper illuminates situations that meshed with the
themes of mathematical literacy and learning in the third space. A wide variety of issues that presented themselves repeatedly throughout the summer, and in the data, have been ignored in this paper - especially notions of adolescent identity formation and the constraints of a mandated curriculum - because they simply did not fit the major themes I decided to pursue in ways that I felt comfortable presenting. Having said that, I believe the following sections accurately portray my reflections and actions, as well as my students' reactions, throughout the summer as we worked toward developing third spaces in which mathematical literacy could be pursued.

## Exploring the critical incidents

In an effort to examine the question - How do I create culturally sensitive third spaces in which students can become mathematically literate? - it was imperative that I settle on a definition of third space. Building on the work of Bhabha (1990, 1994), Soja (1996) and Gutiérrez and her colleagues (Gutiérrez, Rymes, and Larson 1995; Gutiérrez, Baquedano-López, and Tejeda 1997, 1999), Moje et al. note that:
[A] third space focused on cultural, social, and epistemological change ... is one in which everyday resources are integrated with the disciplinary learning to construct new texts and literacy practices, ones that merge the different aspects of knowledge and ways of knowing offered in a variety of different spaces. $(2004,44)$

This conception of third space influenced my thinking as I sought to assist students in the development of mathematical literacy throughout my time with them. Moje and her colleagues make it clear that third spaces require teachers to integrate students' everyday resources, experiences, and ways of knowing into daily instruction. This definition aligned well with my philosophy of teaching and became a reflective focal point as I planned my instruction and interactions with the children. As I will discuss later, it became quite clear that the more I planned for a third space, the less successful I was. Rather, creating an environment in which third spaces could spontaneously develop was much more productive. In the following sections, I highlight several critical incidents that describe the ways in which I created an environment that would set the stage for the development of a third space.

## Creating an environment for the pursuit of a third space

As my first priority, I have always attempted to create an environment in which students feel safe, respected, and confident in their abilities as learners. In addition, it is imperative that all students are contributing members of the classroom community. Wheatley and Reynolds note 'It is vital that the teacher negotiate a set of social norms which foster curiosity, creativity and sense making ... [T]he teacher can play a vital role in negotiating, not legislating, a learning environment which encourages students to become mathematically powerful’ (1999, 32). Teaching summer school presented an interesting challenge in that we were given only six weeks to establish a community of learners, create relationships that were constructive and reciprocal, and build trust and a sense of security amongst all members of the classroom.

To further complicate the situation, the students with whom I worked came from four different elementary schools, and I received their names and telephone numbers only four days before they
entered the classroom for the first time. I had fantastical visions that I might have the time and opportunity to visit each child's home, meet the parents, and talk with her/his teacher prior to the start of summer school. Unfortunately, this was not the case. Furthermore, although I was able to make telephone contact with several of the students' homes, even communicating via telephone proved difficult. Because I am monolingual, I was forced to relay messages through siblings, cousins, and the students themselves. Rarely was I able to make direct contact with parents. This was disheartening and disappointing for me as I had hoped to create some sort of network of trust and understanding prior to the first day of summer school.

Because of these difficulties, I knew that during the first few days I would have to put mathematics on the back burner as the students and I got to know and understand one another. While this was true, I was amazed with how quickly the mathematics came into play. In the following sections, I present a progression of activities and events that contributed to the classroom environment and community that was developed throughout the summer.

## Me Posters

Several activities and ideas helped me as I worked with my students to create an atmosphere that fostered the development of third spaces in our mathematics classroom. On the first day, I had each child create a 'Me Poster' to share with the class. The posters were collections of words and pictures that represented the interests of, and influences on, each child. The posters were full of ideas that would prove useful later in the summer. The students shared information about family members, pets, hobbies, sports, music, travel, and food (Me Posters, 19 June 2006).

The students had little trouble constructing the posters highlighting their interests. However, when it came time to share their creations with the rest of the class, few volunteers emerged. It became clear that a new process was necessary. I quickly changed the method for sharing. Rather than telling the group about herself/himself, each person would simply place her/his poster in the middle of the circle for the class to observe. After looking at the poster for a few moments, students asked the poster's creator questions about her/his work. Once the discussion started, the students became more comfortable talking about themselves. Although many were still shy, the dialogue began to flow a bit more readily.

Beyond simply introducing the students to one another and myself, the Me Posters provided me with a wealth of knowledge about my students. The posters allowed me to instigate conversations with individuals and small groups. Furthermore, the Me Posters presented me with the background information necessary to construct problem-solving activities that directly related to the students, their interests, and their 'funds of knowledge' (González, Moll, and Amanti 2005).

