## ENSNARING THE ELUSIVE EODERMDROME

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We wish to introduce the recreational aspects of the eodermdrome (e-o-derm'-drom), which is a recently formulated concept dealing with the structure of language units such as letters and words. Although this concept may eventually lead to a numerical way to compare the structural differences between languages and to trace structural development in a single language (for details, see the first and third references), the majority of individuals who learn about eodermdromes evince less interest in their potential scholarly ramifications than in the delightful task of creating them. We find that no sooner do many bright and playful people learn what we are seeking, than they plunge into the search to discover new and better and more amusing ones. This fact is encouraging to us. After all, we naturally are enthusiastic, but the efforts of our wives, girl friends, other friends, colleagues, and acquaintances buoy us irrepressibly. The more sobering side to our efforts is our recognition that other people inveitably discover eodermdromes that are more clever than ones we found. Although we are properly humbled, nevertheless, we are not only undaunted, but we avidly call attention to improved specimens whenever found (giving proper credit, of course). A number of examples are included in this article. We wish unabashedly to solicit further examples from this readership, who will indubitably put our faltering beginnings to shame. Send contributions to the editor as well as to Gary Bloom, Computer Science Department, The City College of New York, New York, NY 10031. We will gladly make copies of the references and future reports available to those who are interested.

Because of our unusual orthographic needs for the word, we confess to giving 'eodermdrome' a mixed etymology enlisting the aid of the Compact Edition of the Oxford English Dictionary (1971). There we found that 'eode' is an Old English past tense form of go. 'Erme' is an obsolete verb from Old English whose transitive form means to make miserable, harass, vex and whose intransitive or reflexive form means to grieve or make sorry. Einally, 'dromos' is, of course, Greek for race course. Thus, one may view an eodermdrome as a course on which to go to be made miserable.

Unlike the name 'palindrome' which is itself not a palindrome, we chose the name 'eodermdrome' because it is an eodermdrome. We
will first explain the properties involved in defining eodermdromes and then give their definition. In the eleven-letter word 'eodermarome', there are five distinct letters. Write these five letters in a circle, and draw a line from the first letter, $E$, to the second, $O$, as is shown in the first part of the figure below. Without lifting the pencil from the paper, continue the line to the third letter, $D$, as shown in the second part. Continuing to the next letter in the word returns you to $E$ and completes a triangle as shown in the third part. Still without lifting the pencil, trace out the remaining sequence of seven letters. The final line is traced from $M$ back to $E$ and closes the path, so that the final diagram is shown in the fourth part. There are properties of this diagram that we shall see make 'eodermdrome' an eodermdrome.


Not every diagram obtained by assigning letters in a word or phrase to points and by connecting them as indicated above results in an eodermdrome. In general, any such diagram is termed a spelling net. For example, the figure below shows the spelling net for the phrase 'spelling net'. Some differences between the two figures are immediately apparent. The double $L$ in 'spelling net' gives rise to a self-loop; there is also a double line because NG, the same letter-pair as GN, occurs twice in the phrase. (Note that blanks between words are ignored.)


There is another obvious difference between the two figures: the latter is drawn with none of its lines crossing each other, whereas the former has multiple line-crossings, five of them to be exact. It is a less obvious fact that this figure cannot be redrawn in any way that shows the same letter neighboring pairs with no line-crossings. The figure to the right shows the best redrawing of the spelling net in which just one line-crossing occurs. If the line between the solid nodes were removed, no lines would cross. If a spelling-net cannot be redrawn in some way so that no pair of lines crosses, it is called non-planar. An eodermdrome is defined to be a nonplanar spelling net.

Although we are interested in all eodermdromes for linguistic purposes, we have thus far concentrated on minimal eodermdromes for our recreational pleasure. By this term, we
 mean an eodermdrome that has as few lines in it as possible. Clearly, such an eodermdrome has no selfloops or multiple lines since these can always be removed without destroying the non-planar property. It can easily be shown that a spelling net with five distinct letters which contains all ten pairings of the five letters exactly once, as in the word 'eodermdrome', is the uniquely smallest eodermdrome. In the remainder of this article, we will concentrate solely on the minimal eodermdrome. We will examine what eleven-letter phrases can be constructed in terms of the allowable patterns, and we will indicate what various people have achieved so far. We will also make comments setting these recreational objects into relationship with the somewhat similar concepts of anagrams and palindromes.

## Minimal Eodermdromes

Although we are not qualified to comment on its artistic merit, or lack thereof, we present the following poem, each of whose lines is comprised of a minimal eodermdrome:

TEARS AT REST

Stray satyrs,
Dense and sad, Tip tan paint

Teaser's tart
Pursues prep...
Yearly relay.

Sweat wastes.
Science sins...
Ah... rather tea.

