

AMALGAMATE, CHEMIST!

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When Douglas Hofstadter took over Martin Gardner's column in Scientific American, he changed the name from MATHEMATICAL GAMES to METAMAGICAL THEMAS. The new name is an **anagram** of the old one, that is, a permutation or reordering of the same set of letters. This naturally suggests the question of how easy is it to find an anagram on these letters which "makes sense".

Some sets of letters (such as XJRMQ) will form no English words or phrases at all, though the die-hard may find the call letters of a Mexican radio station! Some sets of letters (such as those in the word FLANK) will form only one word; some will form exactly two (NIGHT, THING); some will form several (POST, POTS, SPOT, STOP, TOPS, OPTS). How many "meaningful" expressions can in fact be formed from the 17-letter set AAAACEEGH1LMMMSTT, the letters in MATHEMATICAL GAMES?

First, the purely mathematical question: how many distinct permutations of these 17 letters are there, whether they make any sense or not? If all 17 letters were different (which they clearly are not), the number of permutations would be $17!$, pronounced "seventeen factorial," and defined to be $17! = 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 355,687,428,096,000$. Because there are repeats among the letters, we must divide this number by the number of ways of permuting identical letters among themselves (since interchanging identical letters produces no visible change in the resulting sequence). The four A's can be permuted in $4! = 4 \times 3 \times 2 \times 1$ or 24 ways, the three M's in $3 \times 2 \times 1$ or 6 ways, the two E's in 2 ways, and the two T's in 2 ways. Thus the total number of distinct sequences of the 17 letters in MATHEMATICAL GAMES becomes $17! / (4! \times 3! \times 2! \times 2!) = 617,512,896,000$. This number is increased if we have the option of introducing spaces between words and punctuation marks wherever we wish in the sequence.

How many of these mathematically distinct sequences are likely to make any sense as English? This is a question for Information Theory, which reformulates it as follows: What is the **entropy** of written English, regarded as a source of letters of the alphabet? (Entropy is a measure of the permitted degree of randomness.) If written English had an entropy as large as $\log_2 26 = 4.7..$ bits per letter, that would mean that essentially every² sequence of letters is "meaningful". We know this is far from the case, and the best estimates of the entropy of written English are in the range of one to two bits per letter. This would suggest that there are between 2^N and 2^{2N} meaningful English messages which are N letters long, and in particular, between $2^{17} = 131,072$ and $2^{34} =$

17,179,869,184 English "messages" 17 letters in length. I suspect, for sequences of 17 letters, the truth is closer to the lower than the upper estimate. In any case, this would only be approximated for large values of N, and would not correspond to only the permutations of a single set of letters. However, most of these messages, for large values of N, would be permutations of sets of letters with the typical letter frequencies of English.

The letters in MATHEMATICAL GAMES are not very typical: three M's but no N's, R's, or D's; four A's but only two E's and no O's, etc. My own estimate is that there are perhaps 3000 "meaningful" sequences of these letters (where punctuation and spaces can be inserted at will, and only fragments of ideas or phrases need be expressed), give or take a factor of 5. And even this estimate is sensitive to how strict or liberal we are with our notion of what constitutes "English". Thus, METAMAGICAL THEMAS consists of two "words" which are not in any standard English dictionary of which I am aware. METAMAGICAL is a Hofstadter coinage whose meaning we are to deduce by analogy with METAMATHEMATICAL and METAPHYSICAL; and THEMAS is an improper plural of THEMA. (Theme and scheme lead to themes and schemes; but from thema and schema come themata and schemata.) As we allow more and more foreign words, variant spellings, abbreviations, initials, contractions and proper names, the entropy of "English" increases, and we get more and more "meaningful" anagrams.

To illustrate what is possible, I have listed 100 "meaningful" anagrams of MATHEMATICAL GAMES. I have created a new literary form for this purpose, which I will call *serve verse*. An example of *serve verse* can be called a *mope poem*. A brief, self-referential *mope poem* is:

Revés: Sever verse, / Veers; serve.

My *serve verse* creation on MATHEMATICAL GAMES is divided into sections on the general themes of metamathematics, geology, zoology, sororities, athletics, culinary arts, and medicine. The metamathematics section is concerned with such things as THEMATIC LEMMAS; the geology section (or is it mineralogy?) with AMALGAMS, MAGMA, MALACHITE and STALAGMITE. The zoology section is obsessed with CAMELS and MAMMALS. The sorority section has GAMMA, ETA, THETA, SIGMA and CHI at its disposal, and seems to contain a vicious *ad feminem* attack against one EMMA GALE SMITH. The athletics section refers to TEAMMATES, GAMES, SETS, MATCHES and to ATHLETICS itself. The culinary section mentions STALE MEAT, STEAM HEAT, HAMS and CLAMS, and the medical section alludes to TEETH and MEASLES. Readers are encouraged to invent even more imaginative anagrams of their own.