## 'Hey, that's me': creating effective problem-solving activities

Based on the criteria for recommendation into the program, I knew the types of problems with which my students would have difficulties. Furthermore, the pre-assessment created by the district allowed me to hone in on the ways in which my students attempted to solve the given problems and the knowledge they already possessed. Using these two data sources (the criteria for recommendation and the pre-assessment) as my guides, and the students' Me Posters as the lens
through which I was beginning to see, I constructed problems that drew on the students' interests while focusing on their areas of need.

A text I use with my pre-service mathematics methods class, Making sense: teaching and learning mathematics with understanding (Hiebert et al. 1997), provided me with guidance as I constructed mathematical tasks for my students. In their work, the authors note that the nature of classroom tasks must deal with three essential features. First, they must make the mathematics problematic. That is, the tasks must create within the student the will to find a solution. Secondly, the tasks must connect with the students' understandings and experiences. Hiebert and his colleagues note 'Students must be able to use the knowledge and skills they already have to begin developing a method for completing the task' (1997, 8). Finally, the tasks must engage the students in thinking about the mathematics. Without inspiring a commitment to do the work, the tasks are simply mandates from a higher power.

As stated previously, the Me Posters and the district-mandated pre-assessment provided me with information about my students as I constructed the problem-solving activities at the start of the summer. I knew that the tasks would catch the attention of my students if they (the students) were the subjects of the problems, but one interesting critical incident came early in our time together. Below, I relay the moment as captured in my teacher journal:

Before school today, I posted the problem we'd be discussing on chart paper. Based on a discussion we'd had about Zachary's ${ }^{8}$ fishing trip this coming weekend, I constructed the problem:

Zachary and his cousin went fishing. Zachary caught $\qquad$ fish. His cousin caught $\qquad$ fish. How many more fish did Zachary catch than his cousin?
$(18,12)(23,16)(64,38)(104,87){ }^{9}$
As the class walked in, several of them stopped to read the problem. Most of them simply read the words and then moved on. However, Zachary saw the problem and immediately announced to the class, 'Hey, that's me!'

For me, that moment alone validated the work I do as a teacher! All of the kids enjoy seeing their names in the problems. They love to discuss the ways the problems could be true or the ways the problems could become ridiculous (usually by someone bragging about how many points they can score or how much money they spend). However, Zachary has taken a while to break out of his shell. To see his excitement about a math problem was extremely gratifying. It makes the extra effort of creating new problems each day, for each class, completely worth it. (Teacher journal, 30 June 2006)

## Dreon and his ratio tables: sharing strategies

One of the characteristics of problem-based classrooms that teach for understanding (Carpenter et al. 1999) is the use of class discussions. These discussions allow students to share their understandings, explore the work of their peers, and engage in a reciprocal dialogue with fellow students. Carpenter et al. state:

After the teacher has selected a problem, posed it to the children, and waited a suitable period of time, she or he asks the children individually to report how they solved the problem to the class or a small group. When one child has reported a solution strategy, the teacher asks another child to report how he or she solved the problem. The teacher may ask how the two solutions are alike or different and if anyone solved it in a different way. (1999, 88)

In contrast to traditional classrooms in which mathematics is copied or memorized, classrooms in which reform mathematics (Van de Walle 2004) is practiced encourage students to communicate their understandings, explore novel ideas, and construct new meaning. This type of mathematics instruction may be new, and intimidating, to many students who have enjoyed the anonymity of more traditional classrooms. In contrast to traditional notions of mathematics instruction, Van de Walle writes:

The teacher's role is to create [a] spirit of inquiry, trust, and expectation. Within that environment, students are invited to do mathematics ... The focus is on students actively figuring things out, testing ideas and making conjectures, developing reasons and offering explanations ... Reasoning is celebrated as students defend their methods and justify their solutions. $(2004,14)$
his 'spirit of inquiry' showed itself many times throughout the summer. Another critical incident rests in Dreon's use of ratio tables. On the survey I had students complete, Dreon responded to the statement ‘School math is the only kind of math I know' by checking 'false’ and stating 'I learned how to do multipacation [sic] outside of school. My cousin taught me ratio table’ (Dreon, survey, 5 July 2006).