Credit to the authors of individual lines is given later; the responsibility for assembling and ordering these lines is solely ours (blame us, not them, if the muse is bruised).

To be sure, each line generates the same spelling net, consisting of the five distinct letters so arranged that each of the possible ten letter
pairings occurs exactlv once. Moreover, when we trace through the phrases to generate their spelling nets, it is apparent that each phrase must start and end with the same letter, which must also occur one further time in the phrase. In generating the remaining lines in the net, it is clear that of the four lines connecting the non-initial letters to their neighbors, two must come from other letters in the tracing and two must lead to other letters in the tracing. Consequently, there are two occurrences of each of the non-initial letters in a minimal eodermdrome.

Despite these similarities, minimal eodermdromes do differ from each other structurally. To see this, we compare the formation of a few of the lines in the poem. Let us number the distinct letters for an eodermdrome in the order in which they are used. Thus, in the first line of the poem $S=1, T=2, R=3, A=4$ and $Y=5$. Consequently, this line can be represented numerically by the order in which its letters appear: stray satyrs / 12345 142531. Now, look at the letter numbering sequence for the sixth line of the poem: yearly relay / 123451 42531. This is the same sequence as before. On the other hand, consider the third line: tip tan paint/12314534251. This is a completely different ordering for tracing through the letters. For clarity, we recommend that the reader actually write the integers 1 through 5 in a circle and draw the lines between the numbers as they would be generated by tracing the diagrams in the given orders. No matter how the numbers are placed on the paper, the intrinsic differences between the patterns is obvious. As shown in the figure below, 'stray satyrs' and 'vearly relav' have patterns that start by drawing a pentagon (1-2-3-4-5-1), whereas 'tip tan paint' starts with a triangle (1-2-3-1).


It is important to keep in mind that the numbers indicate the precise order in which the letters are traced through, or alternatively, that they serve as an exact encoding of the letters once the numbers are as signed. Consequently, an apparent minor modification of 'stray satyrs' gives a very different sequence, even though it also is an eodermdrome: satyr strays / 12345135241 . Although both this example and the earlier one start with pentagons, the letters that occur after the pentagon is formed are traced in a different order. That is, if the initial pentagons for the two phrases are drawn identically (say, clockwise with none of the lines crossing), then the star figures inside the pentagons will be traced out in different orders for 'stray satyrs' and 'satyr strays'.

Incidentallv, it is interesting to note that the pattern for ${ }^{1}$ tip tan paint' is identical to that for 'eodermdrome'.

If one traces through the lines of the poem, one discovers other patterns. In fact, there are 22 possible letter-tracing patterns for minimal eodermdromes. All 22 patterns are presented in the table below, along with some of the best examples of which we are currently aware and the discoverers of those examples. The order in which the patterns are presented is lexicographic, starting with 12314253451 and ending with 12345315241.

12314253451 get great rag (PW,IW), sin sciences (JK), ah... rather tea (GB)
12314254351 ah...tar her tea (GB)
12314352451 hot hat coach (PB), and at den, tea (DW)
12314354251 Tarters east (GB)
12314524351 rear pie pair (PW)
1231453425 l tip tan paint (JK), end; eat Dante (DW), red rum due, Mr. (PW), sea starters (GB)
12341352451 shoes on hens (PW)
1234135425 l kiosk on sink (LC/RA)
12341524531 tractor coat (SB)
12341542531 treat; bar bet (PW), dread bar bed (PW), dent, date, and... (DW)
12342513541 torn out runt (LC/RA)
12342514531 dreary dav, Ed (AT), topcoat, Capt. (SB)
12342531451 dense and sad (PW), tan paint pit (PW,GB), ten meant mat (PW)
12342531541 spin poisons (SB), . star teasers (SB, ML)
12342541351 pursues prep (SB), teaser's tart (SB)
12342541531 order neon, do ( $P W$ ), starter seas (GB)
12345135241 giant gating (AM), the art ...er, hat (GB), satyr strays (GB)
12345142531 stray satyrs (JB), salt is tails (IW), rue o' trouter (GB), yearly relay (AT)
12345241351 tour for Tuft (GG), hours or hush (GG), science sins (GB)
12345241531 sweat wastes ( $P W$ ), $\tan$ dead tent (DW)
12345314251 tears at rest (PW)
12345315241 early, re: Yale (GB)
GB-G. Bloom, Closter, New Jersey; JB - J. Braun, New York, New York; PB - P. Brown, New York, New York; SB - S. Burr, Morristown, New Jersey; GG - G. Grafton, Colchester, England; JK - J.
Kennedy, Colchester, England; AM - A. McAuley, Colchester, England; AT - A. Terrana, New York, New York: DW - D. West, Princeton, New Jersey; IW - I. Wolf, New York, New York; PW - P. Wexler, Colches ter, England; ML - M. Lecomber, Colchester, England; LE/RH - L. Ehlert/R. Howell, Dedham, England