For several centuries, one of the forms of cryptography actually employed to conceal the content of diplomatic messages was the use of permutation ciphers, where the letters of the message were scrambled according to a procedure known only (it was hoped) to the sender and the intended receiver. Without the correct rule,

as we have seen, the same letters might be unscrambled to reveal any "secret" message from MAGMA HEATS CLIMATE to SAM, THE MAG-IC TAMALE.

Mathematical Games

Thematic lemma saga
Meet math as magical
Magic sheet: lama mat
Image chats at lemma

Amalgamatic Themes

Amalgamate, chemist!
Chemist ate amalgam
Magma heats climate
Magi select mahatma
Malachite set magma
Stalagmite came, ham
Almagest: teach imam

This Camel Ate Magma

Get this, Ma'am: a camel
Tight as a camel, Emma
Camel meat? I'm aghast!
As a camel might mate
I get mahatma's camel
Castigate mammal, eh?
Cage atheist mammal
Tia cages the mammal
That's ice-age mammal
The camel's gait, Ma'am
Age maims that camel
Camel stigmata? Ahem!

Hag Claims Teammate

Chi Gamma's late mate
Asthmatic Emma Gale
Met me, claims Agatha
Gamma Chi stalemate

Mag's Mealtime Chat

Masticate ham leg, Ma
Chi Gamma: taste meal!
Stage claim: ham meat
Came as tamale might

Mama Mia! The Last Egg!

Aim, match, tag measles
Mica teeth amalgams

Metamagical Themas

I get the lemma - a scam
Elastic math game, Ma!
The same magical mat
G.I. lemmata schemata

The Magmatical Seam

Misamalgamate, Chet!
Alchemist ate magma
Mahatma's climate, e.g.
Michael, taste magma
Malaesthetic magma
Ah, magmatic Maltese
Megalith caste, Mama

Magma? Camels Hate It

Get this, ace: a mammal!
It aches? Get a mammal!
Aghast, Mac? Mealtime!
I get a chaste mammal
Che agitates mammal
Acetate mammal? Sigh!
Hattie cages mammal
I teach stage mammal
Mammal at "eight aces"
The ice mammal's gate
Ice ages that mammal
Legit schemata, Ma'am

"Hate Mate," Claims Mag

Theta Gamma's malice
(Emma Gale Smith: a cat?)
Me? Clammiest Agatha!
Eta Chi Gamma metals

Mag's Calm Teatime? Ha!

Magic ham? Stale meat!
Teatime: ham, clam, gas
Malt, sage ethic, Ma'am
Sam, the magic tamale

Ethical, Sam? Get Mama!

I calm the stage-mama
Asthma: age claim met

Athletic Games, Ma'am

Michaelmas team tag
 A late game mismatch
 Claim gate at Thames
 Aim, Al: game, set, match
 Sham cage title, Ma'am
 Met Lhasa magic team
 Mime act: Hamlet saga
 1 match eaglets, Ma'am
 Team game: Cal/Smith
 Hit game castle, Ma'am

Thalamic Team Games

1 gash calm teammate
 A lame gate mismatch
 Time Malaga matches
 Lit matches age Mama
 Claim tag? Shame, team!
 Act the lamaism game
 Ham act: get me salami
 1 tag the camel's mama
 I get Tech's alma mama
 Agile mama cast them

Editor's Note: One can ask what is the likelihood that any of the sentences given above have somewhere appeared in English writings in a meaningful (not wordplay) context. Obviously, this is a very stringent (and unverifiable!) measure of the plausibility of an anagrammed phrase; it is quite possible that none of these phrases (save the already-noted METAMAGICAL THEMAS) would pass. Put in other terms, it is not difficult for the reader to pick out the original phrase on which the anagrams were based. Cryptographers using permutation ciphers would be well-advised to select a message using convoluted grammar, with an anagram that sounds more plausible than the intended message.

OMNI GAMES

This paperback anthology of the best brain-teasers from the last five years of Scot Morris's Omni magazine Games column is devoted to an amazing variety of subjects: besides the usual mathematics and logic puzzles, it contains physics problems, geographical oddities, electronic calculator tricks, bar bets, juggling, and a tribute to Martin Gardner. The emphasis is on (1) problems that look difficult but can be solved easily with the proper insight (aha!), (2) problems that look easy but contain hidden traps, and (3) problems that look difficult and in fact are, but which lead to curious facts. Though many problems are golden oldies, Scot Morris writes about them in a lively and entertaining way reminiscent of Martin Gardner, and often succeeds in bringing in new angles. The book is enriched by reader comment and emendations to the original Omni material. A substantial part of the book is devoted to problems related to logology: four unsolved Enigma-style conundrums by Bishop Wilberforce, the Beale cipher, two tough spelling quizzes, the "world's hardest" word quiz (containing much Word Ways material), mnemonics, rebuses, anagrams, and the first crossword puzzle. The book is available from Holt, Rinehart and Winston for \$11.95. Let's hope that this is the start of a long series.