Interestingly, while Dreon reported on his survey that his cousin had taught him about ratio tables, the pre-assessment completed at the start of the summer showed that Dreon was struggling with when, and in what context, to utilize this knowledge. On the pre-assessment, Dreon attempted to use a ratio table to solve a Separate Change Unknown problem (Carpenter et al. 1999). In such a problem, an action takes place and an initial quantity is decreased. In the particular problem Dreon was attempting to solve, a girl had collected 509 shells at a beach. She then gave some of the shells to a friend. Given the fact that 49 shells remained after this action, students were asked to compute the number of shells given to the friend. Typical responses to this type of problem include those presented in Figure 1.

Figure 1 Responses to a Separate Change Unknown problem.
Standard Algorithm -
410
\$609
$-49$
Solution: 460 shells were given to the friend. (Isabella, pre-assessment, 22 June 2006).

Empty Number Line -

(Shells Remaining = 49)
(Initial Quantity $=509$ )
Jumps on Number Line $\rightarrow 400+50+1+9=460$
Solution: 460 shells were given to the friend. (Washington, pre-assessment, 23 June
2006)

## Direct Modeling -


[In this example, the stadent drew 5 hundreds (indicated by large squares) and 9 ones (indicated by small squares). She then 'traded' 1 hundred for 10 tens (indicated by long rectangles). She then set 49 , the number of shells remaining, aside (indicated as shaded). The 460 that remained (unshaded shapes) were the shells given to a friend in the problem.] (Katie, pre-assessment, 22 June 2006)

In contrast to these appropriate strategies that assisted students in achieving correct solutions, Dreon attempted to use a ratio table. While a ratio table is a wonderful strategy in many situations, it cannot be applied to this situation. Dreon began his attempt, and soon realized his error. His work contained the table shown in Figure 2.

Figure 2 Unsuccessful use of a ratio table.

| 1 | 2 | 3 | 4 |  |
| :---: | :---: | :---: | :---: | :--- |
| 49 | 96 | 145 | 194 |  |

While Dreon was unable to use the ratio table in this instance, he kept searching for ways to utilize his knowledge. A few days later, the opportunity presented itself. I offered the class the following problem:

Washington has some soccer balls. Each ball is made of 36 hexagons. He counted 144 total hexagons. How many soccer balls does Washington have?

In response to this problem, Dreon created the ratio table shown in Figure 3. In this instance Dreon uses a ratio table to help organize his thoughts as he uses information he is given (one soccer ball is made of 36 hexagons) and then finds the mathematical patterns that will assist him in achieving the correct solution (four soccer balls). When his solution was discussed later in class, Dreon was very excited. That is, until his words met blank faces. Rather than pushing this strategy, I simply alerted the class to the fact that this was a strategy with which they could experiment, winked at Dreon to validate his efforts, and moved on. However, Dreon was disappointed. His understanding of ratio tables was solidifying internally, but he was unable to celebrate this idea with any of his classmates. Although we had discussed his work and he was validated for his efforts, his peers did not share his enthusiasm.

Figure 3 Successful use of a ratio table.

| 1 | 2 | 4 |  |
| :--- | :---: | :---: | :--- |
| 36 | 72 | 144 |  |

## Solution: 4 soccer balls he had. (Dreon, classwork, 26 June 2006)

What happened next was quite surprising. As the next class came into the room, they looked at the chart paper I had left on display accidentally. Oscar was the first to notice the ratio table, and he immediately asked me to explain it. Knowing that this new group of students was going to be solving a similar problem, I told the group that we would look at the work if we had time later in the period. Although I had forgotten to change the chart paper before class, I purposely left it on display at this point to see what would happen.

As the class was working on their problems for the day, Oscar continued to peek at the chart paper. When presented with a problem in which a ratio table could be used, he took a long look at the chart paper before deciding to use another strategy. However, during class discussion, Oscar attempted to use the strategy. As he fumbled, his classmate, Katie joined him in his attempt to make sense of the strategy. I allowed the discussion to progress until most of the class became disenchanted with their efforts. At that point, I asked the two of them to continue thinking about the strategy for future use.

The following day, I alerted Dreon to the attempts the other class had made in using a ratio table. He smiled, but still seemed disappointed that he was unable to share this strategy with his own classmates. Later that day, while working on their classwork, both Oscar and Katie successfully implemented a ratio table (Oscar and Katie, classwork, 27 June 2006). In our class discussion, both of them were able to construct a ratio table for the following problem:

In order to win his video game, Josef needs to score 300 points. If he wins 8 games, how many points will Josef score?