## Observations and Questions

The above table allows a variety of observations and prompts a multi-
tude of questions. Come of these follow:
a. Most of the eodermdromes in the table contain two vowels in their sets of five letters. The only two exceptions in the table are rear pie pair' and 'rue o' trouter'. Are there any one-vowel or fourvowel minimal eodermdromes?
b. There are five sets of anagrammatical eodermdromes in the list. One set is a triplet: sea's tarters, star teasers, starter seas. The pairs are: ah ... rather tea, ah ... tar her tea; science sins, sin sciences; tan paint pit, tip tan paint; teaser's tart, tears at rest. How close can one come to obtaining all 22 patterns with (reasonably) sensical, anagrammatical minimal eodermdromes?
c. An easier, but related constraint to observe is obtaining sets of minimal eodermdromes is that of obtaining homolexical specimens -that is, eodermdromes formed from the same letter set. For example, in the list of anagrammatic eodermdromes, two of the five sets listed above are homolexical: both the triplet and the final pair use the letters AERST. Obtaining the 22 patterns with homolexical eodermdromes is clearly less difficult than finding anagrammatic ones, but should still be quite challenging.
d. No single word minimal eodermdrome, except the coinage 'eodermdrome', is known to exist. In fact, the only eleven-letter word in either Webster's cecond or Third editions that comes close is 'strumstrums' which has its first letter identical to its last letter as well as having it repeated internally; moreover, each of its other letters appears twice. It may be that words that do not appear in the Webster dictionaries (perhaps newer scientific terms) are minimal eodermdromes. Do anv exist?
e. What minimal eodermdromes occur in other languages? We know of one that is in an old form of Polish: 'wrog warga ow', given to us by M. Hippe of Rzeszow, Poland, that means "the enemy of Warga". It is clear to us that the relative frequencies of the occurrence of minimal eodermdromes in various languages is a direct consequence of the structural rules of the language (see the article in Linguistics) ; consequently, we would be very grateful to receive as many examples in other languages as possible.
f. Just as pairs of minimal eodermdromes can be found that are anagrams of each other, so one can (in theory, at least) find pairs of minimal eodermdromes that are palindromically related. None of the currently-known pairs exhibit this property; we would be delighted to learn of some. In the table below, we show what the letter patterns of such palindromic pairs would have to be. To find these pair patterns, we take a sequence and write it in reverse order; then we renumber it so that each number from left to right represents the order of its occurrence. For example, sequence 12314253451 reverses to 15435241321 , and renumbers to 12342531451 , the thirteenth sequence in the table on the preceding page The eleven palindromic pair sequences are:
$123142.53451 / 12342531451$
$12314254351 / 12342541351$
$12314352451 / 12342531541$
$12314354251 / 12342541531$
12314524351/12345241351
$12341352451 / 12342513541$
$12341354251 / 12342514531$
12341524531/12345314251
12341542531/12345315241
12345135241/12345142531
g. No minimal eodermdrome can be a palindrome; in fact, only the final palindromic pair sequence in the above table has the first five and second five letters as anagrams. It is interesting to consider the spelling nets of anagrams and palindromes to see why this is so. The most rigid structure is that of a palindrome. If it has eleven letters with the first appearing thrice and the others twice, it must have the pattern 12345154321 and its spelling net is the one shown on the right. Note that the 1 must be the central letter, because it occurs three times in the
 word. Indeed, if any minimal eodermdrome is to have its beginning and ending five letters anagrams, the central letter must be identical to the first (and last) letter.

One can further observe that not onlv can a minimal eodermdrome not be a palindrome, but it cannot be even locally palindromic; that is, there can be no double letters, and there can be no letter repeated before and after a single letter (thus ruling out strings of letters like ...ere...). The spelling nets of palindromes and minimal eo. dermdromes make their difference appear dramaticallv: the palindrome has double lines whenever there is any line between a pair of letters, but the minimal eodermdrome has no double lines (no letterpair can appear twice, regardless of order reversal of the letters).
$h$. In the minimal eodermdrome everv letter must occur in exactly one pair with each of the other four letters. Thus, for minimal eodermdromes in English, the letter $O$ is unusable, unless it can be used in a set of proper nouns not followed by $U$. What other structures in English or other languages are ruled out by the structure of the minimal eodermdrome?

Generalizations
One can easily generalize the idea of searching for minimal eodermdromes. For example, one can use words as the basic unit in the construction instead of letters. Thus, one can come up with paragraphs like the following set of instructions to a class of students and alumni for a May graduation ceremonv:

March in students. March past in May. Past students may march.
Working with words instead of letters facilitates (certainly in English) finding examples that seem more difficult to achieve with letters. For
example, it was relatively easy to construct the following word-eodermdrome pair of stanzas that exhibit the final palindromic pair ordering:

Summer-time
Living is easy.
Summer living:
Easy time is summer.