Each of their worksheets show correct computation through ratio tables (Oscar and Katie, classwork, 27 June 2006), and the chart paper from 27 June shows the ratio table constructed during class discussion (Figure 4).

Figure 4 Ratio table constructed during class discussion.


## (Chart paper tablet, 27 June 2006)

As the summer progressed, more and more ratio tables appeared. On 28 June, four students in the second-period class were using ratio tables correctly (Oscar, Alex, Katie, and Alejandro, classwork, 28 June 2006). A week later, these same students continued to use the strategy effectively. Unfortunately, Dreon remained the only student to utilize the ratio table strategy in my first-period class.

However, the shock of the summer came as students completed the post-assessment packets during the final days of school. In completing the assessment, David - one of Dreon's classmates in my first period - used a ratio table to correctly compute the answer to a problem. I was shocked. I immediately returned to my file of completed classwork to search through David's papers. On no occasion had David previously employed this strategy! The tablet of chart paper shows this strategy being presented as a possibility on multiple occasions throughout the semester (chart paper tablet, 28 June 2006, 29 June 2006, 7 July 2006, 10 July 2006), but David had never been the one
to offer the strategy. Dreon was successful! Even though his peers never seemed to validate his knowledge, his repeated attempts to inform his classmates had finally produced results.

## Synthesis

While the previous sections are not examples of third spaces in themselves, they capture the type of environment the students and I developed as we worked together in the classroom setting. I believe that this type of environment is essential to - and a precursor of - the development of third spaces. By encouraging students to take risks, to experiment with new ideas, and to learn from one another, I attempted to create an environment in which mathematics could be seen as a social process. My efforts were fueled by my desire to identify third spaces that would allow students to utilize the wealth of knowledge they had gained in other settings (at home, in their communities, through prior school experiences, etc.) as they navigated the waters of summer school mathematics.

Whether completing and sharing Me Posters, envisioning themselves in contextualized mathematical situations, sharing strategies for solving problems, or capitalizing on these conversations in an individual setting, the students became part of a mathematical community throughout the summer. This is not to say that everything ran smoothly on a daily basis. Frequently, there were identity struggles, issues with cliques, and disenchantment with the mathematics. However, by continually accessing students' knowledge and asking them to share their expertise, an environment of collaboration and exploration was achieved. In the following section, I explore another critical incident that occurred as my students and I continued our mathematical pursuits. Unlike the previous snapshots of classroom events, this critical incident does in fact exemplify what I consider to be a third space.

## The emergence of a third space

In my studies as a graduate student, and in my work as an instructor of mathematics methods, I am surrounded with educational theory. I am constantly reading and writing about - and creating assignments for my pre-service students that incorporate - educational theory. However, I am very aware of the disconnect between educational theory and classroom practice. This study is my attempt to connect the worlds of theory and practice. Rather than subscribing to the false dichotomy that separates theory and practice, this study merges the two by utilizing practitioner research to document ways in which educational theory can come to life in actual classrooms. By examining the literature on third space, and by purposely seeking out instances of those spaces in my classroom, it is my hope that this study accomplishes the stated objective of melding theory and practice.

## Disparejo: experiencing a third space

Throughout the summer, students played a variety of games in which they experimented with mathematical concepts. Often, these games were played in pairs. When asked 'Who should go first?,' I generally responded by having students play 'rock, paper, scissors' to determine who should make the first move in the games.

One day, as I was working with a pair of students, I noticed a group of three students doing some sort of hand jive at their table. They were not playing the game they had chosen, and they were not utilising rock, paper, scissors to identify the order of play. I simply asked what was happening, and one of the students commented 'We're playing Disparejo.' The look on my face must have alerted the students to my confusion because a second student replied, 'There's three of us.' This statement was presented as if it clarified everything. Unfortunately, I had no idea what they were talking about.

Rather than redirecting them to the mathematical game, I simply confessed my ignorance. The explanation that ensued was fascinating. Oscar told me that Disparejo was a game from Mexico. Because rock, paper, scissors only works if two people are playing, Disparejo is a game used to whittle the competition down to those two players. Disparejo ${ }^{10}$ is played when each child puts one fist forward. They chant 'Dis-pa-re-jo' in unison. When the syllable 'jo' is verbalized, each student places a thumb up, down, or to the side. The children whose thumbs are in the majority 'win' and continue to play. Anyone outside of the majority is eliminated. Play continues until only two players remain - at which point rock, paper, scissors commences.

Rather than seeking my input on how rock, paper, scissors was to be played with more than two players, the students accessed their collective funds of knowledge in order to meet the demands of the activity. In addition, by explaining the activity, they created a learning opportunity for me, the teacher. I was extremely proud of the ingenuity shown by the students, and I felt extreme gratification in knowing that the children possessed the confidence to solve problems by utilizing resources with which they were familiar.

Several days later, Disparejo allowed a potentially disastrous situation to become (what I see as) an authentic third space in which the class could utilize knowledge from outside school to facilitate an activity in the classroom. The previous day, Oscar had worn a t-shirt with an image of Che Guevara on the front. After a short discussion, I realized that the students were aware of Che Guevara, but their knowledge about his life and work was quite minimal. 'Si, se puede' was a mantra with which all students were familiar (all of the children were able to discuss recent events concerning illegal immigration ${ }^{11}$ ), but none of the students could elaborate beyond the point of attributing this phrase to Che Guevara.

Taking a page from Gutstein's (2006) work - literally - I presented an activity that dealt with a critical issue I felt needed to be discussed. Because many of my students came from minoritized backgrounds, I wanted to explore the term 'minority' with them through a data-analysis project. In his work, Gutstein presents a picture of a mural in East Los Angeles that depicts Che Guevara and the phrase 'We are not a minority!!’ The activity draws on world population statistics to prove that whites are not a majority. In fact, the label 'minority' has been used by whites throughout our nation's history to disempower large factions of the population.

In preparation for this activity, I asked the students to tell me what they knew about the term 'minority.' My query was greeted with blank faces. This presented a problem. Without a working understanding of the term 'minority,' the activity would be a complete failure. I quickly thought up a short activity that might help facilitate the discussion. I asked the students to form a circle.

Once organized, I asked them to play a game of Disparejo. After each round, I stressed the words 'majority' and 'minority' as students were eliminated and play continued. After the game, I asked students how playing Disparejo related to the lesson. Immediately, responses were recorded on individual students' papers. From there, the lesson proceeded.

Moje et al. remind us:
It is important to acknowledge the many different funds of knowledge (Moll, Veléz-Ibañéz, \& Greenberg, 1989) such as homes, peer groups, and other systems and networks of relationships that shape the oral and written texts young people make meaning of and produce as they move from classroom to classroom and from home to peer group, to school, or to community. It is equally important to examine the ways these funds, or networks and relationships, shape ways of knowing, reading, writing, and talking ... that they use to try to learn in ... schools. $(2004,38)$

Had I not been privy to the students' use of the game Disparejo earlier in the summer, I would not have had the opportunity to incorporate this knowledge into a class lesson. Because I was aware of the game and my students' excitement about it, Disparejo facilitated the development of a genuine third space in our classroom. Rather than attempting to define terms in the abstract for my students, I was able to participate in a contextualized examination of mathematical vocabulary with them.

## Discussion

At the start of the paper, I asked the question 'How do I create culturally sensitive third spaces in which students can become mathematically literate?’ In the instance above, typical classroom interactions occurred. In the past, I might have missed the opportunity that the game Disparejo presented. I might have overlooked the value of the mathematical knowledge present in informal exchanges between students. However, through my reading of theory, I have had the opportunity to reflect on the idea of third space. As a practitioner, I am influenced by these readings, and I purposely set out to identify third spaces within the classroom context. Action research enabled me to take on this task. Without the theory, these moments may have been missed. Without attention to practice, my reading of theory would have remained abstract and decontextualized.

If we return to Moje's definition of third space, the game Disparejo and its impact on the 'We are not a Minority!!' lesson shows how students' everyday resources can be integrated into their learning within formal educational settings. Following Moje and her colleagues, I have attempted to illuminate one example of how '... new texts and literacy practices ... merge the different aspects of knowledge and ways of knowing offered in a variety of different spaces’ (Moje et al. 2006, 44). Through this study, I was able to examine my practices as an educator. By attending to my students' cultural knowledge and by incorporating unconventional/informal mathematical practices into my lessons, I was able to rethink my definition of mathematical literacy. This study taught me to look beyond the things I am used to seeing as a mathematics teacher, to analyze my students' mathematical literacy in new ways.

It is important to note at this point that Disparejo was spontaneously utilized as a way to clarify a vocabulary term with which the students were unfamiliar. It was not until later that afternoon while writing in my journal that I realized I was dealing with an example of third space. Furthermore, my earlier attempts to 'create' third spaces were less than successful and frustrating for me. The children, through the game Disparejo, taught me that a classroom community built to focus on learning - rather than teaching - provides the opportunity for third spaces to develop. While this study was an opportunity for me to examine and improve my teaching, it was not until I stopped trying to create third spaces that one actually developed. Philosophically, I believe that third spaces can be achieved; however, as a teacher, it may be more realistic to simply attempt to create the conditions in which third spaces can be realized.