Summer is time...
Easy living ...
Summer easy.
Is living-time summer?

Obviously, the ease we had in finding an example was related to the poetic license that we allowed ourselves. If one were to restrict their constructions to totally grammatical, or at least vernacular, English, examples could be constructed only with much more difficulty.

Once one begins thinking of using words as the units in minimal eodermdromes, one opens the door to constructing examples in languages in which the basic units of writing are words (or word roots) instead of letters. K, Kajiwara of Kyoto, Japan has given us an (old) Chinese example:

## CHI WŬ WEI Z.U WÜ CHI WEI WÚ WŨ ZU CHI

A phonetic transliteration of the Chinese characters into Japanese gives:

## CHI GO YUI SOKU MU CHI YUI MU GO SOKU CHI

This rather philosophical example translates as: "That I know self denotes: Just to fill nothing and to know nothing is to satisfy myself." In general, we would like very much to discover similar examples. More importantly, we would like to obtain a feel for the relative difficulty in finding such examples in various languages. We have several conjectures about this, but after all it's great to have some evidence, so that we have something with which to test the theory.

General Eodermaromes
There are many questions and almost no answers for eodermdromes in general. In this last section, we indicate how to recognize when a spelling net is an eodermdrome. Then we ask two of the numerous questions that we wonder about.
a. How does one recognize that a spelling net represents an eodermdrome without spending an excessive amount of time drawing and redrawing it to see if it is non-planar? This is most easily shown with an example. In the first figure at the top of the next page, we show the spelling net for the word UNPROSPEROUSNESS drawn so that one pair of lines crosses. Rather than redrawing the spelling net again, let us reduce it. In the second figure, we have thrown away the self-loop SS, the double line RO, and the unneeded line US (as we shall see). If any letter remaining had only a single line to it, we would then throw away that letter and line as well; this would continue until no letter had only a single line to it. In the third figure, we have contracted the net by collapsing some (in this case, two) of the

neighboring pairs of points into one point. At this juncture we are done, for we have contracted the spelling net that we started with to a complete graph on five points in which each of the five points is connected to each of the other four. A spelling net is non-planar whenever it can be reduced to either of the graphs shown below by (1) throwing away lines and points, and (2) contracting points. A non-planar spelling net was what we defined to be an eodermdrome.

b. What single words in a language are eodermdromes? The only work that has been done on this auestion is the fascinating study of $A$. Ross Eckler's on "Dictionary Eodermdromes" elsewhere in this is sue. We would very much also like to see lists of eodermdromic words in languages other than English.
c. What phrases in a language are eodermdromes? This question is most interesting when asked about some class of objects in the language. For example, someone might want to inspect the spelling nets of anagrams. If there were many eodermdromes among them, one might further refine the classification of anagrams by determining whether in the "best" drawing of their unreduced spelling nets, one, two, three, .. pairs of lines must cross. (Indeed, with any
large set of eodermdromes to consider, a classification according to the minimum number of line crossings would be valuable to us.)

As a closing comment, let us mention again that there is a "serious" side to all of this, and that we will really be quite grateful to recreational linguists for any and all examples and ideas of which we learn.

## REFERENCES

Bloom, G.S., A. Gewirtz, J.W. Kennedy, and P.J. Wexler, "Eodermdromes: a graph-theoretical tool for linguistics", to appear in The Proceedings of the 4 th International Conference on the Theory and Applications of Graphs, Kalamazoo, Michigan, May 6-9, 1980, McGraw-Hill, New York
Eckler, A.R., "Dictionary Eodermdromes", Word Ways, August 1980 Kennedy, J. W., P.J. Wexler, and G. S. Bloom, "Linguistic complexity and minimal eodermdromes", to appear in Linguistics (1980)

THE TERMINAL MAN, UNMASKED
"The Terminal Man" in the August 1974 Word Ways listed 45 surnames beginning with two or more $Z_{s}$, culled from various American telephone directories. It was suggested that most of these names were invented by their owners in order to claim the last entry in the directory, and had no existence apart from this. The last of all surnames was then Zachary Zzzzra of San Francisco. The October 29, 1979 issue of Time magazine reports that when he was nudged out of last place by one Zelda Zzzzwramp, he quickly inflated his surname to Zzzzzzzzzra. According to the magazine, he is in reality a painting contractor named Bill Holland, who uses his telephone name as an advertising gimmick. Alas, there are disadvantages: he has received calls in the middle of the night from Australia, and his phone bill is inflated by people at coin phones who illegally charge their calls to his number!