For me, the data sources used and the critical incidents examined highlight the ways in which cultural sensitivity can create opportunities for third spaces to develop. By sharing their knowledge (i.e. Dreon's ration tables) and interests (Me Posters), the students allowed me to create problemsolving activities that were relevant and contextualized. Through these activities, a classroom culture of sharing knowledge - whether sanctioned by the official curriculum or not - was developed. Because of this, the students and I were able to rethink ways in which home knowledge and school knowledge could be integrated to form powerful learning opportunities. The development of a third space through the game Disparejo highlights one way in which a third space was utilized to facilitate the acquisition of literacy in one area of mathematics.

This study provided me with the opportunity to explore issues of mathematical literacy and the development of third spaces in depth. As I conducted the study, third-space theory came to life, and my practice improved. As a result, the expertise and life experiences of my students were validated, the disjuncture between theory and practice was addressed, and my beliefs about the importance of documenting the work of classroom practitioners were solidified.

## Conclusions

For me, this paper has been more than a simple description of classroom events that fit neatly into a model of mathematical literacy. Zeichner states that one indicator of quality in self-study work is addressed when we ask ' $[\mathrm{H}]$ ow does [this] study build upon existing work in an area $\ldots$ and make a contribution to our knowledge about that area?’ (2007, 38). Working from this assumption that all educational research should build on, and contribute to, the greater body of knowledge, I have constructed a practitioner research study that was influenced by the educational literature base. Furthermore, it is my hope that this study has added to the literature by documenting the complexities of third spaces in actual classrooms. By attending to and utilizing third spaces, teachers can assist students in their pursuit of mathematical literacy. On the other hand, without attention to the possibilities that can generate third spaces, teachers ignore valuable resources that may facilitate the attainment of mathematical literacy - or any other school success.

In an educational arena that promotes competition over cooperation, correct answers over correct thinking, and the filling of bubbles over the fueling of passion, third spaces provide educators with opportunities to instigate change. If we truly wish for every child to become mathematically literate, third spaces must be nurtured in every classroom. Until every child recognizes the 'real
world’ application of 'school math,' this work must continue. This study is one small step, but it is buttressed by lessons learned through many stumbles along the way. I hope that others will join me as I continue my journey.

## Appendix 1



## Notes

1. All names are pseudonyms chosen by the participants.
2. I use the term 'student-teacher' to represent any uncertified, pre-service teacher who is pursuing certification through the university.
3. See Compton-Lilly (2007) for an in-depth discussion of alternative discourses and how they apply to teacher research.
4. The district and school names have been changed for the purposes of confidentiality.
5. Scores of three (proficient) and four (advanced) are other evaluative ratings teachers can assign to students on the report cards.
6. Retrieved from district web page, 12 June 2006.
7. For the first week of summer school, Kavon was the eighth member of the second-period class. After moving to another area of town, a bus service was no longer available for Kavon.
8. Again, all names have been changed for purposes of confidentiality. The names used in this report are pseudonyms chosen by the students themselves.
9. Each problem I created offered multiple number choices with which the students could work. Rather than placing the numbers in the problems, I listed the number choices below for the students to insert into the problem. In doing so, the children were asked to pick 'just right' numbers numbers with which the students, themselves, felt they could be successful. Rather than requiring all children to use the same numbers, this process allowed me, as the teacher, to assess the various ways in which the students were approaching the problem rather than monitoring their work for correct answers. Furthermore, with each set of numbers, I attempted to assess a specific mathematical concept. Students came to realize this as we discussed why certain numbers were chosen. Rather than looking at the size of the numbers, we also discussed the different concepts the various numbers addressed. For example, by choosing the numbers 104 and 87, I was trying to push the students who consistently resorted to the traditional subtraction algorithm but had difficulty ‘subtracting across a zero’ to find alternative ways to approach this problem.
10. The Spanish word 'disparejo' roughly translates to the English terms 'uneven, unequal, disparate, and/or different’ (http://education.yahoo.com/reference/dict_en_es/ [accessed 10 August 2006]).
11. For further information on the upheaval surrounding proposed legislation concerning illegal immigration, please see del Barco (2006), Hendricks (2006) and Swarns (2005).

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